GATE

2019

SIGNAL AND SYSTEM

ELECTRICAL ENGINEERING

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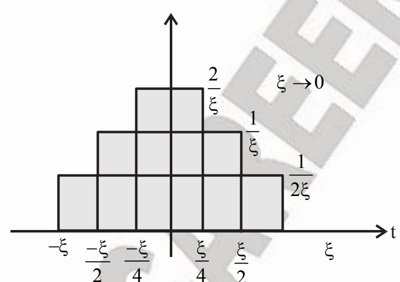
CHAPTER - 1
INTRODUCTION

1.1 SIGNAL
Signal is a function of one or more independent variables contain information about some behavior or natural phenomenon.
Example. Speech, Video, Audio, T.V. Signal, Current, Voltage, RF signal etc.

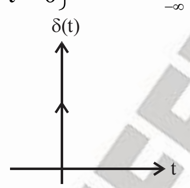
1.2 SYSTEM
Interconnection of various physical elements which are formed to get the desired response.
Example. High Pass filter, Low pass filter, Automobile Car, Mobile, Tablet, etc.

1.3 IMPORTANT SIGNALS
1.3.1 Continuous Time Unit Impulse Signal
The unit Impulse function $\delta(t)$ is known as Dirac Delta function.

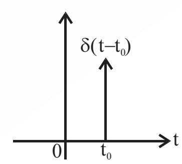


Unity area over an infinitesimal Time interval

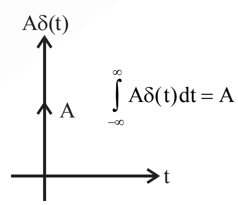
$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \qquad \int_{-\infty}^{\infty} \delta(t)dt = 1$$



Continuous Time Unit Impulse Function



Time Continuous Unit Impulse Function



Scaled Unit Impulse Function

WORKBOOK

Example 1. What are even and odd fart of x(t) = δ(t)

Solution.

$$\delta_e(t) = \frac{\delta(t) + \delta(-t)}{2}$$

$$\delta_0(t) = \frac{\delta(t) - \delta(-t)}{2}$$

δ(t) is even function δ(t) = δ(-t)

$$\delta_e(t) = \frac{2\delta(t)}{2} = \delta(t)$$

$$\delta_0(t) = \frac{\delta(t) - \delta(t)}{2} = 0$$

Example 2. What are even and odd parts of x(t) = u(t)

Solution.

$$u_e(t) = \frac{u(t) + u(-t)}{2} = \frac{1}{2}$$

$$u_0(t) = \frac{u(t) + u(-t)}{2} = \frac{1}{2}\text{Sgn}\,t$$

$$u(t) = \frac{1}{2} + \frac{1}{2}\text{Sgnt}$$

Example 3. What is R.M.S value of $x(t) = A_1 \cos(\omega t+\phi_1) + A_2 \cos(\omega t+\phi_2)$

Solution.

Power of signal x(t) will be

$$P = \frac{A_1^2}{2} + \frac{A_2^2}{2}$$

$$\text{R.M.S value} = \sqrt{\frac{A_1^2}{2} + \frac{A_2^2}{2}}$$

Example 4. Calculate the energy and power of signal

$x(n)=-(0.5)^n\, u(n)$

Solution.

$$E = \sum_{n=-\infty}^{\infty} \left[-(0.5)^n u(n)\right]^2$$

$$= \sum_{n=0}^{\infty} (0.25)^n = \frac{1}{1-0.25} = \frac{4}{3}$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} [x(n)]^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$P = \frac{1}{2N+1} \cdot \frac{4}{3} = 0$$

ASSIGNMENT

1. Random signal can be modeled by
(a) Different equation
(b) Difference equation
(c) Statistical parameters
(d) Integral

2. Even signal satisfies
(a) $x(-t) = x(t)$
(b) $x[-n] = -x[n]$
(c) $x(n+1) = ax[n] + b$
(d) $\frac{dx(t)}{dt} = c$

3. Odd signal satisfies
(a) $x(-t) = x(t)$
(b) $x[-n] = -x[n]$
(c) $x(n+1) = ax[n] + b$
(d) $\frac{dx}{dt}(t) = c$

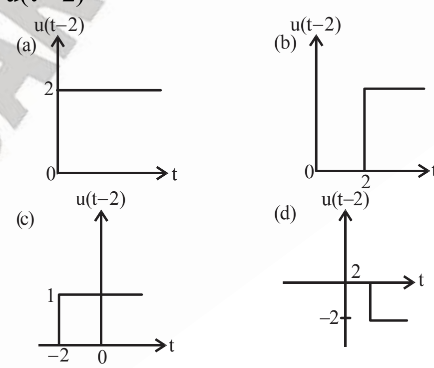
4. Any signal $x(t)$ can be expressed as
(a) $x_e(t) + x_0(t)$
(b) $x_e(t) - x_0(t)$
(c) $\frac{x_e(t)}{x_0(t)}$
(d) $x_e(t) \times x_0(t)$
Where $x_e(t)$ and $x_0(t)$ are even and odd parts of the signal $x(t)$.

5. Periodic signals are
(a) $x(t+T) = x(t)$
(b) $x(t-T) = x(t)$
(c) $x(n+mN) = x[n]$
(d)All the above

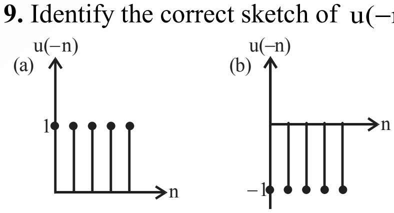
6. Energy signals are the signals with
(a) $0 < E < \infty, P = 0$
(b) $0 < E < \infty, P = \infty$
(c) $0 < P < \infty, E = \infty$
(d) $0 < P < \infty, E = 0$
Where E and P are average energy and power of the signals $x(t)$ or $x[n]$.

7. Power signals are the signals with
(a) $0 < E < \infty, P = 0$
(b) $0 < E < \infty, P = \infty$
(c) $0 < P < \infty, E = \infty$
(d) $0 < P < \infty, E = 0$

8. Identify the correct sketch of unit step signal $u(t-2)$



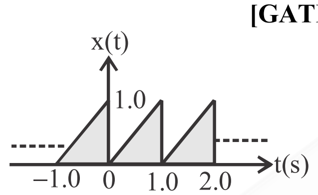
9. Identify the correct sketch of $u(-n)$



GATE QUESTIONS

1. A periodic signal x(t) is shown in the figure. The fundamental frequency of the signal x(t) in Hz is _____.

[GATE - 2017]



2. If a continuous-time signal $x(t) = \cos(2\pi t)$ is sampled at 4Hz, the value of the discrete time sequence x(n) at n = 5 is

[GATE - 2017]

(a) –0.707 (b) –1
(c) 0 (d) 1

3. Two sequences $x_1[n]$ and $x_2[n]$ have the same energy. Suppose $x_n[n] = \alpha\, 0.5^n u[n]$, where α is a positive real number and u[n] is the unit step sequence. Assume

$$x_n[n] = \begin{cases} \sqrt{1.5} & \text{for } n = 0,1 \\ 0 & \text{otherwise.} \end{cases}$$

Then the value of α is _____.

[GATE - 2015]

4. For a periodic signal $v(t) = 30 \sin 100t + 10 \cos 300t + 6 \sin(500t + \pi/4)$, the fundamental frequency in rad/s

[GATE - 2015]

(a) 100 (b) 300
(c) 500 (d) 1500

5. The Dirac delta function $\delta(t)$ is given as

[GATE - 2006]

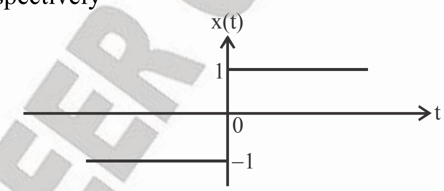
(a) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(b) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(c) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

(d) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

6. The function x(t) is shown in figure. Even and odd part of a unit step function u(t) are respectively



[GATE - 2005]

(a) $\frac{1}{2}, \frac{1}{2}x(t)$ (b) $\frac{1}{2}, \frac{1}{2}x(t)$
(c) $\frac{1}{2}, -\frac{1}{2}x(t)$ (d) $\frac{1}{2}, -\frac{1}{2}x(t)$

7. The power in the signal

$s(t) = 8\cos\left(20\pi t - \frac{\pi}{2}\right) + 4\sin(15\pi t)$ is:

[GATE - 2005]

(a) 40 (b) 41
(c) 42 (d) 82

8. Consider the sequence

$x[n] = (4 - 5j1 + j24)$

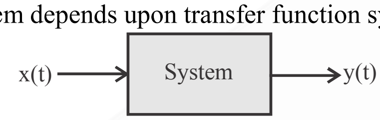
The conjugate anti-symmetric part of the sequence is

[GATE - 2004]

(a) $[-4 - j2.5,\ j2,\ 4 - j2.5]$
(b) $[-j2.5,\ 1,\ j2.5]$
(c) $[-j2.5,\ j2,\ 0]$
(d) $[-4,\ 1,\ 4]$

CHAPTER - 2
SYSTEM

2.1 SYSTEM
A system is a mathematical model of physical process that relates the Inputs (or excitation) signal to the output (or response) of signal.
The response or output of system depends upon transfer function system.



Mathematically, the functional Relationship between I/P and O/P may be written as:

$$y(t) = f\{x(t)\}$$

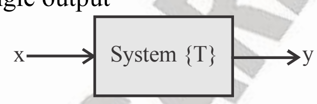
$$y(t) = T[x(f)]$$

Where T implies transformation and gives a mapping to be done on x(t) to get y(t)

2.1.1 Symbolically, we can write

$$x(t) \xrightarrow{s} y(t)$$

Multiple input and/or output signals are possible. But we will restrict our attention for most part in this course to the single Input single output



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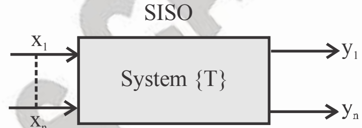
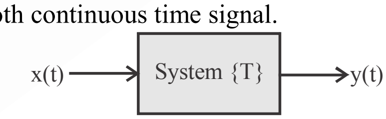


MIMO

Examples of system. Filters, amplifiers, communication channels, T.V. set are various example of electrical system.

2.2 TYPES OF SYSTEMS
1. Continuous-Time System
2. Discrete-Time System

1. Continuous-Time System
Continuous - Time system may be defined as also continuous. This means that Input and output of continuous time system are both continuous time signal.



Example. Audio, Video Amplifier, Power supplies etc.
Simple Practical example of continuous time – system is Low Pass Filter

GATE QUESTIONS

1. Consider a single input single output discrete- time system with x[n] as input and y[n] as output, where the two are related as

$$y[n] = \begin{cases} n\,|x[n]|, & \text{for } 0 \le n \le 10 \\ x[n] - x[n-1], & \text{otherwise} \end{cases}$$

Which one of the following statements is true about the system ?

[GATE - 2017]

(a) It is causal and stable
(b) It is causal but not stable
(c) It is not causal but stable
(d) It is neither causal nor stable

2. Consider and LTI system with magnitude response $|H(f)| = \begin{cases} 1 - \frac{|f|}{20}, & |f| \le 20 \\ 0, & |f| > 20 \end{cases}$ and phase response Arg[H(f)]=2f.
If the input to the system is

$$x(t) = 8\cos\left(20\pi t + \frac{\pi}{4}\right) + 16\sin\left(40\pi t + \frac{\pi}{8}\right) + 24\cos\left(80\pi t + \frac{\pi}{16}\right)$$

Then the average power of the output signal y(t) is___

[GATE - 2017]

3. The transfer function of a causal LTI system is H9s) = 1/s. if the input to the system is x(t) = [sin(t)/πt]u(t); where u(t) is a unit step function. The system output y(t) as $t \to \infty$ is__

[GATE - 2017]

4. The signal x(t) = sin(1400πt), where t is in seconds, is sampled at a rate of 9000 samples per second. The sampled signal is the input to an ideal lowpass filter with frequency response H(f) as follows:

$$H(f) = \begin{cases} 1, |f| \le 12\text{kHz} \\ 0, |f| > 12\text{kHz} \end{cases}$$

What is the number of sinusoids in the output and their frequencies in kHz?

[GATE - 2017]

(a) Number = 1, frequency = 7
(b) Number = 3, frequencies = 2,7,11
(c) Number = 2, frequencies = 2, 7
(d) Number = 2, frequencies = 7, 11

5. An LTI system with unit sample response h(n) = 5δ[n] – 7δ [n –1] + 7δ[mn–3] – 5δ[n –4] is a

[GATE - 2017]

(a) Low pass filter
(b) High pass filter
(c) Band pass filter
(d) Band stop filter

6. The input x(t) and the output y(t) of a continuous time system are related as

$y(t) = \int_{t-T}^{t} x(u)du$. The system is

[GATE - 2017]

(a) Linear and time variant
(b) Linear and time invariant
(c) Non Linear and time variant
(d) Nonlinear and time invariant

7. Consider $g(t) = \begin{cases} t - \lfloor t \rfloor, t \ge 0 \\ t - \lceil t \rceil, \text{otherwise} \end{cases}$,

Where $t \in R$
Here, $\lfloor t \rfloor$ represent the largest integer less than or equal to t and $\lceil t \rceil$ denotes the smallest integer greater than or equal to t. The coefficient of the second harmonic component of the fourier series representing g(t) is _____

[GATE - 2017]

8. Consider the signal x(t) = cos(6πt) + sin(8πt), where t is in seconds. The Nyquist sampling

CHAPTER - 3
LINEAR TIME-INVARIANT SYSTEM

3.1 INTRODUCTION
A system is a mathematical model of physical process that relates the Inputs (or excitation) signal to the output (or response) of signal. We have discussed the Several Basic Properties of systems, two properties namely Linearity and Time - invariance plays a important role in analysis of signals and system. If a system has both linearity and time - invariance properties, then this system is called Linear -Time Invariant system (LTI system).

