

GATE

2019

**FLUID MECHANICS
& MACHINERY**

MECHANICAL ENGINEERING



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GATE-2019: Fluid Mechanics & Machinery | Detailed theory with GATE & ESE previous year papers and detailed solutions.

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CHAPTER - 1
PROPERTIES OF FLUIDS

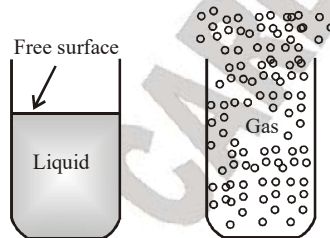
1.1 INTRODUCTION

The fluid a substance in the liquid or gas phase is referred to as a fluid.

A solid can resist an applied shear stress by deforming, whereas a fluid deforms continuously under the influence of a shear stress, no matter how small. In solids, stress is proportional to strain but in fluids, stress is proportional to strain rate. When a constant shear force is applied, a solid eventually stops deforming at some fixed strain angle, whereas a fluid never stops deforming and approaches a constant rate of strain.

The normal component of a force acting on a surface per unit area is called the normal stress, and the tangential component of a force acting on a surface per unit area is called shear stress (fig). In a fluid at rest, the normal stress is called pressure. A fluid at rest is at a state of zero shear stress. When the walls are removed or a liquid container is tilted, a shear develops as the liquid moves to re-establish a horizontal free surface.

In a liquid, groups of molecules can move relative to each other, but the volume remains relatively constant because of the strong cohesive forces between the molecules. As a result , a liquid takes the shape of the container it is in and it forms a free surface in a larger container in a gravitational field. A gas, on the other hand, expands until it encounters the walls of the container and fills the entire available space. This is because the gas molecules are widely spaced, and the cohesive forces between them are very small. Unlike liquids, a gas in an open container cannot form a free surface (fig.)



Unlike a liquid, a gas does not form a free surface, and it expands to fill the entire available space.

1.2 CONTINUUM

A fluid is composed of molecules which may be widely spaced apart, especially in the gas phase. Yet it is convenient to disregard the atomic nature of the fluid and view it as continuous, homogeneous matter with no holes that is continuum. The continuum idealization allows us to treat properties as point functions and to assume that the properties vary continually in space with no jump discontinuities. This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules. The continuum idealization is implicit in many statements we make, such as “the density of water in a glass is the same at any point.”

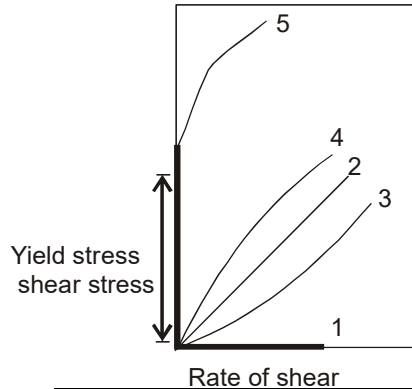
1.3 DENSITY AND SPECIFIC GRAVITY

Density is defined as mass per unit volume

$$\text{Density } \rho = \frac{m}{V} \text{ (kg / m}^3\text{)}$$

GATE QUESTIONS

1. Group I contains the types of fluids while Group II contains the shear stress –rate of shear relationship of different types of fluids , as shown in the figure



Group I	Group II
A. Newtonian fluid	(i) Curve 1
B. Pseudo plastic fluid	(ii) Curve 2
C. Plastic fluid	(iii) Curve 3
D. Dilatant fluid	(iv) Curve 4
	(v) Curve 5

The correct match between Group I and Group II is

[GATE - 2016]

- (a) A-ii, B-iv, C-i, D-v
- (b) A-ii, B-v, C-iv, D-1
- (c) A-ii, B-iv, C-v, D-iii
- (d) A-ii, B-i, C-iii, D-iv

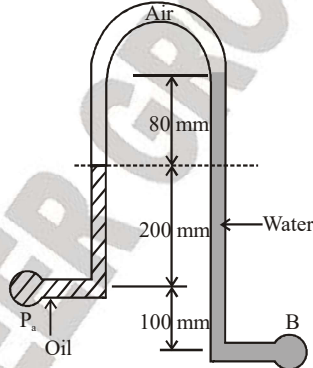
2. Oil (kinematic viscosity , $\nu_{oil} = 1 \times 10^{-5} m^2/s$) flow through a pipe diameter with a velocity of $10 m/s$ $\nu_w = 0.89 \times 10^{-6} m^2/s$ diameter flowing through a model pipe of diameter 10mm for satisfying the dynamic similarity , the velocity of water (in m/s) is _____

[GATE - 2016]

3. An inverted U-tube manometer is used to measure the pressure difference between two pipes A and B, as shown in the figure .Pipe A is carrying oil (specific gravity =0.8) and pipe B is

carrying water .The densities of air and water are $1.16 kg/m^3$ and $1000 kg/m^3$, respectively. The pressure difference between pipes A and B is _____ kPa.

Acceleration due to gravity $g = 10 m/s^2$



[GATE - 2016]

4. The difference in pressure (in N/m^2) across an air bubble of diameter 0.001 m immersed in water (surface tension = 0.072 N/m) is _____.

[GATE - 2014]

5. The dimension for kinematics viscosity is

[GATE - 2014]

- (a) L/MT
- (b) L/T^2
- (c) L^2/T
- (d) ML/T

6. The necessary and sufficient condition for a surface to be called as a ‘free surface’ is

[GATE - 2006]

- (a) No stress should be acting on it
- (b) Tensile stress acting on it must be zero
- (c) Shear stress acting on it must be zero
- (d) No point on it should be under any stress

7. In the inclined manometer shown in the figure below, the reservoir is large. Its surface may be assumed to remain at a fixed elevation . A is connected to a gas pipeline and the deflection noted on the inclined glass tube is 100 mm.

ESE OBJ QUESTIONS

1. Which of the following statements are correct?

- (i) Depression of mercury in a capillary tube is dependent on density and surface tension.
 (ii) Modelling of flow-induced drag on a ship is done invoking both of Froude number 30 and Reynolds number.
 (iii) Flow of fluid in a narrow pipe is related to both Reynolds number and Cauchy number.
 (iv) Formation and collapse of a soap bubble is analyzed through employing surface tension and external pressure.
 (v) Flow over the downstream slope of an ogee spillway can be affected by surface tension
 Select the correct answer using the codes given below :

[CE ESE - 2017]

- (a) i, ii and iv only (b) i, iii and v only
 (c) ii, iii and iv only (d) iii, iv and v only

2. A spherical waterdrop of 1mm in diameter splits up in air into 64 smaller drops of equal size. The surface tension coefficient of water in air is 0.073 N/m. The work required in splitting up the drop is

[ME ESE - 2017]

- (a) 0.96×10^{-6} J (b) 0.69×10^{-6} J
 (c) 0.32×10^{-6} J (d) 0.23×10^{-6} J

3. What is the intensity of pressure in the following SI units, when specific gravity of mercury is 13.6 and the intensity of pressure is 400KPa

[ME ESE - 2015]

- (a) 0.3bar of 4.077m of water or 0.299m of Hg
 (b) 4bar or 5.077m of water or 0.399 m of Hg
 (c) 0.3bar or 5.077m of water or 0.599m of Hg
 (d) 4bar or 4.077m of water or 0.299m of Hg

4. The surface tension in a soap bubble of 50 mm diameter with its inside pressure being 2.5 N/m² above the atmosphere pressure is

[CE ESE - 2015]

- (a) 0.0125 N/m (b) 0.0156 N/m
 (c) 0.2 N/m (d) 0.0312 N/m

5. Manometer is a device used for measuring

[ME ESE - 2014]

- (a) Velocity at a point in a fluid
 (b) Pressure at a point in a fluid
 (c) Discharge of a fluid
 (d) None of the above

6. The velocity distribution in a laminar flow adjacent to a solid wall is given by $u = 3.0 \sin(5\pi y)$. The viscosity of the fluid is 5 poise. What is the shear stress at a section (i) $y = 0.05$ m; (ii) $y = 0.12$ m?

