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CHAPTER - 1

PROPERTIES OF FLUIDS

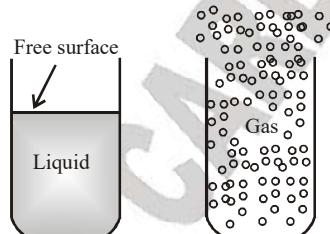
1.1 INTRODUCTION

The fluid a substance in the liquid or gas phase is referred to as a fluid.

A solid can resist an applied shear stress by deforming, whereas a fluid deforms continuously under the influence of a shear stress, no matter how small. In solids, stress is proportional to strain but in fluids, stress is proportional to strain rate. When a constant shear force is applied, a solid eventually stops deforming at some fixed strain angle, whereas a fluid never stops deforming and approaches a constant rate of strain.

The normal component of a force acting on a surface per unit area is called the normal stress, and the tangential component of a force acting on a surface per unit area is called shear stress (fig). In a fluid at rest, the normal stress is called pressure. A fluid at rest is at a state of zero shear stress. When the walls are removed or a liquid container is tilted, a shear develops as the liquid moves to re-establish a horizontal free surface.

In a liquid, groups of molecules can move relative to each other, but the volume remains relatively constant because of the strong cohesive forces between the molecules. As a result , a liquid takes the shape of the container it is in and it forms a free surface in a larger container in a gravitational field. A gas, on the other hand, expands until it encounters the walls of the container and fills the entire available space. This is because the gas molecules are widely spaced, and the cohesive forces between them are very small. Unlike liquids, a gas in an open container cannot form a free surface (fig.)



Unlike a liquid, a gas does not form a free surface, and it expands to fill the entire available space.

1.2 CONTINUUM

A fluid is composed of molecules which may be widely spaced apart, especially in the gas phase. Yet it is convenient to disregard the atomic nature of the fluid and view it as continuous, homogeneous matter with no holes that is continuum. The continuum idealization allows us to treat properties as point functions and to assume that the properties vary continually in space with no jump discontinuities. This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules. The continuum idealization is implicit in many statements we make, such as “the density of water in a glass is the same at any point.”

1.3 DENSITY AND SPECIFIC GRAVITY

Density is defined as mass per unit volume

$$\text{Density } \rho = \frac{m}{V} (\text{kg} / \text{m}^3)$$

CHAPTER - 2

FLUID STATICS

2.1 INTRODUCTION

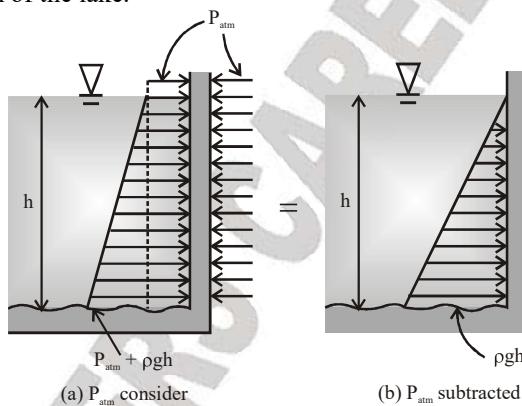
Fluid statics deals with problems associated with fluids at rest.

In fluid statics, there is no relative motion between adjacent fluid layers, and thus there are no shear (tangential) stresses in the fluid trying to deform it. The only stress we deal with in fluid statics is the normal stress, which is the pressure, and the variation of pressure is due only to the weight of the fluid.

The design of many engineering systems such as water dams and liquid storage tanks requires the determination of the forces acting on their surfaces using fluid statics.

2.2 HYDROSTATIC FORCES ON SUBMERGED PLANE SURFACES

A plate (such as a gate valve in a dam, the wall of a liquid storage tank, or the hull of a ship at rest) is subjected to fluid pressure distributed over its surface when exposed to a liquid. On a plane surface, the hydrostatic forces form a system of parallel forces, and we often need to determine the magnitude of the force and its point of application, which is called the centre of pressure. In most cases, the other side of the plate is open to the atmosphere (such as the dry side of a gate), and thus atmospheric pressure acts on both sides of the plate, yielding a zero resultant. In such cases, it is convenient to subtract atmospheric pressure and work with the gage pressure only. For example, $P_{\text{gage}} = \rho gh$ at the bottom of the lake.



When analyzing hydrostatic forces on submerged surfaces, the atmospheric pressure can be subtracted for simplicity when it acts on both sides of the structure.

Consider the top surface of a flat plate of arbitrary shape completely submerged in a liquid, as shown in fig. together with its normal view. The plane of this surface (normal to the page) intersects the horizontal free surface at angle θ , and we take the line of intersection to be the x-axis (out of the page). The absolute pressure above the liquid is P_0 , which is the local atmospheric pressure P_{atm} if the liquid is open to the atmosphere (but P_0 may be different than P_{atm} if the space above the liquid is evacuated or pressurized). Then the absolute pressure at any point on the plate is

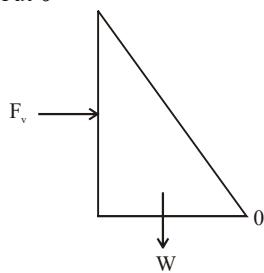
$$P = P_0 + \rho gh = P_0 + \rho gy \sin \theta$$

Sol. 105.(c)

Stability of a freely falling object when centre of gravity lies below its meta-centre

Sol. 106.(d)

Moment about 0



$$F_v \times \frac{h}{3} = \frac{W \times 2b}{3}$$

$$W \times h \times \frac{1}{2} \times \frac{h}{3} = \frac{bh}{2} \times 1 \times 2.56w \times \frac{2}{3}$$

$$B = 0.625h$$

Sol. 107.(c)

- (a) **Centre of pressure:** point of application of hydrostatic pressure force
- (b) **Centre of gravity:** point of application of the weight of the body
- (c) **Centre of buoyancy:** point of application of the weight of displaced liquid
- (d) **Meta centre:** point about which the body starts oscillating when tilted by a small angle

Sol. 108.(d)

When piece of metal and liquid have same specific gravity then metal piece will be considered as the part of liquid

Sol. 109.(c)

Since the water of bucket will exert force of buoyancy equal to the volume displaced by the iron block. Hence, equal and opposite force will be exerted by iron block on water hence the reading of the spring balance will increase

Sol. 110.(c)

$$T = 2\pi \sqrt{\frac{K_G^2}{g \cdot GM}} = 2\pi \sqrt{\frac{(9)^2}{9.81 \times 0.750}} = 20.85 \text{ sec}$$

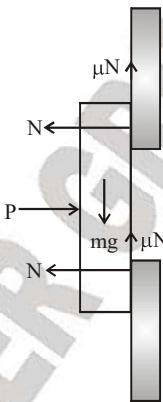
Sol. 111.(b)

Both A and R is true, but R does not give sufficient explanation for phenomenon at A Location of meta-centre and centre of buoyancy decide about floating of a body

Sol. 112.(b)

Pressure force

$$= \gamma A \bar{x} = 10 \times 1000 \times 2 \times 2.5 \times 1 = 50 \text{ kN}$$



$$\sum F_x = 0$$

$$\therefore 2N = P = 50$$

$$\therefore N = 25 \text{ kN}$$

Now, $\sum F_y = 0$

$$\sum F_y = 0$$

$$Mg = 2\mu N$$

$$\therefore \mu = \frac{500 \times 10}{2 \times 25 \times 1000} = 0.1$$

Sol. 113.(b)

Depth of centre of pressure is proportional to second moment of area about the water surface , and depth of centre of gravity i.e,

$$h = \bar{x} + \frac{I_G \sin^2 \theta}{A \bar{x}}$$

Where, h is depth of pressure

\bar{x} is centroid of surface area

A is surface area

I_G is moment of inertia of the area about an axis passing through the centroid of the area

θ is inclination of surface from horizontal centre of area of immersed surface lies above the centre of pressure

Sol. 114.(b)

CHAPTER - 4

FLUID DYNAMICS, FLOW OVER NOTCHES & WEIRS

4.1 INTRODUCTION

This chapter deals with three equations commonly used in fluid mechanics: the mass, Bernoulli, and energy equations. The mass equation is an expression of the conservation of mass principle. The Bernoulli equation is concerned with the conservation of kinetic, potential, and flow energies of a fluid stream and their conversion to each other in regions of flow where net viscous forces are negligible and where other restrictive conditions apply. The energy equation is a statement of the conservation of energy principle.