> We study LTI system because of the fact that the most of practical and physical system can be modeled in form of Linear-Time (invariant) System

In this chapter we develop the fundamental Input-Output relationships for system having these properties and it will be shown that Input-output Relationship for LTI system is described of a convolution operation.
Importance of convolution operation if one knows the output of unit Impulse then output for general input can be calculated.

3.2 CHARACTERISTICS OF LINEAR TIME - INVARIANT(LTI) SYSTEM
Both continuous-time and discrete-time linear time invariant (LTI) system exhibit one important characteristics that the superposition theorem can be applied to find the response y(t) to a given input x(t).

3.2.1 Important steps to adopted to find response of LTI system using superposition
1. Resolve the input function x(t) in terms of simple or basic function like impulse function for which response can be easily evaluated.
2. Determine Response of LTI system for simple or Basic functional individually.
3. Using superposition theorem, find the sum of individual response which will become overall response y(t) of function x(t) from above, to find the response of LTI system to given function first we have to find the response of LTI system to an unit impulse called as unit impulse response of LTI system.

3.2.2 Unit Impulse Response [$h(t)_n$ or h/n]
Impulse response of continuous time or discrete-time LTI system is output of system due to an unit impulse input applied at time t = 0 or n = 0.

GATE QUESTIONS

1. Let the input be u and the output be y of a system, and the other parameters are real constants. Identify which among the following systems is not a linear system:

[GATE - 2018]

(a) $\frac{d^3y}{dt^2} + a_1\frac{d^2y}{dt^2} + a_2\frac{dy}{dt} + a_3 y$

$$= b_3 u + b_2\frac{du}{dt} + b_1\frac{d^2u}{dt^2}$$

(b) $y(t) = \int_0^t e^{\alpha(t-\tau)}\beta u(\tau)\,d\tau$

(c) y = au + b , b ≠ 0

(d) y =a u

2. Let z(t) = x(t)*y(t). Where "*" denotes convolution. Let c be a positive real-valued constant.

Choose the correct expression for z(ct).

[GATE - 2017]

(a) c.x(ct)*y(ct) (b) x(ct)*y(ct)
(c) c.x(t)*y(ct) (d) c.x(ct)*y(t)

3. Consider the system with following input-oputput relation $y[n] = (1 + (-1)^n)x[n]$

Where , x[n] is the input and y[n] is the output. The system is

[GATE - 2017]

(a) Invertible and time invariant
(b) Invertible and time varying
(c) Non-invertible and time invariant
(d) Non-invertible and time varying

4. The result of the convolution

$x(-t) * \delta(-t-t_0)$ is

[GATE - 2015]

(a) $x(t+t_0)$ (b) $x(t-t_0)$
(c) $x(-t+t_0)$ (d) $x(-t-t_0)$

5. The impulse response of an LTI system can be obtained by

[GATE - 2015]

(a) Differentiating the unit ramp response
(b) Differentiating the unit step response
(c) Integrating the unit ramp response
(d) Integrating the unit step response

6. For linear tune invariant systems, that are Bounded Input Bounded Output stable, winch one of the following statements is TRUE?

[GATE - 2014]

(a) The impulse response will be integrable, but may not be absolutely integrable.
(b) The unit impulse response will have finite support.
(c) The unit step response will be absolutely integrable.
(d) The unit step response will be bounded.

7. Consider an LTI system with transfer function $H(s) = \frac{1}{s(s+4)}$

If the input to the system is cos(3t) and the steady state output is A sin(3t + α), then the value of A is

[GATE - 2014]

(a) 1/30 (b) 1/15
(c) 3/4 (d) 4/3

8. Consider an LTI system with impulse response $h(t) = e^{-5t}\,u(t)$. If the output of the system is $y(t) = e^{-3t}\,u(t) = -\,e^{-5t}\,u(t)$ then the input, x(t), is given by

[GATE - 2014]

(a) $e^{-3t}u(t)$ (b) $2e^{-3t}u(t)$
(c) $e^{-5t}u(t)$ (d) $2e^{-5t}u(t)$

9. Two systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by

[GATE - 2013]

(a) Product of $h_1(t)$ and $h_2(t)$

CHAPTER - 4

LINEAR-TIME INVARIANT-2 SYSTEM

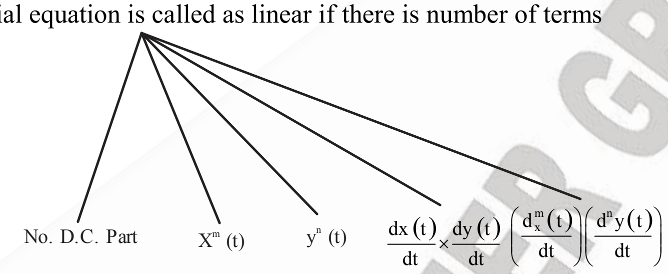
4.1 LINEAR CONSTANT CO-EFFICIENT DIFFERENTIAL EQUATIONS (LCC DE)

A general n^{th} -order linear - constant co-efficient differential equation is given by

$$\sum_{K=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{K=0}^{M} b_K \frac{dk_x(t)}{dt^k}$$

Above equation is representation of continuous system.

1. A differential equation is called as linear if there is number of terms



Example. $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 5 = x(t)$

It is Non-Linear differential equation because a d.c. term present in it.

2. Differential equation is said to be time-invariant if all the co-efficient of differential equation are const.

So, $\frac{a_n d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + ...y(t) = b_0 x(t)$

Represents the linear constant co-efficient differential equation.

> LCCDE equation is used to analyse the LTI system or we can say if any system is LTI system then it can be represented in differential equation by LCCDE

4.1.1 System described by Difference Equations

The role of differential equation in describing continuous-time system is played by difference equations for discrete-time system.

4.2 LINEAR CONSTANT COEFFICIENT DIFFERENCE EQUATIONS (LCCDE)

The discrete time counterpart of general differential equation is the nth order linear cost. Co-efficient difference equation is given by

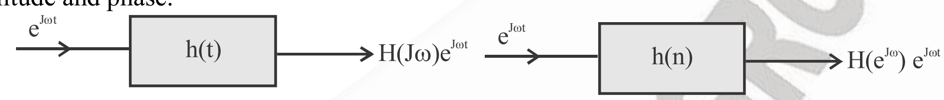
$$\sum_{K=0}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$$

CHAPTER - 5
CONTINUOUS-TIME FOURIER SERIES

5.1 INTRODUCTION

In this chapter we explore an alternative representation of signals and LTI system. Here we represent signal as linear combination of complex exponentials.

The complex exponential in study of LTI system is important from the fact that the response of an LTI system to a complex exponential input is same complex exponential with only change in amplitude and phase.



Till now all signals are draw with respect to time.

That means 't' was considered as variable. The representation of signal w.r.t. to time is called time domain representation.

Since the time domain representation of signal is net sufficient for its analysis. For sake of analysis we will use the frequency domain representation.

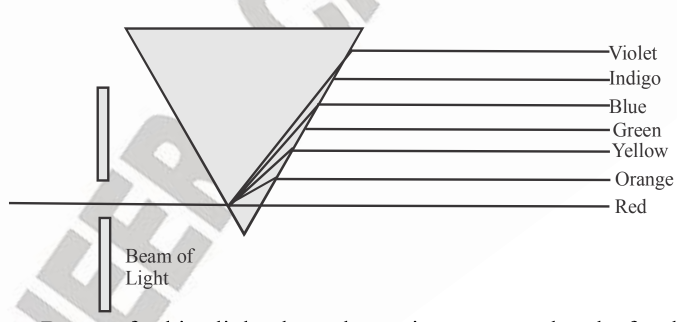
In frequency domain representation the variable is frequency 'f' rather than 't'.

The signal represented in frequency domain is called as "spectrum" the spectrum consists two graphs.

1. Amplitude spectrum occurs in $|M(j\omega)|\,Vs\,\omega$
2. Phase spectrum occurs in $|\phi(j\omega)|\,Vs\,\omega$

Example. Why frequency domain representations is important?

Solution.



Now when we pass Beam of white light through a prism, we get band of colours produced by prism. It means that light Beam passing through a prism analysed it into its colour component without any change. The output colour can be normally specify as spectrum or analysis of light into color is nothing, but frequency analysis.

For analysis the signal, the various parameters are evaluated as

1. Amplitude
2. Frequency context
3. Power and energy densities
4. Periodicity

Periodicity can be obtained in time-domain but frequency content is not evaluated in time-do main. Hence these signal has to be transformed in frequency domain with the help of fourier series,

CHAPTER - 6
FOURIER TRANSFORM

6.1 INTRODUCTION
In previous chapter we have discussed the Fourier series which is tool used to analyse a periodic-time signal in frequency domain.
Disadvantage of Fourier series is it cannot analyse the non-periodic signals.
So Fourier develops a new tool to analyse the non-periodic or a periodic signal in frequency domain known a Fourier transform.

> Fourier transform can be used to analyse both periodic and non-periodic signals.

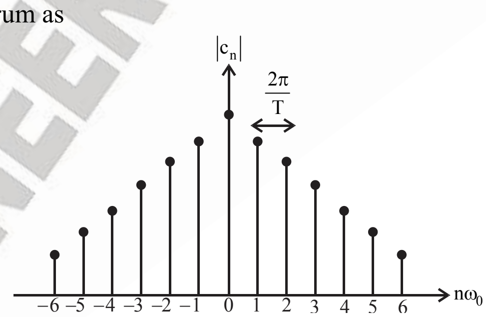
6.2 ANALYSIS OF NON-PERIODIC FUNCTION OVER ENTIRE INTERVAL
A non-periodic signal may assume as limiting case of periodic signal where the period of signal approaches infinity. Such a signal form by replacing fundamental time period T $\rightarrow \infty$ let us consider a periodic function x(t) having period T. The complex Fourier series representation of function may be written.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{Jn\omega_0 t} \qquad \ldots \text{(i)}$$

$$\omega_0 = \frac{2\pi}{T}$$

$$c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-Jn\omega_0 t\, dt} \qquad \ldots\text{(ii)}$$

The typical discrete spectrum as



Spacing between successive harmonics will be

$$\omega_0 = \frac{2\pi}{T} = \Delta\omega(\text{say})$$

ESE OBJ QUESTIONS

1. The Fourier Transform of $e^{-\frac{x^2}{2}}$ is

[EE ESE - 2018]

(a) $\frac{1}{2}.e^{-\frac{\omega^2}{2}}$ (b) $e^{-\frac{\omega^2}{2}}$

(c) $\frac{\pi}{2}$ (d) $\sqrt{\pi}$

2. Fourier series of any periodic signal x(t) can be obtained if

1. $\int_0^T |x(t)|dt < \infty$

2. Finite number of discontinuities within finite time interval t

3. Infinite number of discontinuities

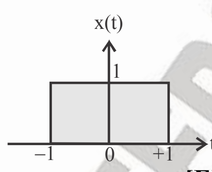
Select the correct answer using the codes given below:

[EE ESE - 2017]

(a) 1, 2 and 3 (b) 1, and 3 only

(c) 1 and 2 only (d) 2 and 3 only

3. The laplace transform of the below function is



[EE ESE - 2017]

(a) $\omega \sin \omega$ (b) $\frac{2\sin\omega}{\omega}$

(c) $\frac{\omega}{\sin\omega}$ (d) $\frac{\cos\omega}{\omega}$

4. Fourier series of any periodic signal x(t) can be obtained if

1. $\int_0^T |x(t)|dt < \infty$

2. Finite Number of discontinuities within finite time interval t

3. Infinite number of discontinuities

Select the correct answer using the codes given below:

[EC ESE - 2017]

(a) 1, 2 and 3 (b) 1 and 3 only

(c) 1 and 2 only (d) 2 and 3 only

5. The fourier transform of a rectangular pulse is

[EC ESE - 2012]

(a) Another rectangular pulse

(b) Triangular pulse

(c) sinc function

(d) Impulse function

6. Which one of the following is Dirichelt condition ?

[EC ESE - 2010]

(a) $\int_{t_1}^{\infty} |x(t)| < \infty$

(b)Signal x(t) must have a finite number of maxima and minima in the expansion interval

(c)x(t) can have an infinite number of finite discontinuities in the expansion interval

(d)$x^2(t)$ must be absolutely summable

7. If f(t) is an even function, then what is its Fourier transform F(jω) ?

[EC ESE - 2008]

(a) $\int_0^{\infty} f(t)\cos(2\omega t)dt$

(b) $2\int_0^{\infty} f(t)\cos(\omega t)dt$

(c) $2\int_0^{\infty} f(t)\sin(\omega t)dt$

(d) $2\int_0^{\infty} f(t)\sin(2\omega t)dt$

CHAPTER - 7
CORRELATION AND FILTERING ACTION

7.1 CORRELATION
Correlation is used to find similarity between two signals.
There are two type of correlation
1. Auto-Correlation
It is used to find similarity between two same signals
2. Cross-Correlation
It is used to find similarity between different signal.

7.1.1 Auto-Correlation
7.1.1.1 Autocorrelation Function
It gives the measure of similarity, match or coherence between a signal and a delayed function.
A signal may be energy signal or power signal.

7.1.1.2 Autocorrelation Function of Energy Signal
Autocorrelation function of this signal may be obtained by integrating the product of signal x(t) and delayed version of its.

$$R_{xx}(\tau)=\int_{-\infty}^{\infty} x(t)x(t-\tau)dt$$

Where τ is called is searching parameter.