[CE ESE - 2014]

- (a) 16.7 N/m² and 6.1 N/m²
 (b) 33.4 N/m² and Zero
 (c) 16.7 N/m² and 12.3 n/m²
 (d) 16.7 N/m² and Zero

7. **Statement (I):** As temperature increases, viscosity of air decreases.

Statement (II): As temperature increases, activity of the air molecules increases.

[CE ESE - 2013]

Codes:

- (a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)
 (b) Both Statement (I) and Statement (II) are individually true but Statement (II) is NOT the correct explanation of Statement (I)
 (c) Statement (I) is true but Statement (II) is false
 (d) Statement (I) is false but Statement (II) is true.

8. **Statement (I):** Fluid pressure is a scalar quantity.

Statement (II): Fluid thrust always acts downwards.

CHAPTER - 2
FLUID STATICS

2.1 INTRODUCTION

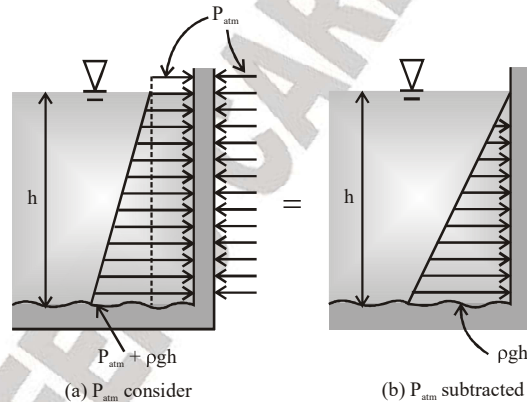
Fluid statics deals with problems associated with fluids at rest.

In fluid statics, there is no relative motion between adjacent fluid layers, and thus there are no shear (tangential) stresses in the fluid trying to deform it. The only stress we deal with in fluid statics is the normal stress, which is the pressure, and the variation of pressure is due only to the weight of the fluid.

The design of many engineering systems such as water dams and liquid storage tanks requires the determination of the forces acting on their surfaces using fluid statics.

2.2 HYDROSTATIC FORCES ON SUBMERGED PLANE SURFACES

A plate (such as a gate valve in a dam, the wall of a liquid storage tank, or the hull of a ship at rest) is subjected to fluid pressure distributed over its surface when exposed to a liquid. On a plane surface, the hydrostatic forces form a system of parallel forces, and we often need to determine the magnitude of the force and its point of application, which is called the centre of pressure. In most cases, the other side of the plate is open to the atmosphere (such as the dry side of a gate), and thus atmospheric pressure acts on both sides of the plate, yielding a zero resultant. In such cases, it is convenient to subtract atmospheric pressure and work with the gage pressure only. For example, $P_{\text{gage}} = \rho gh$ at the bottom of the lake.



When analyzing hydrostatic forces on submerged surfaces, the atmospheric pressure can be subtracted for simplicity when it acts on both sides of the structure.

Consider the top surface of a flat plate of arbitrary shape completely submerged in a liquid, as shown in fig. together with its normal view. The plane of this surface (normal to the page) intersects the horizontal free surface at angle θ , and we take the line of intersection to be the x-axis (out of the page). The absolute pressure above the liquid is P_0 , which is the local atmospheric pressure P_{atm} if the liquid is open to the atmosphere (but P_0 may be different than P_{atm} if the space above the liquid is evacuated or pressurized). Then the absolute pressure at any point on the plate is

$$P = P_0 + \rho gh = P_0 + \rho gy \sin \theta$$

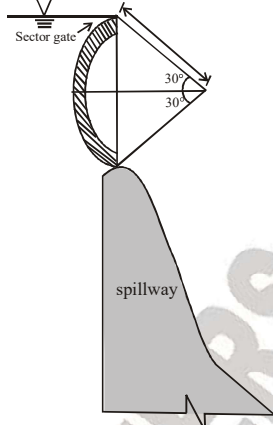
GATE QUESTIONS

1. For the stability of a floating body the

[GATE - 2017]

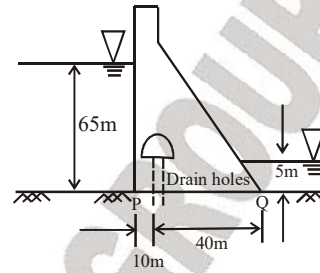
- (a) Centre of buoyancy must coincide with the centre of gravity
- (b) Centre of buoyancy must be above the centre of gravity
- (c) Centre of gravity must be above the centre of buoyancy
- (d) Metacentre must be above the centre of gravity.

2. A section gate is provided on a spillway as shown in the figure. Assuming $g = 10\text{m/s}^2$, the resultant force per meter length (expressed in kN/m) on the gate will be _____



[GATE - 2016]

3. A concrete gravity dam section is shown in the figure. Assuming unit weight of water as 10kN/m^3 and unit weight of concrete as 24kN/m^3 , the uplift force per unit length of the dam (expressed in kN/m) at PQ is _____



[GATE - 2016]

4. A triangular gate with a base width of 2m and a height of 1.5m lies in a vertical plane. The top vertex of the gate is 1.5m below the surface of a tank which contains oil of specific gravity 0.8. Considering the density of water and acceleration due to gravity to be 1000 kg/m^3 and 9.81 m/s^2 , respectively, the hydrostatic force (in kN) exerted by the oil on the gate is _____.

[GATE - 2015]

5. For a completely submerged body with centre of gravity 'G' and centre of buoyancy 'B'. The condition of stability will be _____

[GATE - 2014]

- (a) G is located below B
- (b) G is located above B
- (c) G and B are coincident
- (d) Independent of the locations of G and B

6. An aluminum alloy (density 2600 kg/m^3) casting is to be produced. A cylindrical hole of 100 mm diameter and 100 mm length is made in the casting using sand core (density 1600 kg/m^3). The net buoyancy force (in newton) acting on the core is _____.

[GATE - 2014]

7. A spherical balloon with a diameter of 10m, shown in the figure below is used for advertisements. The balloon is filled with helium ($R_{\text{He}} = 2.08\text{ kJ/kg. K}$) at ambient conditions of 15°C and 100 kPa . Assuming no