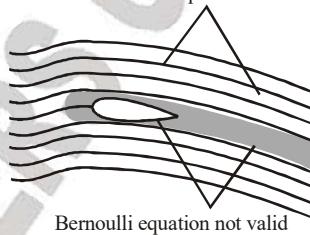
4.2 THE BERNOULLI EQUATION

The Bernoulli equation is an approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible (fig.) Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics. In this section, we derive the Bernoulli equation by applying the conservation of linear momentum principle, and we demonstrate both its usefulness and its limitations.

The key approximation in the derivation of the Bernoulli equation is that viscous effects are negligibly small compared to inertial, gravitational, and pressure effects. Since all fluids have viscosity (there is no such thing as an “inviscid fluid”), this approximation cannot be valid for an entire flow field of practical interest.

Care must be exercised when applying the Bernoulli equation since it is an approximation that applies only to inviscid regions of flow. In general, frictional effects are always important very close to solid wall (boundary layers) and directly down stream of bodies (wakes). Thus, the Bernoulli approximation is typically useful in flow regions outside of boundary layers and wakes, where the fluid motion is governed by the combined effects of pressure and gravity forces.

Bernoulli equation Valid



The Bernoulli equation is an approximate equation that is valid only in inviscid regions of flow where net viscous forces are negligibly small compared to inertial, gravitational, or pressure forces. Such regions occur outside of boundary layers and wakes.

4.2.1 Derivation of the Bernoulli Equation

Consider the motion of a fluid particle in a flow field in steady flow. Applying Newton's second law (which is referred to as the linear momentum equation in fluid mechanics) in the s-direction on a particle moving along a streamline gives

$$\Sigma F_s = m a_s \quad \dots(i)$$

In regions of flow where net frictional forces are negligible, there is no pump or turbine, and there is no heat transfer along the streamline, the significant forces acting in the s-direction are the pressure (acting on both sides) and the component of the weight of the particle in the s-direction (Fig.) Therefore, equation (i) becomes

CHAPTER - 5

FLOW THROUGH PIPES

5.1 FLOW TYPES OF FLOW-REYNOLDS' EXPERIMENT

Reynolds related the inertia to viscous forces and arrived at a dimensionless parameter.

$$\text{Re or } N_R = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{F_i}{F_v} \quad \text{Re or } N_R = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{F_i}{F_v}$$

According to Newton's second law of motion the inertia force F_i is given by

$F_i = \text{mass} \times \text{acceleration}$

$= \rho \times \text{volume} \times \text{acceleration}$

$= \rho \times L^3 \times (L/T^2) = (\rho L^2 V^2)$

Similarly viscous force F_v is given by Newton's law of viscosity as

$F_v = \tau \times \text{area}$

$$= \mu \frac{\partial V}{\partial y} \times L^2 = (\mu VL)$$

$$\therefore \text{Re or } N_R = \frac{(\rho L^2 V^2)}{\mu VL} = \frac{\rho VL}{\mu}$$

This dimensionless parameter is called Reynolds number, in which ρ and μ are respectively the mass density and viscosity of the flowing fluid, V is the characteristic (or representative) velocity of flow and L is the characteristic linear dimension. In the case of flow through pipes the characteristic linear dimension L is taken as the diameter D of the pipe and the characteristic velocity is taken as the average velocity V of flow of fluid. Thus Reynolds number becomes $(\rho DV/\mu)$ or (VD/v) where $(\mu/\rho) = v$, is kinematic viscosity of the flowing fluid. The Reynolds number is therefore a very useful parameter in predicting whether the flow is laminar or turbulent. One may predict that the flow will be laminar if Reynolds number is less than 2000 and turbulent if it is greater than 4000.

5.2 LAWS OF FLUID FRICTION

1. Law of fluid friction for laminar flow

The frictional resistance in the laminar flow is as follows

- (i) Proportional to the velocity of flow
- (ii) Independent of the pressure
- (iii) Proportional to the area of surface in contact
- (iv) Independent of the nature of the surface in contact
- (v) Greatly affected by variation of the temperature of the flowing fluid.

2. Laws of fluid friction for turbulent flow

The frictional resistance in the case of turbulent flow is as follows

- (i) Proportional to velocity where the index n varies from 1.72 to 2.0
- (ii) Independent of the pressure
- (iii) Proportional to the density of the flowing fluid
- (iv) Slightly affected by the variation of the temperature of the flowing fluid
- (v) Proportional to area of surface in contact
- (vi) Dependent on the nature of the surface in contact

CHAPTER - 6

BOUNDARY LAYER THEORY

6.1 INTRODUCTION

When a real fluid flows past a solid boundary, a layer of fluid which comes in contact with the boundary surface adheres to it on account of viscosity. Since this layer of fluid cannot slip away from the boundary surface it attains the same velocity that of the boundary. In other words, at the boundary surface there is no relative motion between the fluid and the boundary. This condition is known as no slip condition. Thus at the boundary surface the layer of fluid undergoes retardation. This retarded layer of fluid further causes retardation for the adjacent layers of the fluid, thereby developing a small region in the immediate vicinity of the boundary surface in which the velocity of flowing fluid increase gradually from zero at the boundary surface to the velocity of the main stream. This region is known as boundary layer. In the boundary layer region since there is a larger variation of velocity in a relatively small distance, there exists a fairly large velocity gradient ($\partial v / \partial y$) normal to the boundary surface. As such in this region of boundary layer even if the fluid has small viscosity, the corresponding shear stress $\tau = \mu (\partial v / \partial y)$, is of appreciable magnitude. The flow may thus be considered to have two regions, one close to the boundary in the boundary layer zone in which due to larger velocity gradient appreciable viscous forces are produced and hence in this region the effect of viscosity is mostly confined and second outside the boundary layer zone in which the viscous forces are negligible and hence the flow may be treated as non-circular or inviscid. The concept of boundary layer was first introduced by L. Prandtl in 1904 and since then it has been applied to several fluid flow problems.