7.1.1.3 Relationship between Auto Correlation and Convolution

$$R_{xx}(\tau)=x(t)\otimes x(-t)|_{\text{Replace } t=\tau}$$

7.1.1.4 Properties of Autocorrelation Function
1. Auto correlation is an even function.

$$R_{xx}(\tau)=R_{xx}(-\tau)$$

2. It τ is increased in either direction, the auto correlation reduces, As τ reduces auto correlation increase and it maximum at $\tau = 0$ i.e. at origin mathematically,

$R_{xx}(\tau)<R_{xx}(0)$ for all τ.

and $\lim_{\tau\to\infty} R_{xx}(\tau)=0$

3. Autocorrelation function at $\tau = 0$ gives energy of signal

i.e. $R_{xx}(\tau)=\int_{-\infty}^{\infty} x(t)x(t-\tau)dt$

Substituting $\tau = 0$

$$R_{xx}(0)=\int_{-\infty}^{\infty} x^2(t)dt$$

$$R_{xx}(0)=\text{Energy of signal}$$

CHAPTER - 8
LAPLACE TRANSFORM

8.1 INTRODUCTION

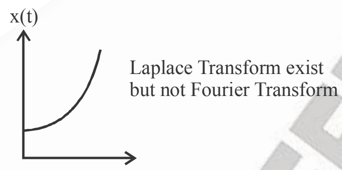
In previous chapters, we have seen the tools such as Fourier series and Fourier Transform to analyse the signals. Now the Laplace Transform is another mathematical tool which is used for analysis of signals and system. Infect, Laplace Transform provides broader characterization of signal and systems compared to Fourier Transform.

The Laplace Transform can be used where Fourier Transform cannot be used.

Laplace Transform can be used for analysis of unstable systems whereas Fourier Transform has several limitation.

Example.

for given $x(t) = e^{3t.U(t)}$



8.1.1 Definition of Laplace Transform

For general continuous time signal x(t)

The Laplace Transform x(s) is defined as.

$$x(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

Where s is generally a complex variable and is expressed as

S = σ + Jω

Also known as complex frequency

Where σ is real part and ω is imaginary part.

For convenience we will sometime denotes the Laplace Transform in operator form al $\angle[x(t)]$ and denote the transform relationship between x(t) and x(s)

As $x(t) \xleftrightarrow{L.T} x(s)$

e^{-st} is kernel of function

It may be noted that integration is taken from 0 to ∞. Therefore, this is called Bilateral Laplace Transform.

Similar if x(t) is zero for t < 0

Then Laplace may be defined as

$$\boxed{x(s) = \int_{0}^{\infty} x(t)e^{-st}at}$$

Where s = σ + Jω

Integration is taken from 0 to ∞. This is called as unilateral/one-sided Laplace Transform.

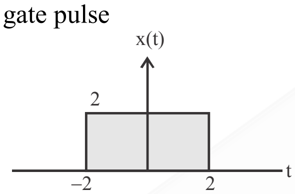
WORKBOOK

Example 1. Find the Laplace of $x(t) = e^{-2t}U(t) - e^{-3t}U(-t)$

Solution.

Laplace Transform is undefined as no common ROC.

Example 2. Find the Laplace Transform of following gate pulse



Solution.

$$x(t) = 2u(t+2) - 2u(t-2)$$

$$x(s) = \frac{2e^{2s}}{s} - \frac{2e^{-2s}}{s}$$

ROC: Entire S-plane finite duration signal

Example 3. Let $x(t) = \alpha s(t) + s(-t)$

$S(t) = \beta e^{-3t}U(t)$ where U(t) is step function and

$$x(s) = \frac{9}{s^2-9} \text{ for } -3 < 6 < 3$$

Find α and β ?

Solution.

$$L[s(t)] = \frac{\beta}{s+3} \quad 6 > -3$$

$$L[s(-t)] = \frac{+\beta}{-s+3} \quad -6 > -3$$

Replace $s \to -s$

$$L[s(-t)] = \frac{\beta}{3-s} \quad \sigma < 3$$

So, $x(t) = \alpha\, s(t) + s(-t)$

$$x(s) = \alpha L[s(t)] + L[s(-t)]$$

$$\frac{9}{s^2-9} = \frac{\alpha\beta}{s+3} + \frac{\beta}{3-s} \quad \text{for} -3 < \sigma < 3$$

$$\frac{9}{s^2-9} = \frac{\alpha\beta}{s+3} - \frac{\beta}{s-3}$$

$$\frac{9}{s^2-9} = \frac{\alpha\beta(s-3)}{s^2-9} - \beta(s+3)$$

$$\frac{9}{s^2-9} = \frac{s(\alpha\beta-\beta)-3\alpha\beta-3\beta)}{s^2-9}$$

By comparison

$\alpha\beta - \beta = 0 \quad \& -3(\alpha\beta+\beta) = 9$

$\alpha\beta = \beta$ ----- (1)

From (1) $-3(2\beta) = 9$

and $\alpha = 1$ $\quad\beta = -9/6 = -3/2$

Example 4. Consider two 2 right sided signal x(t), y(t) related as $\frac{d}{dt}x(t) = -2y(t) + \delta(t)$ and $\frac{dy(t)}{dt} = 2x(t)$

Find x(s) and y(s)?

Solution.

$$\frac{d}{dt}x(t) = -2y(t) + \delta(t)$$

$$sx(s) = -2y(s) + 1$$

$$\frac{d}{dt}y(t) = 2x(t)$$

$$sy(s) = 2x(s)$$

$$sx(s) = \frac{-4}{5}x(s) + 1$$

$$x(s)[s^2+4] = s$$

$$x(s) = \frac{s}{s^2+4}$$

$$y(s) = \frac{2}{s^2+4}$$

Example 5. Find the T.F of system described by given equation.

i.e. $\frac{dy(t)}{dt} + 4y(t) + 3\int_{-\infty}^{t} y(\tau)d\tau = x(t)$

ASSIGNMENT

1. Laplace transform analysis helps in
(a) Solving integral differential equation.
(b) Converts differential equation into algebraic equation
(c) Converts integral equation into algebraic equation
(d) All the above

2. Laplace transform $x(s)$ of signal $x(t)$ is
(a) $\int_{-\infty}^{\infty} x(t)e^{-st}dt$ (b) $\int_{-\infty}^{\infty} x(t)e^{-st}dt$
(c) $\int_{-\infty}^{\infty} e^{-st}dt$ (d) $\int_{-\infty}^{\infty} x(t)e^{-st}dt$

3. Bilateral and unilateral Laplace transform differs in terms of
(a) Lower limit of integration
(b) Upper limit of integration
(c) They are same
(d) Bilateral transform does not exist.

4. Laplace transform of $u(t)$ is
(a) $\frac{1}{s}$ (b) s
(c) 1 (d) s^2

5. Laplace transform of $x(t) = t$ is
(a) $\frac{2}{s^2}$ (b) $\frac{1}{s^2}$
(c) s^2 (d) $\frac{1}{s}$

6. Region of the convergence of x(S) contain
(a) Zeros (b) Poles
(c) No zeros (d) No pole

7. Inverse Laplace transform of $\frac{1}{(s+a)^2}$ is
(a) $tu(t)$ (b) $te^{-at}u(t)$
(c) $e^{-at}u(t)$ (d) $ae^{-at}u(t)$

8. $x(t) = \cos\omega_0 t\, u(t)$
(a) $\frac{s}{s^2+\omega_0^2}$ (b) $\frac{1}{s^2+\omega_0^2}$
(c) $\frac{\omega_0}{s^2+\omega_0^2}$ (d) $\frac{\omega_0}{s^2}$

9. Laplace transform of signal $x_1(t)$ convolving with $x_2(t)$ is
(a) $x_1(s) * x_2(s)$ (b) $x_1(s)x_2(s)$
(c) $x_1(s)/x_2(s)$ (d) $x_1(t)x_2(t)$

10. For causal continuous-time LTI system, ROC is in the
(a) Left of all system poles
(b) Right of all system poles
(c) Right of all zeros
(d) Left of all zeros

11. If the system is causal and stable, the system poles must lie
(a) On the jω axis
(b) On the left half of s-plane
(c) On the right half of s-plane
(d) Both (a) and (c)

12. Laplace transform of $\frac{d}{dt}x(t)$ is
(a) $sx(s)$ (b) $\frac{x(s)}{s} - x(0^-)$
(c) $sx(s) - x(0^-)$ (d) $x(s) - x(0^-)$

13. Laplace transform of $\int_0^t x(\tau)d\tau$ is
(a) $\frac{1}{s}x(s)$ (b) $sx(s)$
(c) $s^2x(s)$ (d) $x(s)/s^2$

GATE QUESTIONS

1. The solution of the differential equation

$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=0$ with $y(0)=y'(0)=1$ is

[GATE - 2015]

(a) $(2-t)e^{t}$ (b) $(1+2t)e^{-t}$
(c) $(2+t)e^{-t}$ (d) $(1-2t)e^{t}$

2. The bilateral Laplace transform of a function $f(t)=\begin{cases}1 \text{ if } a\le t\le b\\ 0 \text{ otherwise}\end{cases}$ is

[GATE - 2015]

(a) $\frac{a-b}{s}$ (b) $\frac{e^{z}(a-b)}{s}$
(c) $\frac{e^{-az}-e^{-bz}}{s}$ (d) $\frac{e^{z(a-b)}}{s}$

3. Let the signal f(t) = 0 outside the interval $[T_1,T_2]$, where T_1 and T_2 are finite. Furthermore, $|f(t)| < \infty$. The region of convergence (ROC) of the signal's bilateral Laplace transform F(s) is

[GATE - 2015]

(a) A parallel strip containing the jΩ axis
(b) A parallel strip no containing the jΩ axis
(c) The entire s-plane
(d) A half plane containing the jΩ axis

4. Let $x(t) = \alpha\, s(t) + s(-t)$ with $s(t) = \beta e^{-4t}u(t)$, where u(t) is unit step function. If the bilateral Laplace transform of x(t) is

$X(s)=\frac{16}{s^2-16}\; -4<\text{Re}\{s\}<4;$

Then the value of β is ________.

[GATE - 2015]

5. Consider the differential equation $\frac{dx}{dt}=10-0.2x$ with initial condition x(0) = 1. The response x(t) for t > 0 is

[GATE - 2015]

(a) $2-e^{-0.2t}$ (b) $2-e^{0.2t}$
(c) $50-49e^{-0.2t}$ (d) $50-49e^{0.2t}$

6. The output of a standard second-order system for a unit step input is given as $y(t)=1-\frac{2}{\sqrt{3}}e^{-t}\cos\left(\sqrt{3}t-\frac{\pi}{6}\right)$. The transfer function of the system is

[GATE - 2015]

(a) $\frac{2}{(s+2)(s+\sqrt{3})}$ (b) $\frac{1}{s^2+2s+1}$
(c) $\frac{3}{s^2+2s+3}$ (d) $\frac{4}{s^2+2s+4}$

7. Input x(t) and output y(t) of an LTI system are related by the differential equation $y''(t)-y'(t)-6y(t)=x(t)/$. If the system is neither causal nor stable, the impulse response h(t) of the system is

[GATE - 2015]

(a) $\frac{1}{5}e^{3t}u(-t)+\frac{1}{5}e^{-2t}u(-t)$

(b) $-\frac{1}{5}e^{3t}u(-t)+\frac{1}{5}e^{-2t}u(-t)$

(c) $\frac{1}{5}e^{3t}u(-t)-\frac{1}{5}e^{-2t}u(t)$

(d) $-\frac{1}{5}e^{3t}u(-t)-\frac{1}{5}e^{-2t}u(t)$

8. Consider the function $g(t)= e^{-t}\sin(2\pi t)u(t)$ where u(t) is the unit step function. The area under g(t) is _________

[GATE - 2015]

9. The stable linear time invariant (LTI) system has a transfer function $H(s)=\frac{1}{s^2+s-6}$.

ESE OBJ QUESTIONS

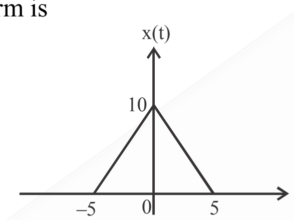
1. The laplace transform of $f(t) = t^n e^{-at} u(t)$ is

[EE ESE - 2018]

(a) $\frac{(n+1)!}{(s+\alpha)^{n+1}}$ (b) $\frac{n!}{(s+\alpha)^{n}}$

(c) $\frac{(n-1)!}{(s+\alpha)^{n+1}}$ (d) $\frac{n!}{(s+\alpha)^{n+1}}$

2. If x(t) is as shown in the figure, its Laplace transform is



[EE ESE - 2018]

(a) $\frac{2e^{+5s} + 2e^{-5s}}{s^2}$ (b) $\frac{2e^{+5s} - 4 + 2e^{-5s}}{s^2}$

(c) $\frac{2e^{+5s} - 2 + 2e^{-5s}}{s^2}$ (d) $\frac{2e^{+5s} + 4 - 2e^{-5s}}{s^2}$

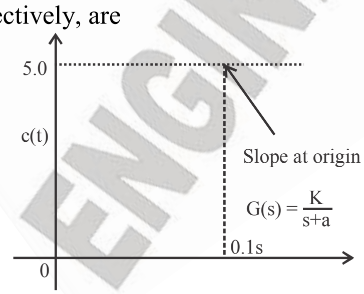
3. The unit – impulse response of a system is $16e^{-2t} - 8e^{-t}$ .Its unit – step response is

[EE ESE - 2018]

(a) $8 + e^{-t} - 4e^{-2t}$ (b) $8 + e^{-t} + 4e^{-2t}$

(c) $8e^{-t} - 8e^{-2t}$ (d) $e^{-t} - 4e^{-2t}$

4. A unit –step input to a first – order system G(s) yields a response as shown in the figure. This can happen when the values of K and a, respectively, are

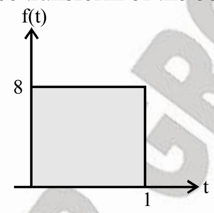


[EE ESE - 2018]