ESE OBJ QUESTIONS

1. An ocean liner, 240m long and 24m wide, displaces 654 MN of sea-water ($\rho = 1025 \text{ kgf/m}^3$). The second moment of inertia of the water plane about its fore-aft axis is $\frac{2}{3}$ of that of the circumscribing rectangle. The position of the centre of buoyancy is 2.30m below the centre of gravity. How high is the meta centre above the centre of buoyancy (to the nearest)
- [CE ESE - 2017]
- (a) 49m (b) 53m
(c) 58m (d) 65m
2. Consider the following statements pertaining to stability of floating bodies:
1. A floating body will be stable when the centre of gravity is above the centre of buoyancy.
 2. The positions of metacentres corresponding to different to different axes of rotation are generally different for the same floating object.
 3. For cargo ships, the metacentric height varies with loading.
- Which of the above statements are correct?
- [ME ESE - 2017]
- (a) 1, 2 and 3 (b) 1 and 2 only
(c) 1 and 3 only (d) 2 and 3 only
3. **Statement (I):** Depth of centre of pressure of any immersed surface is independent of the density of the liquid.
Statement (II): Centre of area of the immersed body lies below the centre of pressure.
- [ME ESE - 2017]
- (a) Both statements (I) and Statement (II) are individually true and Statement (II) is the correct explanation of statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.
4. Uniform fluid occurs when
- [ME ESE - 2016]
- (a) At every point the velocity is identical in magnitude and direction at any given instance
(b) The flow is steady
(c) Discharge through a pipe is constant
(d) Conditions do not change with at any time
5. What is the specific gravity of a marble stone, which weighs 400N in air and 200N in water ($g = 10 \text{ m/s}^2$)?
- [ME ESE - 2015]
- (a) 8 (b) 6
(c) 4 (d) 2
6. Consider the following statements :
1. If a small upward displacement is given to a floating body, it results in the reduction of the buoyant force acting on the body
 2. A slight horizontal displacement does not change either the magnitude or the location of the buoyant force
- Which of the above statement is /are correct?
- [ME ESE - 2015]
- (a) Both 1 and 2 (b) 1 only
(c) 2 only (d) Neither 1 nor 2
7. A tank of length, breadth and height in the ratio of 2:1:2 is full of water. The ratio of hydrostatic force at the bottom to that on any larger vertical surface is
- [ME ESE - 2015]
- (a) 1 (b) 4
(c) 2 (d) 3
8. A mercury water manometer has a gauge difference of 0.8 m. The difference in pressure measured in meters of water is
- [CE ESE - 2015]
- (a) 0.8 (b) 1.06
(c) 10.05 (d) 8.02

CHAPTER - 3***FLUID KINEMATICS*****3.1 INTRODUCTION**

The Science which deals with the geometry of motion of fluids without reference to the forces causing the motion is known as hydro kinematics or simply kinematics. Thus kinematics involves merely the description of the motion of fluids in terms of space-time relationship. On the other hand the science which deals with the action of the forces in producing or changing motion of fluids is known as hydro kinematics or simply kinetics.

There are in general two methods by which the motion of a fluid may be described. These are the Lagrangian method and the Eulerian method.

In the Lagrangian method any individual fluid particle is selected, which is pursued throughout its course of motion and the observation is made about the behavior of this particle during its course of motion through space. In the Eulerian method any point in the space occupied by the fluid is selected and observation is made of whatever changes of velocity, density and pressure which take place at that point. Out of these two methods the Eulerian method is commonly adopted in fluid mechanics and therefore the same is used in the following analysis.

3.2 VELOCITY OF FLUID PARTICLES

If ds is the distance travelled by a fluid particle in time dt then the velocity V of the fluid particle at this point may be expressed as

$$V = \lim_{dt \rightarrow 0} \frac{ds}{dt}$$

The velocity is a vector quantity and hence it has magnitude as well as direction. Therefore the velocity V at any point in the fluid can be resolved into three components u , v and w along three mutually perpendicular directions x , y and z respectively. Thus if dx , dy and dz are the components of the displacement ds in x , y and z directions respectively, then

$$u = \lim_{dt \rightarrow 0} \frac{dx}{dt}, v = \lim_{dt \rightarrow 0} \frac{dy}{dt} \text{ and } w = \lim_{dt \rightarrow 0} \frac{dz}{dt}$$

Since the velocity V at any point in a flowing mass of fluid in general depends on x , y and z , i.e., the coordinate position of the point under consideration and the time t . Hence the velocity V and its components, u , v and w may be expressed in terms of the following functional relationships

$$\left. \begin{aligned} V &= f_1(x, y, z, t) \\ u &= f_2(x, y, z, t) \\ v &= f_3(x, y, z, t) \\ w &= f_4(x, y, z, t) \end{aligned} \right\}$$

In vector notation velocity V may be expressed in terms of its components as $V = iu + jv + kw$

3.3 TYPES OF FLUID FLOW

According to different consideration fluid flows may be classified in several ways as indicated below:

1. Steady flow and unsteady flow
2. Uniform flow and Non-uniform flow
3. One-dimensional flow, Two-dimensional flow and Three-dimensional flow
4. Rotational flow and Ir-rotational flow

GATE QUESTIONS

1. For a two-dimensional flow, the velocity field is $\vec{u} = \frac{x}{x^2+y^2}\hat{i} + \frac{y}{x^2+y^2}\hat{j}$, where

\hat{i} and \hat{j} are the basis vectors in the x0y Cartesian coordinate system. Identify the CORRECT statements from below

1. The flow is incompressible
2. The flow is unsteady
3. y – component of acceleration, $a_y =$

$$a_y = \frac{-y}{(x^2+y^2)^2}$$

4. x- component of acceleration, $a_x =$

$$a_x = \frac{-(x+y)}{(x^2+y^2)^2}$$

[GATE - 2016]

- | | |
|-------------|-------------|
| (a) 2 and 3 | (b) 1 and 3 |
| (c) 1 and 2 | (d) 3 and 4 |

2. For a certain two-dimensional incompressible flow, velocity field is given by $2xy\hat{i} - y^2\hat{j}$. The streamlines for this flow are given by the family of curves

[GATE - 2016]

- (a) $x^2y^2 = \text{constant}$
- (b) $xy^2 = \text{constant}$
- (c) $2xy - y^2 = \text{constant}$
- (d) $xy = \text{constant}$

3. The velocity components of a two dimensional plane motion of a fluid are

$$u = \frac{y^3}{3} + 2x - x^2y \text{ and } v = xy^2 - 2y - \frac{x^3}{3}$$

The correct statement is:

[GATE - 2015]

- (a) Fluid is incompressible and flow is irrotational
- (b) Fluid is incompressible and flow is rotational
- (c) Fluid is compressible and flow is irrotational
- (d) Fluid is compressible and flow is rotational

4. In a two-dimensional steady flow field, in a certain region of the x-y plane, the velocity component in the x-direction is given by $v_x = x^2$ and the density varies as $\rho = \frac{1}{x}$. Which of the

following is a valid expression for the velocity component in the y-direction v_y ?

[GATE - 2015]

- | | |
|--------------------------|-------------------------|
| (a) $v_y = -\frac{x}{y}$ | (b) $V_y = \frac{x}{y}$ |
| (c) $V_y = -xy$ | (d) $V_y = xy$ |

5. A particle moves along a curve whose parametric equations are: $x = t^3 + 2t$, $y = 3e^{-2t}$ and $z = 2 \sin(5t)$, where x, y and z show variations of the distance covered by the particle (in cm) with time t (in s). The magnitude of the acceleration of the particle (in cm/s^2) at $t = 0$ is

[GATE - 2014]

6. A plane flow has velocity components $u = \frac{x}{T_1}$, $v = -\frac{y}{T_2}$ and $w = 0$ along x, y and z

directions respectively, where $T_1 (\neq 0)$ and $T_2 (\neq 0)$ are constants having the dimension of time. The given flow is incompressible if

[GATE - 2014]

- | | |
|---------------------------|----------------------------|
| (a) $T_1 = -T_2$ | (b) $T_1 = -\frac{T_2}{2}$ |
| (c) $T_1 = \frac{T_2}{2}$ | (d) $T_1 = T_2$ |

7. For an incompressible flow field, V , which one of the following conditions must be satisfied?

[GATE - 2014]

- | | |
|--|--|
| (a) $\nabla \cdot \vec{V} = 0$ | (b) $\nabla \cdot \vec{V} = 0$ |
| (c) $(\vec{V} \cdot \nabla) \vec{V} = 0$ | (d) $\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} = 0$ |

CHAPTER - 4***FLUID DYNAMICS, FLOW OVER NOTCHES & WEIRS*****4.1 INTRODUCTION**

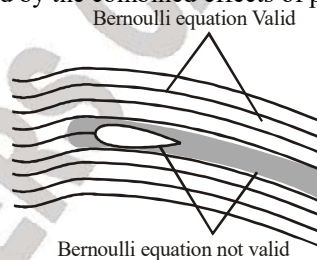
This chapter deals with three equations commonly used in fluid mechanics: the mass, Bernoulli, and energy equations. The mass equation is an expression of the conservation of mass principle. The Bernoulli equation is concerned with the conservation of kinetic, potential, and flow energies of a fluid stream and their conversion to each other in regions of flow where net viscous forces are negligible and where other restrictive conditions apply. The energy equation is a statement of the conservation of energy principle.

