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GATE-2019: General Aptitude | Detailed theory with GATE previous year papers and detailed solutions.

First Edition: 2016

Price of Book: *INR 820/-*

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SECTION-A

QUANTITATIVE APTITUDE

CHAPTER - 1
NUMBER SYSTEM

INTRODUCTION

On the basis of the knowledge of the digits and numbers. we study Arithmetic. Arithmetic is the science that treats of numbers and of the methods of computing by means of them. A number expresses how many times a unit is taken. A unit denotes a single thing, as one man, one rupee, one metre, one kilogram etc. It is known that in Hindu-Arabic System, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 that are called digits to represent any number.

Hence we begin to study this subject with the chapter Number System**.** Supposing that applicants are well aware of numbers, we are going to discuss them briefly.

Natural Number

Numbers which we use for counting the objects are known as natural numbers. They are denoted by 'N'.

N = {1, 2, 3, 4, 5, ...}

Whole Number

When we include 'zero' in the natural numbers, itis known as whole numbers. They are denoted by ‘W’.

W = {0, 1, 2, 3, 4, 5, ...}

Place Value or Local Value and Face Value or Intrinsic Value

The value of digit in a number depends upon its positions well as upon the symbol.

The value depending upon the symbol which is peculiarly its own, is called its simple or intrinsic value. It is also called Face Value.

The value which the digit has in consequence of its position in a line of figure is called its place value or local value.

For example, in 5432, the intrinsic value of 4 is 4 units but its local value is 400.

Greatest Number and Least Number

In forming the greatest number we should have the greatest digit ie 9 in all the places. For example, greatest number of five digits will consist of five nines and it will be 99999.

In forming the least number we should have the least digit at all the places. Zero is the least digit but it cannot occupy the extreme left place. Hence we will put the next higher digit ie 1 in the extreme left and the remaining digits will be zeros. For example, least number of five digits will be 10000.

Even Number

The number which is divisible by 2 is known as even number. For example, 2, 4, 6, 8, 10, 12, 24, 28, . etc are even numbers.

It is also of the form 2n {where n = natural number}.

Odd Number

The number which is not divisible by 2 is known as odd number. For example, 3, 9, 11, 17, 19, ... etc are odd numbers. It is also of the form (2n+1) {where $n \in W$}

WORKBOOK

Example 1. What is the difference between greatest number of five digits and the least number of five digits?

Solution.

In forming the greatest number, we should have the greatest digit i.e. 9 in all places. Thus the greatest number of five digits will consist of five nines and it will be 99999.

In forming the least number, we should have the least digit at all places. Zero is the least digit but it cannot occupy the extreme left place. Hence, we will put the next higher digit ie 1 in the extreme left and the remaining four digits will be zeros. Hence, the number will be 10000.

$\therefore$ required difference = 99999 = 10000 = 89999

Example 2. Form the greatest and the least numbers with the digits 2, 7, 9, 0, 5 and also find the difference between them.

Solution.

The greatest number will have the digits in descending order from left to right. Thus the greatest number is 97520.

The least number will have the digits in ascending order from left to right, though zero cannot occupy the extreme left placed Hence the least number is 20579.

$\therefore$ required difference = 97520 - 20579 = 76941

Example 3. Without performing the operation of division, test whether 8050314052 is divisible by 11.

Solution.

Sum of the digits in odd places

= 8+5 + 3 + 4 + 5 = 25

Sum of the digits in even places =0+0+1+0+2=3

Difference of the two sums = 25 – 3 = 22, which is divisible by11.

Therefore, 8050314052 are divisible by 11.

Example 4. Is 136999005 divisible by 13?

Solution.

136 999 005

Adding up the first and the third sets, we get 136 + 5 = 141

Now, their .difference = 999 – 141 = 858

Since 858 ÷ 13 = 66. Hence, the number is divisible by 13.

Example 5. Sum of the eleven consecutive numbers is 2761. Find the middle number.

Solution.

Suppose middle number = x

$\therefore$Numbers will be, x – 5, x – 4, x – 3, x – 2, x – 1, x, x + 1, x+2, x+ 3, x+ 4 and x + 5. Sum of these numbers = 11x = 2761 $\therefore$ x = $\dfrac{2761}{11}$ = 251

Example 6. In the number 28654, find the intrinsic or face value and place value or local value of digit 6.

Solution.

Intrinsic value of 6 = 6 units

Local value of 6 = 600 (Six hundred)

Example 7. The quotient arising from the division of 24446 by a certain number is 79 and the remainder is 35; what is the divisor?

Solution.

Divisor × Quotient = Dividend – Remainder.

$\therefore$ 79 × Divisor = 24446 – 35 = 24411.

$\therefore$ Divisor = 24411 ÷ 79 = 309

Example 8. What least number must be added to2716321 to make it exactly divisible by3456?

Solution.

On dividing 2716321 by 3456, we get 3361 as remainder.

$\therefore$ Number to be added = 3456 – 3361 = 95.

```
3456)  2716321 (785
       24192
       29712
       27648
       20641
       17280
        3361
```

ASSIGNMENT - I

1. Find the product of place value and face value of 5 in 65231
(a) 28000 (b) 25000
(c) 27000 (d) 26000

2. Find the sum of all even numbers from 100 to 175
(a) 2218 (b) 5216
(c) 5206 (d) 5200

3. If $\frac{4}{5}$ of a number is 36. Then, find $\frac{3}{5}$ of the number
(a) 27 (b) 25
(c) 22 (d) 21

4. When 17^{200} is divided by 18, then find the remainder
(a) 1 (b) 4
(c) 5 (d) 3

5. The sum of two numbers is twice their difference. If one of the numbers is 10, the other number is
(a) $3\frac{1}{3}$ (b) 30
(c) 30 or $-3\frac{1}{3}$ (d) 30 or $3\frac{1}{3}$

6. If one-fifth of one-third of one-half of number is 15, then find the number.
(a) 450 (b) 430
(c) 440 (d) 420

7. The sum of two numbers is 85 and their difference is 9. What is the difference of their squares?
(a) 765 (b) 845
(c) 565 (d) 645

8. When a two-digit number is multiplied by the sum of its digits, 405 is obtained. On multiplying the number written in reverse order of the same digits i.e., by the sum of digits, 486 is obtained. Find the number
(a) 81 (b) 45
(c) 36 (d) 54

9. The sum of the digits of a two digit number is 9. If 9 is added to the number, then the digits are reversed. Find the number
(a) 36 (b) 63
(c) 45 (d) 54

10. Ashok had to do a multiplication. Instead of taking 35 as one of the multipliers, he took 53. As a result, the product went up by 540. What is the new product?
(a) 1050 (b) 1590
(c) 1440 (d) None of these

11. If a price of rod is 3000 m and we have to supply some lampposts. One lamppost is at each end the distance between two consecutive lamppost is 75 m. Find the number of lampposts required.
(a) 41 (b) 39
(c) 40 (d) 36

12. A number, when divided by 119 leaves the remainder 19. If the same number is divided by 17, the remainder will be
(a) 19 (b) 10
(c) 7 (d) 2

13. A number is of two digits. The position of digits is interchanged and new number is added to the original number. The resultant number is always divisible by
(a) 8 (b) 9
(c) 10 (d) 11

14. Find the number nearest to 2559 which is exactly divisible by 35
(a) 2535 (b) 2555
(c) 2540 (d) 2560

15. A number when divided by 5 leaves a remainder 3. What is the remainder when the square of the same number is divided by 5?

GATE QUESTIONS

1. Consider a sequence of number a_1, a_2, a_3 ......, a_n where a_n = $a_n = \frac{1}{n} - \frac{1}{n+2}$, integer $n > 0$. What is the sum of the first 50 terms?

[GATE - 2018]

(a) $\left(1+\frac{1}{2}\right)-\frac{1}{50}$ (b) $\left(1+\frac{1}{2}\right)+\frac{1}{50}$

(c) $\left(1+\frac{1}{2}\right)-\left(\frac{1}{51}+\frac{1}{52}\right)$ (d) $1-\left(\frac{1}{51}+\frac{1}{52}\right)$

2. $\underbrace{\frac{a+a+a+......+a}{\text{n times}}}$ $= a^2b$ and $\frac{b+b+b+......+b}{\text{m times}} = ab^2$, where a, b, n and m are natural numbers. What is the value of $\left(\frac{m+m+m+....+m}{\text{n times}}\right)\left(\frac{n+n+n+....+n}{\text{m times}}\right)$?

[GATE - 2018]

(a) $2a^2b^2$ (b) a^4b^4

(c) $ab(a + b)$ (d) $a^2 + b^2$

3. For what values of k given below is $\frac{(k+2)^2}{k-3}$ an integer?

(GATE - 2018)

(a) 4, 8, 18 (b) 4, 10, 16

(c) 4, 8, 28 (d) 8, 26, 28

4. The three roots of the equation f(x) = 0 are x = {–2, 0, 3}. What are the three values of x for which f(x–3) = 0?

(GATE - 2018)

(a) –5, –3, 0 (b) –2, 0, 3

(c) 0, 6, 8 (d) 1, 3, 6

5. Functions, F (a, b) and G(a, b) are defined as follows:
$F(a,b) = (a-b)^2$ and $G(a, b) = |a-b|$, where $|x|$ represents the abosolute value of x.
What would be the value of G(F(1,3), G(1,3)?

(GATE - 2018)

(a) 2 (b) 4

(c) 6 (d) 36

6. What is the value of $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+.....?$

[GATE - 2018]

(a) 2 (b) $\frac{7}{4}$

(c) $\frac{3}{2}$ (d) $\frac{4}{3}$

7. If the number 715 ? 423 is divisible 3 (? denotes the missing digit in the thousandths place), then the smallest whole number in the place of ? is _______.

[GATE - 2018]

(a) 0 (b) 2

(c) 5 (d) 6

8. If a and b are integers and $a + a^2b^3$ is odd, then

[GATE - 2018]

(a) a and b odd (b) a and b even

(c) a even b odd (d) a odd b even

9. A House Number has to be allotted with the following Conditions
1. If the Number is a multiple of 3 it will lie between 50 to 59
2. The Number will not be multiple of 4 it will lie between 60 to 69
3. The Number will not be multiple of 6 it will lie between 70 to 79.
Identify the House No.

[GATE - 2018]

(a) 54 (b) 65

(c) 66 (d) 76

10. What is the smallest natural number which when divided by 20 & by 42 & 76 leaves a remainder '7' is ______ ?

CHAPTER - 2
AVERAGES

INTRODUCTION

In general average is the central value of the given data. For example if the heights of three persons A, B and C be 90 cm, 110 cm and 115 cm respectively, then the average height of A, B and C together will be $\frac{90+110+115}{3} = 105\text{cm}$.

So we can say that the height of each person viz. A, B and C is near about 105 cm. Thus in layman's language it can be said that everyone is almost 105 cm tall.

Basically the average is the arithmetic mean of the given data. For example if the x_1, x_2, x_3,x_4… x_n be any 'n' quantities (i.e., data), then the average (or arithmetic mean) of these 'n' quantities.

$= \frac{x_1 + x_2 + x_3 + ...x_n}{n}$

Properties of Averages

1. The average of any two or more quantities (or data) necessarily lies between the lowest an highest values of the given data. i.e., if x_ℓ and x_h be the lowest and highest (or greatest) values of the given data (x_1, x_2, …. x_ℓ, … x_h , … x_n) then x_ℓ< Average < x_h; $x_1 \neq x_h$

i.e. $x_\ell < \frac{(x_1 + x_2 + x_3 + x_\ell .... + x_h ... + x_n)}{n} < x_h$

2. If each quantity is increased by a certain value 'K' then the new average is increased by K.
3. If each quantity is decreased by a certain value K, then the new average is also decreased by K.
4. If each quantity is multiplied by a certain value K, then the new average is the product of old average with K.
5. If each quantity is divided by a certain quantity 'K' the new average becomes $\frac{1}{K}$ times of the initial average, where K≠ 0.
6. If 'A' be the average of x, , x_2, x_m, ... y_l , y_2, ...,y_n. where x_l,x_2,...,$x_m$be the below A and y_1, y_2, y_3,..., $y_n$be the above A, then

$(A – x_l) + (A – x_2)+...(A – x_m)$
$= (y_i– A) + (y_2–A)+...(y_n–A)$

i.e a, the surplus above the average is always equal the net deficit below average.

CHAPTER - 3
PERCENTAGES

PERCENTAGE AND ITS APPLICATION
A fraction with denominator 100 is called a per cent. Per cent is an abbreviation for the latin word "percentum" meaning "per hundred" or "hundreds" and is denoted by symbol %.

> A fraction with denominator 10 is called as decimal. Since per cent is a form of fraction, we can express per cent as fractions (or decimals) and vice-versa.

Conversion of a Fraction into Percentage
To convert a fraction into a percentage, multiply the fraction by 100 and put "%" sign.

Conversion of a Percentage into a fraction
To convert a percentage into a fraction, replace the % sign with $\frac{1}{100}$ and reduce the fraction to simplest form.

Conversion of a Percentage into a Ratio
To convert a percentage into a ratio, first convert the given percentage into a fraction in simplest form and then to a ratio.

Conversion of a Ratio into a Percentage
To convert a ratio into a percentage, first convert the given ratio into a fraction then to a percentage.

Conversion of a Percentage into a Decimal
To convert a percentage into a decimal remove the % sign and move the decimal point two places to the left.

Conversion of a Decimal into a Percentage
To convert a decimal into a percentage, move the decimal point two place to the right (adding zeros if necessary) and put % sign.

1.Work out some more examples so that all these thing rest on your figure tips.
Remember $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=....=50\%$ etc
Learn and practice all the values given below.

ASSIGNMENT - I

1. 150% of 15 + 75% of 75=?
(a) 75.75 (b) 78.75
(c) 135 (d) 281.25

2. (9%of 386)*(6.5% of 14(d)=?
(a) 328.0065 (b) 333.3333
(c) 325.1664 (d) 340.1664
(e) None

3. 40% of ? =240
(a) 60 (b) 6000
(c) 960 (d) 600
(e) None

4. (37.1% of 480)-(?% of 280)=(12% of 32.2)
(a) 37.6 (b) 39.6
(c) 49.8 (d) 52.4
(e) None

5. 60=?% of 400
(a) 6 (b)12
(c) 15 (d) 20
(e) None

6. 80% of 50 % of 250% of 34=?
(a) 38 (b) 40
(c) 42.5 (d) 43
(e) None

7. (50+50% of 50)=?
(a) 50 (b) 75
(c) 100 (d) 150

8. How is ½% expressed as a decimal fraction?
(a) 0.0005 (b) 0.005
(c) 0.05 (d) 0.5

9. How is ¾ expressed as percentage?
(a) 0.75% (b) 7.5%
(c) 60% (d) 75%

10. 0.02=?%
(a) 20 (b) 2
(c) 0.02 (d) 0.2

11. The fraction equivalent to 2/5% is
(a) 1/40 (b) 1/125
(c) 1/250 (d) 1/500

12. What percent of 7.2 kg is 18 gms?
(a) 0.025% (b) 0.25%
(c) 2.5% (d) 25%

13. Which number is 60% less than 80?
(a) 48 (b) 42
(c) 32 (d) 12

14. A number exceeds 20% of itself by 40. The number is
(a) 50 (b) 6
(c) 80 (d) 320

15. What percent is 3% of 5%?
(a) 15% (b) 1.5%
(c) 0.15% (d) 60%

16. If 37 ½ % of a number is 900, then 62 ½ % of the number is:
(a) 1200 (b) 1350
(c) 1500 (d) 540

17. A number increased by 37 ½ % gives 33. The number is
(a) 22 (b) 24
(c) 25 (d) 27

18. If the average of a number, its 75% and its 25% is 240, then the number is
(a) 280 (b) 320
(c) 360 (d) 400

19. Hari's income is 20% more than Madhu's income. Madhu's income is less than Hari's income by
(a) 15% (b) 16 2/3 %
(c) 20% (d) 22%

CHAPTER - 4
CI/SI/INSTALMENTS

INTRODUCTION
Simple Interest is nothing but the fix percentage of the principal (invested/borrowed amount of money)
Some key words used in the concept of interest
Principal (P): It is the sum of money deposited/loaned etc. also known as capital

Interest: It is the money paid by borrower, calculated on the basis of principal.

Time(T/n): This is the duration for which money is lent/borrowed.

Rate of Interest (r/R): It is the rate at which the interest is charged on principal.
Amount (A) = Principal + Interest

Simple Interest: When the interest is calculated uniformly only on the principal for the given time period.