(a) 10 and 10 (b) 5 and 10

(c) 10 and 5 (d) 5 and 5

5. The laplace transform of the below function is



[EE ESE - 2017]

(a) $F(s) = 8s\,(1 - e^{-s})$ (b) $F(s) = \frac{8}{s}(1 + e^{-s})$

(c) $F(s) = 8s(1 + e^{-s})$ (d) $F(s) = \frac{8}{s}(1 - e^{-s})$

6. The function which has its fourier transform , Laplace transform and Z- transform unity is

[EC ESE - 2012]

(a) Gaussian (b) Impulse

(c) Sinc (d) Pulse

7. $H(e^{j\omega})$ is the frequency response of a discrete time LTI system and $H_1(e^{j\omega})$ is the frequency response of its inverse function. Then

[EC ESE - 2012]

(a) $H(e^{j\omega})H_1(e^{j\omega}) = 1$

(b) $H(e^{j\omega})H_1(e^{j\omega}) = \delta(\omega)$

(c) $H(e^{j\omega}) * H_1(e^{j\omega}) = 1$

(d) $H(e^{j\omega}) * H_1(e^{j\omega}) = \delta(\omega)$

8. With the following equations, the time invariant systems are

1. $\frac{d^2y(t)}{dt^2} + 2t\frac{d}{dt}y(t) + 5y(t) = x(t)$

2. $y(t) = e^{-2x(t)}$

3. $y(t) = \left[\frac{d}{dt}x(t)\right]^2$

CHAPTER - 9
Z-TRANSFORM

9.1 INTRODUCTION
Z-Transform which is discrete-time counterpart of Laplace Transform.
It may be observed that Laplace Transform is an extension of continuous-time Fourier. Transform because of fact that Laplace Transform may be applied to broader class of signals than Fourier Transform. Just for instances, there are several signal s for which the Fourier transform does not converges but Laplace Transform converses.
Similarly Z-Transform is introduced to represent discrete-time sequences in Z-domain (Z is complex variable). Also to analyse the difference equations that describes the linear time-invariant (LTI systems) and converts into algebraic equation. Thus simplifying further analysis.
In general Z-Transform of discrete signal x(n) is expressed as

$$x(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Generally denoted a

$$x(z) = z[x(n)]$$

$x(n)$ & $x(z)$ form a Z-Transform pair

$$x(n) \overset{7}{\longleftrightarrow} x(z)$$

It may be noted that Z-Transform is an infinite power series. It may exists only for those values of Z for which series converges.
x(z) is a complex number and a function of complex variable Z.
In polar form, $Z = re^{J\omega}$
With r gives magnitude of z, $|z|$
ω gives phase of z, $\angle z$

So, $x(re^{J\omega}) = \sum_{n=-\infty}^{\infty} x(n)\left(re^{-J\omega}\right)^n$

$$x(re^{J\omega}) = \sum_{n=-\infty}^{\infty} \left[x(n)r^{-n}\right]e^{-J\omega n}$$

We see that $x(re^{J\omega})$ is discrete time Fourier Transform of sequence x(n) multiplied by real exponential r^{-n}
i.e., $x(re^{J\omega}) = F\left[x(n)r^{-n}\right]$
The exponential weighting r^{-n} may decaying or growing with increasing n depended on whether it is greater than or less than unity.
Now if r = 1 or |z| = 1
The expression thus reduces to discrete Fourier Transform of input sequence.

$$x(z)\Big|_{z=e^{J\omega}} = x(e^{j\omega}) = DTFT[x(n)] \quad \ldots(i)$$

9.2 Z-PLANE OR Z-DOMAIN
Here we transforming discrete time sequence x(n) into x(z)
Where $Z = re^{J\omega}$

ASSIGNMENT

1. Z-transform helps to convert
(a) Differential equation into algebraic equation
(b) Difference equation into algebraic equation
(c) Solve integral differential equation
(d) All the above

2. For causal signal $x[n]$, z -transform X(z) is

(a) $X(z)=\sum_{n=-\infty}^{\infty}x[n]z^{n}$

(b) $X(z)=\sum_{n=0}^{\infty}x[n]z^{-n}$

(c) $X(z)=\sum_{n=-\infty}^{\infty}x[n]z^{-n}$

(d) $X(z)=\sum_{n=0}^{\infty}x[n]z^{n}$

3. ROC is defined as
(a) Range of values for which z-transform converges
(b) Range of value for which z-transform diverges
(c) Range of values of n for which series converges
(d) Range of values of n for which series diverges.

4. If the lower limit of ROC is less than upper limit of ROC for $X(z)=\sum_{n=-\infty}^{\infty}x[n]z^{-n}$, then the series
(a) Diverges (b) Converges
(c) Can't say (d) Zero

5. ROC of $x[n]$ contains
(a) Poles (b) Zeros
(c) No poles (d) No zero

6. Z-transform of $\delta[n]$ is
(a)1 (b)zero
(c) $\delta[z]1^{-n}$ (d)all the above

7. Z-transform of $x[-n]$ is
(a) $-X(z)$ (b) $X(-z)$
(c) $X\left[\frac{1}{Z}\right]$ (d) $\frac{1}{X(z)}$

8. Z-transform of $x[n-n_0]$ is
(a) $z^{-n_0}X[z]$ (b) $z^{n_0}X[z]$
(c) $X(z+z_0)$ (d) $X[z_0]$

9. ROC of the z-transform of unit step sequence is
(a) $|z|<1$ (b) $|z|>1$
(c) Real part of z (d) $|z|=0$

10. Convolution of two sequences $x_1[n]$ and $x_2[n]$ is
(a) $X_1(z)*X_2(z)$
(b) $X_1(z)X_2(z)$
(c) $X_1(z)+X_2(z)$
(d) $X_1(z)/X_2(z)$

11. Inverse z-transform of $\frac{az}{(z-a)^2}$ is
(a) $a^2u[n]$ (b) $a^nu[n]$
(c) $2a^2u[n]$ (d) $na^nu[n]$

12. Inverse z-transform of $X[z/a]$ is
(a) $x\left[\frac{n}{a}\right]$ (b) $x[n]/a$
(c) $a^nx[n]$ (d) $ax[n]$

13. z-transform of nx is
(a) $\frac{dX(z)}{dz}$ (b) $z\frac{dX(z)}{dz}$

GATE QUESTIONS

1. Consider two discrete-time signals $X_1(n) = \{1,1\}$ and $\{1,2\}$, for n = 0.1

The Z-transform of the convoluted sequence x(b) = $x_1(n) * x_2(n)$ is

[GATE - 2017]

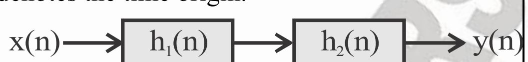
(a) $1 + 2z^{-1} + 3z^{-1}$ (b) $z^2 + 3z + 2$

(c) $1 + 3z^{-1} + 2z^{-2}$ (d) $z^{-2} + 3z^{-3} + 2z^{-4}$

2. Consider a causal and stable LTI(system with rotational transfer function H9z), whose corresponding impulse response begins at n = 0. Further more, $H(1) = \frac{5}{4}$. The poles of H(z) are $P_k = \frac{1}{\sqrt{2}} \exp\left(j\frac{(2k-1)\pi}{4}\right)$ for k = 1, 2, 3, 4. The zeros of H(z) are all at z = 0. Let $g(n) = j^n h(n)$. The value of g(8) equals _____. (Give the answer up to three decimal places).

[GATE - 2017]

3. A cascade system having the impulse responses $h_1(b) = \{\underset{\uparrow}{1}, -1\}$ and $h_2(n) = \{\underset{\uparrow}{1}, 1\}$ is shown in the figure below, where symbol ↑ denotes the time origin.



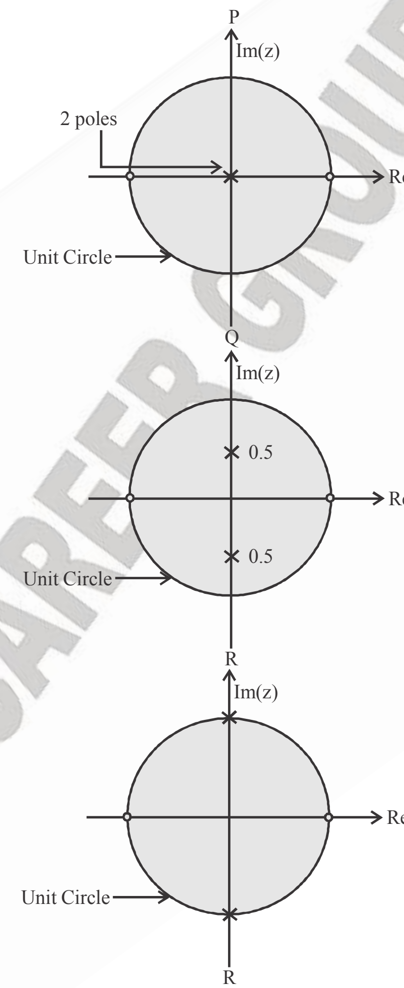
The input sequence x(n) for which the casecade system produces an output sequence $y(n) = \{\underset{\uparrow}{1}, 2, 1, -1, -2, -1\}$ is

[GATE - 2017]

(a) $x(n) = \{\underset{\uparrow}{1}, 2, 1, 1\}$ (b) $x(n) = \{\underset{\uparrow}{1}, 1, 2, 2\}$

(c) $x(n) = \{\underset{\uparrow}{1}, 1, 1\}$ (d) $x(n) = \{\underset{\uparrow}{1}, 2, 2, 1\}$

4. The pole-zero plots of three discrete-time systems P, Q and R on the z-plane are shown below.



Which one of the following is TRUE about the frequency selectivity of these systems?

[GATE - 2017]

(a) All three are high-pass filters

(b) All three are band-pass filters

(c) All three are low-pass filters

(d) P is a low-pass filter, Q is a band-pass filter and R is a high - pass filter.

CHAPTER - 10

DISCRETE TIME FOURIER TRANSFORM

10.1 INTRODUCTION

Basically the Fourier Transform of periodic finite energy signal is called DTFT mathematical.

$$x(e^{J\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-J\omega n}$$

or also denoted as x(Ω)

Where Ω is discrete frequency

10.1.1 Periodic Nature of DTFT

Since Ω is discrete frequency.

Then Substituting ω = ω + 2π k

$$x\left[e^{J(\omega+2\pi k)}\right] = \sum_{n=-\infty}^{\infty} x(n)e^{-J(\omega+2\pi k)n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)e^{-J\omega n} \quad e^{-J2\pi kn}$$

By using Euler's identity

$$e^{-J2\pi kn} = \cos(2\pi kn) - J\sin(2\pi kn)$$

= 1 – J0

So, $x\left(e^{J(\omega+2\pi k)}\right) = \sum_{n=-\infty}^{\infty} x(n)e^{-J\omega n}$

So, $x\left(e^{J(\omega+2\pi k)}\right) = x\left(e^{J\omega}\right)$

or

x(Ω+2πk)=x(Ω)

Thus DTFT is periodic nature with a period of 2π. We DTFT is restricted to 0 to 2π or – π to π.

> DTFT is continuous frequency Ranging from - ∞ to ∞ because of a periodic time function.

Inverse discrete time Fourier Transform:

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\left(e^{J\omega}\right)e^{J\omega n}d\omega$$

Thus we can say x(n) and $x(e^{J\omega})$ form a Fourier Transform pair.

GATE QUESTIONS

1. Let X[k] = k + 1, 0 ≤ k ≤ 7 be 8-point DFT of a sequence x[n], where $X[k]=\sum_{n=0}^{N-1}x[n]e^{-j\pi nk/N}$. The value (correct to two decimal places) of $\sum_{n=0}^{3}x[2n]$ is _______

[GATE - 2018]

2. Let h[n] be the impulse response of a discrete-time linear time invariant (LTI) filter. The impulse response is given by $h(0)=\frac{1}{3}; h[1]=\frac{1}{3}; h[2]=\frac{1}{3};$ and h[n] = 0 for n < 0 and n > 2.
Let H (ω) be the Discrete-Time Fourier transform (DTFT) of h[n], where ω is the normalized angular frequency in radians. Given that $H(\omega_0) = 0$ and $0 < \omega_0 < \pi$, the value of ω_0 (in radians) is equal to _____.

[GATE - 2017]

3. Let $\tilde{x}[n]=1+\cos\left(\frac{nn}{8}\right)$ be a periodic signal with period 16. Its DFS coefficients are defined by $a_k=\frac{1}{16}\sum_{n=0}^{15}\tilde{x}[n]\exp\left(-j\frac{\pi}{8}kn\right)$ for all k. The value of the coefficient a_{31} is ________.

[GATE - 2015]

4. The discrete – time signal $x[n]\longleftrightarrow X[z]=\sum_{n=0}^{\infty}\frac{3^n}{2+n}z^{2n}$, where ↔ denotes a transform – pair relationship, is orthogonal to the signal

[GATE - 2008]

(a) $y_1[n]\leftrightarrow Y_1(z)=\sum_{n-0}^{\infty}\left(\frac{2}{3}\right)^n z^{-n}$

(b) $y_2[n]\leftrightarrow Y_2(z)=\sum_{n-0}^{\infty}\left(5^n-n\right)z^{-(2n+1)}$

(c) $y_3[n]\leftrightarrow Y_3(z)=\sum_{n-0}^{\infty}2^{-|n|}z^{-n}$

(d) $y_4[n]\leftrightarrow Y_4(z)=2z^4+3z^{-2}+1$

5. A 5-point sequence x [n] is given as
X[−3]=1, x[−2]=1, x[−1]=0, x[0]=5, x[1]=1
Let $X(e^{j\omega})$denote the discrete-time Fourier transform of X[n].The value of $\int_{-\pi}^{\pi}X(e^{j\omega})d\omega$ is

[GATE - 2007]

(a) 5 (b) 10π
(c) 16 π (d) 5+j10π

6. Let $x(n)=\left(\frac{1}{2}\right)^n u(n), y(n)=x^2(n)$, and $Y(e^{j\omega})$ be the Fourier transform of y(n). Then $Y(e^{j0})$ is.