6.1.1 Following Boundary Conditions May Be Noted

Essential Boundary condition

1. at $x = 0$ (leading edge), thickness of boundary layer = 0 i.e, $\delta = 0$
2. at $y = 0$, $u = 0$
3. at $y = \delta$, $u = V_2$ = Free stream velocity = constant
4. at $y = \delta$, $\frac{du}{dy} = 0$

Desirable Boundary conditions

$$\text{At } y = \delta, \frac{du}{dy} = 0, \frac{d^2u}{dy^2} = 0$$

- (i) When a fluid flows past a flat plate, the velocity at leading edge is zero and retardation of fluid increases as more and more of the plate is exposed to flow. Hence boundary layer thickness increase as distance from leading edge increases
- (ii) Up to certain distance from the leading edge, flow in boundary layer is laminar irrespective of the fact that flow of approaching stream is laminar or turbulent
- (iii) As the depth of laminar boundary layer increases, it cannot dissipate the effect of instability in flow and hence transition to turbulent boundary layer is more
- (iv) Thus thickness of turbulent boundary layer is more
- (v) Change of boundary layer from Laminar to turbulent is affected by
 - (a) Roughness of plate
 - (b) Plate curvature

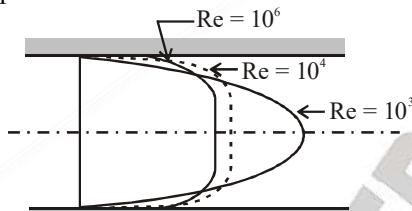
CHAPTER - 7

TURBULENT FLOW IN PIPES

7.1 INTRODUCTION

As stated earlier if Reynolds number is greater than 4000 the flow is turbulent. The velocity distribution in turbulent flow is relatively uniform and the velocity profile of turbulent flow is much flatter than the corresponding laminar flow parabola for the same mean velocity.

In the case of turbulent flow the velocity fluctuations influence the mean motion in such a way that an additional shear (or frictional) resistance to flow is caused. This shear stress produced in turbulent flow is in addition to the viscous shear stress and it is termed as turbulent shear stress which may be evaluated as explained in the next section.



(i) Velocity distribution in laminar and turbulent flow

7.2 RELATION BETWEEN SHEAR AND PRESSURE GRADIENTS IN LAMINAR FLOW

Consider a free body of fluid having the form of an elementary parallelopiped of length δ_x , width is δ_z , thickness δ_y .

The magnitudes of shear stresses on the layers abcd and a'b'c'd' will be different. Thus if τ represents the shear stress on the layer abcd then the shear stress on the layer a'b'c'd' is equal to

$$\left(\tau + \frac{\partial \tau}{\partial y} \delta y \right).$$

For two – dimensional steady flow there will be no shear stresses on the vertical faces abb'a' and cdd'c'. Thus the only forces acting on the parallelepiped in the direction of flow x will be the pressure and shear forces. The net shear force acting on the parallelepiped.

$$= \left[\left(\tau + \frac{\partial \tau}{\partial y} \delta y \right) \delta x \delta z - \tau \delta x \delta z \right] = \frac{\partial \tau}{\partial y} \delta x \delta y \delta z$$

If the pressure intensity on face add'a' is p , and since there exists a pressure gradient in the direction of flow, the pressure intensity on the face bcc'b' will be $\left(p + \frac{\partial p}{\partial x} \delta x \right)$. The net pressure force acting on the parallelopiped.

$$= \left[p \delta y \delta z - \left(p + \frac{\partial p}{\partial x} \delta x \right) \delta y \delta z \right] = - \left(\frac{\partial p}{\partial x} \right) \delta x \delta y \delta z$$

For steady and uniform flow, there being no acceleration in the direction of motion, the sum of these forces in the x-direction must be equal to zero. Thus

CHAPTER - 8

OPEN CHANNEL FLOW

UNIFORM FLOW

8.1 DEFINITION OF AN OPEN CHANNEL

An ‘open channel’ may be defined as a passage in which liquid flows with its upper surface exposed to atmosphere. In open channels the flow is due to gravity; thus the flow conditions are greatly influenced by the slope of the channel.

8.2 COMPARISON BETWEEN OPEN CHANNEL AND PIPE FLOW

The important points of difference between the two types of flows are given below:

S.No.	Aspects	Open Channel Flow	Pipe Flow
1.	Cause of flow	Gravity force (provided by sloping bottom)	The pipe runs full and the flow, in general, takes place at the expense of hydraulic pressure ; the pressure continuously decreases in the direction of flow.
2.	Geometry of Cross – section	Open channel may have any shape; triangular, rectangular, trapezoidal, parabolic, circular etc.	Pipes generally round in cross – section ... cross – section of flow is fixed, since the flowing liquid entirely fills the pipe section.
3.	Surface roughness	Varies between wide limits; the hydraulic roughness varies with depth of flow.	Roughness co –efficient varies from a low value to a very high value , depending upon the material of the pipe.
4.	Piezometric head	$(z + y)$, where y is the depth of flow; H.G.L. coincides with the water surface.	$\left(z + \frac{p}{w}\right)$, where p is the pressure in the pipe, H.G.L. does not coincide with water surface.
5.	Velocity distribution	The maximum velocity occurs at a little distance below the water surface. The shape of the velocity profile is dependent on the channel roughness.	The velocity distribution is symmetrical about the pipe axis, maximum velocity occurring at the pipe centre and the velocity at the pipe wall reducing to zero.

8.2.1 Types of Channels

The various types of channels are:

1. Natural channel

It is the one which has irregular sections of varying shapes, developed in a natural way.

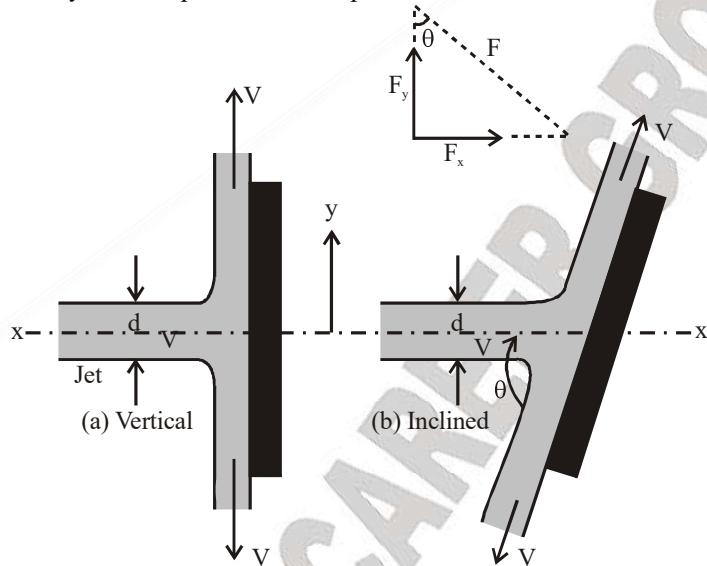
Example. Rivers, streams etc.

CHAPTER - 9

FLUID MACHINERY

9.1 IMPACT OF JETS

A jet of fluid emerging from a nozzle has some velocity and hence it possesses a certain amount of kinetic energy. If this jet strikes an obstruction placed in its path, it will exert a force on the obstruction. This impressed force is known as impact of the jet and it is designated as hydrodynamic force, in order to distinguish it from the forces due to hydrostatic pressure. Since a dynamic force is exerted by virtue of fluid motion, it always involves a change of momentum, unlike a force due to hydrostatic pressure that implies no motion.



Fluid jet striking stationary flat plate

9.2 FORCE EXERTED BY FLUID JET ON STATIONARY FLAT PLATE

1. Flat Plate Normal to the Jet

Let a jet of diameter d and velocity V issue from a nozzle and strike a flat plate as shown in figure below. The plate is held stationary and perpendicular to the centre line of the Jet. The jet after striking the plate will leave it tangentially i.e., the jet will get deflected through 90° . If the plate is quite smooth the friction between the jet and the plate may be neglected. Further if there is no energy loss in the flow because of impact of the fluid jet, and the difference in elevation between the incoming and outgoing jets is neglected; the application of Bernoulli's equation indicates that the jet will move on and off the plate with the same velocity V . However, if some energy loss occurs the velocity of the fluid leaving the plate will be slightly less than V .