Compound Interest: In this case for every next period of time the interest is charged on the total previous amount (which is the sum of principal and interest charged on it so far.) i.e. every time we calculate successive increase in the previous amount.

Important Formulae
Simple Interest (SI)

$$SI = \frac{P \times r \times t}{100}$$

P = principal
r = rate of Interest (in %)
t = time period (yearly, half yearly etc.)

Amount (A) = $P + \frac{Prt}{100} = P\left(1 + \frac{rt}{100}\right)$

> Out of the five variables A, Si, P, r, t we can find any one of these, if we have the requisite information

Conversion of Time Period – Rate of interest

Given (r%)	Given (t)	Required (r%)	Required (t)
r % annual	t years	$\frac{r}{2}(\%)$half – yearly	2t
r % annual	t years	$\frac{r}{4}(\%)$ quartely	4t
r % annual	t years	$\frac{r}{12}(\%)$ monthly	12t

Compound Interest (Cl)

WORKBOOK

Example 1. Find the simple interest on Rs. 1000 at 12% per ____ 5 years.

Solution.

$$SI = \frac{Prt}{100} = \frac{1000 \times 12 \times 5}{100} = Rs.600$$

Total amount = P + SI = 1000 + 600 = Rs. 1600}

Example 2. Find the simple interest on Rs. 800 at 7% per annum Rs. 700 at 16% p.a. and on Rs. 500 at 4% p.a. for 2 years.

Solution.

$$SI = \frac{P_1 r_1 t_1}{100} + \frac{P_2 r_2 t_2}{100} + \frac{P_3 r_3 t_3}{100}$$

$$= \frac{800 \times 7 \times 2}{100} + \frac{700 \times 16 \times 2}{100} + \frac{500 \times 4 \times 2}{100}$$

= 112 + 224 + 40 = Rs. 376

Example 3. A sum of money (P) doubles in 10 years. In how many years it will be treble at the same rate of simple interest ?

Solution.

A = 2P

$\therefore$ SI = P (SI = 2P – P)

$$P = \frac{P \times r \times 10}{100}$$

$\Rightarrow$ r = 10%

So, the new amount = 3P

But the new SI = 2P = (3P – P)

$$2P = \frac{P \times 10 \times t}{100} \quad (r = 10\%)$$

T = 20 years

Example 4. A sum of money in 3 years becomes 1344 and in 7 years it becomes Rs. 1536. What is the principal sum where simple rate of interest is to be charged ?

(a) 4000 (b) 1500

(c) 1200 (d) 2800

Solution.

It would be very time saving if we do it by unitary method.

1536 – 1344 = Rs. 192

CHAPTER - 5
PROFIT, LOSS AND DISCOUNT

THEORY AND CONCEPTS
In day – to – day life we sell and purchase the things as per our requirement. A customer can get things in the following manner.
Manufacture (or producer) $\rightarrow$ Whole – seller (dealer) $\rightarrow$ (Shopkeeper) Retailer (or sales person) $\rightarrow$ customer

Terminology
Cost price (CP): The money paid by the shopkeeper to the manufacture or whole – seller to buy the goods is called the cost price (CP) of the goods purchased by the shopkeeper.

> If an article is purchased for some amount and there are some additional expenses on transportation labour, commission etc., these are to be added in the cost price. Such expenses are called overhead expenses or overheads

Selling Price (SP): The price at which the shopkeeper sells the goods is called the selling price (SP) of the goods sold by the shopkeeper.
Profit: If the selling price of an article is more than its cost price, then the dealer (or shopkeeper) makes a profit (or gain) i.e. Profit = SP – CP; SP > CP
Loss: If the selling price of an article is less than its cost price, then the dealer suffer a loss.
i.e loss = CP – SP ; CP > SP

Important Formulae
(i) Profit = SP – CP (ii) Loss = CP – SP

(iii) Profit percentage = $\dfrac{\text{profit}}{\text{cost price}} \times 100$

(iv) Loss percentage = $\dfrac{\text{loss}}{\text{cost price}} \times 100$

(v) $\text{SP} = \left(\dfrac{100+\text{gain}\%}{100} \times \text{CP}\right) = \left(\dfrac{100-\text{loss}\%}{100} \times \text{CP}\right)$

(vi) $\text{CP} = \left(\dfrac{100}{100+\text{gain}\%} \times \text{SP}\right) = \left(\dfrac{100}{100-\text{loss}\%} \times \text{SP}\right)$

(vii) SP = (100 + k)% of CP; when profit = k% of CP
(viii) SP = (100 – k)% of CP; when loss = k% of CP

> Profit or loss is always calculated on the basis of cost price unless otherwise mentioned in the problem.

CHAPTER - 6
TIME AND WORK

CONCEPT OF EFFICIENCY

Suppose a person can complete a particle work in 2 days then we can say that each day he does half of the work or 50% work each day. Thus it is clear that his efficiency is 50% per day. Efficiency is generally considered with respect to the time. The time can be calculated either in days, hours minutes or months etc. So if a person completes his work in 4 days, then his efficiency (per day) is 25%. Since each day he works $1/7^{th}$ of the total work (i.e. 25% of the total work).
I would like to mention that the calculation of percentage and conversion of ratios and fractions into percentage and vice versa is the prerequisite for this chapter
Now, if a person can complete a work in n days then his one day's work = 1/n
And his one day's work in terms of percentage is called his efficiency.
Also if a person can compete 1/n work in one day, then he can complete the whole work in n days.

Relation between Work of 1 unit of Time and Percentage Efficiency

A person can complete his work in n days, then his one day's work = 1/n, his percentage efficiency = $\frac{1}{n}\times 100$

No. of days/ hours etc. required to complete the whole work	Work of 1 day/hour	Percentage efficiency
n	1/n	100/n
1	1/1	100%
2	½	50%
3	1/3	33.33% = $33\frac{1}{3}$%
4	¼	25%
5	1/5	20%
6	1/6	16.66% = $16\frac{2}{3}$%
7	1/7	14.28% = $14\frac{2}{7}$%
8	1/8	12.5%
9	1/9	11.11% = $11\frac{1}{9}$%
10	1/10	10%

This table is very similar to the percentage fraction table given in the chapter of percentage. This table just manifests as a model for efficiency conversion.
Basically for faster and smarter calculation you have to have your percentage calculation very smart.

CHAPTER - 7
RATIO, PROPORTIONAL AND VARIATION

RATIO
The comparison between two quantities in terms of magnitude is called the ratio, i.e. e., it tells us that the one quantity is how many times the other quantity.
For example, Amit has 5 pens and Sarita has 3 pens. It means the ratio of number of pens between Amit and Sarita is 5 is to 3. It can be expressed as '5 : 3'.

It should be noted that in a ratio, the order of the terms is very important. For example, in the above illustration the required ratio is 5: 3 while 3: 5 is wrong.
So the ratio of any two quantities is expressed as a/b or a : b.
The numerator 'a' is called the **antecedent** and denominator 'b' is called as **consequent**

Rule of Ratio
The comparison of two quantities is meaningless if they are not of the same kind or in the same units (of length, volume currency etc). We do not compare 8 boys and 6 cows or 15 cities and 5 toys or 5 metres and 25 centimetres. Therefore, to find the ratio of two quantities (of the same kind), it is necessary to express them in same units.

1. We do not compare 8 boys and 6 cows, but we can compare the number (8) of boys and number (6) of cows. Similarly, we cannot compare the number (15) of litres and the number (5) of toys etc.
2. Ratio has no units.

Properties of Ratios
1. The value of a ratio does not change when the numerator and denominator both are multiplied by same quantities i.*e.*,

$$\frac{a}{b}=\frac{ka}{kb}=\frac{\ell a}{\ell b}=\frac{ma}{mb} \text{ etc.}$$

e.g., $\frac{3}{4}=\frac{6}{8}=\frac{9}{12}$.... ... etc. have the same ratio.

2. The value of a ratio does not alter (or change) when the numerator and denominator both are divided by same quantities i.e.,

$$\frac{a}{b}=\frac{a/k}{b/k}=\frac{a/\ell}{b/\ell}=\frac{a/m}{b/m} \text{ etc.}$$

Example. $\frac{3}{4}=\frac{3/2}{4/2}=\frac{3/3}{4/3}=\frac{3/4}{4/4}$ ... etc. have the same ratio.

CHAPTER - 8
TIME, SPEED AND DISTANCE

INRODUCTION
This chapter includes
(a) Motion in a straight line
(b) Circular motion and races
(c) Problems based on trains, boats, rivers and clocks etc.

Concept of Motion
When a body moves from a point A to another point B at a distance of D with a particular speed (S).
The relation between T, S and D is as follows:
$T \times S = D$
i.e, Time × Speed = Distance
Therefore, when D is constant,
$T \propto \frac{1}{S}$
And when T is constant, $D \propto S$
And when S is constant, $D \propto T$

> The relation of proportionality is very important

Formulae: Distance = Speed× Time
$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$
$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

> To solve the problem all the units involved in the calculation must be uniform i.e, either all of them be in metres and second or in kilometers and hours etc

Conversion of Unit
$$1\text{km}/\text{h} = \frac{5}{18}\text{m}/\text{s}$$
$$1\text{m}/\text{s} = \frac{18}{5}\text{km}/\text{h}$$
[1 km = 1000 m, 1h = 60 min, 1 min = 60s]

CHAPTER - 9
PERMUTATION & COMBINATION

INTRODUCTION
In recent days questions from Permutation/Combination is a regular feature of various competitive exams. And another importance of this chapter is that in most of the problems of probability we have to take the help of this chapter. To solve a problem of Permutation and Combination your approach to the question is very important. You should be very clear about the concept of Permutation and Combination and your approach should be logical rather than Mathematical. General I students make mistake in these types of problems because of 1 their poor concept. So try to read the question carefully and understand it first then solve them in a logical way using some Mathematical formulae.

Difference between Permutation and Combination
Permutation means the number of ways of arranging-, 'n' different things taken 'r' at a time. And it is denoted' as ${}^nP_r = \frac{n!}{(n-r)!}$ and combination means the number of selections that can be made out of 'n' elements taking 'r' at a time and is denoted as ${}^nC_r = \frac{n!}{r!(n-r)!}$. (Are you confused?

Let us explain the symbols first:

Suppose a number is given by $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$.

This number is denoted as 8! Or ($\underline{|8}$ (i.e. 8 factorial). So, if number 'x' is multiplied by all natural numbers less than x, then it is said to be ‘x’ factorial and denoted as x!

So, $x! = x(x-1)(x-2)(x-3) \ldots \times 1$

Now, come to the definition part. The definition of' permutation says that the number of ways of arrangement of n different things taking r at a time is known as permutation

i.e. ${}^nP_r = \frac{n!}{(n-r)!}$

Example

Suppose you have three books. Quicker Math (QM), Analytical Reasoning (AR) and English Is Easy (EE), In how many different ways can you arrange these books in a self? The following are the number of ways of arrangement

1. QM on the bottom, AR in the middle and EE at the top
2. QM on the bottom EE in the middle and AR at the top
3. AR on the bottom, QM in the middle and EE at the top
4. AR on the bottom, EE in the middle and QM at the top
5. EE on the bottom, AR in the middle and QM at the top
6. EE on the bottom, QM in the middle and AR at the top

So, we can arrange these books in six ways. "Now the problem is; here there are three books and we can count the number of ways of arrangement. But if the number of books are 10 then can we count like this?

CHAPTER - 10
PROBABILITY

INTRODUCTION
Probability is a concept which numerically measures the degree of uncertainty and therefore the degree of certainty of the occurrence of events.
In simple words the chances of happening or not happening of an event is known as probability.

Some Important Definition
Difference between "trial" and "event'. Tossing a coin is a trial and getting a head/tail is an event.

Random experiment: If a trial conducted under identical condition then the outcomes are not unique and these trials are called random experiment. All the possible outcomes are known as Sample Spaces or Exhaustive no. of cases.

Equally likely Events: Two or more events are called equally likely if any of them cannot be expected to occur in preference to the other. For example, in tossing a coin anything can occur, i.e there is equal chances of getting a head or getting a tail. (But what was the case in the film Sholey? That was not an equally likely event.)

Mutually exclusive Event: If happening of one event excludes the happening of the other event in a single experiment then that is said to be mutually exclusive events.
For example, in tossing of a coin if head will occur tail cannot occur at the same time.

Independent Events: If two or more events occur in such a way that the occurrence of one does not affect the occurrence of the other. They are said to be independent events.

Dependent Event: If occurrence of one event influences the occurrence of the other then the second event is said to be dependent on the other.
Example. If from a pack of playing cards two cards are drawn one after the other then the 2nd draw is dependent of the first.

Mathematical definition of Probability
If there are 'n' number of exhaustive,, mutually exclusive and equally likely cases (sample space) and of them m are favourable cases of an event A, then

$$P(A) = \frac{\text{no. of favourable cases}}{\text{no. of sample space}}$$

And probability of not happening of that event is $P(\overline{A}) = 1 - P(A)$

Simple approach to Probability
Let us assume that chances of happening of an event and chances of not happening of that event is in the ratio a : b

Then probability of happening of that event $= \dfrac{a}{a+b}$

CHAPTER - 11
DATA INTERPRETATION

INTRODUCTION

In our daily life, we come across figures, statistics and statements of all sorts. These could be anything ranging from. India's exports of various commodities to different countries to the travel plans of any executive. In fact, rarely we can do without facts and figures. Figures, statistics, statements, etc relating to any event are termed as data.

Bills, receipts, vouchers, readings while conducting an experiment, production of cars in India etc, are all examples of what constitute data.

But data, as such, is of very little use unless it is organised. Bills and receipts are of little use unless they are organised in a proper form, such as journals, ledgers etc. Data, when organised in a form from which we can make interpretations, is information. In fact, the very objective of any data is to assist us in obtaining the required information.

This act of organising and interpreting data to get meaningful information is called data interpretation.

Effective Organisation and Presentation of Data

As has already been emphasised, haphazard data makes little sense and is of no use. Top management rarely find enough time to go through entire details of any report, be it the daily production report or the sales forecast. Hence, what is required, is to effectively present the data in such a manner that they are able to draw upon the information, which they require with the least effort. Thus, Effective organization and presentation of data is of prime importance.

Decision-making is seldom done without any survey or research. Hence, interpretation analysis of the data thus obtained are most important for the decision-making process.

Comparison of Data, interpretation and Quantitative Aptitude

Each of the problems in Quantitative Aptitude questions has a basic concept and there is a specific methodology available to tackle them. Data Interpretation requires only the concept of arithmetic and statistics. It mostly deals with the comparison of numbers, arid is not formulae-based.

In Quantitative Aptitude, the data is normally given whereas in Data Interpretation, culling out the requisite data is the first step.

Types of Data Interpretation

The numerical data pertaining to any situation can be presented in the form of

1.Tables: It is the easiest way of presenting data but it does not show trends effectively.

2.Line Graphs: It is easy to spot trends in the given data, though it is difficult to read the actual values.

3.Bar Graphs: The data is, shown in blocks and direct comparison of actual values is very easy.

4.Pie-Charts: Data that is expressed as percentages is best represented in pie-charts.

5.Caselet Form: It is the most difficult and raw form for data interpretation.

6.Geometrical Diagrams: Knowledge of geometry, such as formulae for circumference, area of circle etc helps in

CHAPTER - 12
PIE – GRAPH

INTRODUCTION

In pie-graph, the total quantity in question is distributed over a total angle 360°, which is one complete circle or pie. Unlike the bar and line graphs, where the variables can be plotted on two coordinates x and y, here the data can be plotted with respect to any one parameter. Hence its usage is restricted. It is best used when data pertaining to share of various parties^ of a particular quantity are to be shown. This method of data 1i interpretation is useful for representing shares of proportions^ or percentage of various elements with respect to the total »' quantity. Following types of pie-graph are frequently asked in various competitive exams.

1. Bar Graph

A bar is a thick line whose width is shown merely for attention, These are really just one-dimensional as only the length of the bar matters and not the width. Bars may be horizontal or vertical. The respective figures are normally written at the end of each bar to facilitate easy interpretation. Otherwise, the figures are written only on the parallel axis. Some of the main bar graphs are

(a) Simple – Bar Graph
(b) Sub-divided or Component Bar Graph
(c) Multiple Bar Graph

(i) Simple Bar Graph

Less us see some examples of Simple Bar Graph.