4.2 THE BERNOULLI EQUATION

The Bernoulli equation is an approximate relation between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow where net frictional forces are negligible (fig.) Despite its simplicity, it has proven to be a very powerful tool in fluid mechanics. In this section, we derive the Bernoulli equation by applying the conservation of linear momentum principle, and we demonstrate both its usefulness and its limitations.

The key approximation in the derivation of the Bernoulli equation is that viscous effects are negligibly small compared to inertial, gravitational, and pressure effects. Since all fluids have viscosity (there is no such thing as an “inviscid fluid”), this approximation cannot be valid for an entire flow field of practical interest.

Care must be exercised when applying the Bernoulli equation since it is an approximation that applies only to inviscid regions of flow. In general, frictional effects are always important very close to solid wall (boundary layers) and directly down stream of bodies (wakes). Thus, the Bernoulli approximation is typically useful in flow regions outside of boundary layers and wakes, where the fluid motion is governed by the combined effects of pressure and gravity forces.



The Bernoulli equation is an approximate equation that is valid only in inviscid regions of flow where net viscous forces are negligibly small compared to inertial, gravitational, or pressure forces. Such regions occur outside of boundary layers and wakes.

4.2.1 Derivation of the Bernoulli Equation

Consider the motion of a fluid particle in a flow field in steady flow. Applying Newton’s second law (which is referred to as the linear momentum equation in fluid mechanics) in the s -direction on a particle moving along a streamline gives

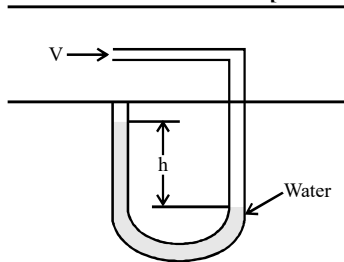
$$\Sigma F_s = ma_s \quad \dots(i)$$

In regions of flow where net frictional forces are negligible, there is no pump or turbine, and there is no heat transfer along the streamline, the significant forces acting in the s -direction are the pressure (acting on both sides) and the component of the weight of the particle in the s -direction (Fig.) Therefore, equation (i) becomes

GATE QUESTIONS

1. The arrangement shown in the figure measures the velocity V of a gas of density 1 kg/m^3 flowing through a pipe. The acceleration due to gravity is 9.81 m/s^2 . If the manometric fluid is water (density 1000 kg/m^3) and the velocity V is 20 m/s , the differential head h (in mm) between the two arms of the manometer is _____.

[GATE - 2017]



2. A circular pipe has a diameter of 1 m, bed slope of 1 in 1000, and Manning's roughness coefficient equal to 0.01. It may be treated as an open channel flow when it is flowing just full, i.e., the water level just touches the crest. The discharge in this condition is denoted by Q_{full} . Similarly, the discharge when the pipe is flowing half-full, i.e., with a flow depth of 0.5 m, is denoted by Q_{half} . The ratio $\frac{Q_{full}}{Q_{half}}$ is _____.

[GATE - 2015]

- (a) 1
- (b) $\sqrt{2}$
- (c) 2
- (d) 4

3. A nozzle is so shaped that the average flow velocity changes linearly from 1.5 m/s at the beginning to 15 m/s at its end in a distance of 0.375 m . The magnitude of the convective acceleration (in m/s^2) at the end of the nozzle is _____.

[GATE - 2015]

4. Group-I lists a few devices while Group-II provides information about their uses. Match the devices with their corresponding use.

Group-I

- A. Anemometer
- B. Hygrometer
- C. Pitot Tube
- D. Tensimeter

Group-II

- (i) Capillary potential of soil water
- (ii) Fluid velocity at a specific point in the flow stream
- (iii) Water vapour content of air
- (iv) Wind speed

[GATE - 2014]

Codes:

- (a) A-i, B-ii, C-iii, D-iv
- (b) A-ii, B-i, C-iv, D-iii
- (c) A-iv, B-ii, C-i, D-iii
- (d) A-iv, B-iii, C-ii, D-i

5. A venturimeter, having a diameter of 7.5 cm at the throat and 15 cm at the enlarged end, is installed in a horizontal pipeline of 15 cm diameter. The pipe carries an incompressible fluid at a rate of $30 \text{ litres per second}$. The difference of pressure head measured in terms of the moving fluid in between the enlarged and the throat of the venturimeter is observed to be 2.45 m . Taking the acceleration due to gravity as 9.81 m/s^2 , the coefficient of discharge of the venturimeter correct up to two places of decimal) is _____.

[GATE - 2014]

6. A venturimeter having a throat diameter of 0.1 m is used to estimate the flow rate of a horizontal pipe having a diameter of 0.2 m . For an observed pressure difference of 2 m of water head and coefficient of discharge equal to unity, assuming that the energy losses are negligible, the flow rate (in m^3/s) through the pipe is approximately equal to _____.

CHAPTER - 5**FLOW THROUGH PIPES****5.1 FLOW TYPES OF FLOW-REYNOLDS' EXPERIMENT**

Reynolds related the inertia to viscous forces and arrived at a dimensionless parameter.

$$\text{Re or } N_R = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{F_i}{F_v} \quad \text{Re or } N_R = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{F_i}{F_v}$$

According to Newton's second law of motion the inertia force T_i is given by

$$F_i = \text{mass} \times \text{acceleration}$$

$$= \rho \times \text{volume} \times \text{acceleration}$$

$$= \rho \times L^3 \times (L/T^2) = (\rho L^2 V^2)$$

Similarly viscous force F_v is given by Newton's law of viscosity as

$$F_v = \tau \times \text{area}$$

$$= \mu \frac{\partial v}{\partial y} \times L^2 = (\mu VL)$$

$$\therefore \text{Re or } N_R = \frac{(\rho L^2 V^2)}{\mu VL} = \frac{\rho VL}{\mu}$$

This dimensionless parameter is called Reynolds number, in which ρ and μ are respectively the mass density and viscosity of the flowing fluid, V is the characteristic (or representative) velocity of flow and L is the characteristic linear dimension. In the case of flow through pipes the characteristic linear dimension L is taken as the diameter D of the pipe and the characteristic velocity is taken as the average velocity V of flow of fluid. Thus Reynolds number becomes $(\rho DV/\mu)$ or (VD/ν) where $(\mu/\rho) = \nu$, is kinematic viscosity of the flowing fluid. The Reynolds number is therefore a very useful parameter in predicting whether the flow is laminar or turbulent. One may predict that the flow will be laminar if Reynolds number is less than 2000 and turbulent if it is greater than 4000.

5.2 LAWS OF FLUID FRICTION**1. Law of fluid friction for laminar flow**

The frictional resistance in the laminar flow is as follows

- (i) Proportional to the velocity of flow
- (ii) Independent of the pressure
- (iii) Proportional to the area of surface in contact
- (iv) Independent of the nature of the surface in contact
- (v) Greatly affected by variation of the temperature of the flowing fluid.