The quantity of fluid striking the plate $Q = (\pi d^2/4) \times V = aV$, where a is the area of cross-section of the jet. Thus the mass of fluid issued by the jet per second is $m = \rho Q = \rho aV$; where ρ represents the mass density of the fluid. Since $p = (w/g)$, where w is the specific weight of the fluid, the mass m may also be expressed as $m = (wV/g)$.

CHAPTER - 10

DIMENSIONAL ANALYSIS & MODEL STUDIES

10.1 INTRODUCTION

1. Dimensional analysis is a mathematical technique for solving engineering problems. It makes uses of the dimensions of variables on which the problem depends.
2. A physical phenomenon can be expressed by an equation giving relationship between different quantities. Such quantities are dimensional and non-dimensional .
3. Dimensional analysis helps in determining a systematic arrangement of the variables in the physical relationship, combining dimensional variables to form non-dimensional parameters.
4. It has becomes an important tool for analyzing fluid flow problems.

10.2 SYSTEMS OF DIMENSIONS

1. The various physical quantities can be expressed in terms of fundamental quantities.
2. The fundamental (or primary) quantities are: mass (M), length (L), time (T), and temperature (θ).
3. The quantities which are expressed in terms of the fundamental-quantities are called derived (or secondary) quantities, e.g. velocity, area, acceleration etc.
4. The expression for a derived quantity in terms of the primary quantities is called the dimension of the physical quantity.
5. The two common system of dimensioning a physical quantity are; M-L-T and F-L-T system of units where F is force. The dimensions of various quantities used in both the system are given in table on the next page.

10.3 DIMENSIONAL HOMOGENEITY

1. Dimensional homogeneity states that every term in an equation when reduced to fundamental dimensions must contain identical powers of each dimension.
2. A dimensionally homogenous equation is independent of the fundamental units of measurement if the units there in are consistent.
3. Let us consider the velocity equation,

$$V = \sqrt{2gh}$$

$$[LT^{-1}] = [2 \times LT^{-2} \times L]^{1/2} = [L^2 T^{-2}]^{1/2} = [LT^{-1}]$$

Similarly, $h_f = \frac{4flv^2}{2gd}$

$$[L] = \frac{[L][LT^{-1}]^2}{[LT^{-2}][L]} = [L]$$

Dimensional homogeneity is bases on the Fourier's principle of homogeneity.

Table: Dimensions of Various Physical Quantities

Physical quantity	Symbol	Dimensions	
		M-L-T system	F-L-T system
1. Fundamental Quantities			
Mass	M	M	$FL^{-1}T^2$
Length	L	L	L


WORKBOOK

Example 1. If 5m^3 of a certain oil weight 4000kg(f) . Calculate the specific weight. Mass density and specific gravity of this oil.

Solution.

$$\text{Specific weight of oil} = \frac{\text{Weight}}{\text{Volume}}$$

$$= \frac{400\text{kg(f)}}{5\text{m}^3} = 800\text{kg(f)}/\text{m}^3$$

Mass density of oil

$$= \frac{\text{Specific weight of oil}}{\text{Acceleration due to gravity}}$$

$$= \frac{800\text{kg(f)}/\text{m}^3}{9.81\text{m/sec}^2} = 81.55\text{msl}/\text{m}^3$$

Specific gravity of oil

$$= \frac{\text{Specific weight of oil}}{\text{Specific weight of water}}$$

$$= \frac{800\text{kg(f)m}^3}{1000\text{kg(f)}/\text{m}^3} = 0.8$$

Example 2. If m^3 of a certain oil weights 40kN , calculate the specific weight, mass density and specific gravity of this oil.

Solution.

Specific weight of oil

$$= \frac{\text{Weight}}{\text{Volume}} = \frac{40 \times 100\text{N}}{5\text{m}^3} = 8000\text{N}/\text{m}^3$$

Mass density of oil

$$= \frac{\text{Specific weight of oil}}{\text{Acceleration due to gravity}}$$

$$= \frac{8000\text{N}/\text{m}^3}{9.81\text{m/s}^2} = 815.49\text{kg}/\text{m}^3$$

Specific gravity of oil

$$= \frac{\text{Specific weight of oil}}{\text{Specific weight of water}}$$

$$= \frac{8000\text{N}/\text{m}^3}{9810\text{N}/\text{m}^3} = 0.815$$

Example 3. A plate 0.0254 mm distant from a fixed plate, moves at 61 cm/sec and requires a force of $0.2\text{ kg(f)}/\text{m}^2$ to maintain this speed. Determine the dynamic viscosity of the fluid between the plates.

Solution.

From equation shear stress

$$\tau = \frac{F}{A} = \mu \frac{dv}{dy} = \mu \frac{V}{Y}$$

$$\tau = \frac{F}{A} = 0.2\text{kg(f)}/\text{m}^2$$

$$V = 61\text{cm/sec} = 0.61\text{m/sec}$$

$$\text{and } Y = 0.0254\text{mm} = 2.54 \times 10^{-5}\text{ m}$$

By substituting in the above equation, we get

$$0.2 = \mu \times \frac{0.61}{2.54 \times 10^{-5}}$$

$$\mu = \frac{0.2 \times 2.54 \times 10^{-5}}{0.61} \text{kg(f)-sec/m}^2$$

$$= 8.328 \times 10^{-6} \text{kg(f)-sec/m}^2$$

$$= 8.328 \times 10^{-10} \text{kg(f)-sec/cm}^2$$

Example 4. At a certain point in castor oil the shear stress is 0.216N/m^2 and the velocity gradient 0.216s^{-1} . If the mass density of castor oil is 959.42 kg/m^3 , find kinematic viscosity.

Solution.

From equation shear stress

$$\tau = \mu \left(\frac{dv}{dy} \right)$$

$$\tau = 0.216\text{N}/\text{m}^2; \left(\frac{dv}{dy} \right) = 0.216\text{s}^{-1}$$

By substitution, we get $0.216 = \mu (0.216)$

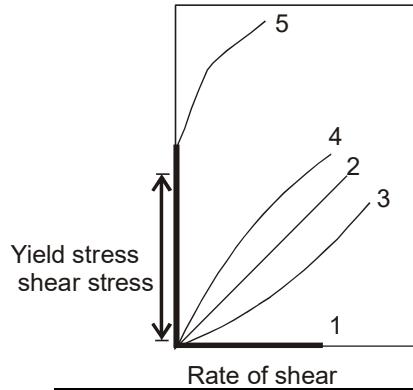
$$\therefore \mu = 1\text{N.s/m}^2$$

∴ Kinematic viscosity

$$\nu = \frac{\mu}{\rho} = \frac{1}{959.42} = 1.042 \times 10^{-3} \text{m}^2/\text{s}$$

GATE QUESTIONS

1. Group I contains the types of fluids while Group II contains the shear stress → rate of shear relationship of different types of fluids , as shown in the figure



Group I	Group II
A. Newtonian fluid	(i) Curve 1
B. Pseudo plastic fluid	(ii) Curve 2
C. Plastic fluid	(iii) Curve 3
D. Dilatant fluid	(iv) Curve 4
	(v) Curve 5

The correct match between Group I and Group II is

[GATE - 2016]

- (a) A-ii, B-iv, C-i, D-v
- (b) A-ii, B-v, C-iv, D-1
- (c) A-ii, B-iv, C-v, D-iii
- (d) A-ii, B-i, C-iii, D-iv

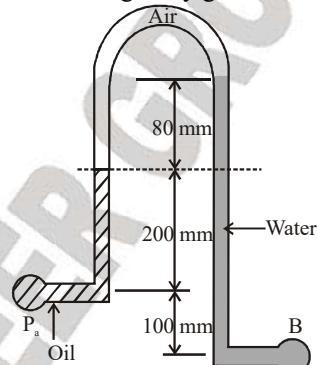
2. Oil (kinematic viscosity , $\nu_{oil} = 1 \times 10^{-5} \text{ m}^2/\text{s}$) flow through a pipe diameter with a velocity of 10 m/s $v_w = 0.89 \times 10^{-6} \text{ m}^2/\text{s}$) diameter flowing through a model pipe of diameter 10 mm for satisfying the dynamic similarity , the velocity of water (in m/s) is _____

[GATE - 2016]

3. An inverted U-tube manometer is used to measure the pressure difference between two pipes A and B, as shown in the figure .Pipe A is carrying oil (specific gravity =0.8) and pipe B is

carrying water .The densities of air and water are 1.16 kg/m^3 and 1000 kg/m^3 , respectively. The pressure difference between pipes A and B is _____ kPa.