(ii) Sub-divided or Component Bar Graph

The sub-divided bar diagram is used where the total magnitude of the given variable is to be divided into various parts of sub-classes. The bars are drawn proportional in length to the total and divided in the ratio of their components. Let us see the examples given below.

(iii) Multiple Bar Graph

CHAPTER - 13
STATEMENT AND CONCLUSION

INTRODUCTION

We have discussed numerous types of problems and concepts associated with them in the preceding chapters. Here, some miscellaneous types of questions have been given that require the knowledge of basic concepts like Arguments, **Assumptions, Inferences,** Statement-**Conclusion,** Premises, **Cause-Effect,** etc. We have discussed these concepts in earlier chapters. Here, we will discuss only "Strengthening and **Weakening** Arguments." Without study of this, our study of logical reasoning would be incomplete.

Strengthening and Weakening Arguments

In **Chapter** (An Introduction to Logic) of this book, we have studied how arguments work. We must recall that arguments are based on **(1)** certain premises; these premises act as a support and further, the argument makes **(2)** certain assumptions; these assumptions are implicit, they are not stated and they also provide support, and using the support of these two, the argument reaches (3) certain **conclusion,** This can be shown diagrammatically as in the figure given below:

Premise (Support) + Hidden Assumption (Hidden support) = Conclusion

This is how an argument work

We know that a standard argument consists of the following three stages;

(a) The stated premises

(b) The hidden assumptions

(c) The conclusions

This means that

1. An argument would be strengthened if

(a) the stated premises are supported by some more facts of the same nature, or

(b) the hidden assumptions are supported by a fact of the same nature, or

(c) the conclusion itself is supported by a fact of the same nature AND

2. An argument would be weakened if

(a) the stated premises are **contradicted** by some contradicting facts, or

(b) the hidden assumptions are attacked by some contradicting facts, or

(c)the conclusion itself is directly **contradicted** by some contradicting facts.

This means that if we have an argument by example, this argument would be strengthened (weakened) if

(a) we prove that the example itself is totally correct (incorrect),or

(b) we support (contradict) the assumption, or

(c)we support (contradict) the conclusion directly or by some other means.

This would be clear from the following example: Statement:

We must follow the policy of non-violence because Gandhiji used to practise it.

Analysis: Let us first make a complete post-mortem of this argument:

Type: *Argument by example.*

Premise (support): Gandhiji used to practise nonviolence.

CHAPTER - 14
CLOCK AND CALENDAR

INTRODUCTION TO CALENDAR
The Solar Year consists of 365 days, 5 hours, 48 minutes, In the calendar known as Julian Calendar, arranged in 47 BC by Julius Caesar, the year was taken as being of $365\frac{1}{4}$ days and in order to get rid of the odd quarter of a day, an extra or intercalary day was added once in every fourth year and this was called Bissextile or Leap Year, The calendar so arranged is known as the Old Style, and is now used only in Russia. But as the Solar Year is 11 minutes 12 seconds less than a quarter of a day, it followed in a course of years that the Julian Calendar became inaccurate by several days and in 1582 AD this difference amounted to 10 days, Pope Gregory XIII determined to rectify this and devised the calendar now known as the Gregorian Calendar. He dropped or cancelled these 10 days - October 5th being called October 15th and made centurial years leap years only once in 4 centuries - so that whilst 1700, 1800 and 1900 were to be ordinary years. 2000 would be a leap year. This modification brought the Gregorian System into such close exactitude with the Solar Year that there is only a difference of 26 seconds which amounts to a day in 3323 years. This is the New Style. It was ordered by an Act of Parliament to be adopted in England in 1752. 170 years after its formation and is now used throughout the civilized world with the single exception already named. The difference between the two styles will remain 13 days until AD 2100.
In India Vikrami and a number of other calendars were being used till recently. In 1952, a Committee was appointed to examine the different calendars and suggest an accurate and uniform calendar for the whole of India. On the basis of its report, Government of India adopted the National Calendar based on Saka era with Chaitra as its first month. The days of this calendar have permanent correspondence with the days of the Gregorian Calendar. Chaitra 1 falling on March 22 in an ordinary year and March 21 in a Leap Year.

Leap and Ordinary Year
Every year which is exactly divisible by 4 such as 1988, 1992, 1996 etc is called a **leap year.**
Also every 4th century is a **leap year.** The other centuries, although divisible by 4, are not leap years. Thus, for a century to be a leap year, it should be exactly divisible by 400. For example:
1. 400, 800, 1200, etc are leap years since they are exactly divisible by 400.
2. 700, 600, 500 etc are not leap years since they are not exactly divisible by 400.

Number of Odd Days
"Today is 15 August 1995." And you are asked to find the day of week on 15 August 2001.
If you don't know the method, it will prove a tough job for you. The process of finding it lies in obtaining the number of odd days. So, we should be familiar with **odd** days,
The number of days more than the complete number of weeks in a given period, are called ***odd days.***

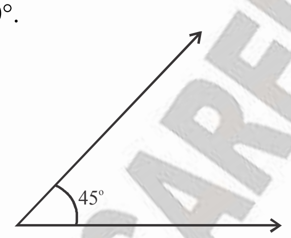
How to Find Number of Odd Days
An **ordinary year** has 365 days. If we divide 365 by 7, we get, 52 as quotient and 1 as remainder. Thus, we may say that an ordinary year of 365 days has 52 weeks and 1 day. Since, the remainder day is left odd-out we call it **odd day.**
Therefore, an ordinary year has 1 odd day.

CHAPTER - 15
GEOMETRY

The chapter of Geometry and mensuration have iad their share in various competition examinations. For doing well in questions based on this topic, student should be familiar with the very basics of various two dimentional and three dimensional solid figures.
To grasp easily the given topic of Geometry and mensuration, we have divided the theory in five parts.
(i) Angles, Parallel lines & Transverse.
(ii) Triangles and Quadrilaterals
(iii) Mensuration and Solid Geometry
(iv) Circles and its properties
(v) Coordinate Geometry and Trigonometry

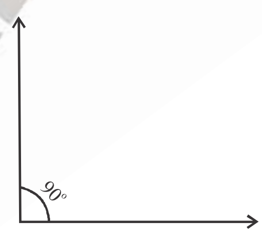
ANGLES, PARALLEL LINES AND TRANSVERSE
When two lines meet at common point they form angle.

Types
1. Acute Angle: Angle less than 90°.



2. Obtuse Angle: Angle more than 90° but less than 180°

3. Right Angle: Angle equal to 90°.



4. Supplementary Angle: When sum of two angles is equal to **180°** then angles are said to be supplementary.

CHAPTER - 16
MISCELLANEOUS

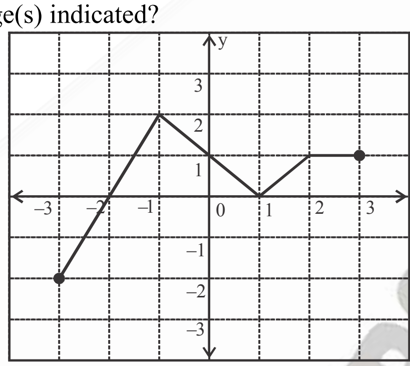
1. The temperature T in a room varies as a function of the outside temperature T_0 and the number of persons in the room p, according to the relation $T = K(\theta_p + T_0)$, where θ is K are constants. What would be the value of θ given the following data?

T_0	p	T
25	2	32.4
30	5	42.0

[GATE - 2018]

(a) 0.8 (b) 1.0
(c) 2.0 (d) 10.0

2. What of the following function(s) in an accurate description of the graph for the range(s) indicated?



(i) $y = 2x + 4$ for $-3 \le x \le -1$
(ii) $y = |x - 1|$ for $-1 \le x \le 2$
(iii) $y = ||x| - 1|$ for $-1 \le x \le 2$
(iv) $y = 1$ for $2 \le x \le 3$

[GATE - 2018]

(a) (i), (ii) and (iii) only
(b) (i), (ii) and (iv) only
(c) (i) and (iv) only
(d) (ii) and (iv) only

3. For non-negative integers, a, b, c, what would be the value of a + b + c if log a + log b + log c = 0?

[GATE - 2018]

(a) 3 (b) 1
(c) 0 (d) –1

4. In manufacturing industries, loss is usually taken to be proportional to the square of the deviation from a target. If the loss is Rs. 4900 for a deviation of 7 units, what would be the loss in Rupees for a deviation of 4 units from the target?

[GATE - 2018]

(a) 400 (b) 1200
(c) 1600 (d) 2800

5. Given that $\frac{\log P}{y - z} = \frac{\log Q}{z - x} = \frac{\log R}{x - y} = 10$ for $x \ne y \ne z$, what is the value of the product PQR?

[GATE - 2018]

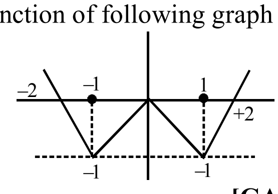
(a) 0 (b) 1
(c) xyz (d) 10^{xyzccc}

6. P, Q, R and S crossed a lake in a boat that can hold a maximum of two persons, with only one set of oars. The following additional facts are available.
(i)The boat held two persons on each of the three forward trips across lake and one person on each of the two return trips.
(ii)P is unable to row when someone else is in the boat.
(iii)Q is unable to row with anyone else except R.
(iv)Each person rowed for at least one trip.
(v)Only one person can row during a trip.
Who rowed twice?

[GATE – 2018]

(a) P (b) Q
(c) R (d) S

7. Find function of following graph



[GATE – 2018]

(a) $||x|+1|- 2$ (b) $||x| - 1| -1$

SECTION-B
REASONING

CHAPTER - 1
ANALOGY

INTRODUCTION

'Analogy' means 'Correspondence'.

In questions based on analogy, a particular relationship is given and another similar relationship has to be identified from the given alternatives.

Verbal Analogy

In this analogy relationship between two given words is established and then applied to other words. The type of relationship may vary, so, while attempting such questions first step is to identify the type of relationship.

Kinds of Relationships With Examples

A. Instrument and Measurements

1. Thermometer : Temperature
(Thermometer is an instrument used to measure temperature)
2. Barometer: Pressure
3. Odometer: Speed .
4. Balance : Mass
5. Rain Gauge : Rain
6. Ammeter : Current
7. Seismograph : Earthquakes
8. Anemometer: Wind
9. Scale : length
10. Sphygmomanometer : Blood Pressure
11. Hygrometer: Humidity
12. Screw Gauge : Thickness
13. Taseometer : Strains

B. Quantity and Unit

1. Mass : Kilogram
2. Force : Newton
3. Resistance : Ohm
4. Angle : Radians
5. Potential: Volt
6. Current: Ampere
7. Pressure : Pascal
8. Temperature : Degrees
9. Conductivity: Mho
10. Length : Meters
11. Energy : Joule
12. Volume : Litre
13. Time : Seconds
14. Work: Joule
15. Luminosity : Candela
16. Area : Hectare
17. Power : Watt
18. Magnetic field : Oersted

C. Individual and Groups

1. Soldiers : Army (group of soldiers is called Army)
2. Flowers : Bouquet
3. Singer: Chorus
4. Fish : Shoal
5. Riders : Cavalcade
6. Man : Crowd
7. Nomads : Horde
8. Grapes : Bunch
9. Artist: Troupe
10. Sheep : Flock
11. Bees ; Swarm
12. Sailors : Crew
13. Cattle : Herd

D. Animals and Young one

1. Cow : Calf
8. Horse : Pony/colt

CHAPTER - 2
DISTANCE AND DIRECTION

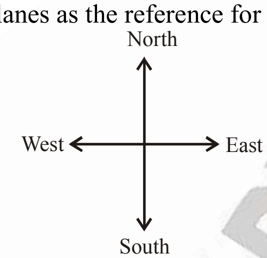
INTRODUCTION

These questions are introduced in reasoning tests to gauge the **'sense of direction'** of the candidate. But as the reasoning tests have become frequent in competitive examinations, the usage of such questions has been increased. Today, direction tests are not only used in reasoning tests for checking 'sense-of-direction', but logical comprehension of particular situations also.

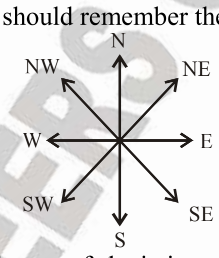
Here in the examples, you will be acquainted with the type of questions that are likely to be asked in the examination. Exercise of this chapter will serve as an exhaustive practice exercise to achieve the desired speed in comprehending and solving the problems.

Tips for Solving Questions Based on Sense of Directions

1. Always try to use the direction planes as the reference for all the questions.



2. Now, as the statement of the question progresses, you should also proceed over this reference plane only.
3. Always mark the starting point and end-point different from the other points.
4. Always be attentive while taking right and/or left turns.
5. Mark distances, with a scale (if your rough diagrams confuse you).
6. To solve this type of questions you should remember the following diagrams:



The figure above shows the standard way of depicting the four **main directions** and the **four** cardinal **directions:** North (N), South (S), East (E), West (W) and North East (NE), North West (NW), South West(SW), South East (SE).

7. One should be aware of basic geometric rule, such as Pythagoras Theorem.

Pythagoras Theorem $\Rightarrow AC^2 = AB^2 + BC^2$

$\therefore \quad AC = \sqrt{AB^2 + BC^2}$

Where, AABC is a right-angled triangle.

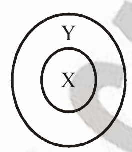
CHAPTER - 3
LOGICAL VENN-DIAGRAM

INTRODUCTION
Venn-diagrams are named after a British Mathematician, John Venn who developed the idea of using diagrams to represent sets.

Sets
A set is a well-defined collection of objects. The objects of a set are called its elements or members. For example, a set of animals can include monkeys, leopards, rabbits, jackals, dogs, cats etc. These individual animals are elements of the set of animals.

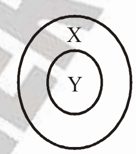
Venn-Diagrams
In these tests a relationship is to be established between two or more elements or members represented by diagrams. The items represented by the diagrams may be individuals, a particular group or class of people (items), etc. In other words, venn-diagrams are diagrammatic representation of sets, using geometrical figures like, circle, triangles, rectangles etc. Each geometrical figure represents a group. The area common to two or more figures represents those elements which are common to two or more groups. There are various models in venn-diagrams which we see as we progress in this chapter. There are mainly three standard ways in which the relation could be made by the venn-diagram as given below.

1. All X are Y



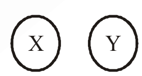
This diagram represents a category that is completely included by the other.

Example. 'All stars twinkle' is represented by the above diagram; where X = Stars and Y = Twinkle. Suppose, if we have an example which says, 'Only stars twinkle', it would be represented as follows:



Here, X = Stars and Y=Twinkle
['Only stars twinkle' would mean that 'Nothing else twinkles'.
or 'All that twinkles are stars'.]

2. No X are Y



This diagram represents a category that is completely exclusive of the others.

Example. 'No stars twinkle' is represented by the above diagram. Where X = Stars and Y = Twinkle.

CHAPTER - 4
SYLLOGISM

INTRODUCTION

Syllogism is originally a word given by the Greeks which means *'inference'* or *'deduction'*.

Definitions of Some Important Terms

The terms defined below are used in the well defined method for solving the problems on syllogism.

Proposition

A proposition is a sentence that makes a statement and gives a relation between two terms. It consists of three parts

(a) The subject

(b) The predicate

(c) The relation between the subject and the predicate

Example.

(i) All coasts are beaches.

(ii) No students are honest.

(iii) Some documents are secret

(iv) Some cloths are not cotton.

Subject and Predicate

A subject is that part of the proposition about which something is being said. A predicate, on the other hand, *m* is that term of the proposition which is stated about or related to the subject.

Thus, for example, in the four propositions mentioned above, 'coasts', 'students', 'documents' and 'cloths' are subjects while 'beaches', 'honest', 'secret' and 'cotton' are predicates.

Categorical Propositions

A categorical proposition makes a direct assertion. It has no conditions attached with it. For example, "All S are P", "No S are P", "Some S are P" etc are categorical propositions, but "If S, then P" is not a categorical proposition.

Types of Categorical Propositions

1. Universal Proposition

Universal propositions either fully include the subject or fully exclude it.

Examples

(i)All coasts are beaches.

(ii)No Students are honest.

Universal propositions are further classified as:

(i) Universal Positive Proposition

A proposition of the form "All S are P", for example, "All coasts are beaches",is called a universal positive proposition. And it is usually denoted by a letter "A".

(ii) Universal Negative Proposition

A proposition of the form "No S are P", for example, "No students are honest", is called a universal negative proposition. And it is usually denoted by a letter "E".