2. Laws of fluid friction for turbulent flow

The frictional resistance in the case of turbulent flow is as follows

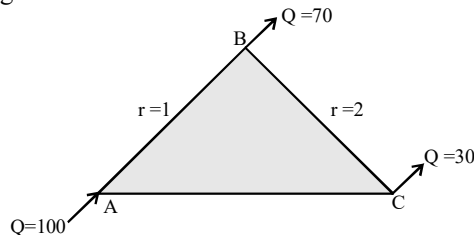
- (i) Proportional to velocity where the index n varies from 1.72 to 2.0
- (ii) Independent of the pressure
- (iii) Proportional to the density of the flowing fluid
- (iv) Slightly affected by the variation of the temperature of the flowing fluid
- (v) Proportional to area of surface in contact
- (vi) Dependent on the nature of the surface in contact

GATE QUESTIONS

1. Water is pumped at a steady uniform flow rate of $0.01 \text{ m}^3/\text{s}$ through a horizontal smooth circular pipe of 100 mm diameter. Given that the Reynolds number is 800 and g is 9.81 m/s^2 , the head loss (in meter, up to one decimal place) per km length due to friction would be _____

[GATE - 2017]

2. A triangular pipe network is shown in the figure.



The head loss in each pipe is given by $h_f = rQ^{1.8}$, with the variables expressed in a consistent set of units. The value of r for the pipe AB is 1 and for the pipe BC is 2. If the discharge supplied at the point A (i.e., 100) is equally divided between the pipes AB and AC, the value of r (up to two decimal places) for the pipe AC should be _____

[GATE - 2017]

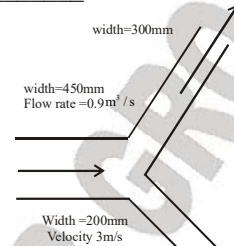
3. For steady flow of a viscous incompressible fluid through a circular pipe of constant diameter, the average velocity in the fully developed region is constant. Which one of the following statements about the average velocity in the developing region is TRUE ?

[GATE - 2017]

- It increases until the flow is fully developed.
- It is constant and is equal to the average velocity in the fully developed region
- It decreases until the flow is fully developed.
- It is constant but is always lower than the average velocity in the fully developed region.

4. A channel of width 450mm branches into sub-channels having width 300mm and 200mm

as shown in figure. If the volumetric flow rate (taking unit depth) of an incompressible flow through the main channel is $0.9 \text{ m}^3/\text{s}$ and the velocity in the sub-channel of width 200mm is 3 m/s , the velocity in the sub channel of width 300mm is _____



[GATE - 2016]

5. For steady incompressible flow through a closed-conduit of uniform cross-section, the direction of flow will always be

[GATE - 2015]

- From higher to lower elevation
- From higher to lower pressure
- From higher to lower velocity
- From higher to lower piezometric head

6. Two reservoirs are connected through a 930 m long, 0.3 m diameter pipe, which has a gate valve. The pipe entrance is sharp (loss coefficient = 0.5) and the valve is half-open (loss coefficient = 5.5). The head difference between the two reservoirs is 20 m. Assume the friction factor for the pipe as 0.03 and $g = 10 \text{ m/s}^2$. The discharge in the pipe accounting for all minor and major losses is _____ m^3/s .

[GATE - 2015]

7. A pipe of 0.7 m diameter has a length of 6 km and connects two reservoirs A and B. The water level in reservoir A is at an elevation 30 m above the water level in reservoir B. Halfway along the pipe line, there is a branch through

CHAPTER - 6***BOUNDARY LAYER THEORY*****6.1 INTRODUCTION**

When a real fluid flows past a solid boundary, a layer of fluid which comes in contact with the boundary surface adheres to it on account of viscosity. Since this layer of fluid cannot slip away from the boundary surface it attains the same velocity that of the boundary. In other words, at the boundary surface there is no relative motion between the fluid and the boundary. This condition is known as no slip condition. Thus at the boundary surface the layer of fluid undergoes retardation. This retarded layer of fluid further causes retardation for the adjacent layers of the fluid, thereby developing a small region in the immediate vicinity of the boundary surface in which the velocity of flowing fluid increases gradually from zero at the boundary surface to the velocity of the main stream. This region is known as boundary layer. In the boundary layer region since there is a larger variation of velocity in a relatively small distance, there exists a fairly large velocity gradient ($\partial v/\partial y$) normal to the boundary surface. As such in this region of boundary layer even if the fluid has small viscosity, the corresponding shear stress $\tau = \mu (\partial v/\partial y)$, is of appreciable magnitude. The flow may thus be considered to have two regions, one close to the boundary in the boundary layer zone in which due to larger velocity gradient appreciable viscous forces are produced and hence in this region the effect of viscosity is mostly confined and second outside the boundary layer zone in which the viscous forces are negligible and hence the flow may be treated as non-viscous or inviscid. The concept of boundary layer was first introduced by L. Prandtl in 1904 and since then it has been applied to several fluid flow problems.

6.1.1 Following Boundary Conditions May Be Noted

Essential Boundary condition

1. at $x = 0$ (leading edge) , thickness of boundary layer = 0 i.e, $\delta = 0$
2. at $y = 0$, $u = 0$
3. at $y = \delta$, $u = V_2 = \text{Free stream velocity} = \text{constant}$
4. at $y = \delta$, $\frac{du}{dy} = 0$

Desirable Boundary conditions

$$\text{At } y = \delta , \frac{du}{dy} = 0, \frac{d^2u}{dy^2} = 0$$

- (i) When a fluid flows past a flat plate, the velocity at leading edge is zero and retardation of fluid increases as more and more of the plate is exposed to flow. Hence boundary layer thickness increases as distance from leading edge increases
- (ii) Upto certain distance from the leading edge, flow in boundary layer is laminar irrespective of the fact that flow of approaching stream is laminar or turbulent
- (iii) As the depth of laminar boundary layer increases, it cannot dissipate the effect of instability in flow and hence transition to turbulent boundary layer is more
- (iv) Thus thickness of turbulent boundary layer is more
- (v) Change of boundary layer from Laminar to turbulent is affected by
 - (a) Roughness of plate
 - (b) Plate curvature

GATE QUESTIONS

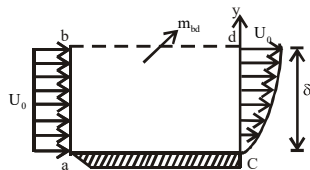
1. For a steady incompressible laminar flow between two infinite parallel stationary plates, the shear stress variation is

[GATE - 2017]

- (a) Linear with zero value at the plates
- (b) Linear with zero value at the centre
- (c) Quadratic with zero value at the plates
- (d) Quadratic with zero value at the centre

2. A steady laminar boundary layer is formed over a flat plate as shown in the figure. The free stream velocity of the fluid is U_0 . The velocity profile at the inlet a-b is uniform, while that at a downstream location c-d is given by $u =$

$$U_0 \left[2 \left(\frac{y}{\delta} \right) - \left(\frac{y}{\delta} \right)^2 \right]$$



The ratio of the mass flow rate, m_b , leaving through the horizontal section b-d to that entering through the vertical section a-b is _____

[GATE - 2016]

3. A fluid (Prandtl number, $Pr = 1$) at 500K flows over a flat plate of 1.5m length, maintained at 300K. The velocity of the fluid is 10m/s. Assuming kinematic viscosity, $\nu = 30 \times 10^{-6} \text{m}^2/\text{s}$, the thermal boundary layer thickness (in mm) at 0.5m from the leading edge is _____

[GATE - 2016]

4. An incompressible fluid flows over a flat plate with zero pressure gradient. The boundary layer thickness is 1 mm at a location where the Reynolds number is 1000. If the velocity of the fluid alone is increased by a factor of 4, then the

boundary layer thickness at the same location, in mm will be

[GATE - 2012]

- (a) 4
- (b) 2
- (c) 0.5
- (d) 0.25

Linked Statement for Q.5 & Q.6

An automobile with projected area 2.6 m^2 is running on a road with a speed of 120 km per hour. The mass density and the kinematic viscosity of air are 1.2 kg/m^3 and $1.5 \times 10^{-5} \text{ m}^2/\text{s}$, respectively. The drag coefficient is 0.30.