Acceleration due to gravity $g = 10 \text{ m/s}^2$



[GATE - 2016]

4. The difference in pressure (in N/m^2) across an air bubble of diameter 0.001 m immersed in water (surface tension = 0.072 N/m) is _____.

[GATE - 2014]

5. The dimension for kinematics viscosity is

[GATE - 2014]

- (a) L / MT
- (b) L / T^2
- (c) L^2 / T
- (d) ML / T

6. The necessary and sufficient condition for a surface to be called as a 'free surface' is

[GATE - 2006]

- (a) No stress should be acting on it
- (b) Tensile stress acting on it must be zero
- (c) Shear stress acting on it must be zero
- (d) No point on it should be under any stress

7. In the inclined manometer shown in the figure below, the reservoir is large. Its surface may be assumed to remain at a fixed elevation . A is connected to a gas pipeline and the deflection noted on the inclined glass tube is 100 mm.

ESE OBJ QUESTIONS

- 1.** The normal stresses within an isotropic Newtonian fluid are related to
- Pressure
 - Viscosity of fluid
 - Velocity gradient
- Which of the above are correct?
- [ME ESE - 2018]
- | | |
|------------------|------------------|
| (a) 1 and 2 only | (b) 1 and 3 only |
| (c) 2 and 3 only | (d) 1, 2 and 3 |
- 2.** The surface tension in a soap bubble of 20 mm diameter, when the inside pressure is 2.0 N/m² above atmospheric pressure, is
- [CE ESE - 2018]
- | | |
|----------------------------|-------------------------------|
| (a) 0.025 N/m | (b) 0.0125 N/m |
| (c) 5×10^{-3} N/m | (d) 4.25×10^{-3} N/m |
- 3.** A jet of water has a diameter of 0.3 cm. The absolute surface tension of water is 0.072 N/m and atmospheric pressure is 101.2 kN/m². The absolute pressure within the jet of water will be
- [CE ESE - 2018]
- | | |
|-------------------------------|-------------------------------|
| (a) 101.104 kN/m ² | (b) 101.152 kN/m ² |
| (c) 101.248 kN/m ² | (d) 101.296 kN/m ² |
- 4.** A glass tube of 2.5 mm internal diameter is immersed in oil of mass density 940 kg/m³ to a depth of 9mm. If a pressure of 148 N/m² is need to from a bubble which is just released. What is the surface tension of the oil?
- [CE ESE - 2018]
- | | |
|---------------|---------------|
| (a) 0.041 N/m | (b) 0.043 N/m |
| (c) 0.046 N/m | (d) 0.050 N/m |
- 5.** Which of the following statements are correct?
- Depression of mercury in a capillary tube is dependent on density and surface tension.
- (ii) Modelling of flow-induced drag on a ship is done invoking both of Froude number 30 and Reynolds number.
- (iii) Flow of fluid in a narrow pipe is relatable to both Reynolds number and Cauchy number.
- (iv) Formation and collapse of a soap bubble is analyzed through employing surface tension and external pressure.
- (v) Flow over the downstream slope of an ogee spillway can be affected by surface tension
- Select the correct answer using the codes given below :
- [CE ESE - 2017]
- | | |
|-------------------------|------------------------|
| (a) i, ii and iv only | (b) i, iii and v only |
| (c) ii, iii and iv only | (d) iii, iv and v only |
- 6.** A spherical waterdrop of 1mm in diameter splits up in air into 64 smaller drops of equal size. The surface tension coefficient of water in air is 0.073 N/m. The work required in splitting up the drop is
- [ME ESE - 2017]
- | | |
|-----------------------------|-----------------------------|
| (a) 0.96×10^{-6} J | (b) 0.69×10^{-6} J |
| (c) 0.32×10^{-6} J | (d) 0.23×10^{-6} J |
- 7.** What is the intensity of pressure in the following SI units , when specific gravity of mercury is 13.6 and the intensity of pressure is 400KPa
- [ME ESE - 2015]
- | |
|---|
| (a) 0.3bar of 4.077m of water or 0.299m of Hg |
| (b) 4bar or 5.077m of water or 0.399 m of Hg |
| (c) 0.3bar or 5.077m of water or 0.599m of Hg |
| (d) 4bar or 4.077m of water or 0.299m of Hg |
- 8.** The surface tension in a soap bubble of 50 mm diameter with its inside pressure being 2.5 N/m² above the atmosphere pressure is
- [CE ESE - 2015]
- | | |
|----------------|----------------|
| (a) 0.0125 N/m | (b) 0.0156 N/m |
| (c) 0.2 N/m | (d) 0.0312 N/m |

SOLUTIONS

Sol.1. (d) $\tau_{\text{dependson}} \rightarrow \text{viscosity } (\mu)$ $\rightarrow \text{velocity gradient } \left(\frac{d\mu}{dy} \right)$ $\rightarrow \text{Pressure } (\tau_0)$ **Sol.2. (c)**

$$\Delta P = \frac{8\sigma}{D}$$

$$2 \text{ N/m}^2 = \frac{8\sigma}{0.032}$$

Sol.3. (c)

$$d_j = 0.3 \text{ cm}$$

$$\sigma_{\text{water}} = 0.072 \text{ N/m}$$

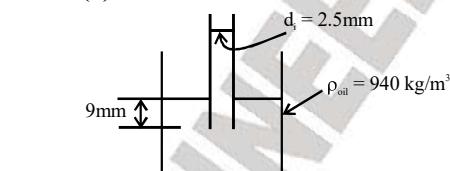
$$P_{\text{atm}} = 101.2 \text{ N/m}^2$$

$$\Delta P = \frac{2\sigma}{D} \text{ (for jet)}$$

$$P_{\text{jet}} - P_{\text{atm}} = \frac{2 \times 0.072 \text{ N/m}}{0.3 \times 10^{-2}}$$

$$= 48 \text{ N/m}^2 = 0.048 \text{ KN/m}^2$$

$$P_{\text{jet}} = 101.2 + 0.048 = 101.248 \text{ KN/m}^2$$

Sol.4. (a)

$$(P_i - P_0) = \frac{4\sigma}{D}$$

$$\text{Now } P_0 = 940 \times 9.81 \times 9 \times 10^{-3} = 82.99 \text{ N/m}^2$$

$$\Rightarrow 148 - 82.99 = \frac{4 \times \sigma}{2.5 \times 10^{-3}}$$

$$\Rightarrow \sigma = \frac{65.01 \times 2.5 \times 10^{-3}}{4} \approx 0.041 \text{ N/m}$$

Sol.5. (a)

$$\text{(i) Depression of mercury in capillary tube} \\ = \frac{4\sigma \cos \theta}{\text{rmsd}}$$

(ii) In ship model we Froude number and reynolds number both

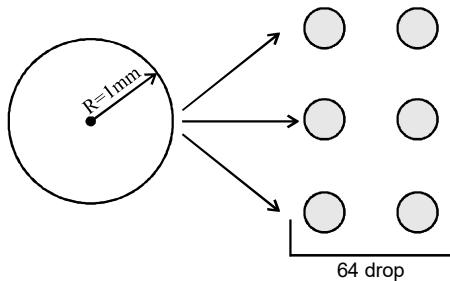
$$\frac{\rho V D}{\mu} = C \quad \frac{V}{\sqrt{g} y} = C$$

(iii) Flow of a fluid in a narrow pipe is relatable to both Reynolds number and weber number .