CHAPTER - 5
PUZZLE

INTRODUCTION
From practical experience and the general trends, it can be asserted that the questions on **"Puzzle"** can be generally classified into the following:
1.Simple problems of categorization
2.Arrangement problems
3.Comparison problems
4.Blood relations
5.Blood relations and professions
6.Conditional selection
7.Miscellaneous problems.
In this lesson, you shall be given fast – working and efficient methods for all the types of problems above. Before that, however, let us see what is the pattern of each of these types. But to begin with, we will give you some general tips and rules that should be applied by you for all the types mentioned above. These rules can be considered as the preliminary steps that should be taken before you really being solving the problem.

Some Preliminary Steps
1. First of all, take a quick glance at the question. This would need not more than a couple of seconds. After performing this step you would develop a general idea as to what the general theme of the problem is.
2. Next, determine the usefulness of each of the information and classify them accordingly into 'actual information' or 'useful secondary information' or 'negative information' as the case may be. This can be done in the following way:

(i) Useful Secondary Information
Usually the first couple of sentences of the given data are such that they give you some basic information that is essential to give you the general idea of the situation. These can be classified as *useful secondary information.* For example, in Ex, 2 the following sentence makes up 'useful secondary information': “Six persons A, B, C, D, E and F .... three in each”
(ii) Actual Information
Whatever remains after putting aside the useful secondary information can be categorized as actual information. While trying to solve a problem, one should begin with the actual information while the useful secondary information should be borne in mind.
(iii) Negative Information
A part of the actual information may consist of negative sentences or negative information. A negative information does not inform us anything exactly but it gives a chance to eliminate a possibility. Sentences like "B is not the mother of A” or “H is not a hill-station” are .called negative information.
As we shall see, negative information, like useful secondary information, does not help us directly in reaching an answer. Usually we have to analyse the (non negative) actual information. The negative information and the useful secondary information are supplementary data and they are used to reach a definite conclusion.

CHAPTER - 6
CODING-DECODING

INTRODUCTION
Coding is a system of signals. This is a method of transmitting information in the form of codes or signals without it being known by a third person. The person who transmits the code or signal, is called the sender and the person who receives it, is called the receiver. Transmitted codes or signals are decoded on the other side by the receiver—this is known as decoding.

In this type of test secret messages or words have to be deciphered or decoded. They are coded according to a definite pattern or rule which should be identified first. Then the same rule could be applied to decipher another coded word or message. Now, we care presenting a detail study of various standard forms of coding. Study them carefully and then solve the practice exercises.

Types of Coding-Decoding
We will be discussing the following types of coding-decoding one by one in greater detail.
1. Letter Coding
1. Coding based on direct letter
2. Coding based on Numerals
3. Coding based on symbols and numbers
4. Coding based on 'Group of Words'
5. Coding based on Substitution

1. Letter Coding
Letter Coding In this section, we are going to deal with types of questions, in which the letters of a word are replaced by certain other letters according to a specific pattern/rule to form a code. You are required to detect the coding pattern/rule and answer the question(s) that follow, based on that coding pattern/rule.

1. If more than one codes are given then the required code can be derived from the question itself and you will not need to solve it mathematically .e.g, In a certain code LOCATE is written as 981265 and SPARK as 47230, the code for CASKET can be derived by common letters in LOCATE and SPARK.
2. For a word in which a letter repeats at those same pattern repeats for 2nd letter in the word itself. e.g., TASTE has code SZRSD, in this case code for T is S in both cases so if the coding pattern is -1 for T it will be same for all the letters.

2. Coding based on Direct Letter
In direct letter coding system, the code letters occur in the same sequence as the corresponding letters occur in the words. This is basically a substitution method.

3. Coding based on Symbols and Numbers
In these types of questions, either numerical code values are assigned to a word or alphabetical code letters are assigned to the numbers.

CHAPTER - 7
RANKING AND NUMBER TEST

Type-1. To find out the Position of a Person in the ROW from L.H.S/R.H.S.
To find out the position of a person in a row from right hand side and left hand side = Number of persons in the row +1 – position of the person from the other side.

Type-2. To find out the Number of Persons in the row

Case-1. Position of a person from L.H.S. as well as R.H.S. to find out the number of persons in the row, add up both the positions of the given person and reduce the value by 1.

Type-3. Number Test

In this type of questions, generally a set, group or series of numerals is given and the candidate is asked to trace out numerals following certain given conditions or lying at specific mentioned positions after shuffling according to a certain given pattern.

Type-4. Time Sequence Test

CHAPTER - 8
MATHEMATICAL OPERATIONS

INTRODUCTION

In these types of questions the mathematical operations like +, –, ×, ÷ are represented by symbols. Sometimes the operands like =, ≠, >, <, ≤, ≥ are also represented by some fictitious symbols in the mathematical equation. The candidate is required to substitute these fictitious symbols with the actual signs (mathematical operand) and solve the equation using BODMAS principle.

In the following example, the value can be found by following the BODMAS RULE- i.e., Bracket, of, Division, Multiplication, Addition and Subtraction.

For example, $(8 \times 3) \div 8 - 4 + 2 \times 4 = ?$

$= 24 \div 8 - 4 + 2 \times 4$ (Solving Bracket)

$= 3 - 4 + 2 \times 4$ (Solving Division)

$= 3 - 4 + 8$ (Solving Multiplication)

$= 3 + 8 - 4 = 7$ (Solving Addition and Subtraction)

Type-1. Problem Solving By Substitution

In this type, we are provided with substitutes for various mathematical symbols or numerals followed by the question involving calculation of an expression or choosing the correct/incorrect equation.

Type-2. Sign Language

Type-3. Deriving the Appropriate Conclusion

In this type of questions, certain relationships between different sets of elements are given, using either the real symbols or substituted symbols. The candidate is required to analyse the given statements and then decide which of the relations given as alternatives follows from those given in the statements.

Rules helpful in solving such problems

Rule 1. First see, if the two inequalities have a common term. Go to next step only if they have the common term (otherwise don't).

Rule 2. If the common term is greater than or equal to (≥) on terms, and less than or equal to ('≤') other one, i..e, if it is greater than or equal to both (or less than or equal to both), a combination is not possible.

Rule 3. Combine the two inequalities and draw a conclusion by letting the middle term disappear. The conclusion will normally have a '>' (or a '<') sign strictly, unless the '≥' sign (or '≤') appears twice in the combined inequality.

Rule 4. The relationship represented by sing '≥' or '≤' can only the established between two terms, if and only if th common term is preceded as well as succeeded by the same sign.

Rule 5. If the common terms is preceeded by '≥' and followed by >i.e, $A \geq B > C$, then the relation between A and C can only be: $A > C$, because common terms is only preceeded by '≥' and is not followed by the same sign again.

The solution requires that we should follow the following steps

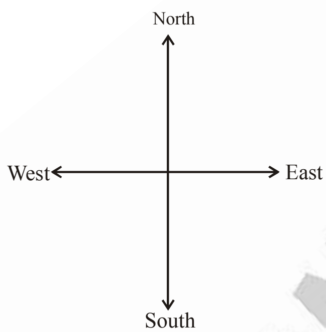
Step-1. From the given equation, first of all, take one symbol or coded relation and change the same with the inequality sign in all questions.

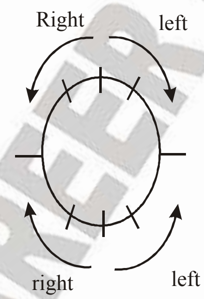
CHAPTER - 9
SITTING ARRANGEMENT

Under this topic the questions are provided in the form of puzzles involving certain number of items. The candidate is require to analyse the given information, condense it in a suitable from and answer the questions asked.

Type-1. Person Sitting in a Circle around A Table
In the questions of type above the persons are sitting either around a table or circle. In either of the condition, the person are facing the center. The important point to be remembered is that the left side of the person who is facing North, is just opposite of one, sitting opposite to him, who is facing South.

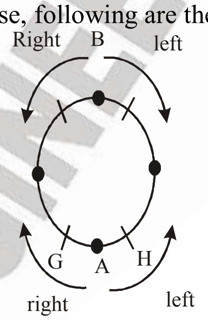
Directions



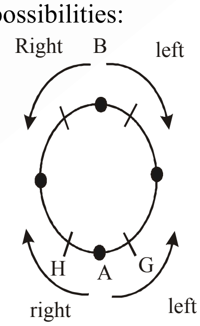


From the diagram above, we observe that A is facing North and B is facing South. Also the left side of A is just opposite to Right side of B. Similarly right side of A is just opposite of left side of B.

Procedure
Whenever, we are presented with this kind of problem, the first step should be locate the 'Fulcrum' i.e., the position around which we can locate the other positions. The next step is to draw the circle of the table the start the process of allocating the position. In almost all the questions, the position of one person in relation to two other persons is given. We find that two different positions are possible. Let us say, we are given that A is sitting between G and H, just opposite to B. In such a case, following are the two possibilities:



Case-I



Case-II

In case I, G is to the left of A and in the case II G is to the right of A.

CHAPTER - 10
INPUT AND OUTPUT

In this type of questions, a message comprising of randomized letters/words or number or a combination of both is given as the input followed by steps of rearrangement to give sequential outputs. The candidate is required to trace out the pattern is given rearrangement and then determine the desire output step, according as is asked in the questions.
Patterns to look for in the given sequence
1. Arranging the given words in the forward/reverse alphabetical order.
2. Arranging the given numbers in ascending/descending order.
3. Writing a particular set of words in the reverse order, stepwise.
4. Changing places of words/ numbers according to a set pattern.

The above points are possible criteria which you should look for to determine the pattern in a given rearrangement. In this, in order to find number of steps, write the number below the digit/letter if it is to be arranged. However, if it is already arranged, then number it above and after count the number below the letter/digit, which reveals the number of steps, as shown in example below.

CHAPTER - 11
CUBE AND CUBOID

Introduction

Cube is a solid body which has 6 faces, 12 edges (AB, BC, CD, AD, AE, BF, DH, CG, EF, FG, GH and EH) and 8 corners (A, B, C, D, E, F, G and H). Each face of the cube is square in shape and all faces are congruent squares. Hence, if the edge length of the cube is 'a' units, each edge has the length 'a' units.

Volume of a cube of edge length 'a' units = a3 cubic units.

By the term 'unit cube', we mean a cube with edge length I unit

Volume of a unit cube = (13∴ = 1) 1 cubic unit

Volume of a cube of edge length 'a' units = sum of the volumes of the unit cubes used to from the given cube

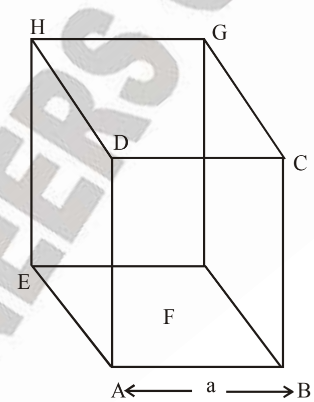
= 1 + 1+ ....(a3 times)

= a3 cubic units

Hence, if a cube of edge length 'a' units is divided into unit cubes the number of unit cubes will be equal to the volume of the cube, i.e., a3

Example. if a cube of edge length 4 cm is divided into unit cubes, then the number of unit cubes will be (4)3 = 64. If a cube of edge length 6 cm is divided into unit cubes, the number of unit cubes will be (6)3 = 216

> In general, a cube of edge length 'a' units can be divided into 'a3 ' unit cubes i.e. the number is equal to the volume of the cube



Now, out of the a3 of their a3 unit cubes, there are 4 different types of cubes 4 different types of cubes:

(i) Cubes with three face visible (ii) Cubes with two face visible

(iii) Cubes with one face visible (iv) Cubes with no face visible

The cubes with three faces visible are the cubes at the corners. Hence, the number of cubes whose three faces are visible is equal to the number of corners, i.e. 8

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CONTENTS

CHAPTER - 1
LINEAR ALGEBRA

1.1 INTRODUCTION

Linear Algebra and matrix theory occupy an important place in modern mathematics and has applications in almost all branches of engineering and physical sciences. An elementary application of linear algebra is to the solution of a system of linear equations in several unknowns, which often result when linear mathematical models are constructed to represent physical problems. Nonlinear models can often be approximated by linear ones. Other applications can be found in computer graphics and in numerical methods.
In this chapter, we shall discuss matrix algebra and its use in solving linear system of algebraic equations $A\hat{x} = b$ and in solving the eigen value problem $A\hat{x} = \lambda\hat{x}$.

1.2 ALGEBRA OF MATRICES

1.2.1 Matrix Definition

A system of mn numbers arranged in the form of a rectangular array having m rows and n columns is called an matrix of order m × n.
If $A = [a_{ij}]_{m\times n}$ be any matrix of order m × n then it is written in the form:

$$A = [a_{ij}]_{m\times n} = \begin{bmatrix} a_{11} & a_{12} \ldots\ldots\ldots a_{1n} \\ a_{21} & a_{22} \ldots\ldots\ldots a_{2n} \\ \ldots & \ldots\ldots\ldots\ldots\ldots \\ \ldots & \ldots\ldots\ldots\ldots\ldots \\ a_{m1} & a_{m2} \ldots\ldots\ldots a_{mn} \end{bmatrix}$$

Horizontal lines are called rows and vertical lines are called columns.

1.2.2 Types of Matrices

1. Square Matrix

An m × n matrix tor which m = n (The number of rows is equal to number of columns) is called square matrix. It is also called an n-rowed square matrix. i.e. The elements a_{ij} + I =j, i.e. a_{11}, a_{22} .... are called **DIAGONAL ELEMENTS** and the line along which they lie is called **PRINCIPLE DIAGONAL** of matrix. Elements other than a_{11}, a_{22}, etc are called off-diagonal elements i.e. $a_{ij}| \neq j$.

Example. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 9 & 8 & 3 \end{bmatrix}_{3\times 3}$ is a square Matrix

A square sub matrix of a square matrix A is called a **"principle sub-matrix"** it its diagonal elements are also the diagonal elements of the matrix A.

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ASSIGNMENT

1. The rank of the matrix
$$\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 3 & 6 & 9 & 12 \\ 0 & 6 & 10 & 15 & 20 \end{bmatrix} \text{ is}$$
(a) Zero (b) 1
(c) 2 (d) 3

2. A square matrix a is invertible if and only if
(a) It has non zero element
(b) Determinant of A is zero
(c) Determinant of A is non zero
(d) Has all elements not equal to zero

3. If A is a matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then
(a) $A(\text{Adj } A) = |A|I$ (b) $|A^{-1}| = (|A|)^{-1}$
(c) $|\text{adj } A^{-1}| = |A|$ (d) $|\text{adj } A| = |A^{-1}|$

4. If A $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 1 & 2 \\ -2 & 3 \\ 3 & 1 \end{bmatrix}$ C= $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$
Are matrices, then the order of $(5A-3B)C$ is
(a) 5×1
(b) 2×1
(c) 3×1
(d) Matrix does not exist

5. The matrix $\begin{bmatrix} 0 & 3 & 5+2i \\ -3 & 0 & -9 \\ -5 & 9 & 0 \end{bmatrix}$
(a) Symmetric matrix
(b) Skew-symmetric matrix
(c) Hermitian matrix
(d) skew-Hermitian matrix

6. Let A be square matrix and A^t be its transpose matrix then $A-A^t$ is
(a) Symmetric matrix
(b) Skew-symmetric matrix
(c) Zero matrix
(d) Identity matrix

7. The rank of the matrix: $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ is
(a) 1 (b) 2
(c) 3 (d) 4

8. The system of linear equation.
$x+2y+3z=\lambda x$
$3x+y+2z=\lambda y$
$2s+3y+z=\lambda z$
has a non-zero solution when λ equals
(a) 2 (b) 4
(c) 6 (d) 8

9. If $A = \begin{pmatrix} 0 & \alpha \\ \beta & 0 \end{pmatrix}$ then $A^3 + A = 0$ whenever
(a) $\alpha\beta = 0$ (b) $\alpha\beta = 1$
(c) $\alpha\beta \neq 0$ (d) $\alpha\beta = -1$

10. If $A = \begin{pmatrix} 1 & 0 & -1 \\ -2 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$ then inverse of matrix A will be :
(a) $\begin{pmatrix} -1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & -2 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$
(c) $\begin{pmatrix} 0 & 0 & 1 \\ 2 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 2 \\ 1 & 2 & -1 \end{pmatrix}$

11. Consider the equation AX = B where
$A = \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}$ and B = $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ then

-

GATE QUESTIONS

1. For the given orthogonal matrix Q,

$$Q = \begin{bmatrix} \frac{3}{7} & \frac{2}{7} & \frac{6}{7} \\ -\frac{6}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{6}{7} & -\frac{3}{7} \end{bmatrix}$$

The inverse is

[GATE - 2018]

(a) $\begin{bmatrix} \frac{3}{7} & \frac{2}{7} & \frac{6}{7} \\ -\frac{6}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{6}{7} & -\frac{3}{7} \end{bmatrix}$

(b) $\begin{bmatrix} -\frac{3}{7} & -\frac{2}{7} & -\frac{6}{7} \\ \frac{6}{7} & -\frac{3}{7} & -\frac{2}{7} \\ -\frac{2}{7} & -\frac{6}{7} & \frac{3}{7} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{3}{7} & -\frac{6}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \\ \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \end{bmatrix}$

(d) $\begin{bmatrix} -\frac{3}{7} & \frac{6}{7} & -\frac{2}{7} \\ -\frac{2}{7} & -\frac{3}{7} & -\frac{6}{7} \\ -\frac{6}{7} & -\frac{2}{7} & \frac{3}{7} \end{bmatrix}$

2. The rank of the following matrix is $\begin{pmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{pmatrix}$

[GATE - 2018]

(a) 1
(b) 2
(c) 3
(d) 4

3. The matrix $\begin{pmatrix} 2 & -4 \\ 4 & -2 \end{pmatrix}$ has

[GATE - 2018]

(a) Real eigenvalues and eigenvectors
(b) Real eigenvalues but complex eigenvectors
(c) Complex eigenvalues but real eigenvectors
(d) Complex eigenvalues and eigenvectors

-

4. Consider a matrix P whose only eigenvectors are the multiples of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Consider the following statements:
(i)P does not have an inverse.
(ii)P has a repeated eigenvalue.
(iii)P cannot be diagonalized.
Which one of the following options is correct ?