[GATE - 2006]

(a) $\frac{1}{4}$ (b) 2
(c) 4 (d) $\frac{4}{3}$

7. x[n] = 0; n< - 1, n > 0, x[– 1] = – 1, x[0] = 2 is the input and
y[n] = 0; n < – 1, n > 0, x[–1] = –1, = y[1], y[0] = 3, y[2] = – 2 is the output of a discrete -time LTI system. The system impulse response h[n] will be

[GATE - 2006]

(a) h[n] = 0; n < 0, n > 2, h[0] = 1, h[1] = h[2] = –1
(b) h[n] = 0; n < – 1, n > 1, h [–1] = 1, h[0] = h[1] = 2
(c) h[n] = 0; n > 3, h[0] = –h[1] = 2, h[2] = 1
(d) h[n] = 0; n < - 2, n> 1, h[–2] = h [1] = h[–1] = –[0] = 3

CHAPTER - 11
DISCRETE TIME FOURIER SERIES

11.1 INTRODUCTION

Discrete time Fourier series is used to analyses the discrete periodic signals.

A discrete time x(n) is said to periodic is there is smallest positive integer 'N' for which it is satisfied.

x(n + n) = x(n) for all 'n'

11.1.1 Discrete Fourier Series Representation

The discrete Fourier series Representation of periodic sequence x(n) with fundamental time period N is given by

$$x(n) = \sum_{K=0}^{N-1} c_k e^{\frac{J2\pi nk}{N}}$$

$$(K = 0, 1, 2 ...... N-1)$$

The FS representation of x(n) consists of N harmonically related exponential function.

$e^{\frac{J2\pi kn}{N}}$ K = 0, 1, 2 .... N – 1

Where c_k is the Fourier Series co-efficient.

It is given by

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{\frac{-2\pi kn}{N}}$$

Here input sequence x(n), FS coefficient c_k both are periodic.

11.1.2 Comparison between Continuous Time Fourier Series and Discrete Time Fourier Series

CTFS	DTFS
$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{Jn\omega_0 t}$	$x(n) = \sum_{K=0}^{N-1} c_k e^{\frac{J2\pi kn}{N}}$
$c_n = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-Jn\omega_0 t}$	$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{\frac{-J2\pi nk}{N}}$
C_n is discrete and periodic	C_n is Periodic and Discrete

11.1.3 Convergence of Discrete Fourier Series

Since x(n) is discrete Fourier Series is a Finite series because summation limits are from K = 0 to N – 1. So, I_n compassion to continuous-time case, there is no convergence issue with discrete Fourier series.

11.1.4 Discrete Fourier Series of Arbitrary Periodic Sequence x(n)

W define Twiddle factor) Phase factor

ASSIGNMENT

1. Discrete-time signal x[n] will be periodic if
(a) x[n+N] = x[n]
(b) x[n+N] = – x[n]
(c) $x[n+N]=\frac{1}{x[n+N]}$
(d) x[n+N] = 1

2. Convergence of discrete Fourier series
(a) Always guaranteed
(b) Conditional convergence
(c) Is not an issue
(d) Non convergent

3. Discrete Fourier series is dual if
(a) $c[n]\xleftrightarrow{DFS}\frac{1}{N_0}x[-k]$
(b) $c[n]\xleftrightarrow{DFS}x[k]$
(c) $c[n]\xleftrightarrow{DFS}x[-k]$
(d) $c[n]\xleftrightarrow{DFS}N_0x[k]$

4. If x[n] is real and even, then its discrete Fourier series coefficient c_k will be
(a) Real (b) Odd
(c) Both (a)and (b) (d) Imaginary

5. If x[n] is real and odd, then its discrete Fourier series coefficient c_k will be
(a) Real (b) Odd
(c) Imaginary (d) Both (a) & (c)

6. Find the discrete Fourier series for each of the following periodic sequences
(a) $x[n] = \cos(0.1\pi n)$
(b) $x[n] = \sin(0.1\pi n)$
(c) $x[n] = 2\cos(1.6\pi n) + \sin(2.4\pi n)$

7. Consider the sequence
$$x[n]=\sum_{k=-\infty}^{\infty}\delta[n-4k]$$
(a) Sketch x[n].
(b) Find the Fourier coefficients c_k of x[n].

8. Determine the discrete Fourier series representation for each of the following sequences
(a) $x[n]=\cos\frac{\pi}{4}n$
(b) $x[n]=\cos\frac{\pi}{3}n+\sin\frac{\pi}{4}n$
(c) $x[n]=\cos^2\left(\frac{\pi}{8}n\right)$

CHAPTER - 12
DISCRETE AND FAST FOURIER TRANSFORM

12.1 INTRODUCTION

In previous chapter we have mainly studied about signal analysis in frequency domain by Fourier Tools and for a discrete time sequence the Fourier Tool used is generally a discrete time Fourier Transform.

$$x\left(e^{J\omega}\right)=\sum_{n=-\infty}^{\infty}x\left(n\right)e^{-J\omega n} \qquad \ldots(i)$$

We know that $x(e^{J\omega})$ is Fourier Transform of discrete time signal x(n).

The frequency analysis of discrete-time signals are usually and most conventional performed on digital computer. To perform frequency analysis of discrete time x(n). We convert time-domain sequence to an equivalent frequency-domain representation such a representation is given by Fourier Transform $x(e^{J\omega})$ or $H(\omega)$.

Since $x(\omega)$ is continuous function of frequency 'ω'. The range of ω is from 0 to 2π or $-\pi$ to π.

Since this calculation is not possible to computer $x(\omega)$ on digital computer because range of summation (Equation (i)) is from "$-\infty$ to ∞".

So, if we make Range finite then it is possible to do these calculation on digital computed.

12.2 FREQUENCY DOMAIN SAMPLING AND RECONSTRUCTION OF DISCRETE-TIME SIGNALS

We recall that a periodic signals have continuous spectrum. If we consider such an a periodic discrete-time sequence x(n) with Fourier Transform.

$$x\left(e^{J\omega}\right)=\sum_{n=-\infty}^{\infty}x\left(n\right)e^{-J\omega n}$$

But DTFT is continuous in nature and periodic with period of 2π and unique information in frequency domain in only one period i.e. "0 to 2π" and DTFT range of from "$-\infty$ to ∞".

A finite range of sequence is obtained by extracting a particular portion from infinite sequence [x(n)] now $x(\omega)$ is continuous and periodic.

A discrete finite sequence is obtained by sampling $x(\omega)$ periodically in frequency at specing $D\omega$ radians between two samples.

For uniqueness of information only samples in fundamental period is necessary.

For convenience, we take N equidistant samples in interval $0 < \omega \le 2\pi$.

Total Range = 2π (one period)

Total samples = N

$D\omega$ (spacing between samples) $=\dfrac{2\pi}{N}$

GATE QUESTIONS

1. The DFT coefficient out of five DFT coefficients of a five – point real sequence are given as: X(0) = 4, X(1) = 1 – j1 and X(3) = 2 + j2. The zero – the value of the sequence x(n)x(0) is.

[GATE - 2017]

(a) 1 (b) 2
(c) 3 (d) 4

2. The Discrete Fourier Transform (DFT) of the 4-point sequence
X[n] = {x[0], x[1], x[2], x[3]} = {3, 2, 3, 4} is
X[k] = {X[0], X[1], X[2], X[3]} = {12, 2j, 0, –2j}
If $X_1[k]$ is the DFT of the 12-point sequence
$x_1[n]$ = {3, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0},
The value of $\left|\dfrac{x_1[8]}{X_1[11]}\right|$ is ____.

[GATE - 2016]

3. Two sequence $[a,b,c]$ and $[A,B,C]$ are related as,

$$\begin{bmatrix}A\\B\\C\end{bmatrix}=\begin{bmatrix}1&1&1\\1&W_3^{-1}&W_3^{-2}\\1&W_3^{-2}&W_3^{-4}\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} \text{ where } W_3=e^{j\frac{2\pi}{3}}$$

If another sequence [p.q.r] is derived as,

$$\begin{bmatrix}p\\q\\r\end{bmatrix}=\begin{bmatrix}1&1&1\\1&W_3^{1}&W_3^{2}\\1&W_3^{2}&W_3^{4}\end{bmatrix}\begin{bmatrix}1&0&0\\0&W_3^{2}&0\\0&0&W_3^{4}\end{bmatrix}\begin{bmatrix}A/3\\B/3\\C/3\end{bmatrix},$$

then the relationship between the sequences [p.q.r] and [a.b.c] is

[GATE - 2015]

(a) [p.q.r]=[b.a.c] (b) [p.q.r]=[b.c.a]
(c) [p.q.r]=[c.a.b] (d) [p.q.r]=[c.b.a]

4. Consider two real sequences with time-origin marked by the bold value,
$x_1[n]$ = {1,2,3,0}, $x_2[n]$ = {1,3,2,1}
Let $X_1(k)$ and $X_2(k)$ be 4-point DFTs of $x_1[n]$ and $x_2[n]$, respectively.
Another sequence $x_3[n]$ is derived by taking 4-point inverse DFT of $X_3(k)=X_1(k)X_2(k)$.
The value of $x_3[2]$ is ______.

[GATE - 2015]

5. The N – point DFT X of sequence x[n], 0 ≤ n ≤ N – 1 is given by

$$X[k]=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}x[n]e^{-j\frac{2\pi}{N}nk},\ \ 0\le k\le N-1.$$

Denote this relation ax X =DFT (x) . For N = 4, which one of the following sequence satisfies DFT (DFT(x)) = x?

[GATE - 2014]

(a) x = [1 2 3 4] (b) x = [1 2 3 2]
(c) x = [1 3 2 2] (d) x = [1 2 2 3]

6. The DFT of a vector [a b c d] is the vector [α β λ δ]. Consider the product

$$[p\,q\,r\,s]=[a\,b\,c\,d]\begin{bmatrix}a&b&c&d\\d&a&b&c\\c&d&a&b\\b&c&d&a\end{bmatrix}.$$

The DFT of the vector [p q r s] is scaled version of

[GATE - 2013]

(a) $[\alpha^2\ \beta^2\gamma^2\delta^2]$
(b) $[\sqrt{\alpha}\sqrt{\beta}\sqrt{\gamma}\sqrt{\delta}]$
(c) $[\alpha+\beta\ \beta+\delta\ \delta+\gamma\ \gamma+\alpha]$
(d) $[\alpha\ \beta\ \gamma\ \delta]$

7. The first six points of the 8-point DFT of a real valued sequence are 5,1-j3,0,3-j4,0 and 3+j4.. The last two points of the DFT are respectively

[GATE - 2011]

(a) 0,1-j3 (b) 0,1+j3

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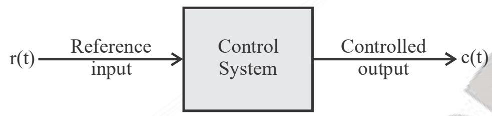
CONTENTS

CHAPTER - 1
INTRODUCTION TO CONTROL SYSTEM

1.1 INTRODUCTION
A control System is a combination of elements arranged in a planned manner where in each element causes an effect to produce a desired output. This cause and effect relationship is governed by a mathematical relation.



If the input of system is controlled in desired manner, the system is called control system.
Any system can be characterized mathematically by
1. Transfer function 2. State model

$$\text{Transfer function} = \frac{\text{L.T. of output}}{\text{L.T. of input}} = \frac{L[c(t)]}{[c(s)]} = \frac{C(s)}{R(s)}\Bigg|_{\text{initial conditions}=0}$$

Transfer function is also called impulse response of the system.

1. Disturbances
The signal that has some adverse effect on output of system called disturbances if it is generated inside called internal distributes if it is other called out external disturbances.

2. Plant
It is defined as the portion of system when is to be controlled it is also called process.

3. System
A system is an arrangement or component such that it gives proper output to given input e.g. classroom example of physical system.

4. Control System
It is an arrangement of different physical component such that it gives the desired output for the given input by means of regulate or control either direct or indirect.

5. Controllers
It is the element of system it say, may be external to system it controls the plant or process.

6. Performance Specifications
Control system are designed to perform specific task. The requirement imposed on control system are usually spelled out as performance specifications. These specifications may be given transact response requirement maximum overshoot settling time is step response.
1. Steady state requirement (steady state error) or may be given in terms of frequency response.
2. Specification of the control system must be given before the design process begins.
3. Most important part of control system design is to sate the performance specification precisely so that they will yield on optional control system for the given purpose.
Mathematical modeling of control system regular must be able to model dynamic system in mathematical terms and analyse their dynamic characteristics.

GATE QUESTIONS

1. The Nyquist stability criterion and the Routh criterion both are powerful analysis tools for determining the stability of feedback controllers. Identify which of the following statements is FALSE:

[GATE - 2018]

(a) Both the criteria provide information relative to the stable gain range of the system.

(b) The general shape of the Nyquist plot is readily obtained from the Bode magnitude plot for all minimum-phase systems.

(c) The Routh criterion is not applicable in the condition of transport lag, which can be readily handled by the Nyquist criterion

(d) The closed-loop frequency response for a unity feedback system cannot be obtained from the Nyquist plot.