(iv) In soap bubble

$$\Delta P = \frac{8\sigma}{d}$$

(v) Flow over the downstream slope of an ogee spillway can not be affected by surface tension. So option (a) I , ii and iv only.

Sol.6. (b)

Volume before splitting = volume after splitting

$$\frac{4}{3} \pi R^3 = n \times \frac{4}{3} \pi r^3$$

$$r = \frac{R}{n^{1/3}} = \frac{0.5}{(64)^{1/3}} = \frac{0.5}{4} = 0.125 \text{ mm}$$

$$W = \sigma(\Delta A) \\ = 0.073 \times (n \times 4\pi r^2 - 4\pi R^2) \\ = 0.073 \times 4\pi [64 \times (0.125)^2 - (0.5)^2]$$


WORKBOOK

Example 1. In the accompanying figure, fluid A is water, fluid B is oil of specific gravity 0.85, $Z = 0.7\text{m}$ and $y = 1.5\text{m}$. Compute pressure difference between m and n.

Solution.

Let the height of the common surface above the point m be x . Since pressure head at T = pressure head at T'; we have

$$\frac{p_m}{w} - x - (Z \times 0.85) = \frac{p_n}{w} - (Z + x - y)$$

$$\text{or } \frac{p_m}{w} - \frac{p_n}{w} = y - Z(1 - 0.85)$$

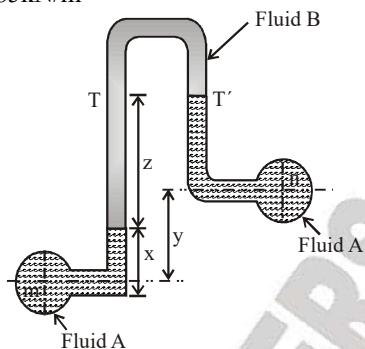
$$= 1.5 - 0.7(0.15) = (1.5 - 0.105)$$

$$= 1.395\text{m of water}$$

$$\text{or } (p_m - p_n) = \frac{1.395 \times 1000}{10^4} = 0.1395\text{kg (f/cm}^2\text{)}$$

$$\text{or } (p_m - p_n) = 1.395 \times 9.810$$

$$= 13.685\text{kN/m}^2$$



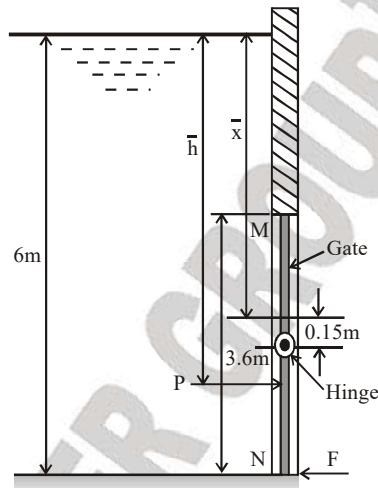
Example 2. A 3.6m by 1.5m wide rectangular gate MN is vertical and is hinged at point 0.15 m below the centre of gravity of the gate. The total depth of water is 6m. What horizontal force must be applied at the bottom of the gate to keep closed?

Solution.

Total pressure acting on the plane surface of the gate is given by

$$P = wA\bar{x}$$

$$A = (3.6 \times 1.5)\text{m}^2$$



$$\text{and } \bar{x} = (6 - 1.8) = 4.2\text{m}$$

∴ By substitution

$$P = 1000 \times (3.6 \times 1.5) \times 4.2$$

$$= 22680 \text{ kg(f)}$$

The depth of centre of pressure is given by

$$\bar{h} = \bar{x} + \frac{I_G}{Ax}$$

$$= 4.2 + \frac{\frac{1}{12} \times 1.5 \times (3.6)^3}{(1.5 \times 3.6) \times 4.2}$$

$$= (4.2 + 0.257) = 4.457\text{m}$$

Let F be the force required to be applied at the bottom of the gate to keep it closed.

By taking moments of all the forces about the hinge and equating to zero for equilibrium, we get

$$F(1.8 - 0.15) - 22680(0.257 - 0.15) = 0$$

$$\therefore F = \frac{22680 \times 0.107}{1.65} = 1471\text{kg(f)}$$

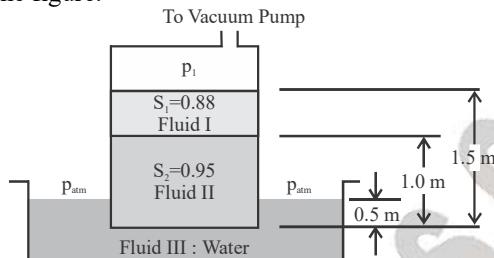
Example 3. A vertical gate closes a horizontal tunnel 5m high and 3m wide running full with water. The pressure at the bottom of the gate is 196.2kN/m^2 [$12\text{kg(f/cm}^2\text{)}$]. Determine the total

— GATE QUESTIONS —

1. A closed tank contains 0.5 m thick layer of mercury (specific gravity =13.6) at the bottom. A 2.0 m thick layer of water lies above the mercury layer. A 3.0 thick layer of oil (specific gravity = 0.6) lies above the water layer. The space above the oil layer contains air under pressure. The gauge pressure at the bottom of the tank is 196.2 kN/m^2 . The density of water is 1000 kg/m^3 and the acceleration due to gravity is 9.81 m/s^2 . The value of pressure in the air space is

- [GATE - 2018]
 (a) 92.214 kN/m^2 (b) 95.644 kN/m^2
 (c) 98.922 kNm^2 (d) 99.321 kN/m^2

2. A three-fluid system (immiscible) is connected to a vacuum pump. The specific gravity values of the fluids (S_1, S_2) are given in the figure.



The gauge pressure value (in kNm^2 , up to two decimal places) of p_1 is _____

[GATE - 2018]

3. A closed tank contains 0.5 m thick layer of mercury (specific gravity =13.6) at the bottom. A 2.0 m thick layer of water lies above the mercury layer. A 3.0 thick layer of oil (specific gravity = 0.6) lies above the water layer. The space above the oil layer contains air under pressure. The gauge pressure at the bottom of the tank is 196.2 kN/m^2 . The density of water is 1000 kg/m^3 and the acceleration due to gravity

is 9.81 m/s^2 . The value of pressure in the air space is

[GATE - 2018]

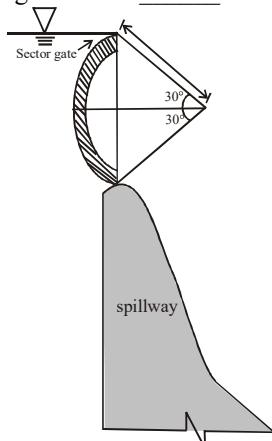
- (a) 92.214 kN/m^2 (b) 95.644 kN/m^2
 (c) 98.922 kNm^2 (d) 99.321 kN/m^2

4. For the stability of a floating body the

[GATE - 2017]

- (a) Centre of buoyancy must coincide with the centre of gravity
 (b) Centre of buoyancy must be above the centre of gravity
 (c) Centre of gravity must be above the centre of buoyancy
 (d) Metacentre must be above the centre of gravity.