[GATE - 2018]

(a)Only i and iii are necessarily true
(b)Only ii is necessarily true
(c)Only i and ii are necessarily true
(d)Only ii and iii are necessarily true

5. Consider a matrix $A = uv^T$ where $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Note that v^T denotes the transpose of v. The largest eigenvalue of A is ________.

[GATE - 2018]

6. Let M be a real 4 × 4 matrix. Consider the following statements:
S1: M has 4 linearly independent eigenvectors.
S2: M has 4 distinct eigenvalues
S3: M is non-singular (invertible).
Which one among the following is TRUE?

[GATE - 2018]

(a) S1 implies S2
(b) S2 implies S1
(c) S1 implies S3
(d) S3 implies S2

7. Consider matrix $A = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$ and vector $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. The number of distinct real values of k for which the equation AX = 0 has infinitely many solution is __________

[GATE - 2018]

8. Which one of the following matrices is singular?

CHAPTER - 2
CALCULUS

2.1 LIMIT
2.1.1 Definition
A number 00001 A is said to be limit of function f (×) at × = a if for any arbitrarily chosen positive integer $\in$, however small but not zero there exist a corresponding number δ greater than zero such that: $|f(x)-A|<\in$ or all values of × for which 0 < |× − a| < ∂ where | × − a| means the absolute value of (× – a) without any regard to sign.

2.1.2 Right and Left Hand Limits
If × approaches a from the right, that is, from larger value of × than a, the limit of f as defined before is called the right hand limit of f(×) and is written as:

$\underset{x\to a+0}{\text{Lt}} f(x)$ or f(a+0) or $\underset{x\to a^+}{\text{Lt}} f(x)$

Working rule for finding right hand limit is, put a + h for × in f(×) and make h approach zero.

In short, we have, $f(a+0) = \lim\limits_{h\to 0} f(a+h)$

Similarly if × approaches a from left, that is from smaller values of × than a, the limit of f is called the left hand limit and is written as:

$\underset{x\to a-0}{\text{Lt}} f(x)$ or f(a-0) or $\underset{x\to a^-}{\text{Lt}} f(x)$

In this case, we have f(a–0) = $\lim\limits_{h\to 0} f(a\text{-}h)$

In both right hand and left hand limit of f, as $x\to a$ exist and are equal in value, their common value, evidently, will be the limit of f as $x\to a$. If however, either or both of these limits do not exist, the limit of f as x → adoes not exist. Even if both these limits exist but are not equal in value then also the limit of f as x → a does not exist.

$\therefore$ when $\underset{x\to a}{\text{Lt}} f(x) = \underset{x\to a^-}{\text{Lt}} f(x)$

then $\underset{x\to a}{\text{Lt}} f(x) = \underset{x\to a^+}{\text{Lt}} f(x) = \underset{x\to a^-}{\text{Lt}} f(x)$

Limit of a function can be any real number, ∞ or $-\infty$. It can sometimes be ∞ or □ ∞, which are also allowed values for limit of a function.

Various Formulae

These formulae are sometimes useful while taking limits.

1. $(1+x)^n = 1 - nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$

2. $(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots.$

3. $a^x = 1 + \times \log a + \frac{x^2}{2!}(x\log a)^2 + \frac{x^3}{3!}(x\log a)^3 + \ldots$

-

WORKBOOK

Example 1. What is the value of $\lim_{x\to 0}\frac{\sin\left[\frac{4}{3}x\right]}{x}$?

Solution.
We have

$$\lim_{x\to 0}\frac{\sin\left[\frac{4}{3}x\right]}{x} = \lim_{\frac{4}{3}x\to 0}\frac{4}{3}\frac{\sin\left[\frac{4}{3}x\right]}{\frac{4}{3}x}$$

$$=\frac{4}{3}\lim_{\frac{4}{3}x\to 0}\frac{\sin\left[\frac{4}{3}x\right]}{\frac{4}{3}x} = \frac{4}{3}\times 1=\frac{4}{3}$$

Example 2. What is the value of

$$\lim_{x\to 0}\frac{x^3-6x^2+11x-6}{x^2-6x+8}?$$

Solution.

When $x\to 2, \frac{x^3-6x^2+11x-6}{x^2-6x+8}=\frac{0}{0}$

Hence, we apply L'Hospital's rule,

$$\lim_{x\to 2}\frac{3x^2-12x+11}{2x-6}=\frac{3(2)^2-12(2)+11}{2(2)-6}$$

$$=\frac{12-24+111}{-2}=\frac{-1}{-2}=\frac{1}{2}$$

Example 3. If a function is given by

$$f(x)=\begin{cases}\frac{\sin x}{x}+\cos x & x\neq 0\\ 2, & x=0\end{cases}$$

Find out whether or not f(x) is continuous at x = 0.

Solution.
We have
L.H.L at x = 0

$$=\lim_{x\to 0}f(x)=\lim_{h\to 0}f(o-h)=\lim_{h\to 0}f(-h)$$

$$=\lim_{h\to 0}\left[\frac{\sin(-h)}{-h}+\cos(-h)\right]=1+1=2$$

R.H.L. at x = 0

$$=\lim_{x\to 0}f(x)=\lim_{h\to 0}f(o+h)=\lim_{h\to 0}f(h)$$

$$=\lim_{h\to 0}\left[\frac{\sin h}{h}+\cos h\right]=1+1=2$$

Also, we k now that f(0) = 2.

Thus, $\lim_{h\to 0^-}f(x)=\lim_{h\to 0^+}f(x)=f(0)$.

Hence, f(x) is continuous at x = 0.

Example 4. Discuss the continuity of the function f(x) at x = ½, where

$$f(x)=\begin{cases}1/2^{-x}, & x\le x<1/2\\ 1, & x=1/2\\ 3/2^{-x}, & 1/2<x\le 1\end{cases}$$

Solution.
We have

L.H.L. at $x=\frac{1}{2}$

$$=\lim_{x\to 1/2^-}f(x)\lim_{x\to 1/2}\left(\frac{1}{2}-x\right)=\frac{1}{2}-\frac{1}{2}=0$$

R.H.L. $x=\frac{1}{2}$

$$=\lim_{x\to 1/2^+}f(x)\lim_{x\to 1/2}\left(\frac{3}{2}-x\right)=\frac{3}{2}-\frac{3}{2}=1$$

Since, $\lim_{x\to 1/2^-}f(x)\neq\lim_{x\to 1/2^+}f(x)$

Hence, f(x) not continuous at $x=\frac{1}{2}$.

Example 5. Discuss the continuity of f(x) = 2x – |x| at x = 0.

Solution.
We have

$$f(x)=2x-|x|=\begin{cases}2x-x, & \text{if} \quad x\ge 0\\ 2x-(-x), & \text{if} \quad x<0\end{cases}$$

$$\Rightarrow f(x)=\begin{cases}x, & \text{if} \quad x\ge 0\\ 3x, & \text{if} \quad x<0\end{cases}$$

Now,

-

ASSIGNMENT

1. $\lim_{x\to 0} x \log_x$ equals

(a) 1 (b) 0

(c) 1/2 (d) 1/3

2. If x = r cos θ, y = r sin θ; then the value of $\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}$ is

(a) 0 (b) 1

(c) $\frac{\partial r}{\partial x}$ (d) $\frac{\partial x}{\partial y}$

3. $\lim_{n\to\infty}\frac{I^2+2^n+....+n^2}{n^3}$ equals.

(a) 1/4 (b) 1/2

(c) 1/6 (d) 1/3

4. The value of the integral $\iint xy\,dx\,dy$. Taken over the region bounded by the two axes and the straight line x + y = 1.

(a) 1/20 (b) 1/24

(c) 1/30 (d) 1/40

5. For the function f(x) = |x| language's mean value theorem does not hold in the interval

a) [–1,0] (b) [0,1/2]

(c) [0,1] (d) [–1,1]

6. The value of $\int_0^1\int_0^1 (x^2+y^2)dx\ dy$ is

(a) 1 (b) 0

(c) 1/3 (d) 2/3

7. The point of inflexion of curve $y = x^{5/2}$ is

(a) (1,1) (b) (0,0)

(c) (1,0) (d) (0,1)

8. The value of

$$\lim_{n\to\infty}\left[\frac{n^{1/2}}{n^{3/2}}+\frac{n^{1/2}}{(n+3)^{3/2}}+.....\frac{n^{1/2}}{(n+3)(n-1)^{3/2}}\right]$$

(a) $\int_0^1\frac{dx}{(1+3x)^{3/2}}$ (b) $\int_0^\infty\frac{dx}{(1+3x)^{3/2}}$

(c) $\int_0^1\frac{dx}{(1+3x)^{3/1}}$ (d) None

9. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then the value of $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u$ is

(a) $\frac{3}{(x+y+z)^3}$ (b) $\frac{-9}{(x+y+z)^2}$

(c) $\frac{9}{(x+y+z)}$ (d) $\frac{3}{(x+y+z)^2}$

10. The value of $\int_0^{\pi/2}\frac{(\cos x-\sin x)dx}{1+\sin x\cos x}$

(a) 1 (b) 1/2

(c) 0 (d) 2

11. Let $f(x)=\begin{cases} x\sin\frac{1}{x} \text{ if } x\neq 0 \\ \text{if } x=0\end{cases}$. Then at $x=0$, f is

(a) Continuous but not differentiable

(b) Not continuous

(c) Differentiable

(d) Neither continuous nor differentiable

12. The function f(x,y) may have a maxima or minima at a point if at that point -

(a) $\left[\frac{\partial^2 f}{dx^2}\cdot\frac{\partial^2 f}{\partial y}-\left(\frac{\partial^2 f}{\partial xdy}\right)^2\right]>0$

(b) $\left[\frac{\partial^2 f}{dx^2}\cdot\frac{\partial^2 f}{\partial y^2}-\left(\frac{\partial^2 f}{\partial xdy}\right)^2\right]<0$

(c) $\left[\frac{\partial^2 f}{dx^2}\cdot\frac{\partial^2 f}{\partial y^2}-\left(\frac{\partial^2 f}{\partial xdy}\right)^2\right]=0$

(d) None of these

-

GATE QUESTIONS

1. The value (up to two decimal places) of a line integral $\int_c \vec{F}(\vec{r}).\,\vec{dr}$, for $\vec{F}(\vec{r}) = x^2\hat{i} + y^2\hat{j}$ along C which is a straight line joining (0, 0) to (1, 1) is ______

[GATE - 2018]

2. Taylor series expression of $f(x) = \int_0^x e^{-\left(\frac{r^2}{2}\right)} dt$ around x = 0 has the form. The coefficient a_2 (correct to two decimal places) is equal to ______

[GATE - 2018]

3. At the point x = 0, the function f(x) = x^3 has

[GATE - 2018]

(a) Local maximum
(b) Local minimum
(c) Both local maximum and minimum
(d) Neither local maximum nor local minimum

4. A cantilever beam of length 2 m with a square section of side length 0.1 m is loaded vertically at the free end. The vertical displacement at the free end is 5 mm. The beam is made of steel with Young's modulus of 2.0×10^{11} N/m^2. The maximum bending stress at the fixed end of the cantilever is

[GATE - 2018]

(a) 20.0 MPa (b) 37.5 MPa
(c) 60.0 MPa (d) 75.0 MPa

5. The value of $\lim_{x \to 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$

[GATE - 2017]

(a) is 0 (b) is –1
(c) is 1 (d) Does not exit

6. If $f(x) = R\sin\left(\frac{\pi x}{2}\right) + S.f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2R}{\pi}$, then the constants R and S are respectively.

[GATE - 2017]

(a) $\frac{2}{\pi}$ and $\frac{16}{\pi}$ (b) $\frac{2}{\pi}$ and 0
(c) $\frac{4}{\pi}$ and 0 (d) $\frac{4}{\pi}$ and $\frac{16}{\pi}$

7. An integral I over a counter clock wise circle C is given by

$$I = \oint_C \frac{z^2 - 1}{z^2 + 1} e^z dz$$

If C is defined as |z| = 3, then the value of I is

[GATE - 2017]

(a) $-\pi i \sin(1)$ (b) $-2\pi i \sin(1)$
(c) $-3\pi i \sin(1)$ (d) $-4\pi i \sin(1)$

8. The minimum value of the function $f(x) = \frac{1}{3}x(x^2 - 3)$ in the interval $-100 \le x \le 100$ occurs at x = ____

[GATE - 2017]

9. The value of the contour integral in the complex – plane $\oint \frac{z^3 - 2z + 3}{z - 2} dz$ along the contour |z| = 3, taken counter - clockwise is

[GATE - 2017]

(a) $-18\pi i$ (b) 0
(c) $14\pi i$ (d) $48\pi i$

10. Let $g(x) = \begin{cases} -x, & x \le 1 \\ x + 1 & x \ge 1 \end{cases}$ and

$f(x) = \begin{cases} 1 - x, & x \le 0 \\ x^2 & x > 0 \end{cases}$

-

CHAPTER - 3
DIFFERENTIAL EQUATION

3.1 INTRODUCTION
Differential equations are fundamental in engineering mathematics since many of the physical laws and relationships between physical quantities appear mathematically in the form of such equations.
The transition from a given physical problem to its mathematical representation is called modeling. This is of great practical interest to engineer, physicist or computer scientist. Very often, mathematical models consist of a differential equations or system of simultaneous differential equations, which needs to be solved. In this chapter we shall look at classifying differential equations and solving them by various standard methods.

3.2 DIFFERENTIAL EQUATIONS OF FIRST ORDER
3.2.1 Definitions
A differential equation is an equation which involves derivatives or differential coefficients or differentials. Thus the following are all examples of differential equations.

1. $x^2dx + y^2\,dy = 0$

2. $\frac{d^2x}{dt^2} + a^2x = 0$

3. $y = x\frac{dy}{dx} + \frac{x^2}{dy/dx}$

4. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{-5/3} = a\frac{d^2y}{dx^2}$

5. $\frac{dx}{dt} - wy = a\cos pt,\ \frac{dy}{dt} + wx = a\sin pt$

6. $x^2\frac{\partial z}{\partial x} + t\frac{\partial z}{\partial y} = 3z$

7. $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}$

8. An ordinary Differential Equations is that in which all the differential coefficients all with respect to a single independent variable. Thus the equations (a) to (d) are all ordinary differential equations. (e) is a system of ordinary differential equations.

9. A partial Differential Equations is that in which there are two or more independent variables and partial differential coefficients with respect to any of them. The equations (f) and (g) are partial differential equations.
The order of a differential equation is the order of the highest derivative appearing in it, The degree of a differential equation is the degree of the highest derivative occurring in its, after the equation has been expressed in a form free from radicals and fractions as far as the derivatives are concerned.
Thus from the examples above,

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WORKBOOK

Example 1. Determine the order and degree of $\frac{\left[1+\left(dy/dx\right)^2\right]^{3/2}}{d^2y/dx^2}=K$.