2. A system is described by the following differential equation:

$$\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} + x(t), \quad x(0) = y(0) = 0$$

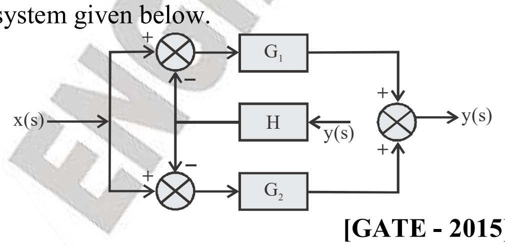
where x(t) and y(t) are the input and output variables respectively. The transfer function of the inverse system is

[GATE - 2017]

(a) $\frac{s+1}{s-2}$ (b) $\frac{s+2}{s+1}$

(c) $\frac{s+1}{s+2}$ (d) $\frac{s-1}{s-2}$

3. Find the transfer function $\frac{Y(s)}{X(s)}$ of the system given below.



[GATE - 2015]

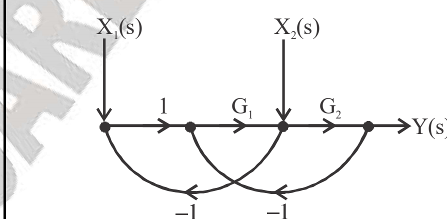
(a) $\frac{G_1}{1+HG_1} + \frac{G_2}{1-HG_2}$

(b) $\frac{G_1}{1+HG_1} + \frac{G_2}{1+HG_2}$

(c) $\frac{G_1+G_2}{1+H(G_1+G_2)}$

(d) $\frac{G_1+G_2}{1-H(G_1+G_2)}$

4. For the signal - flow graph shown in the following expressions is equal to the transfer function $\left.\frac{Y(s)}{X_2(s)}\right|_{X_1(s)=0}$?



[GATE - 2015]

(a) $\frac{G_1}{1+G_2(1+G_1)}$ (b) $\frac{G_2}{1+G_1(1+G_2)}$

(c) $\frac{G_1}{1+G_1G_2}$ (d) $\frac{G_2}{1+G_1G_2}$

5. The impulse response g(t) of a system, G, is a shown in Figure (a). What is the maximum value attained by the impulse response of two cascaded blocks of G as shown in Figure (b)?

ESE OBJ QUESTIONS

1. The open – loop transfer function of a system is $\frac{10K}{1+10s}$

When the system is converted into a closed – loop with unity feedback, the time constant of the system is reduced by a factor of 20. The value of K is

[EE ESE - 2018]

(a) 1.9 (b) 1.6

(c) 1.3 (d) 1.0

2. The effects of feedback on stability and sensitivity are

[EC ESE - 2015]

(a)Negative feedback improves stability and system response is less sensitive to external inputs and parameter variations.

(b)Feedback does not affect stability but system response is sensitive to disturbances and parameter variations.

(c)Feedback does not affect stability response is sensitive to disturbances and parameter variations

(d)Negative feedback affects stability and system response is more sensitive disturbances and parameter variations.

3. The D.C. gain and steady state error for step input for G (s) = $\frac{s+1}{s^2+s+1}$ are:

[EC ESE - 2013]

(a) 1 and 1 (b) 0 and 1

(c) 1 and 0 (d) 0 and 0

4. In control systems, excessive bandwidth is NOT employed because:

[EC ESE - 2013]

(a) Noise is proportional to bandwidth

(b) It leads to low relative stability

(c) It leads to slower response

(d) Noise is proportional to the square of the bandwidth

SOLUTIONS

Sol.1. (a)

$$\text{OLTF} = \frac{10k}{1+10s}$$

$Z_1 = 10$

$$Z_2 = \frac{10}{20} = 0.5$$

$$\text{CLTF} = \frac{10k}{10k+1+10s}$$

$$Z_2 = \frac{10}{10k+1} = 0.5$$

$$\frac{10}{0.5} = 10k+1$$

k = 1.9

Sol.2. (a)

Sol.3. (c)

$$G(s) = \frac{s+1}{s^2+s+1}$$

$$G(s) =\Big|_{s=0} = \frac{0+1}{0+0+1} = 1$$

$$e_{ss} = \text{Steady State Error} = \frac{1}{1+k_p}$$

$$k_p =^{t}_{s\to 0} G(s)H(s) = 0$$

Sol.4. (a)

Noise Power = $\eta_0\beta$

Noise Power × Bandwidth × B

CHAPTER - 2
MATHEMATICAL MODES OF PHYSICAL SYSTEMS

2.1 INTRODUCTION
1.A physical system is collection of physical objects connected together to serve an objective.
2.Idealizing assumptions are always made for the purpose of analysis and synthesis of systems. An idealized physical system is called a physical mode.
3.Once a physical model of a physical system is obtained, the next step is to obtain a mathematical model which is the mathematical representation of the physical model through use of appropriate physical laws.

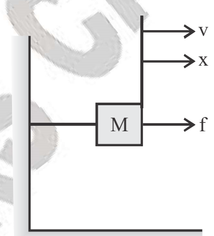
2.2 MECHANICAL SYSTEMS
A mechanical system which is modeled using the three ideal elements would yield a mathematical model which is an ordinary differential equation. All mechanical systems are divided into two parts:

2.2.1 Mechanical Translational System
In this type of mechanical system input is the forced (F) and the output is linear displacement (x) or linear velocity (v). The three ideal elements are:

1. Mass Element

$$F = M\frac{d^2x}{dt^2}$$

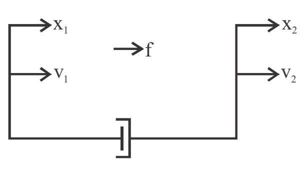
or $F = M\frac{dv}{dt}$



2. Damper Element

$$F = f\frac{d}{dt}(x_1 - x_2) = f\frac{dx}{dt}$$

where $x_1 - x_2$ x
or $F = f(v_1 - v_2) = fv$
where $v = v_1 - v_2$

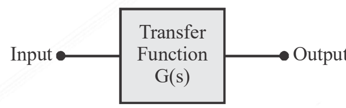


3. Spring Element

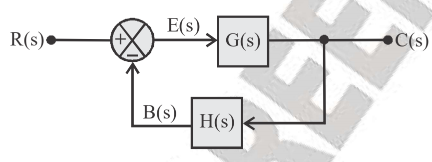
$F = K(x_1 - x_{2=}) = Kx$

CHAPTER - 3
BLOCK DIAGRAM ALGEBRA

3.1 BLOCK DIAGRAM
It is a pictorial representation of function performed by each component and of flow of signals. Such a diagram depicts the inter-relationship that exists among various components differing from a purely abstract mathematical model. The block diagram has the advantage of indicating more realistically the signal flows of actual system.

3.2 ELEMENT OF BLOCK DIAGRAM


This represents the elements of a block diagram. The arrow heads pointing towards the block diagram indicate the input and the arrowheads leaving the block represent output. Such arrows are represented to as signal.



Open Loop Transfer Function

1. Open Loop Transfer Function

B(s) = C(s) H(s)

$$\frac{B(s)}{E(s)} = G(s)H(s)$$

2. Feed Forward Transfer Function

$$\frac{C(s)}{E(s)} = G(s)$$

If H(s) = 0 then

G(s) = G(s) H(s) $\therefore$ H(s) = 1

3. Closed Loop Transfer Function

C(s) = G(s) E(s)

E(s) = R(s) – B(s)

E(s) = R(s) – H(s) C(s)

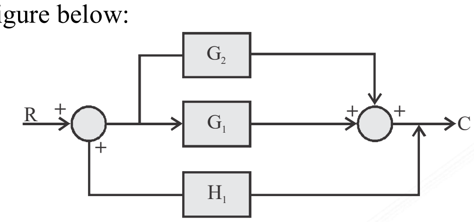
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

4. Branch Point

A branch point is a point from which the signal from the block goes concurrently to other block or summing points.

ASSIGNMENT

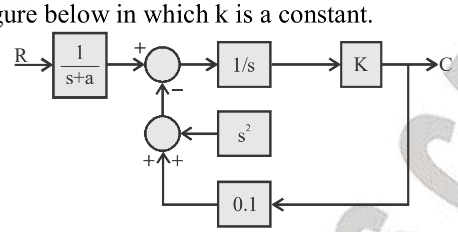
1. Determine C/R from the system shown in figure below:



(a) $\frac{G_1+G_2}{1-G_1H_1+G_2H_1}$ (b) $\frac{G_1}{1-G_1H_1}$

(c) $\frac{G_1+G_2}{1-G_1H_1-G_2H_1}$ (d) None

2. Find the transfer C/R for the system shown in figure below in which k is a constant.



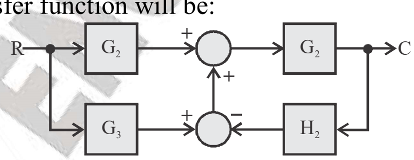
(a) $\frac{1}{(s+a)(s^2+s+0.1k)}$

(b) $\frac{k}{(s+a)(s^2+s+0.1k)}$

(c) $\frac{k}{(s+a)(s^2+s-0.1k)}$

(d) None of these

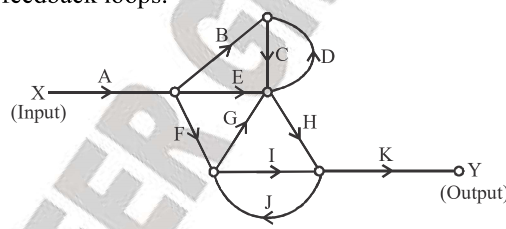
3. Determine C/R for the system given in figure below. Then put $G_3 = G_1\ G_2\ H_2$. Now the new transfer function will be:



(a) $\frac{G_1G_2+G_2G_3}{1+G_2H_2}$ (b) $\frac{G_1G_2+G_2G_3}{1-G_2H_2}$

(c) $G_1\ G_3$ (d) $G_1\ G_2$

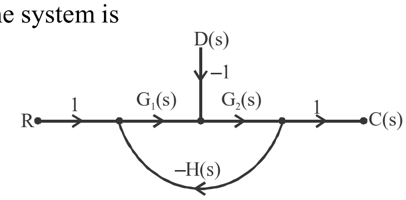
4. The signal flow graph of figure shown below has _______ forward paths and _______ feedback loops.



(a) (4, 4) (b) (4, 5)

(c) (4, 3) (d) (3, 3)

5. The signal flow graph of the system is shown in the given figure. The transfer function $\frac{C(s)}{D(s)}$ of the system is



(a) $\frac{G_1(s)G_2(s)}{1+G_1(s)H(s)}$

(b) $\frac{G_1(s)G_2(s)}{1-G_1(s)G_2(s)H(s)}$

(c) $\frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}$

(d) $\frac{-G_2(s)}{1-G_1(s)G_2(s)H(s)}$

6. In the signal flow graph of figure y/x equal.

CHAPTER - 4
TIME RESPONSE ANALYSIS OF CONTROL SYSTEM

4.1 INTRODUCTION

4.1.1 Types of System
(No. of open loop poles of the system at origin)
Example.

(i) $G(s)=\dfrac{K}{(s+1)(s+2)}$, No pole at origin. So it is type 0.

(ii) $G(s)=\dfrac{K}{s(s+1)(s+1)}$, 1 pole at origin. So type 1.

(iii) $G(s)=\dfrac{K}{s^2(s+1)(s+2)}$, 2 poles at origin. So type 2

Order is the highest coefficient of s in the denominator of closed loop transfer function.

Example. Consider a unity feedback system whose open loop transfer function is

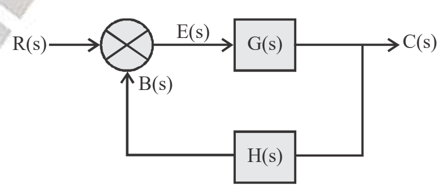
$G(s)=\dfrac{K}{(s+1)(s+2)}$ What is the type and order of the system?

Solution.
The closed loop transfer function is

$$=\frac{K}{s^2+3s+2}$$

So it is a type 0 and order 2 system.

4.2 ERROR ANALYSIS
4.2.1 Steady State Error
A desirable feature of a control system is the faithful following of its input by the output. However, if the actual output of a control system during steady state deviates from the reference input (i.e. desired output, the system is said to possess a steady state error.
As the steady state performance of a control system is assessed by the magnitude of the steady state error possessed by the system and the system input specified as either step or ramp or parabolic.



The magnitude of the steady state error in a closed-loop control system depends on its open-loop transfer function, i.e. G(s) H(s) of the system. The classification of open loop transfer function of a control system is explained below:
GH is loop transfer function
G is open loop transfer function

SOLUTIONS

Sol. 1. (a)

$$e_{ss} = \frac{1}{1+kp}$$

$$1+K_p = \frac{1}{0.2}$$

$k_p = 4$

$$k_p = \lim_{s\to 0} GCS\,H(s) = 4$$

The error due to step i/p is made to zero so type of system would have increased

$$G(s) = \frac{G(S)H(S)}{S},\ K_v = \lim_{s\to 0}\ \&\ \frac{GCSH(S)}{s} = 4$$

$$k_v = \frac{1}{4} = 0.25$$

Sol. 2. (b)

$$\text{CE.}\ \ 1+\frac{25}{s(s+6)} = 0$$

$$s^2+65+25=0$$

$\omega n = 5$

$$\xi = \frac{6}{2\times 5} = 0.6$$

Setting time

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{5\times 0.6} = 1.33\,\text{sec}$$

Sol. 3. (d)

$$\text{CE.}1+\frac{k}{s(s+a)} = 0$$

$S^2 + as + k = 0$

$2\xi\omega n = a$

$$\omega_n = \sqrt{k}$$

$$\xi = \frac{a}{2\sqrt{k}}$$

For undreamed system

$\xi < 1$

$$\frac{a}{2\sqrt{k}} < 0\ \ k > \frac{a^2}{4}$$

$$\sqrt{k} > \frac{a}{2}$$

Sol. 4. (b)

Settling time is defined as the time for the response to react and stay within 2% of its final value.