5. A section gate is provided on a spillway as shown in the figure. Assuming $g = 10 \text{ m/s}^2$, the resultant force per meter length (expressed in kN / m) on the gate will be _____



[GATE - 2016]

6. A concrete gravity dam section is shown in the figure .Assuming unit weight of water as 10 kN/m^3 and unit weight of concrete as


WORKBOOK

Example 1. Two velocity components are given in the following cases, find the third component such that they satisfy the continuity equation.

- (a) $u = x^3 + y^2 + 2z^2$; $v = -x^2y - yz - xy$;
 (b) $u = \log(y^2 + z^2)$; $v = \log(x^2 + z^2)$;

(c) $u = \frac{-2xyz}{(x^2 + y^2)^2}$; $w = \frac{y}{(x^2 + y^2)}$

Solution.

For an incompressible fluid the equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

(a) In this case

$$u = x^3 + y^2 + 2z^2$$

$$\therefore \frac{\partial u}{\partial x} = 3x^2$$

$$\text{and } v = -x^2y - yz - xy$$

$$\frac{\partial v}{\partial y} = -x^2 - z - x$$

\therefore By substitution in the equation of continuity, we get

$$3x^2 - x^2 - z - x + \frac{\partial w}{\partial z} = 0$$

$$\text{or } \frac{\partial w}{\partial z} = x + z - 2x^2$$

$$\text{or } \partial w = (x + z - 2x^2) \partial z$$

By integrating both sides, we get

$$w = \left(xz + \frac{z^2}{2} - 2x^2z \right) + \text{constant of integration}$$

The constant of integration could be a function of x and y , that is $f(x, y)$. Hence the third component is

$$w = \left(xz + \frac{z^2}{2} - 2x^2z \right) + f(x, y)$$

(b) In this case $u = \log(y^2 + z^2)$

$$\therefore \frac{\partial u}{\partial x} = 0$$

$$\text{Also } v = \log(x^2 + z^2)$$

$$\therefore \frac{\partial v}{\partial y} = 0$$

By substitution in the equation of continuity, we get $\frac{\partial w}{\partial z} = 0$

By integration, we get
 $w = f(x, y)$

By symmetry one of the values of the third component can be
 $W = \log(x^2 + y^2)$

(c) In this case

$$u = \frac{-2xyz}{(x^2 + y^2)^2}$$

$$\therefore \frac{\partial u}{\partial x} = \frac{(x^2 + y^2)^2(-2yz) - (-2xyz) \times 2(x^2 + y^2) \times 2x}{(x^2 + y^2)^4}$$

$$= \frac{6x^2yz - 2y^3z}{(x^2 + y^2)^3}$$

$$\text{Also } w = \frac{y}{(x^2 + y^2)}$$

$$\therefore \frac{\partial w}{\partial z} = 0$$

By substituting in the continuity equation, we get

$$\frac{6x^2yz - 2y^3z}{(x^2 + y^2)^3} + \frac{\partial v}{\partial y} + 0 = 0$$

$$\text{or } \frac{\partial v}{\partial y} = \frac{2y^3z - 6x^2yz}{(x^2 + y^2)^3}$$

$$\text{or } \partial_v = \frac{2y^3z - 6x^2yz}{(x^2 + y^2)^2} \partial y$$

By integrating both sides, we get

$$v = \frac{z(x^2 - y^2)}{(x^2 + y^2)^2} + f(x, z).$$


WORKBOOK

Example 1. A 0.25 m diameter pipe carries oil of specific gravity 0.8 at the rate of 120 liters per second and the pressure at a point A is 19.62kN/m² (gage). If the point A is 3.5m above the datum line, calculate the total energy at point A in metres of oil.

Solution.

Total energy in terms of oil is given by

$$= \frac{P}{w} + \frac{V^2}{2g} + Z$$

$$\frac{P}{w} = \frac{19.62 \times 10^3}{9810 \times 0.8} = 2.5 \text{m of oil}$$

By continuity

$$Q = AV$$

$$Q = 120 \times 10^{-3} = 0.12 \text{m}^3/\text{s}$$

$$A = \frac{\pi}{4} \times (0.25)^2 = 0.049 \text{m}^2$$

$$\text{Therefore } V = \frac{Q}{A} = \frac{0.12}{0.049} = 2.45 \text{m/s}$$

$$\text{And } \frac{V^2}{2g} = \frac{(2.45)^2}{2 \times 9.81} = 0.31 \text{m of oil}$$

$$Z = 3.5$$

$$\therefore \text{Total energy} = (2.5 + 0.31 + 3.5) = 6.31 \text{ m of oil.}$$

Example 2. A 0.3m pipe carries water at a velocity of 24.4m/s. At points A and B measurements of pressure and elevation were respectively 361kN/m² and 288kN/m² and 30.5m. For steady flow, find the loss of head between A and B.

Solution.

Total energy in terms of meter is given by

$$= \frac{P}{w} + \frac{V^2}{2g} + Z$$

At point A

$$\frac{P}{w} = \frac{361 \times 10^3}{9810} = 36.80 \text{m of water}$$

$$Z = 30.50 \text{m}$$

$$\therefore \text{Total energy at A} \\ = (36.80 + 30.34 + 30.50) = 97.64 \text{m}$$

At point B

$$\frac{P}{w} = \frac{288 \times 10^3}{9810} = 29.36 \text{m of water}$$

$$\frac{V^2}{2g} = \frac{(24.4)^2}{2 \times 9.81} = 30.34 \text{m of water}$$

$$Z = 33.5 \text{m}$$

$$\therefore \text{Total energy at B} \\ = (29.36 + 30.34 + 33.50) = 93.20 \text{ m}$$

$$\therefore \text{Loss of head} = (97.64 - 93.20) = 4.44 \text{m.}$$

Example 3. The velocity distribution in a pipe

is given by $v = V_{\max} \left(1 - \frac{r}{R}\right)^n$ where R is the radius of the pipe, r is any radius at which the velocity is v and n is a constant index. Find the energy correction factor a for this case. Also determine the value of a when $n = \frac{1}{7}$.

Solution.

The mean velocity of flow V is given by

$$Q = \pi R^2 V = \int_0^R 2\pi r V_{\max} \left(1 - \frac{r}{R}\right)^n dr \\ = \frac{2\pi R^2 V_{\max}}{(n+1)(n+2)}$$

$$\therefore V = \frac{2V_{\max}}{(n+1)(n+2)}$$

$$\text{and } \frac{v}{V} = \frac{(n+1)(n+2)}{2} \left(1 - \frac{r}{R}\right)^n$$

From equation, we have

$$\alpha = \frac{1}{A} \iint_A \left(\frac{v}{V}\right)^3 dA$$

WORKBOOK

Example 1. Find the expression for the drag force on a smooth sphere of diameter D, moving with a uniform velocity v in a fluid of density ρ and dynamic viscosity μ .

Solution.

The drag force F_D is a function of D , v , ρ and μ .

$$F_D = f(D, v, \rho, \mu)$$

$$\text{or } F_D = \phi(D^a, v^b, \rho^c, \mu^d)$$

where ϕ is a non-dimensional constant.