5. The drag force on the automobile is

[GATE - 2008]

- (a) 620 N
- (b) 600 N
- (c) 580 N
- (d) 520 N

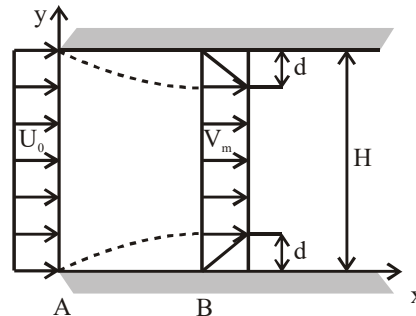
6. The metric horse power required to overcome the drag force is

[GATE - 2008]

- (a) 33.23
- (b) 31.23
- (c) 23.23
- (d) 20.23

Common data for Q. 6 & Q. 7

Consider a steady incompressible flow through a channel as shown below.

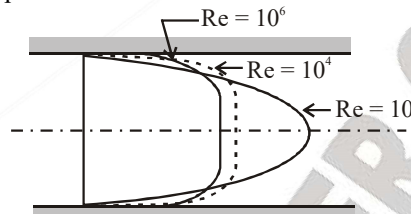


The velocity profile is uniform with a value of U_0 at the inlet section A. The velocity profile at section B downstream is

CHAPTER - 7
TURBULENT FLOW IN PIPES

7.1 INTRODUCTION

As stated earlier if Reynolds number is greater than 4000 the flow is turbulent. The velocity distribution in turbulent flow is relatively uniform and the velocity profile of turbulent flow is much flatter than the corresponding laminar flow parabola for the same mean velocity. In the case of turbulent flow the velocity fluctuations influence the mean motion in such a way that an additional shear (or frictional) resistance to flow is caused. This shear stress produced in turbulent flow is in addition to the viscous shear stress and it is termed as turbulent shear stress which may be evaluated as explained in the next section.



(i) Velocity distribution in laminar and turbulent flow

7.2 RELATION BETWEEN SHEAR AND PRESSURE GRADIENTS IN LAMINAR FLOW

Consider a free body of fluid having the form of an elementary parallelepiped of length δ_x , width is δ_z , thickness δ_y .

The magnitudes of shear stresses on the layers $abcd$ and $a'b'c'd'$ will be different. Thus if τ represents the shear stress on the layer $abcd$ then the shear stress on the layer $a'b'c'd'$ is equal to

$$\left(\tau + \frac{\partial \tau}{\partial y} \delta y \right).$$

For two – dimensional steady flow there will be no shear stresses on the vertical faces $abb'a'$ and $cdd'c'$. Thus the only forces acting on the parallelepiped in the direction of flow x will be the pressure and shear forces. The net shear force acting on the parallelepiped.

$$= \left[\left(\tau + \frac{\partial \tau}{\partial y} \delta y \right) \delta x \delta z - \tau \delta x \delta z \right] = \frac{\partial \tau}{\partial y} \delta x \delta y \delta z$$

If the pressure intensity on face $add'a'$ is p , and since there exists a pressure gradient in the direction of flow, the pressure intensity on the face $bcc'b'$ will be $\left(p + \frac{\partial p}{\partial x} \delta x \right)$. The net pressure force acting on the parallelepiped.

$$= \left[p \delta y \delta z - \left(p + \frac{\partial p}{\partial x} \delta x \right) \delta y \delta z \right] = - \left(\frac{\partial p}{\partial x} \right) \delta x \delta y \delta z$$

For steady and uniform flow, there being no acceleration in the direction of motion, the sum of these forces in the x -direction must be equal to zero. Thus

GATE QUESTIONS

1. Consider fluid flow between two infinite horizontal plates which are parallel (the gap between them being 50mm) .The top plate is sliding parallel to the stationary bottom plate at a speed of 3m/s. The flow between the plates is solely due to the motion of the top plate .The force per unit area (magnitude) required to maintain the bottom plate stationary is _____ N/m² .Viscosity of the fluid $\mu = 0.44\text{kg/m-s}$ and density $\rho = 888\text{kg/m}^3$

[GATE - 2016]

2. With reference to a standard Cartesian (x, y) plane, the parabolic velocity distribution profile of fully developed laminar flow in x-direction between two parallel, stationary and identical plates that are separated by distance, h, is given by the expression

$$u = -\frac{h^2}{8\mu} \frac{dp}{dx} \left[1 - 4 \left(\frac{y}{h} \right)^2 \right]$$

In this equation, the $y = 0$ axis lies equidistant between the plates at a distance $h/2$ from the two plates, p is the pressure variable and μ is the dynamic viscosity term. The maximum and average velocities are, respectively

[GATE - 2014]

(a) $u_{\max} = -\frac{h^2}{8\mu} \frac{dp}{dx}$ and $u_{\text{average}} = \frac{2}{3} u_{\max}$

(b) $u_{\max} = \frac{h^2}{8\mu} \frac{dp}{dx}$ and $u_{\text{average}} = \frac{2}{3} u_{\max}$

(c) $u_{\max} = -\frac{h^2}{8\mu} \frac{dp}{dx}$ and $u_{\text{average}} = \frac{3}{8} u_{\max}$

(d) $u_{\max} = \frac{h^2}{8\mu} \frac{dp}{dx}$ and $u_{\text{average}} = \frac{3}{8} u_{\max}$

3. Consider the turbulent flow of a fluid through a circular pipe of diameter, D. Identify the correct pair of statements.

- I. The fluid is well-mixed
- II. The fluid is unmixed
- III. $Re_D < 2300$

IV. $Re_D > 2300$

- (a) I and III
- (c) II and III

- (b) II and IV
- (d) I and IV

[GATE - 2014]

4. Water flows through a pipe having an inner radius of 10 mm at the rate of 36 kg/hr at 25°C. The viscosity of water at 25°C is 0.001 kg/ms. The Reynolds number of the flow is _____.

[GATE - 2014]

5. For a fully developed flow of water in a pipe having diameter 10 cm, velocity 0.1 m/s and kinematic viscosity $10^{-5} \text{ m}^2/\text{s}$, the value of Darcy friction factor is _____.

[GATE - 2014]

6. In a simple concentric shaft-bearing arrangement, the lubricant flows in the 2 mm gap between the shaft and the bearing. The flow may be assumed to be a plane Couette flow with zero pressure gradient. The diameter of the shaft is 100 mm and its tangential speed is 10 m/s. The dynamic viscosity of the lubricant is 0.1 kg/m.s. The frictional resisting force (in newton) per 100 mm length of the bearing is _____.

[GATE - 2014]

7. Consider laminar flow of water over a flat plate of length 1 m. If the boundary layer thickness at a distance of 0.25 m from the leading edge of the plate is 8 mm, the boundary layer thickness (in mm), at a distance of 0.75 m, is _____.

[GATE - 2014]

8. The maximum velocity of a one-dimensional incompressible fully developed viscous flow, between two fixed parallel plates, is 6 ms^{-1} . The mean velocity (in ms^{-1}) of the flow is _____.