Sol. 5. (a)

$$k_P = \lim_{s\to 0} G(s)$$

$$= k_P = \lim_{s\to 0}\frac{k(s+2)}{s^2(s^2+75+12)} = \infty$$

$$k_v = \lim_{s\to 0} s.G(s)$$

$$= \lim_{s\to 0}\frac{k(s+2)}{s(s^2+75+12)} = \infty$$

$$K.G = \lim_{s\to 0} s^2G(s)$$

$$= \lim_{s\to 0}\frac{k(s+2)}{s^2+75+12} = \frac{2k}{12} = \frac{k}{6}$$

Sol. 6. (c)

For open loop T.F.

Poles are lies at s = 0, 0, –2

Hence repeated poles at origin unstable

For close loop system

$$1+\frac{k(s+1)}{s^2(s+2)} = 0$$

$S^3 + 2s^2 + ks + k = 0$

S^3	1	k
S^2	2	k
S^1	$\frac{2k-k}{2}$	
S^0	k	k>0

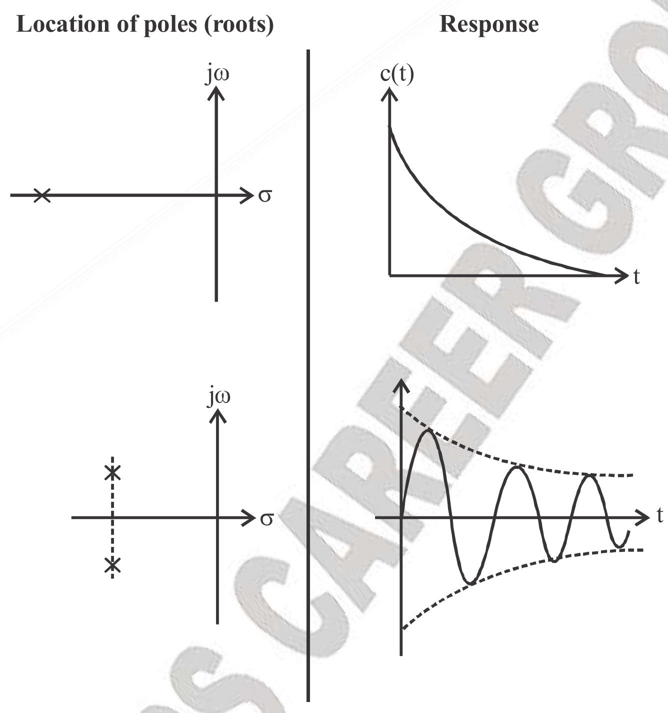
So for k > 0 close loop system is stable.

Sol. 7. (b)

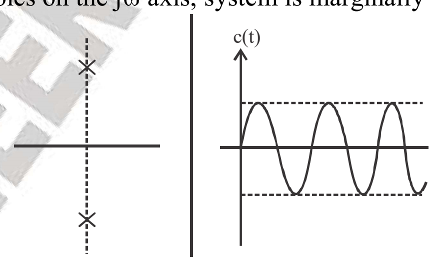
Derivative compensation is phase lead compensation so damping factor (δ) increases ω_n (natural frequency) remains unchanged.

CHAPTER - 5
STABILITY ANALYSIS OF CONTROL SYSTEM

5.1 INTRODUCTION
If all the poles of the system lie in the left half of s plane, then the system is stable.



Case-I.
If there are non – repeated poles on the jω axis, system is marginally stable.



Case-II.
If there are repeated poles of the system on jω axis, system is unstable.

CHAPTER - 6
ROOT LOCUS

6.1 INTRODUCTION
The Routh's criterion gives a satisfactory answer to the question of stability but its adoption to determine the relative stability is not satisfactory and requires trial and error procedure even in the analysis problem.
A simple technique, known as the root locus technique, for finding the roots of the characteristic equation, introduced by W.R. Evans, is extensively used in control engineering practice. This technique provides a graphical method of plotting the locus of the roots in the s-plane as a given system parameter is varied over the complete range of values (may be from zero to infinity). The roots corresponding to a particular value of the system parameter can then be located on the locus or the value of the parameter for a desired root location can be determined from the locus.

6.2 RULES OF DRAWING THE ROOT LOCUS
1. Root locus start from open loop poles and ends on open loop zeros or ∞ with $K = \infty$
Let no. of poles = n (open loop poles)
No. of open loop zeros = m
(i) No. of root loci ending on $\infty = n - m, n > m$
2. Root locus is always symmetrical about real axis.
3. A point on real axis lies on the root locus if no. of poles + zeros to the right of the point are odd.
4. Asymptotes are the paths along which root locus moves towards ∞.
(i) No. of asymptotes = $(n - m)$
(ii) Angle of asymptotes

$$\theta_A = \frac{(2x+1)180°}{n-m}$$

$x = 0, 1, 2, \ldots, n - m - 1$
(iii) Centroid : It is the point of intersection of asymptotes with the real axis.

$$\sigma_A = \frac{\sum(\text{real part of poles}) - \sum(\text{real part of zeros})}{n-m}$$

5. Determination of Breakaway or break in point : On the root locus between two adjacent poles the two poles move towards each other with K=0 and move at a point where K is maximum and the root locus will break away into two parts. This point is called the breakaway point and it is determined by:
Put $\left(\frac{dK}{ds} = 0\right)$ and find out the value of 's'
6. Angle of departure or Angle of arrival
angle made by root locus with real axis when it departs from a complex open loop poles is called angle of departure.

$$\begin{Bmatrix} \phi_D \text{ (angle of departure)} = 180° + \angle GH' \\ \phi_A \text{ (angle of arrival} = 180° - \angle GH') \end{Bmatrix}$$

GH' is value of function excluding the concerned poles at the poles itself

GATE QUESTIONS

1. The range of K for which all the roots of the equation $s^3 + 3s^2 + 2s + K = 0$ are in the left half of the complex s-plane is

[GATE - 2017]

(a) $0 < K < 6$ (b) $0 < K < 16$
(c) $6 < K < 36$ (d) $6 < K < 16$

2. The root locus of the feedback control system having the characteristic equation $s^2 + 6Ks + 2s + 5 = 0$ where $K > 0$, enters into the real axis at

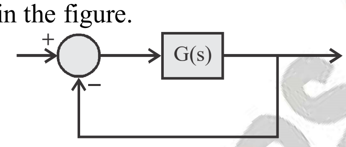
[GATE - 2017]

(a) $s = -1$ (b) $s = -\sqrt{5}$
(c) $s = -5$ (d) $s = \sqrt{5}$

3. A linear time invariant (LTI) system with the transfer function

$$G(s) = \frac{K(s^2 + 2s + 2)}{(s^2 - 3s + 2)}$$

Is connected in unity feedback configuration as shown in the figure.



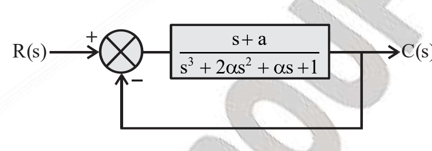
For the closed loop system shown, the root locus for $0 < K < \infty$ intersects the imaginary axis for K = 1.5. the closed loop system is stable for

[GATE - 2017]

(a) $K > 1.5$
(b) $1 < K < 1.5$
(c) $0 < K < 1$
(d) No positive value of K

4. A closed-loop system is shown in the figure. The system parameter α is not known. The condition for asymptotic stability of the closed loop system is

[GATE - 2017]



(a) $\alpha < -0.5$ (b) $-0.5 < \alpha < 0.5$
(c) $0 < \alpha < 0.5$ (d) $\alpha > 0.5$

5. The gain at the breakaway point of the root locus of a unity feedback system with open loop transfer function $G(s) = \frac{Ks}{(s+1)(s-4)}$ is

[GATE - 2016]

(a) 1 (b) 2
(c) 5 (d) 9

6. The forward-path transfer function and the feedback-path transfer function of a single loop negative feedback control system are given as

$$G(s) = \frac{K(s+2)}{s^2 + 2s + 2}$$

and H(s) = 1 respectively. If the variable parameter K is real positive, then the location of the breakaway point on the root locus diagram of the system is ______.

[GATE - 2016]

7. The open-loop transfer function of a unity-feedback control system is

$$G(s) = \frac{K}{s^2 + 5s + 5}$$

The value of K at the breakaway point of the feedback control system's root-locus plot is ________.

[GATE - 2016]

8. The open loop poles of a third order unity feedback system are at 0, −1. −2. Let the frequency corresponding to the point where the root locus of the system transits to unstable

CHAPTER - 7
CONTROLLERS

7.1 INTRODUCTION

While designing a system, the designer selects the reasonable values for the peak overshoot, rise time and the settling time. The designer is never sure of the final design of the system as to whether it is good or not. For example, if the system has been designed for minimum overshoot, the rise time increases and on the other hand if the rise time chosen is small, peak overshoot will be large. A system thus requires modification in order to meet even two independent specifications. This is called compensation and is achieved by the help of proportional, derivative or integral or derivative feedback control. In practice a combination of derivative and integral control is employed.

Let us consider a system whose block diagram is shown in Figure. It has a controller whose output signal will have an effect on the system performance. Its purpose is to measure the error between the output and the desired output.

The transfer function of the controller is

$$K = \frac{Y(s)}{E(s)}$$

Where

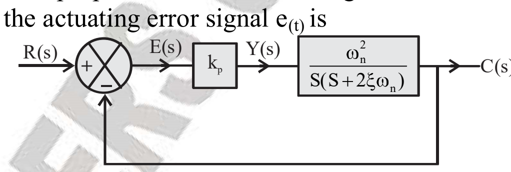
E(s) = R(s) – B(s)

or E(s) = R(s) – H(s)C(s)

this relationship is termed as control action relationship. We will now discuss various control actions as available to the control system engineer for improvement of system performance.

7.2 PROPORTIONAL CONTROL ACTION

In this the actuating signal is proportional to the error signal. The relationship between the output of the controller, $y_{(t)}$ and the actuating error signal $e_{(t)}$ is



$y_{(t)} = Ke_{(t)}$

In Laplace-transform form, it can be written as

$Y_{(s)} = KE_{(s)}$

Or $K_p = \frac{Y(s)}{E(s)}$

7.3 INTEGRAL CONTROL ACTION

In this value of the controller output $y_{(t)}$ is altered at a rate proportional to the error signal $e_{(t)}$. The output $y_{(t)}$. The output $y_{(t)}$ depends upon the integral of the error signed $e_{(t)}$.

ASSIGNMENT

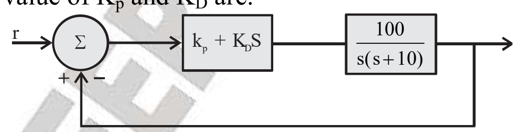
1. Consider the following statements:
1. A Proportional plus derivative controller.
2. Increase the stability of the system
3. Improves the steady-state accuracy
Which of these statements are correct?
(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) 1 and 3

2. The transfer function of simple RC network as a controller is $G_C(s) = \frac{s+z_1}{s+p_1}$. The condition for the RC network to act as a phase lead controller is
(a) $p_1 < z_1$ (b) $p_1 = 0$
(c) $p_1 = z_1$ (d) $p_1 > z_1$

3. The industrial controller having the best steady
state accuracy is
(a) A derivative controller
(b) An integral controller
(c) A rate feed back controller
(d) A proportional controller

4. The transfer function of a phase lead controller is $\frac{1+3Ts}{1+Ts}$. The maximum value of phase provided by this controller is
(a) 90° (b) 60°
(c) 45° (d) 30°

5. A control system with a PD controller is shown in fig. if the velocity error constant K_v = 1000 and the damping ratio ζ = 0.5, then the value of K_p and K_D are.



(a) $K_P = 100$, $K_D = 0.09$
(b) $K_p = 100$, $K_D = 0.9$
(c) $K_p = 10$, $K_D = 0.09$
(d) $K_p = 10$, $K_D = 0.9$

6. A controller transfer function is given by C(s) = (2s + 1)/(09.2s + 1). What is its nature and parameter?
(a) Lag controller, $\alpha = 10$
(b) Lag controller, $\alpha = 2$
(c) Lead controller, $\beta = 0.1$
(d) Lead controller, $\beta = 0.2$

ANSWER KEY

1.	b	**2.**	d	**3.**	b	**4.**	d	**5.**	b	**6.**	c

CHAPTER - 8
FREQUENCY RESPONSE ANALYSIS

8.1 INTRODUCTION

8.1.1 The various Frequency Response Analysis Techniques are

1. Polar plot
2. Nyquist plot
3. Bode plot
4. M & N circles
5. Nicholas chart

8.1.1 Polar Plot

The sinusoidal transfer function G(jω) is a complex function and is given by

$G(j\omega) = \text{Re}\ G(j\omega) + j\ I_m\ G\ (j\omega)$

Or $G(j\omega) = |G(j\omega)|\sqrt{G(j\omega)} = M\sqrt{\phi}$

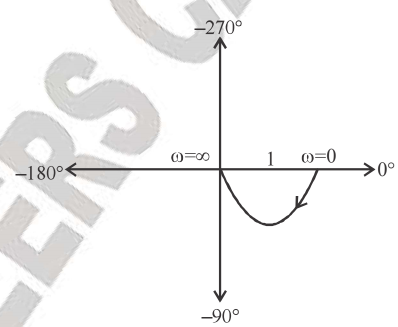
from above equation, it is seen that G(jω) may be represented as a phasor of magnitude M and phase angle ϕ. As the input frequency ω is varied from 0 to∞, the magnitude M and phase angle ϕ change and hence the tip of the phasor G (jω) traces a locus in the complex plane.

The locus thus obtained is known as polar plot.

When a transfer function consists of 'p' poles and 'Z' zeros, and it doesn't consist poles at origin then the polar plot start from 0^0 with some magnitude and terminates at $-90^0 \times (P - Z)$ with zero magnitude.