Solution.
The given differential equation when written as a polynomial in derivatives becomes

$$K^2\left(\frac{d^2y}{dx^2}\right)=\left[1+\left(\frac{dy}{dx}\right)^2\right]^3$$

The highest order differential coefficient in this equation is $\frac{d^2y}{dx^2}$ and its power is 2.
The order is 2 and degree is 2.

Example 2. Solve dy/dx = $(x + y + 1)^2$, if y(0) =0

Solution.
Putting x + y + 1 = t, we get $\frac{dy}{dx}=\frac{dt}{dx}-1$

Thus, the given equation becomes $\frac{dt}{dx}-1=t^2$

or $\frac{dt}{dx}=1+t^2$

Integrating both side, we get

$\int\frac{dt}{1+t^2}=\int dx+c$

or $\tan^{-1} t = x + c$

$\Rightarrow \tan^{-1}(x + y + 1) = x + c$

$\Rightarrow x + y + 1 = \tan (x + c)$

When x = 0, y = 0

1 = tan (c)

$\Rightarrow c = \pi/4$

Thus, the solution is given by x + y + 1 tan $(x + \pi/4)$.

Example 3. Solve the differential equation $(x^2 - y^2)\,dx + 2xy\,dy = 0$, given that y = 1 when x = 1.

Solution.
We have $(x^2 - y^2)dx + 2xy\,dy = 0$

$(x^2 - y^2)dx = - 2xydy$

-

$$\frac{dy}{dx}=-\frac{x^2-y^2}{2xy}=\frac{y^2-x^2}{2xy} \quad \ldots(i)$$

Putting y = vx and $\frac{dy}{dx}=v+x\frac{dv}{dx}$ in equation (i), we get $v+x\frac{dv}{dx}=\frac{v^2x^2-x^2}{2x\cdot vx}$

$\Rightarrow v+x\frac{dv}{dx}=\frac{v^2-1}{2v}$

$\Rightarrow x\cdot\frac{dv}{dx}=\frac{v^2-1}{2v}-v=\frac{v^2-1-2v^2}{2v}=-\left[\frac{v^2+1}{2v}\right]$

$\Rightarrow \frac{2v}{v^2+1}\cdot dv=-\frac{dx}{x}, x\neq 0$

$\Rightarrow \int\frac{2v}{v^2+1}\cdot dv=-\int\frac{dx}{x}$

$\Rightarrow \log\left(v^2+1\right)=-\log|x|+c$

$\Rightarrow \log(v^2 + 1) + \log|x| = \log c$

$\Rightarrow (v^2 + 1)|x| = c$

Now putting v = y/x

$(y^2/x^2 + 1)|x| = c$

$\Rightarrow (x^2 + y^2) = c|x|$

Substituting x = 1 and y = 1, we get

c =2

Putting value of c = 2 in equation (2), we get

$x^2 + y^2 = 2x$ or $x^2 + y^2 = 2(-x)$.

Hence, $x^2 + y^2 = 2x$ is the required solution.

Example 4. Solve the differential equation

$\frac{dy}{dx}-\frac{y}{x}=2x^2, x>0$

Solution.
We know $\frac{dy}{dx}+\left(\frac{-1}{x}\right)y=2x^2$

$\frac{dy}{dx}+Py=Q$, where $P=-\frac{1}{x}$ and $Q = 2x^2$

Now,

I.F. = $\text{I.F.}=e^{\int P\cdot dx}=e^{\int -1/x\cdot dx}=e^{-\log x}=e^{\log x^{-1}=x^{-1}=\frac{1}{x}}$

Multiplying both sides with I.F., we get

ASSIGNMENT

1. Amongst the following differential equations the non-linear equation is
(a) $y^1 + y = x^2$ (b) $(y^1)^2 + y = x$
(c) $y^1 + y = x$ (d) $y^1 + xy = x^2$

2. The solution of $(xy^2 + 1)\,dx + (x^2 y + 1)\,dy = 0$
(a) $x^2y^2 + 2x^2 + 2y^2 = C$
(b) $x^2y^2 + x^2 + y^2 = C$
(c) $x^2y^2 + x + y = C$
(d) $x^2y^2 + 2x + 2y = C$

3. The solution of $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$
(a) $e^y = e^{-x} + \frac{1}{3}x^3 + C$
(b) $e^{-y} = e^x + \frac{1}{3}x^3 + C$
(c) $e^y = e^x + \frac{1}{3}x^3 + C$
(d) None of these

4. The roots of the auxiliary equation of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0$ are
(a) 1, 4 (b) –1, 4
(c) –1, –4 (d) 1, –4

5. An integrating factor for making the differential equation $(xy + 1)y\,dx + (xy - 1)x\,dy=0$
(a) xy (b) 1/xy
(c) y/x (d x/y

6. With C_1 and C_2 as arbitrary constants the general solution of the differential equation $(D^2 - 1)\,y = x^2$ is:
(a) $y= C_1e^x + C_2e^{-x} - x^2$
(b) $y = C_1e^x + C_2e^{-x} +(x^2 +)$
(c) $y = C_1e^x + C_2e^{-x} - 2$
(d) $y = C\,e^x + C_2e^{-x} - (x^2 +2)$

7. The particular integral of $(D^2 + 1)\,y = e^{-x}$ is
(a) $\left(\frac{1}{4} - \frac{x}{2}\right)e^{-x}$ (b) $\left(\frac{1}{4} + \frac{x}{2}\right)e^{-x}$
(c) $\left(\frac{1}{2}e^{-x}\right)$ (d) $-\frac{1}{2}e^{-x}$

8. The differential equation of the system of circles touching the x-axis at origin is
(a) $\left(x^2 - y^2\right)\frac{dy}{dx} - 2xy = 0$
(b) $\left(x^2 - y^2\right)\frac{dy}{dx} + 2xy = 0$
(c) $\left(x^2 + y^2\right)\frac{dy}{dx} - 2y = 0$
(d) $\left(x^2 + y^2\right)\frac{dy}{dx} + 2xy = 0$

9. A particular integral of differential equation $(D^2 + 4)\,y = x$ is
(a) xe^{-2x} (b) x cos 2x
(c) x sin 2x (d) x/4

10. The orthogonal trajectories of the family of parabolas $y = ax^2$ are given by the solution of the differential equation
(a) $\frac{dy}{dx} = \frac{2y}{x}$ (b) $\frac{dy}{dx} = -\frac{2y}{x}$
(c) $\frac{dy}{dx} = \frac{x}{2y}$ (d) $\frac{dy}{dx} = -\frac{x}{2y}$

11. The differential equation $M(x, y)\,dx + N(x, y)\,dy = 0$ is an exact equation
(a) $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x} = 0$ (b) $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 0$
(c) $\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x} = 0$ (d) $\frac{\partial M}{\partial y} - \frac{\partial M}{\partial x} = 0$

12. The general solution of the differential equation $\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = 0$ (C_1, C_2, C_3 and C_4 are arbitrary constant)
(a) $y = (C_1 + C_2x)\sin x + (C_3 + C_4x)\cos x$

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GATE QUESTIONS

1.

2. The solution of the equation $x\frac{dy}{dx}+y=0$ passing through the point (1, 1) is

[GATE - 2018]

(a) x (b) x^2
(c) x^{-1} (d) x^{-2}

3. The position of a particle y(t) is described by the differential equation:

$$\frac{d^2y}{dt^2}=-\frac{dy}{dt}-\frac{5y}{4}$$

The initial conditions are y(0) = 1 and $\left.\frac{dy}{dt}\right|_{t=0}=0$. The position (accurate to two decimal places) of the particle at t = π is _________.

[GATE - 2018]

4. A curve passes through the point (x =1, y = 0) and satisfied the differential equation $\frac{dy}{dx}=\frac{x^2+y^2}{2y}+\frac{y}{x}$. The equation that describes the curve is

[GATE - 2018]

(a) $\ln\left(1+\frac{y^2}{x^2}\right)=x-1$

(b) $\frac{1}{2}\ln\left(1+\frac{y^2}{x^2}\right)=x-1$

(c) $\ln\left(1+\frac{y}{x}\right)=x-1$

(d) $\frac{1}{2}\ln\left(1+\frac{y}{x}\right)=x-1$

5. The solution (up to three decimal places) at x=1 of the differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$ subject to boundary conditions y(0) = 1 and $\frac{dy}{x}=(0)=-1$ is ____

[GATE - 2018]

6. Consider a quadratic equation $x^2 - 13x + 36 = 0$ with coefficients in a base b. The solutions of this equation in the same base b are x = 5 and x = 6. Then b = ________.

[GATE - 2017]

7. The value of the integrals

$$\int_0^1\left(\int_0^1\frac{x-y}{(x+y)^3}dy\right)dx \text{ and}$$

$$\int_0^1\left(\int_0^1\frac{x-y}{(x+y)^3}dx\right)dy \text{ are}$$

[GATE - 2017]

(a) Same and equal to 0.5
(b) Same and equal to –0.5
(c) 0.5 and –0.5, respectively
(d) –0.5 and –0.5, respectively

8. The general solution of the differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-5y=0$$

In terms of arbitrary constants K_1 and K_2 is

[GATE - 2017]

(a) $K_1e^{(-1+6\sqrt{6})x}+K_2e^{(-1-\sqrt{6})x}$

(b) $K_1e^{(-1+\sqrt{8})x}+K_2e^{(-1-\sqrt{8})x}$

(c) $K_1e^{(-2+\sqrt{6})x}+K_2e^{(-2-\sqrt{6})x}$

(d) $K_1e^{(-2+\sqrt{8})x}+K_2e^{(-2-\sqrt{8})x}$

9. Consider the differential equation $(t^2-81)\frac{dy}{dt}+5ty = \sin(t)$ with y(1) = 2π. There

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CHAPTER - 4
PROBABILITY AND STATICS

4.1 PROBABILITY FUNDAMENTALS

4.1.1 Definitions

Sample Space and Event: Consider an experiment whose outcome is not predictable with certainty. Such an experiment is called a random experiment. However, although the outcome of the experiment will not be known in advance, let us suppose that the set of all possible outcomes is known. This set of all possible outcomes of an experiment is known as the sample space of experiment and is denoted by S. Some examples follow.

1. If the outcome of an experiment consist in the determination of the sex of a newborn child, then S = {g, b} where the outcome g means that the child is a girl and b is the boy.

2. If the outcome of an experiment consist of what comes up on a single dice, then S = {1, 2, 3, 4, 5, 6}.

3. If the outcome of an experiment is the order of finish in a race among the 7 horses having post positions 1, 2, 3, 4, 5, 6, 7; then S = {all 7! permutations of the (1, 2, 3, 4, 5, 6, 7)}.

The outcome (2, 3, 1, 6, 5, 4, 7) means, for instances, that the number 2 horse comes in first, then the number 3 horse, then the number 1 horse, and so on.

Any subset E of the sample space is known as Event. That is, an event is a set consisting of some or all of the possible outcomes of the experiment. For example, in the throw of a single dice S = {1, 2, 3, 4, 5, 6} and some possible events are

$E_1 = \{1, 2, 3\}$ $\quad$ $E_2 = \{3, 4\}$ $\quad$ $E_3 = \{1, 4, 6\}$ etc.

If the outcome of the experiment is contained in E, then we say that E has occurred. Always $E \subseteq S$. Since E & S are sets, theorems of set theory may be effectively used to represent & solve probability problems which are more complicated.

Example. If by throwing a dice, the outcome is 3, then events E_1 and E_2 are said to hare occured.
In the child example – (i) If E, = {g}, then E_1 is the event that the child is a girl.
Similarly, if E_2 = {b}, then E_2 is the event that the child is a boy. These are examples of Simple events.

Compound events may consist of more than one outcome. Such as E = {1, 3, 5} for an experiment of throwing a dice. We say event E has happened if the dice comes up 1 or 3 or 5.

For any two events E and F of a sample space S, we define the new event $E \cup F$ to consists of all outcomes that are either in E or in F or in both E and F That is, the event $E \cup F$ will occur if either E or F or both occurs. For instances, in dice example (i) if event E = {1, 2} and F = {3, 4}, then $E \cup F = \{1, 2, 3, 4\}$.

That is $E \cup F$ would be another event consisting of 1 or 2 or 3 or 4. The event $E \cup F$ is called union of event E and the event F Similarly, for any two events E and F we may also define the new event $E \cap F$, called intersection of E and F to consists of all outcomes that are common to both E and F.

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WORKBOOK

Example 1. A box contains 5 white and 10 black balls. Eight of them are placed in another box. What is the probability that the latter box contains 2 white and 6 black balls?

Solution.

The number of balls is 15. The number of ways in which 8 balls can be drawn out of 15 is $^{15}C_8$. The number of ways of drawing 2 white balls = $^5C_2$. The number of ways of drawing 6 black balls = $^{10}C_6$

Total number of ways in which 2 white and 6 red balls can be drawn is $^5C_2 \times {}^{10}C_6$.

$\therefore$ The required probability $= \frac{^5C_2 \times {}^{10}C_6}{^{15}C_8} = \frac{140}{429}$

Example 2. Four cards are drawn at random from a peak of 52 playing cards. What is the probability of getting all the four cards of the same suit?

Solution.

For cards can be drawn from a deck of 52 cards in $^{52}C_4$ ways; there are four suits in a deck, each of 13 cards.

Thus, total number of ways of getting all four cards of same suit is

$$13C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 = 4\left({}^{13}C_4\right)$$

Hence, required probability

$$= \frac{4\left({}^{13}C_4\right)}{^{52}C_4} = \frac{198}{20825}$$

Example 3. The letters of word 'SOCIETY' are placed at random in a row. What is the probability that the three vowels come together?

Solution.

The letter in the word 'SOCIETY' can arranged in 7! Ways. The three vowels can be put together in 3! Ways. And considering these three vowels are one letter, we have 5 letter which can be arranged in 5! Ways.

Thus, favorable number of outcomes = 5! ×3!

Required probability $= \frac{5! \times 3!}{7!} = \frac{1}{7}$.

Example 4. In a race, the odds in favor of the four cars C_1, C_2, C_3, C_4 are 1:4, 1:5, 1:7, respectively. Find the probability that one of them wins the race assuming that a dead heat is not possible.

Solution.

The events are mutually exclusive because it is not possible for all the cars to cover the same distance at the same time. If P_1, P_2, P_3, P_4 are the probabilities of wining for the cars C_1, C_2, C_3, C_4, respectively, then

$$P_1 = \frac{1}{1+4} = \frac{1}{5} \qquad P_2 = \frac{1}{1+5} = \frac{1}{6}$$

$$P_3 = \frac{1}{1+6} = \frac{1}{7} \qquad P_4 = \frac{1}{1+7} = \frac{1}{8}$$

Hence, the chance that one of them wins

$= P_1 + P_2 + P_3 + P_4$

$= \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} = \frac{533}{840}$.

Example 5. Given $P(A) = \frac{1}{4}, P(B) = \frac{1}{3}$ and $P(A \cup B) = 1/2$, then what is the value of $P\left(\frac{A}{B}\right), P\left(\frac{B}{A}\right), P(A \cap B')$ and $P\left(\frac{A}{B'}\right)$?

Solution.

We know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{1}{2} = \frac{1}{4} + \frac{1}{3} - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = \frac{1}{12}$$

Thus, $P\left(\frac{A}{B}\right) = P\frac{(A \cap B)}{P(B)} = \frac{1/12}{1/3} = \frac{1}{4}$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{1/2}{1/3} = \frac{1}{4}$$

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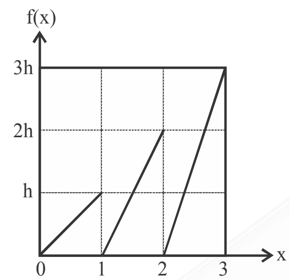
GATE QUESTIONS

1. The graph of a function f(x) is shown in the figure



For f(x) to be valid probability density function, the value of h is

[GATE - 2018]

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) 1 (d) 3

2. Probability (up to one decimal place) of consecutively picking 3 red balls without replacement from a box containing 5 red balls and 1 white ball is ____________

[GATE - 2018]

3. Consider Guwahati (G) and Delhi (D) whose temperatures can be classified as high (H) medium (M) and low (L). Let $P(H_G)$ denote the probability that Guwahati has high temperature. Similarly, $P(M_G)$ and $P(L_G)$ denotes the probability of Guwahati having medium and low temperatures respectively. Similarly, we use $P(H_D)$, $P(M_D)$ and $P(L_D)$ for delhi.
The following table gives the conditional probabilities for Delhi's temperature given Guwahati's temperature.