Using M – L – T system, we have

$$MLT^{-2} = \phi [L^a] [LT^{-1}]^b [ML^{-3}]^c [ML^{-1}T^{-1}]^d$$

$$\text{For M: } c + d = 1 \quad \dots \text{(i)}$$

$$\text{For L: } a + b - 3c - d = 1 \quad \dots \text{(ii)}$$

$$\text{For T: } -b - d = -2 \quad \dots \text{(iii)}$$

There are four unknowns (a , b , c , d) and three equations. Therefore, it is not possible to find the values of a , b , c and d explicitly. Therefore, three of them can be expressed in terms of the fourth variable, which is most important. The role of viscosity μ is very important for drag force and hence a , b , c are expressed in terms of d .

$$\therefore c = 1 - d$$

$$b = 2 - d$$

$$a = 1 - (2 - d) + 3(1 - d) + d = 2 - d$$

$$\text{For } F_D = \phi [D]^{2-d} [v]^{2-d} [\rho]^{2-d} [\mu]^d$$

$$= \phi [D^2 v^2 \rho (D^{-d} v^{-d} \rho^{-d} \mu^d)]$$

$$= \phi \left[\rho D^2 v^2 \left(\frac{\mu}{\rho v D} \right)^d \right] = \rho D^2 v^2 \phi \left(\frac{\mu}{\rho v D} \right)^d$$

Example 2. The resistance R of a supersonic plan during flight depends upon the length of aircraft l , velocity v , are viscosity μ , air density ρ , and bulk modulus of air K . Determine the functional relationship between these variables and the resisting force.

Solution.

$$R = f_1(l, v, \mu, \rho, K)R = f_2(l^2, v^b, \mu^c, \rho^d, K^e)$$

where f_2 is a non-dimensional constant.

Using the M–L–T system, we have

$$[M][L][T]^{-2} = f_2 [L]^a [LT^{-1}]^b [ML^{-1}T^{-1}]^c [ML^{-3}]^d [ML^{-1}T^{-2}]^e$$

$$= f_2 [M]^{c+d+e} [L]^{a+b-c-3d-e} [T]^{-b-c-2e}$$

For dimensional homogeneity, we have

$$\text{For M: } c + d + e = 1$$

$$\text{For L: } a + b - c - 3d - e = 1$$

$$\text{For T: } -b - c - 2e = -2$$

There are five variable and three equations. For a supersonic plant, μ and K are very important variables. Therefore, we express a , b , and d in terms of c and e .

$$d = 1 - c - e$$

$$b = 2 - c - 2e$$

$$a = 1 - (2 - c - 2e) + c + 3(1 - c - e) + e$$

$$= 1 - 2 + c + 2e + c + 3 - 3c - 3r + r$$

$$= 2 - c$$

$$R = F_2 [l]^{2-c} [v]^{2-c-2e} (\mu) c [\rho]^{1-c-e} [K]^e$$

$$= f_2 l^2 v^2 \rho \left(\frac{\mu}{l v \rho} \right)^c \left(\frac{K}{v^2 \rho} \right)^e = l^2 v^2 \rho \phi \left[\left(\frac{\mu}{l v \rho} \right), \left(\frac{K}{v^2 \rho} \right) \right]$$

Example 3. The pressure drop Δp in a pipe diameter D and length l depends on the density ρ and viscosity μ of the following fluid, mean velocity v of flow and average height of protusion t , show that the pressure drop can be expressed in the form.

$$\Delta p = \rho v^2 \phi \left(\frac{1}{D}, \frac{\mu}{v D \rho}, \frac{t}{D} \right).$$

Solution.

$$\Delta p = f_1(D, l, \rho, \mu, v, t) = f_2 [D^a, l^b, \rho^c, \mu^d, v^e, t^f]$$

Where f_2 is a non-dimensional constant.

Using M-L-T system, we have

$$[M][L]^{-1}[T]^2 = f_2 [L]^a [L]^b [ML^{-1}]^c$$

$$[ML^{-1}T^{-1}]^d [LT^{-1}]^e [L]^f$$

$$= f_2 [M]^{c+d} [L]^{a+b-3c-d+e+f} [T]^{-d-e}$$

For dimensional homogeneity, we have

GATE QUESTIONS

1. A 1:50 model of a spillway is to be tested in the laboratory. The discharge in the prototype spillway is $1000 \text{ m}^3/\text{s}$. The corresponding discharge (in m^3/s up to two decimal places) to be maintained in the model, neglecting variation in acceleration due to gravity is _____

[GATE - 2018]

2. In a laboratory, a flow experiment is performed over a hydraulic structure. The measured values of discharge and velocity are $0.05 \text{ m}^3/\text{s}$ and 0.25 m/s , respectively. If the full scale structure (30 times bigger) is subjected to a discharge of $270 \text{ m}^3/\text{s}$, then the time scale (model to full scale) value (up to two decimal places) is _____

[GATE - 2018]

3. The drag force, F_D , on a sphere kept in a uniform flow field depends on the diameter of the sphere D , flow velocity V ; fluid density ρ ; and dynamic viscosity μ . Which of the following options represents the non-dimensional parameters which could be used to analyze this problem?

[GATE - 2015]

- | | |
|---|---|
| (a) $\frac{F_D}{V_D}$ and $\frac{\mu}{\rho V D}$ | (b) $\frac{F_D}{\rho V D^2}$ and $\frac{\rho V D}{\mu}$ |
| (c) $\frac{F_D}{\rho V^2 D^2}$ and $\frac{\rho V D}{\mu}$ | (d) $\frac{F_D}{\rho V^3 D^3}$ and $\frac{\mu}{\rho V D}$ |

4. The relationship between the length scale ratio (L_r) and the velocity scale ratio (V_r) in hydraulic models, in which Froude dynamic similarity is maintained, is

[GATE - 2015]

- | | |
|-----------------------|------------------------|
| (a) $V_r = L_r$ | (b) $L_r = \sqrt{V_r}$ |
| (c) $V_r = L_r^{1.5}$ | (d) $V_r = \sqrt{L_r}$ |

5. Group-I contains dimensionless parameter and Group-II contains ratio.

Group-I

- A. Match number
- B. Reynold number
- C. Weber number
- D. Froude number

Group-II

- (i) Ratio of internal force and gravity force.
- (ii) Ratio of fluid velocity and velocity of sound.
- (iii) Ratio of inertial force and viscous force.
- (iv) Ratio of inertial force and surface tension force.

Correct match of the dimensionless parameter in Group-I with Group-II is

[GATE - 2013]

Code:

- (a) A-iii, B-ii, C-iv, D-i
- (b) A-iii, B-iv, C-ii, D-i
- (c) A-ii, B-iii, C-iv, D-i
- (d) A-iii, B-iii, C-ii, D-iv

6. A phenomenon is modeled using n dimensional variables k primary dimensions. The number of non-dimensional variables is

[GATE - 2010]

- | | |
|-------------|-------------|
| (a) k | (b) n |
| (c) $n - k$ | (d) $n + k$ |

7. A river reach of 2.0 km long with maximum flood discharge of $10000 \text{ m}^3/\text{s}$ is to be physically modeled in the laboratory where maximum available discharge is $0.20 \text{ m}^3/\text{s}$. For a geometrically similar model based on equality of Froude number, the length of the river reach (m) in the model is

[GATE - 2008]

- | | |
|----------|----------|
| (a) 26.4 | (b) 25.0 |
| (c) 20.5 | (d) 18.0 |

8. A 1:50 scale model of a spillway is to be tested in the laboratory. The discharge in the prototype is $1000 \text{ m}^3/\text{s}$. The discharge to be maintained in the model test is

[GATE - 2007]

- | | |
|----------------------------------|---------------------------------|
| (a) $0.057 \text{ m}^3/\text{s}$ | (b) $0.08 \text{ m}^3/\text{s}$ |
| (c) $0.57 \text{ m}^3/\text{s}$ | (d) $5.7 \text{ m}^3/\text{s}$ |

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