[GATE - 2013]

- (a) 2
- (c) 4

- (b) 3
- (d) 5

ESE OBJ QUESTIONS

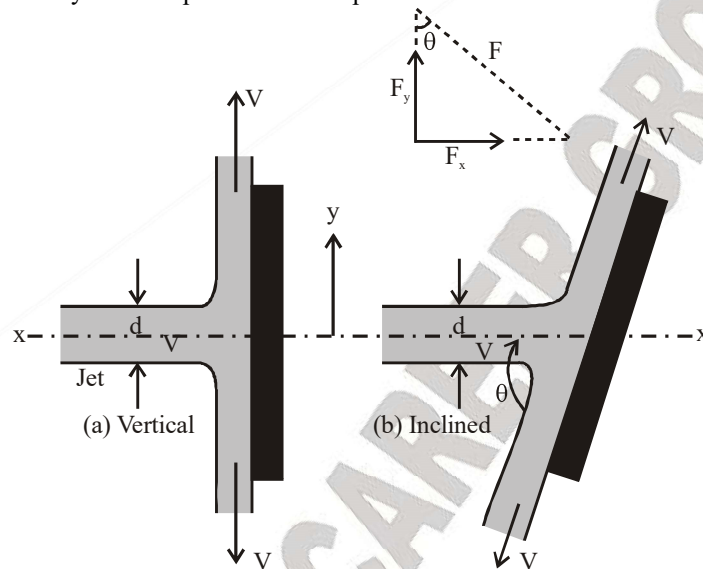
1. A man 65 kg, descends to the ground with the help of a parachute, 18kg. The parachute is hemispherical in shape, 2m diameter. Density of air can be taken as 0.00125 g/cm^3 and its kinematic viscosity as 0.15 stoke. What is the terminal velocity of the parachute?
(Take $C_D = 1.5$ and $g = 1000 \text{ cm/sec}^2$)
[CE ESE - 2017]
- (a) 16.6 m/sec (b) 15.8 m/sec
(c) 15.0 m/sec (d) 14.1 m/sec
2. The laminar flow is characterized by Reynolds number which is
[ME ESE - 2015]
- (a) Equal to critical value
(b) Less than the critical value
(c) More than the critical value
(d) Zero critical value
3. In laminar flow through a circular pipe, the discharge varies
[ME ESE - 2015]
- (a) Linearly with fluid density
(b) Inversely with pressure drop
(c) Directly as square of pipe radius
(d) Inversely with fluid viscosity
4. Air flowing over a flat plate with a free stream velocity of 24m/s, and its kinematic viscosity is $72 \times 10^{-6} \text{ m}^2/\text{s}$. If a particular point, the Reynolds number is 30000, its location from the leading edge is
[ME ESE - 2015]
- (a) 0.05m (b) 0.07m
(c) 0.08m (d) 0.09m
5. In turbulent flows through rough pipes, the ratio of the maximum velocity to the mean velocity is
[CE ESE - 2015]
- (a) 2
(b) 4/3
(c) 1.1
(d) Dependent on the friction factor.
6. Water flows through a smooth circular pipe of diameter D and length L because of a pressure difference ΔP across the length. The volume flow rate is Q and the flow is turbulent with Reynolds number 10^5 . If the pressure difference is increased to $4\Delta P$ the volume flow rate will be
[ME ESE - 2014]
- (a) 2Q
(b) A little more than 2Q
(c) A little less than 2Q
(d) 4Q
7. Laminar flow between closely spaced parallel plates is governed by the consideration of which one of the following pair of forces?
[ME ESE - 2014]
- (a) Pressure and inertial forces
(b) Gravity and inertial forces
(c) Viscous and inertial forces
(d) Pressure and viscous force
8. For fully developed laminar flow through a circular pipe with Reynolds number Re the friction factor is
[ME ESE - 2014]
- (a) Inversely proportional to Re
(b) Proportional to Re
(c) Proportional to square of Re
(d) Independent of Re
9. The loss of head in a pipe carrying turbulent flow varies:
[CE ESE - 2013]
- (a) Inversely as the square of the velocity of flow
(b) Inversely as the square of the diameter of pipe.
(c) Directly as the square of the velocity of flow.
(d) Directly as the velocity of flow.
10. A 0.20 m diameter pipe 20 km long transports oil at a flow rate of $0.01 \text{ m}^3/\text{s}$. Calculate power required to maintain flow if dynamic viscosity and density of oil is 0.08 Pa.s and 900 kg/m^3 respectively.

CHAPTER - 8

FLUID MACHINERY

9.1 IMPACT OF JETS

A jet of fluid emerging from a nozzle has some velocity and hence it possesses a certain amount of kinetic energy. If this jet strikes an obstruction placed in its path, it will exert a force on the obstruction. This impressed force is known as impact of the jet and it is designated as hydrodynamic force, in order to distinguish it from the forces due to hydrostatic pressure. Since a dynamic force is exerted by virtue of fluid motion, it always involves a change of momentum, unlike a force due to hydrostatic pressure that implies no motion.



Fluid jet striking stationary flat plate

9.2 FORCE EXERTED BY FLUID JET ON STATIONARY FLAT PLATE

1. Flat Plate Normal to the Jet

Let a jet of diameter d and velocity V issue from a nozzle and strike a flat plate as shown in figure below. The plate is held stationary and perpendicular to the centre line of the Jet. The jet after striking the plate will leave it tangentially i.e., the jet will get deflected through 90° . If the plate is quite smooth the friction between the jet and the plate may be neglected. Further if there is no energy loss in the flow because of impact of the fluid jet, and the difference in elevation between the incoming and outgoing jets is neglected; the application of Bernoulli's equation indicates that the jet will move on and off the plate with the same velocity V . However, if some energy loss occurs the velocity of the fluid leaving the plate will be slightly less than V .

The quantity of fluid striking the plate $Q = (\pi d^2/4) \times V = aV$, where a is the area of cross-section of the jet. Thus the mass of fluid issued by the jet per second is $m = \rho Q = \rho aV$; where ρ represents the mass density of the fluid. Since $\rho = (w/g)$, where w is the specific weight of the fluid, the mass m may also be expressed as $m = (waV/g)$.

GATE QUESTIONS

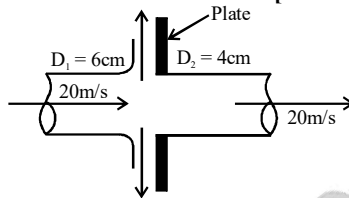
1. If a centrifugal pump has an impeller speed of N (in rpm), discharge Q (in m^3/s) and the total head H (in m), the expression for the specific speed N_s of the pump is given by

[GATE - 2017]

- (a) $N_s = \frac{NQ^{0.5}}{H^{0.5}}$ (b) $N_s = \frac{NQ^{0.5}}{H}$
 (c) $N_s = \frac{NQ^{0.5}}{H^{0.75}}$ (d) $N_s = \frac{NQ}{H^{0.75}}$

2. A 60mm-diameter water jet strikes a plate containing a hole of 40mm diameter as shown in the figure. Part of the jet passes through the hole horizontally, and the remaining is deflected vertically. The density of water is $1000kg/m^3$. If velocities are as indicated in the figure, the magnitude of horizontal force (in N) required to hold the plate is

[GATE - 2017]



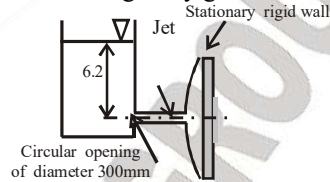
3. A penstock of 1m diameter and 5km length is used to supply water from a reservoir to an impulse turbine. A nozzle of 15cm diameter is fixed at the end of the penstock. The elevation difference between the turbine and water level in the reservoir is 500m. Consider the head loss due to friction as 5% of the velocity head available at the jet. Assume unit weight of water = $10kN/m^3$ and acceleration due to gravity (g) = $10m/s^2$. If the overall efficiency is 80%, power generated (expressed in kW and rounded to nearest integer) is _____

[GATE - 2016]

4. The water jet exiting from a stationary tank through a circular opening of diameter 300mm impinges on a rigid wall as shown in figure. Neglect all minor losses and assume the water

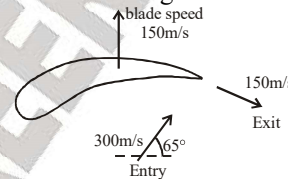
level in the tank to remain constant. The net horizontal force experienced by the wall is _____ kN.