	H_D	M_D	L_D
H_G	0.40	0.48	0.12
M_G	0.10	0.65	0.25
L_G	0.01	0.50	0.49

Consider the first row in the table above. The first entry denotes that if Guwahati has high temperature (H_G) then the probability of Delhi also having a high temperature (H_D) is 0.40; i.e. $P(H_D|H_G)=0.40$. Similarly, the next two entries are $P(M_D|H_G)=0.48$ and $P(L_D|H_G)=0.12$. Similarly for the other rows.
If it is known that $P(H_G) = 0.2$, $P(M_G)=0.5$ and $L(L_G) = 0.3$, then the probability (correct to two decimal places) that Guwahati has high temperature given that Delhi has high temperature is ________.

[GATE - 2018]

4. Two people P and Q, decide to independently roll two identical dice, each with 6 faces. Numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial as a throw of the dice by P and Q. Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is ________.

[GATE - 2018]

5. Let X_1, X_2, X_3 and X_4 be independent normal random variables with zero mean and unit variance. The probability that X4 is the smallest among the four is ___________

[GATE - 2018]

6. For any discrete random variable X, with probability mass function
$P(X = j) = p_j,\ p_j \geq 0,\ j \in \{0, \ldots\ldots N]$, and $\sum_{j=0}^{N} p_j = 1$, define the polynomial function $g_x(z) = \sum_{j=0}^{N} p_j z^j$. For a certain discrete random variable Y, there exists a scalar $\beta \in [0, 1]$ such that $g_y(z) = \{1-\beta + \beta z)^N$. The expectation of Y is

[GATE - 2017]

(a) $N\beta(1 - \beta)$
(b) $N\beta$

-

ESE OBJ QUESTIONS

1. Consider a random variable to which a Poisson distribution is best fitted. It happens that $P_{(x=1)} = \frac{2}{3}P_{(x=2)}$ on this distribution plot. The variance of this distribution will be

(a) 3 (b) 2
(c) 1 (d) $\frac{2}{3}$

[EE ESE - 2018]

2. In a sample of 100 students, the mean of the marks (only integers) obtained by them in a test is 14 with its standard deviation of 2.5 (marks obtained can be fitted with a normal distribution). The percentage of students scoring 16 marks is

(a) 36 (b) 23
(c) 12 (d) 10

(Area under standard normal curve between z = 0 and z = 0.6 is 0.2257; and between z = 0 and z = 1.0 is 0.3413)

[EE ESE - 2018]

3. A bag contains 7 red and 4 white balls. Two balls are drawn at random. What is the probability that both the balls are red?

[ESE - 2017]

(a) $\frac{28}{55}$ (b) $\frac{21}{55}$
(c) $\frac{7}{55}$ (d) $\frac{4}{55}$

4. A random variable X has the density function $f(x) = K\frac{1}{1+x^2}$, where $-\infty < x < \infty$. Then the value of K is

[ESE - 2017]

(a) π (b) $\frac{1}{\pi}$
(c) 2π (d) $\frac{1}{2\pi}$

5. A random variable X has a probability density function

$$f(x) = \begin{cases} kx^n e^{-x}; & x \geq 0 \\ 0; & \text{otherwise} \end{cases} \quad \text{(n is an integer)}$$

with mean 3. The values of {k, n} are

[ESE - 2017]

(a) $\left\{\frac{1}{2}, 1\right\}$ (b) $\left\{\frac{1}{4}, 2\right\}$
(c) $\left\{\frac{1}{2}, 2\right\}$ (d) {1, 2}

6. What is the probability that at most 5 defective fuses will be found in a box of 200 fuses, if 2% of such fuses are defective?

[ESE - 2017]

(a) 0.82 (b) 0.79
(c) 0.59 (d) 0.82

7. 0If X is a normal variate with mean 30 and standard deviato 4, what is probability $(26 \leq X \leq 34)$, given A (z = 0.8) = 0.2881?

[ESE - 2017]

(a) 0.2881 (b) 0.5762
(c) 0.8181 (d) 0.1616

-

CHAPTER - 5
NUMERICAL METHOD

5.1 INTRODUCTION

Mathematical methods used to solve equation or evaluate integrals or solve differential equations can be classified broadly into two types:

1. Analytical Methods

2. Numerical Methods

5.1.1 Analytical Methods

Analytical methods an those which by an analysis of the equation obtain a solution directly as a readymade formulae in terms of say, the coefficients present in the equations.

Example 1. Solve $ax^2 + bx + c$ analytically

Solution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 2. Evaluate $\int x^2$ analytically

Solution.

$$\int_1^2 x^2 dx = \left[\frac{x^3}{3}\right]_1^2 = \frac{2^3 - 1^3}{3} = \frac{7}{3}$$

Example 3. Solve the different equation

$\frac{dy}{dx} - 2y = 0$ with initial condition y(o) = 3.

Solution.

$$\int \frac{dy}{y} = \int 2\ dx$$

$\Rightarrow y = 2x$

$Y = c\ e^{2x}$

$Y(0) = 3$

$\Rightarrow c = 3$

$\therefore y = 3e^{2x}$ is the required analytical solution.

5.1.2 Numerical Methods

Those same problems could also be solved numerically as we shall see in this chapter. In numerical solution, instead of directly writing the answer in terms of some formulae, we perform stepwise calculations using some algorithms or numerical procedures (usually on a computer) and arrive at the same results.

The advantage of numerical methods is that usually these procedures work on a much wider range of problems as compared to analytical solutions which work only on a limited class of problems.

WORKBOOK

Example 1. Solve the following set of equations using Gauss elimination method:

$x + 4y - z = -5$

$x + y - 6z = -12$

$3x - y - z = 4$

Solution.

We have

$x + 4y - z = -5$ …(i)

$x + y - 6x = -12$ …(ii)

$3x - y - z = 4$ …(iii)

Now, performing Equation (ii), Equation (i) and Equation (iii) $- 3 \times$ Equation (i) to eliminate x from Equation (ii) and Equation (iii), we get

$-3y - 5z = -7$ …(iv)

$-13y + 2z = 19$ ….(v)

Now, eliminating y by performing Equation (v) $-\frac{13}{3}\times$ Equation (iv), we get

$\frac{71}{3}z = \frac{148}{3}$ …(vi)

Now, by back substitution, we get

$$z = \frac{148}{71} = 2.0845$$

From Equation. (iv), we get

$$y = \frac{7}{3} - \left(\frac{5}{3}\right)\left(\frac{148}{71}\right) = -\frac{81}{71} = -1.1408$$

From Equation (1), we get

$$x = -5 - 4\left(-\frac{81}{71}\right) + \left(\frac{148}{71}\right) = \frac{117}{71} = 1.6479$$

Hence, x = 1.6479, y = – 1.1408, z = 2.0845.

Example 2. Solve the following system of equations by Crout's method:

$x + y + z = 3$

$2x - y + 3z = 16$

$3x + y - z = -3$

Solution.

We choose $u_{ii} = 1$ and write

A = LU

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 1 & -1 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_{11} & l_{11}u_{12} & l_{11}u_{13} \\ l_{21} & l_{21}u_{22} & l_{21}u_{13} + l_{22}u_{23} \\ l_{31} & l_{31}u_{12} + l_{32} & l_{31}u_{13} + l_{32}u_{23} + l_{33} \end{bmatrix}$$

Equating, we get

$l_{11} = 1, l_{21} = 2, l_{31} = 3$

$l_{11}u_{12} = 1 \Rightarrow u_{12} = 1, l_{11}u_{13} = 1 \Rightarrow u_{13} = 1$

$l_{21}u_{12} + l_{22} = -1 \Rightarrow l_{22} = -3,$

$l_{31}u_{13} + l_{22}u_{23} = 3 \Rightarrow u_{23} = -1/3,$

$l_{31}u_{13} + l_{32}u_{23} + l_{33} = -1 \Rightarrow l_{33} = -\frac{14}{3}$

Thus, we get

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 0 \\ 3 & -2 & -\frac{14}{3} \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

The given system is AX = B. The gives

LUX = B …(i)

Let UX = Y, so from Equation (1), we have

$$LY = B \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 0 \\ 3 & -2 & -\frac{14}{3} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 16 \\ -3 \end{bmatrix}$$

Which gives $y_1 = 3$

$3y_1 - 3y_2 = 16 \Rightarrow 9 - 3y_2 = 16 \Rightarrow y_2 = -\frac{10}{3}$

$$3y_1 - 2y_2 - \frac{14}{3}y_3 = -3 \Rightarrow 9 + \frac{20}{3} - \frac{14}{3}y_3$$

$= -3 \Rightarrow y_3 = 4$

ASSIGNMENT

1. A function f(x) has following values at different values of x

x	60	75	90	105	120
f(x)	28.2	38.2	43.2	40.9	37.7

Find out $f'(x)$ at x = 65
(a) 0.6521 (b) 0.7262
(c) 0.7012 (d) 0.7121

2. The order of error is the Simpson's rule for numerical integration with a step size h s is
(a) h (b) h^2
(c) h^3 (d) h^4

3. We wish to solve $x^2 - 2 = 0$ by Newton Raphson technique. Let the initial guess $bx_0 = 1.0$ subsequent cestimate of x (i.e. x_1) will be
(a) 1.141 (b) 1.5
(c) 2.0 (d) none of these

4. The accuracy of Simpson's rule quadrature for a step size h is
(a) $O(h^2)$ (b) $O(h^3)$
(c) $O(h^4)$ (d) $O(h^5)$

5. The values of a function f(x) are tabulated below :

x	0	1	2	3
f(x)	1	2	1	10

Using Newton's forward difference formula the cubic polynomial that can be fitted to the above data is
(a) $2x^3 + 7x^2 - 6x + 2$
(b) $2x^3 - 7x^2 + 6x - 2$
(c) $x^3 - 7x^2 - 6x^2 + 1x$
(d) $2x^3 - 7x^2 + 6x + 1$

6. Following are he values of a function y(x) :
y(−1) = 5, y(0),y(1) = 8 $\frac{dy}{dx}$ a x = 0 as per Newton's central difference scheme is
(a) 0 (b) 1.5
(c) 2.0 (d) 3.0

7. The Newton Raphson iteration $x_{n+1} = \frac{x_n}{2} + \frac{3}{2x_n}$ can be used to solve the equation
(a) $x^2 = 3$ (b) $x^3 = 3$
(c) $x^2 = 2$ (d) $x^3 = 2$

8. Which of the following statements applies to the bisection method used for finding roots of functions?
(a) Converges within a few iterations
(b) Guaranteed to work for all continuous function
(c) Is faster than the Newton - Raphson method
(d) Requires that there be no error in determining the sign of the function

9. The Newton's Raphson method is sued to find the root of the equation $x^2 - 2 = 0$.
If the iterations are started from −1, the interactions will be
(a) Converge to −1 (b) Converge to $\sqrt{2}$
(c) Converge to $-\sqrt{2}$ (d) None

10. Which of the following statements is TRUE in respect of the convergence of the Newton - Raphson method?
(a) It converges always under all circumstances
(b) it does not converge to a tool where the second differential coefficient changes sign.
(c) It does not converge to a root where the second differential coefficient vanishes.
(d) None

11. The Newton's Raphson iterative formula for finding f(x) = $x^2 - 1$ is
(a) $x_{i+1} = \frac{x_i^2 - 1}{2x_i}$ (b) $x_{i+1} = \frac{x_i^2 + 1}{2x_i}$

GATE QUESTIONS

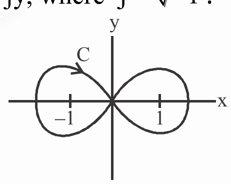
1. The quadratic equation $2x^2 - 3x + 3 = 0$ is to be solved numerically starting with an initial guess as $x_0 = 2$. The new estimate of x after the first iteration using Newton-Raphson method is

[GATE - 2018]

2. Let $r = x^2 + y - z$ and $z^3 - xy + yz + y^3 = 1$. Assume that x and y are independent variables. At (x, y, z) = (2, −1, 1), the value (correct to two decimal places) of $\frac{\partial r}{\partial x}$ is __________

[GATE - 2018]

3. The contour C given below is on the complex plane z = x + jy, where $j = \sqrt{-1}$.



The value of the integral $\frac{1}{\pi j}\oint \frac{dz}{z^2 - 1}$ is ______.

[GATE - 2018]

4. The solution at x = 1, t =1 of the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 25\frac{\partial^2 u}{\partial t^2}$ subject to initial conditions of u(0) = 3x and $\frac{\partial u}{\partial t}(0) = 3$ is ______________

[GATE - 2018]

(a) 1 (b) 2
(c) 4 (d) 6

5. Consider the equation $\frac{du}{dt} = 3t^2 + 1$ with u = 0 at t = 0. This is numerically solved by using the forward Euler method with a step size. $\Delta t = 2$. The absolute error in the solution at the end of the first time step is _____

[GATE - 2017]

6. Starting with x = 1, the solution of the equation $x^3 + x = 1$, after two iterations of Newton Rephason's method (up to two decimal places) is ______

[GATE - 2017]

7. The following table lists an n^{th} order polynomial $f(x) = a_n x^n + a_{n-1} + \ldots + a_1 x + a_0$ and the forward differences evaluated at equally spaced values of x. The order of the polynomial is

x	f(x)	Δf	$\Delta^2 f$	$\Delta^3 f$
−0.4	1.7648	−0.2965	0.089	−0.03
−0.3	1.4683	−0.2075	0.059	−0.0228
−0.2	1.2608	−0.1485	0.0362	−0.0156
−0.1	1.1123	−0.1123	0.0206	−0.0084
0	1	−0.0917	0.0122	−0.0012
0.1	0.9083	−0.0795	0.011	0.006
0.2	0.8288	−0.0685	0.017	0.0132

[GATE - 2017]

(a) 1 (b) 2
(c) 3 (d) 4

8. P(0, 3), Q (0.5, 4), and R(1, 5) are three points on the curve defined by f(x). Numerical integration is carried out using both Trapezoidal rule and Simpson's rule within limits x = 0 and x = 1 for the curve. The difference between the two results will be

[GATE - 2017]

(a) 0 (b) 0.25
(c) 0.5 (d) 1

9. Newton - Raphson method is to be used to find root of equation $3x - e^x + \sin x = 0$. If the initial trial value for the root is taken as 0.333, the next approximation for the root would be _________ (note: answer up to three decimal)

CHAPTER - 6
COMPLEX VARIABLE

6.1 INTRODUCTION

Many engineering problems may be treated and solved by methods involving complex numbers and complex functions. There are two kinds of such problems. The first of them consists of "elementary problems" for which some acquaintance with complex numbers is sufficient. This includes many applications to electric circuits or mechanical vibrating systems.

The second kind consists of more advanced problems for which we must be familiar with the theory of complex analytic functions- "complex function theory" or "complex analysis," for short- and with its powerful and elegant methods. Interesting problems in heat conduction, fluid flow, and electrostatics belong to this category.

We shall see that the importance of complex analytic functions in engineering mathematics has the following two main roots.

1. The real and imaginary parts of an analytic function are solutions of Laplace's equation in two independent variables. Consequently, two-dimensional potential problems can be treated by methods developed for analytic functions.

2. Most higher functions is engineering mathematics are analytic functions, and their study for complex values of the independent variable leads to a much deeper understanding of their properties. Furthermore, complex integration can help evaluating complicated complex and real integrals of practical interest.

6.2 COMPLEX FUNCTIONS

lf for each value of the complex variable 2 (= x + iy) in a given region R, we have one or more values of w (= u +y), then w is said to be a complex function of z and we write w = u(x, y) + iv(x, y) = f(z) where u, v are real functions of x and y.

lf to each value of z, there corresponds one and only one value of w, then w is said to be a single-valued function of 2 otherwise a multi-valued function. For example w = 1/z is a single–valued function and $w = \sqrt{z}$ is a multi–valued function of z. The former is defined at all points of the z–plane except at z = 0 and the latter assumes two values for each value of z except at z = 0.