Density of water is $1000kg/m^3$
 Acceleration due to gravity $g = 10m/s^2$



[GATE - 2016]

5. The blade and fluid particles for an axial turbine are as shown in figure



The magnitude of absolute velocity at entry is $300m/s$ at an angle of 65° to the axial direction while the magnitude of the absolute velocity at exit is $150m/s$. The exit velocity vector has a component in the downward direction. Given that the axial (horizontal) velocity is the same at entry and exit, the specific work (in kJ /kg) is _____

[GATE - 2016]

6. A horizontal jet of water with its cross sectional area of $0.0028 m^2$ hits a fixed vertical plate with a velocity of $5 m/s$. After impact the jet splits symmetrically in a plane parallel to the plane of the plate. The force of impact (n N) of the jet on the plate is

[GATE - 2014]

- (a) 90 (b) 80
 (c) 70 (d) 60

7. A horizontal nozzle of 30 mm diameter discharges a steady jet of water into the atmosphere at a rate of 15 litres per second. The diameter of inlet to the nozzle is 100mm. The jet

CHAPTER - 9

DIMENSIONAL ANALYSIS & MODEL STUDIES

10.1 INTRODUCTION

1. Dimensional analysis is a mathematical technique for solving engineering problems. It makes uses of the dimensions of variables on which the problem depends.
2. A physical phenomenon can be expressed by an equation giving relationship between different quantities. Such quantities are dimensional and non-dimensional .
3. Dimensional analysis helps in determining a systematic arrangement of the variables in the physical relationship, combining dimensional variables to form non-dimensional paramaters.
4. It has becomes an important tool for analyzing fluid flow problems.

10.2 SYSTEMS OF DIMENSIONS

1. The various physical quantities can be expressed in terms of fundamental quantities.
2. The fundamental (or primary) quantities are: mass (M), length (L), time (T), and temperature (θ).
3. The quantities which are expressed in terms of the fundamental-quantities are called derived (or secondary) quantities, e.g. velocity, area, acceleration etc.
4. The expression for a derived quantity in terms of the primary quantities is called the dimension of the physical quantity.
5. The two common system of dimensioning a physical quantity are; M-L-T and F-L-T system of units where F is force. The dimensions of various quantities used in both the system are given in table on the next page.

10.3 DIMENSIONAL HOMOGENEITY

1. Dimensional homogeneity states that every term in an equation when reduced to fundamental dimensions must contain identical powers of each dimension.
2. A dimensionally homogenous equation is independent of the fundamental units of measurement if the units there in are consistent.
3. Let us consider the velocity equation,

$$V = \sqrt{2gh}$$

$$[LT^{-1}] = [2 \times LT^{-2} \times L]^{1/2} = [L^2T^{-2}]^{1/2} = [LT^{-1}]$$

Similarly, $h_f = \frac{4fv^2}{2gd}$

$$[L] = \frac{[L][LT^{-1}]^2}{[LT^{-2}][L]} = [L]$$

Dimensional homogeneity is bases on the Fourier’s principle of homogeneity.

Table: Dimensions of Various Physical Quantities

Physical quantity	Symbol	Dimensions	
		M-L-T system	F-L-T system
1. Fundamental Quantities			
Mass	M	M	FL ⁻¹ T ²
Length	L	L	L

GATE QUESTIONS

1. The drag force, F_D , on a sphere kept in a uniform flow field depends on the diameter of the sphere D , flow velocity V ; fluid density ρ ; and dynamic viscosity μ . Which of the following options represents the non dimensional parameters which could be used to analyze this problem?

- [GATE - 2015]
- (a) $\frac{F_D}{V_D}$ and $\frac{\mu}{\rho VD}$ (b) $\frac{F_D}{\rho VD^2}$ and $\frac{\rho VD}{\mu}$
- (c) $\frac{F_D}{\rho V^2 D^2}$ and $\frac{\rho VD}{\mu}$ (d) $\frac{F_D}{\rho V^3 D^3}$ and $\frac{\mu}{\rho VD}$

2. The relationship between the length scale ratio (L_r) and the velocity scale ratio (V_r) in hydraulic models, in which Froude dynamic similarity is maintained, is

- [GATE - 2015]
- (a) $V_r = L_r$ (b) $L_r = \sqrt{V_r}$
- (c) $V_r = L_r^{1.5}$ (d) $V_r = \sqrt{L_r}$

3. Group-I contain dimensionless parameter and Group-II contains ratio.

Group-I

- A. Match number
 B. Reynold number
 C. Weber number
 D. Froude number

Group-II

- (i) Ratio of internal force and gravity force.
 (ii) Ratio of fluid velocity and velocity of sound.
 (iii) Ratio of inertial force and viscous force.
 (iii) Ratio of inertial force and surface tension force.

Correct match of the dimensionless parameter in Group-I with Group-II is

[GATE - 2013]

Code:

- (a) A-iii, B-ii, C-iv, D-i
 (b) A-iii, B-iv, C-ii, D-i
 (c) A-ii, B-iii, C-iv, D-i
 (d) A-iii, B-iii, C-ii, D-iv

4. A phenomenon is modeled using n dimensional variables k primary dimensions. The number of non-dimensional variables is

- [GATE - 2010]
- (a) k (b) n
 (c) $n - k$ (d) $n + k$

5. A river reach of 2.0 km long with maximum flood discharge of $10000 \text{ m}^3/\text{s}$ is to be physically modeled in the laboratory where maximum available discharge is $0.20 \text{ m}^3/\text{s}$. For a geometrically similar model based on equality of Froude number, the length of the river reach (m) in the model is

- [GATE - 2008]
- (a) 26.4 (b) 25.0
 (c) 20.5 (d) 18.0

6. A 1:50 scale model of a spillway is to be tested in the laboratory. The discharge in the prototype is $1000 \text{ m}^3/\text{s}$. The discharge to be maintained in the model test is

- [GATE - 2007]
- (a) $0.057 \text{ m}^3/\text{s}$ (b) $0.08 \text{ m}^3/\text{s}$
 (c) $0.57 \text{ m}^3/\text{s}$ (d) $5.7 \text{ m}^3/\text{s}$

7. The flow of glycerin (kinematic viscosity $\nu = 5 \times 10^{-4} \text{ m}^2/\text{s}$) as the flowing fluid. If both gravity and viscosity are important, what should be the length scale (i.e., ratio of prototype to model dimensions) for maintaining dynamic similarity?

- [GATE - 2006]
- (a) 1 (b) 22
 (c) 63 (d) 500

8. The height of a hydraulic jump in the stilling pool of 1 : 25 scale model was observed to be 10 cm. The corresponding prototype height of the jump is

- [GATE - 2004]
- (a) Not determinable from the data given
 (b) 2.5 m
 (c) 0.5 m
 (d) 0.1 m