6.2.1 Exponential Function of a Complex Variable

When x is real, we are already familiar with the exponential function $e^x = 1 + \frac{X}{1!} + \frac{x^2}{2!} + ... + \frac{x^n}{n!} + \infty$

Similarly, we define the exponential function of the complex variable z = x + iy. As

$$e^z \text{ or } \exp(z) = 1 + \frac{z}{1!} + \frac{z^2}{2!} + ... + \frac{z^n}{n!} + ...\infty \qquad ... \text{(i)}$$

Putting x = 0 in (i), we get, z = iy and $e^{iy} = 1 + \frac{iy}{1!} + \frac{(iy)^2}{2!} + \frac{(iy)^3}{3!} + \frac{(iy)^4}{4!} + ...\infty$

$$= \left(1 - \frac{y^2}{2!} + \frac{y^4}{4!} - ....\right) + i\left(y - \frac{y^3}{3!} + \frac{y^5}{5!} - .....\right) = \cos y + i \sin y$$

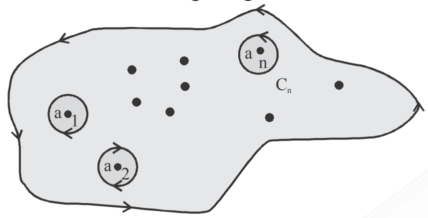
Thus $e^z = e^x \cdot e^{iy} = e^x (\cos y + i \sin y)$

Also $x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$

$\therefore$ Exponential form of $z = (= x + iy) = re^{i\theta}$.

WORKBOOK

Example 1. Prove the residue theorem.
Solution.
Consider the following diagram



Let us surround each of the singular points a_1, a_2, ….., a_n by a small circle such that it encloses no other singular point. These circle c_1, c_2,….c_n together with c form a multiple – connected region in which f(z) is analytic.
Applying Cauchy′s theorem, we have

$$\oint_c f(x)\times dz = \oint_{c_1} f(z)\cdot dz + \oint_{c_2} f(z)\times dz$$

$$+\ldots+\oint_{c_n} f(z)\cdot dz$$

= $2\pi i[\text{Resf}(a_1) + \text{Resf}(a_2)+\ldots+\text{Resf}(a_n)]$
Which is the desired result.

Example 2. Separate $\tan^{-1}(x + iy)$ into real and imaginary parts.
Solution.
Let $\alpha + i\beta = \tan^{-1}(x + iy)$ …(i)
Then $\alpha - i\beta = \tan^{-1}(x - iy)$ …(ii)
Adding equation (1) and (2), we get
$2\alpha = \tan^{-1}(x + iy) + \tan^{-1}(x - iy)$

$$= \tan^{-1}\frac{(x+iy)+(x-iy)}{1-(x+iy)(x-iy)}$$

Therefore, $\alpha = \frac{1}{2}\tan^{-1}\frac{2x}{1-x^2-y^2}$

Subtracting equation (ii) from Equation (i), we get
$2i\beta = \tan^{-1}(x + iy) - \tan^{-1}(x - iy)$

$$= \frac{\tan^{-1}(x+iy)-(x-iy)}{1+(x+iy)(x-iy)}$$

$$= \tan^{-1} i\frac{2y}{1+x^2+y^2}$$

$$= i\tan^{-1}\frac{2y}{1+x^2+y^2} \quad [\because \tan^{-1} iz = i\tanh^{-1} z]$$

$$\beta = \frac{1}{2}\tanh^{-1}\frac{2y}{1+x^2+y^2}$$

Example 3. Show that $f(z) = z^3$ is analytic.
Solution.
Let z = x + iy
$\Rightarrow z^2 = (x + iy)(x + iy) = x^2 - y^2 + ixy$
$\Rightarrow z^3 = (x^2 - y^2 + ixy)(x + iy)$
$= (x^3 - 3xy^2) + (3x^2y - y^3)i$
Now, $u = x^3 - 3xy^2$ and $v = 3x^2y-y^3$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2$$

$$\frac{\partial u}{\partial y} = -6xy \qquad \frac{\partial v}{\partial x} = 6xy$$

$$\frac{\partial v}{\partial y} = 3x^2 - 3y^2$$

So, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

So, Cauchy-Riemann equations are satisfied and also the partial derivatives are continuous at all points. Hence, z^3 is analytic for every z.

Example 4. If w = log z, find dw/dz and determine if w is non - analytic.
Solution.
We have

$$w = u + iv = \log(x+iy) = \frac{1}{2}\log(x^2+y^2)$$

$$+i\tan^{-1} y/x$$

Hence, $u = \frac{1}{2}\log(x^2+y^2)$ and $v = \tan^{-1} y/x$

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{x}{x^2+y^2} \qquad \Rightarrow \frac{\partial u}{\partial y} = \frac{y}{x^2+y^2}$$

$$\Rightarrow \frac{\partial v}{\partial x} = \frac{y}{x^2+y^2} \qquad \Rightarrow \frac{\partial v}{\partial y} = \frac{x}{x^2+y^2}$$

GATE QUESTIONS

1. The residues of a function

$$f(z)=\frac{1}{(z-4)(z+1)^3}$$

[GATE - 2017]

(a) $\frac{-1}{27}$ and $\frac{-1}{125}$ (b) $\frac{1}{125}$ and $\frac{-1}{125}$

(c) $\frac{-1}{27}$ and $\frac{1}{5}$ (d) $\frac{1}{125}$ and $\frac{-1}{5}$

2. If $f(z) = (x^2 + ay^2) + ibxy$ is a complex analytic function of $z = x + iy$, where $I=\sqrt{-1}$, then

[GATE - 2017]

(a) $a = -1, b = -1$ (b) $a = -1, b = 2$
(c) $a = 1, b = 2$ (d) $a = 2, b = 2$

3. The value of the integral

$$\int_{-\infty}^{\infty}\frac{\sin x}{x^2+2x+2}dx$$

Evaluated using contour integration and the residue theorem is

[GATE - 2016]

(a) $-\pi \sin(1)/e$ (b) $-\pi \cos(1)/e$
(c) $\sin(1)/e$ (d) $\cos(1)/e$

4. $f(z) = u(x, y) + iv(x, y)$ is an analytic function of complex variable $z = x + iy$ where $i = \sqrt{-1}$. If $u(x, y) = 2xy$, then $v(x, y)$ may be expressed as_______.

[GATE - 2016]

5. Let $Z = x + iy$ be a complex variable, consider continuous integration is performed along the unit circle in anticlockwise direction. Which one of the following statements is NOT TRUE ?

[GATE - 2015]

(a) The residue of $\frac{z}{z^2-1}$ at $z = 1$ is $\frac{1}{2}$

(b) $\oint_C z^2 dz = 0$

(c) $\frac{1}{2\pi i}.\oint_C \frac{1}{z}dz = 1$

(d) $\bar{z}$ (complex conjugate of z) is an analytical function

6. If C is a circle of radius r with centre z_0, in the complex z-plane and if n is a non - zero integer, then $\oint_C \frac{dz}{(z-z_0)^{n+1}}$

[GATE - 2015]

(a) $2\pi nj$ (b) 0

(c) $\frac{nj}{2\pi}$ (d) $2\pi n$

7. Given $f(z) = g(z) + h(z)$, where f, g , h are complex valued functions of complex variable z. Which one of the following statements is TRUE?

[GATE - 2015]

(a) If f(z) is differentiable at z_0, then g(z) and h(z) are also differentiable at z_0
(b) If g(z) and h (z) are differentiable at z_0, then f(z) is also differentiable at z_0
(c) If f(z) is continuous at z_0 , then it is differentiable at z_0
(d) if f(z) is differentiable at z_0, then so are its real and imaginary parts.

8. Given two complex numbers $z_1 = 5+(5\sqrt{3})i$ and $z_2=\frac{z}{\sqrt{3}}+2i$, the argument of $\frac{z_1}{z_2}$ in degrees is

[GATE - 2015]

(a) 0 (b) 30

(c) 60 (d) 90

9. Consider the following complex function:

$$f(x)=\frac{9}{(x-1)(x+2)^2}$$

Which of the following is one of the residues of the above function

CHAPTER - 7
TRANSFORM THEORY

7.1 INTRODUCTION
The Laplace transform method solve differential equations and corresponding initial and boundary value problems. The process of solution consists of three main steps:
Step-I. The given "hard" problem is transformed into a "simple" equation (subsidiary equation).
Step-II. The subsidiary equation is solved by purely algebraic manipulations.
Step-III. The solution of the subsidiary equation is transformed back to obtain the solution of the given problem.
In this way Laplace transforms reduce the problem of solving a differential equation to an algebraic problem. This process is made easier by tables of functions and their transforms, whose role is similar to that of integral tables in calculus.
This switching from operations of calculus to algebraic operations on transforms is called operational calculus, a very important area of applied mathematics, and for the engineer, the Laplace transform method is practically the most important operation method. It is particularly useful in problems where the mechanical or electrical driving method. It is particularly useful in problems where the mechanical or electrical driving force has discontinuities, is impulsive or is a complicated periodic function, not merely a sine or cosine. Another operational method is the Fourier transform.
The Laplace transform also has the advantage that it solve initial value problems directly, without first determining a general solution. It also solves nonhomogeneous differential equations directly without first solving the corresponding homogeneous equation.
System of ODES and partial differential equations can also be treated by Laplace transforms.

7.2 DEFINITION
Let f(t) be a function of t defined for all positive values of t. Then the Laplace transforms of f(t), denoted by L{f(t)} is defined by $L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$

Provided that the integral exists, s is a parameter which may be a real or complex number.
L{f(t)} being clearly a function of s is briefly written as $\bar{f}(s)$ or as F(s). i.e. L{f(t)} = $\bar{f}(s)$,

Which can be also be written as f(t) = $L^{-1}\{\bar{f}(s)\}$

Then f(t) is called the inverse Laplace transform of $\bar{f}(s)$. The symbol L. Which transforms f(t) into $\bar{f}(s)$, is called the Laplace transformation operator.

Example.
If f(t) = 1

$$L[f(t)] = \int e^{-st}.1dt = \left[\frac{e^{-st}}{-s}\right]_0^\infty = \frac{e^{-\infty} - e^0}{-s} = \frac{1}{s}$$

Similarly Laplace transform of other common function can also be evaluated and is shown below:

7.3 CHANGE OF SCALE PROPERTY
1. $L(1) = \frac{1}{s}\ (s > 0)$

WORKBOOK

Example 1. Determine Laplace transform of $x(t) = e^{at} u(-t)$.

Solution.

Laplace transform of x(t) is given by

$$X(x) = \int_{-\infty}^{\infty} e^{at}u(-t)e^{-st}dt =$$

$$\int_{-\infty}^{0} e^{at}\cdot e^{-st}dt = \int_{-\infty}^{\infty} e^{-t(s-a)}dt \quad = -\frac{1}{(s-a)}e^{-t(s-a)}\Big|_{-\infty}^{0}$$

$$= -\frac{1}{(s-a)} \text{ for Re(s) < a}$$

Example 2. Determine the Laplace transform and associated region of convergence of $x(t) = e^{-2t}[u(t) - u(t-5)]$

Solution.

x(t) can be rewritten as $x(t) = e^{-2t}u(t)-e^{-2t}u(t-5)$ $=e^{-2t}u(t) - e^{-10}\cdot e^{-2(t-5)}u(t-5)$

Now Laplace transform of

$$e^{-2t}u(t) = \frac{1}{s+2} = e^{-5s}\times\frac{1}{s+2}$$

Therefore $X(s) = \dfrac{1}{(s+2)} - \dfrac{e^{-10}e^{-5s}}{(s+2)}$

$$= \frac{1}{(s+2)}\left[1-e^{-5(s+2)}\right] \text{ with Re(s) > - 2}$$

Example 3. Given that Laplace transform of $u(t) = \frac{1}{s}$, what is the Laplace transform of δ(t)?

Solution.

Given that $u(t) \leftrightarrow \frac{1}{s}$

Laplace transform of δ(t) = Laplace transform of $\dfrac{du(t)}{dt}$

Using time differentiation property, we get

$$\delta(t) \leftrightarrow s\cdot\frac{1}{s} = 1$$

Example 4. Determine the inverse Laplace transform and associated ROC of $\dfrac{(Rs+4)}{(s^2+4s+3)}$.

[Assume ROC = - 3 < Re(s) < - 1]

Solution.

Partial fraction expansion,

$$\frac{2s+4}{(s^2+4s+3)} = \frac{1}{(s+1)} + \frac{1}{(s+3)}$$

It is given that Re(s) > - 3

Therefore, inverse Laplace transform of

$$\frac{1}{(s+3)} = e^{-3t}u(t)$$

It is also given that Re(s) < - 1

Therefore, inverse Laplace transform of

$$\frac{1}{(s+1)} = e^{-1t}u(t)$$

Therefore, inverse Laplace transform of $\dfrac{2s+4}{s^2+4s+3}$ is given by

$e^{-3t}u(t) + e^{-t}u(-t)$.

Example 5. Determine the z-transform of $x[n] = a^{-n}u[-n]$.

Solution.

z-Transform of $a^n u[n]$ is equal to $\dfrac{z}{z-a}$ for |z|< a. By using time reversal property, we get

z-Transform of $a^{-n}u[-u] = \dfrac{\frac{1}{z}}{\frac{1}{z}-a} = \dfrac{1}{1-az}$ for $|z| < \dfrac{1}{|a|}$.

Example 6. Determine the inverse z-transform of $\dfrac{az}{(z-a)^2}$.

GATE QUESTIONS

1. The Laplace transform F(s) of the exponential function. $f(t) = e^{at}$ when $t \geq 0$, where a is a constant and $(S - a) > 0$, is

[GATE - 2018]

(a) $\frac{1}{s+a}$ (b) $\frac{1}{s-a}$

(c) $\frac{1}{a-s}$ (d) ∞

2. The laplace transform of te^t is

[GATE - 2017]

(a) $\frac{s}{(s+1)^2}$ (b) $\frac{1}{(s-1)^2}$

(c) $\frac{1}{(s+1)^2}$ (d) $\frac{s}{s-1}$

3. The fourier series of the function,

$f(x) = 0, \quad -\pi < x \leq 0$

$= \pi - x, \ 0 < x < \pi$

In the interval $[-\pi, \pi]$ is

$$f(x) = \frac{\pi}{4} + \frac{2}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + ...\right] + \left[\frac{\sin x}{1} + \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + ...\right]$$

The convergence of the above Fourier series at x = 0 gives

[GATE - 2016]

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$

(c) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = \frac{\pi}{4}$

4. If f(t) is a function defined for all $t \geq 0$, its Laplace transform F(s) is defined as

[GATE - 2016]

(a) $\int_0^{\infty} e^{st} f(t)\,dt$ (b) $\int_0^{\infty} e^{-st} f(t)\,dt$

(c) $\int_0^{\infty} e^{ist} f(t)\,dt$ (d) $\int_0^{\infty} e^{-ist} f(t)\,dt$

5. The bilateral Laplace transform of a function

$f(t) = \begin{cases} 1 & \text{if } a \leq t \leq b \\ 0 & \text{otherwise} \end{cases}$ b

[GATE - 2015]

(a) $\frac{a-b}{s}$ (b) $\frac{e^s(a-b)}{s}$

(c) $\frac{e^{as} - e^{-bs}}{s}$ (d) $\frac{e^{s(a-b)}}{s}$

6. Let $x(t) = \alpha s(t) + s(-t)$ with $s(t) = \beta e^{-4t} u(t)$, where u(t) is a unit - step function. If the bilateral Laplace transform of x(t) is $X(s) = \frac{16}{S^2 - 16} - 4 < \text{Re}\{s\} < 4$, then the value of β is _____.

[GATE - 2015]

7. The Laplace transform of $f(t) = 2\sqrt{t/\pi}$ is $s^{-3/2}$. The Laplace transform of $g(t) = \sqrt{1/\pi t}$ is

[GATE - 2015]

(a) $3s^{-5/2}/2$ (b) $s^{-1/2}$

(c) $s^{1/2}$ (d) $s^{3/2}$

8. Consider a signal defined by

$$x(t) = \begin{cases} e^{j10t} & \text{for } |t| \leq 1 \\ 0 & \text{for } |t| > 1 \end{cases}$$

Its Fourier transform is

[GATE - 2015]

(a) $\frac{2\sin(\omega - 10)}{\omega - 10}$ (b) $2e^{j10\frac{\sin(\omega-10)}{\omega-10}}$

(c) $\frac{2\sin\omega}{\omega - 10}$ (d) $e^{j10\omega\frac{2\sin\omega}{\omega}}$

9. The z-transform of a sequence x[n] is given as $X(z) = 2z + 4 - 4/z + 3/z^2$. If y[n] is the first difference of x[n], then Y(x) is given by