When a transfer consists of poles at origin, then the polar plot starts from $-90^0 \times$ no. of poles at origin with '∞' magnitude and ends at $-90^0 \times (P - Z)$ with zero magnitude

Polar coordinates (|GH|∠GH)



Example. Draw the polar plot for the following transfer function:

$$GH = \frac{1}{j\omega+1} |GH| = \frac{1}{\sqrt{\omega^2+1}}, \angle GH = -\tan^{-1}\omega$$

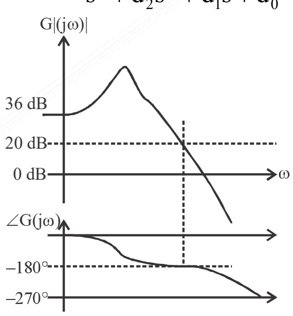
GATE QUESTIONS

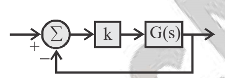
1. For a unity feedback control system with the forward path transfer function $G(s) = \dfrac{K}{s(s+2)}$.
The peak resonant magnitude M_r of the closed-loop frequency response is 2. The corresponding value of the gain K (correct to two decimal places) is _______

[GATE - 2018]

2. The figure below shows the Bode magnitude and phase plots of a stable transfer function

$$G(s) = \frac{n_0}{s^3 + d_2 s^2 + d_1 s + d_0}$$



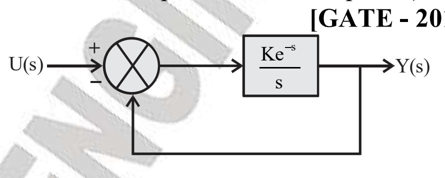


Consider the negative unity feedback configuration with gain k in the feedforward path. The closed loop is stable for $K < k_0$. The maximum value of k_0 is _________.

[GATE - 2018]

3. Consider the unity feedback control system shown. The value of K that results in a phase margin of the system to be 30° is _______. (Give the answer up to two decimal places).

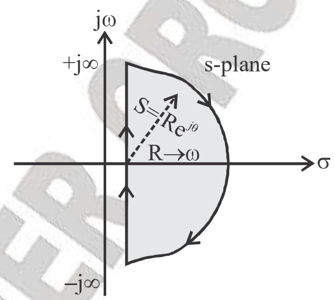
[GATE - 2017]



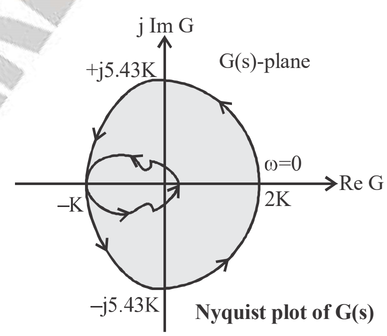
4. A unity feedback control system is characteriszed by the open loop transfer function

$$G(s) = \frac{10K(s+2)}{s^3 + 3s^2 + 10}$$

The Nyquist path and the corresponding Nyquist plot of G(s) are shown in the figures below.



Nyquist plot of G(s)



Nyquist plot of G(s)

If $0 < K < 1$, then number of poles of the closed loop transfer function that lie in the right half of the s-plane is

[GATE - 2017]

(a) 0 (b) 1
(c) 2 (d) 3

5. The Nyquist plot of the transfer function

$$G(S) = \frac{K}{(s^2 + 2s + 2)(s+2)}$$

Does not encircle the point (–1+j0) for K = 10 but does encircle the point (–1 + j0) for

SOLUTIONS

Sol.1. (a)

Sol.2. (b)
For open loop system no of poles in right half of s plane (P) = 1
$n = p^+ - z^+$
For stability $Z^+ = 0$
N = P = 1

Sol.3. (b)
The T.F. of given Bode plot.

$$\text{T.F.} = \frac{k_1\left(\frac{S}{20}+1\right)}{S\left(\frac{S}{2}+1\right)} = \frac{k(s+20)}{s(s+2)}$$

Sol.4. (c)

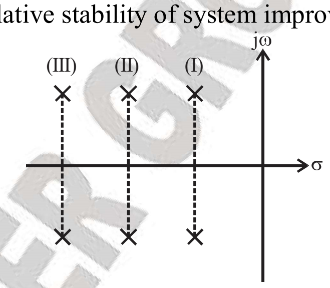
$$\text{T.F.} = \frac{ks^2}{\left(\frac{S}{10}+1\right)^5}$$

Sol.5. (c)
Low – frequency asymptote slope depends upon the poles or zeros at origin.
= (–20) × 2
= –40 dB/decade

Sol.6. (d)
From bode plot we can determine the open loop transfer function but to determine function but to determine the roots of closed – loop control system we have to know G(s) or H(s) separately. So, statement – I is wrong.

Sol.7. (b)
The slop of highest frequency asymptote
= (Z – P) × 20 dB/dec
= (2 – 14) × 20
= –240 dB/dec

Sol.8. (c)
Gain Margin and Phase margin of the system gives relative stability.
Relative stability is analysis of how fast transient has died out in the system. If we moves away from jω axis in left half of s plane then relative stability of system improves.



(iii) is relatively more stable to (ii)
(ii) is relatively more stable to (i).

Sol.9. (d)

Sol.10. (a)

$$G(s)H(s) = \frac{2K}{s(s+1)(s+5)}$$

For marginal stability we need to find frequency of sustained oscillation.
If G(s) H(s) ⇒ s(s + 1) (s + 5) + 2k = 0
⇒ $s^3 + 6s^2 + 5s + 2k = 0$
Now from Rough Huswitz criteria

S^3	1	5
S^2	6	2K
S^1	$\frac{30-2k}{6}$	
S^0	2k	

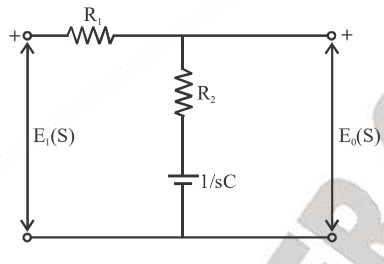
So k = 15
Now we get that k = 15
So $6s^2 + 30 = 0$
$\omega_{oscillation} = \sqrt{5}$ rad / sec

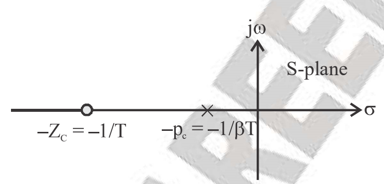
Sol.11. (b)

CHAPTER - 9
COMPENSATORS

9.1 LAG COMPENSATOR

A compensator having the characteristic of a lag network is called a lag compensator. Lag compensation results in a large improvement in steady state performance but results in a slower response due to reduced band width. Lag compensator is essentially a low pass filter and so high frequency noise signals are attenuated.





Transfer function of lag compensator, $G_c(s) = \frac{s+z_c}{s+p_c} = \left(\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}\right)$

9.1.1 Frequency Response of a Lag Compensator

Consider the general form of lag compensator

$$G_c(s) = \frac{s+(1/T)}{s+(1+\beta T)} = \beta\frac{(1+sT)}{(1+s\beta T)}$$

The sinusoidal transfer function of lag compensator is obtained by letting s = jω

$$\therefore G_c(j\omega) = \beta\frac{(1+j\omega t)}{(1+j\omega\beta T)}$$

When ω = 0, $G_c(j\omega)=\beta$

$$G_c(j\omega) = \frac{1+j\omega T}{1+j\omega\beta T} = \frac{\sqrt{1+(\omega T)^2}\angle\tan^{-1}\omega T}{\sqrt{1+(\omega\beta T)^2}\angle\tan^{-1}\omega\beta T} \quad ...(i)$$

The sinusoidal transfer function has two corner frequencies and they are denoted as ω_{c1} and ω_{c2}

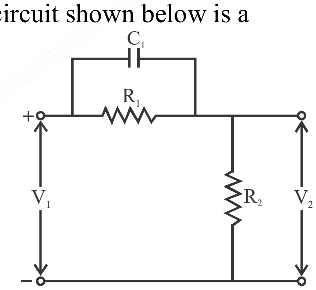
Here, $\omega_{c1} = 1/\beta T$ and $\omega_{c2} = 1/T$

Since, $\beta T > T$, $\omega_{c1} < \omega_{c2}$

ASSIGNMENT

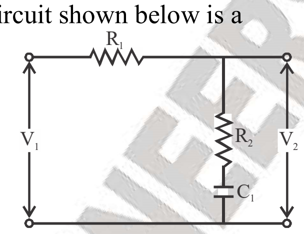
1. A negative feedback control system has a transfer function $G(s) = \frac{k}{s+2}$. We select a compensator $G_c(s) = \frac{s+a}{s}$ in order to achieve zero steady state error for a step input. Select 'a' and 'k' so that the overshoot to a step is approximately 5% and the settling time (with a 2% criterion) is approximately 1 second.
(a) k=8, a = 5.6 (b) k=8, a = 6.6
(c) k=6, a = 5.6 (d) k=6, a =6.6

2. The circuit shown below is a



(a) Lag network (b) Lead network
(c) Lead-lag network (d) None

3. The circuit shown below is a



(a) Lag network (b) Lead network
(c) Lead-lag network (d) None

4. The transfer function of a simple RC network functioning as a controller is $G_c(s) = \frac{(s+z_1)}{(s+p_1)}$. The required condition for the RC network to act as a phase lead controller is
(a) $p_1 < z_1$ (b) $p_1 > z_1$
(c) $p_1 = z_1$ (d) None of these

5. The damping of the system can be increased by using a compensator having a pair of complex roots as
(a) Phase lead (b) Phase lag lead
(c) Phase lag (d) None of these

6. If poles are added in a transfer function it will cause
(a) Lag compensation
(b) Lead compensation
(c) Lead-lag compensation
(d) None of these

7. If zero are added in a transfer function, it will cause
(a) Lag compensation
(b) Lead compensation
(c) Lead-lag compensation
(d) None of these

8. The transfer function of a lead compensator is $G_c(s) = \frac{1+0.12s}{1+0.04s}$. The maximum phase shift that can be obtained from this compensator is
(a) 60^o (b) 45^o
(c) 30^o (d) 15^o

9. Consider the following statements in case of phase lead compensation:
(a) Improvement of gain and phase margins
(b) Less rise time and more settling time
(c) Bandwidth is increased
(d) Affect the steady-state error
Which of these statements are correct?
(a) 1, 2 and 3 (b) 1 and 3
(c) 2 and 3 (d) 2 and 4

10. Consider the following statement in case of phase lag compensation:

GATE QUESTIONS

1. The transfer function C(s) of a compensator is given below:

$$C(s)=\frac{\left(1+\frac{s}{0.1}\right)\left(1+\frac{s}{100}\right)}{(1+s)\left(1+\frac{s}{10}\right)}$$

The frequency range in which the phase (lead) introduce by the compensator reaches the maximum is

[GATE - 2017]

(a) $0.1 < \omega < 1$ (b) $1 < \omega < 10$
(c) $10 < \omega < 100$ (d) $\omega > 100$

2. Which of the following statement is incorrect?

[GATE - 2017]

(a)Lead compensator is used to reduce the settling time.
(b)Lag compensator is used to reduce the steady state error.
(c)Lead compensator may increase the order of a system
(d)Lag compensator always stabilizes an unstable system.

Common data for Q. 3 and Q. 4

The transfer function of a compensator is given as $G_c(s) = \frac{s+a}{s+b}$

3. $G_c(s)$ is a lead compensator if

[GATE - 2012]

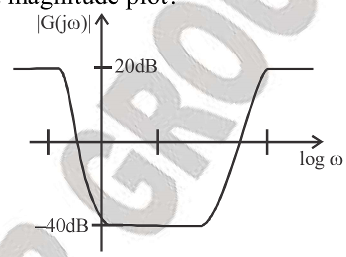
(a) a = 1, b = 2 (b) a = 3, b = 2
(c) a = –3, b = –1 (d) a = 3, b = 1

4. The phase of the above lead compensator is maximum at

[GATE - 2012]

(a) $\sqrt{2}$ rad / s (b) $\sqrt{3}$ rad / s
(c) $\sqrt{6}$ rad / s (d) $1/\sqrt{3}$ rad / s

5. The magnitude plot of a rational transfer function G(s) with real coefficients is shown below. Which of the following compensators has such a magnitude plot?



[GATE - 2009]

(a) Lead compensator
(b) Lag compensator
(c) PID compensator
(d) Lead - lag compensator

6. The transfer functions of two compensators are given below :

$$C_1=\frac{10(s+1)}{(s+10)}, C_2=\frac{s+10}{10(s+1)}$$

Which one of the following statements is correct?

[GATE - 2008]

(a) C_1 is lead compensator and C_2 is a lag compensator
(b) C_1 is a lag compensator and C_2 is a lead compensator
(c) Both C_1 and C_2 are lead compensator
(d) Both C_1 and C_2 are lag compensator

7. The open loop transfer function of a plant is given as $G(s)=\frac{1}{s^2-1}$. If the plant is operated in a unity feedback configuration, then the lead compensator that an stabilize this control system is

[GATE - 2007]

(a) $\frac{10(s-1)}{s+2}$ (b) $\frac{10(s+4)}{s+2}$

CHAPTER - 10
STATE VARIABLE APPROACH

10.1 INTRODUCTION
These are minimal set of variables which can completely determine the behavior of system at any given time.

State model:

$X = AX + BU$ State eqns.

$Y = CX + DU$ Output eqns.

And both equation combined together is called. State model

X – State vector U – Input vector Y – Output vector

A – System matrix B – Input matrix C – Output matrix

D – Transmission matrix Let n $\Rightarrow$ No. of state variables = order of the system

p $\Rightarrow$ No. of outputs m $\Rightarrow$ No. of inputs Order [A] = n x n

Order [B] = n x m Oder [C] = p x n Order [D] = p x m

10.2 DISADVANTAGES OF TRANSFER FUNCTIONS
1.It is defined only under zero initial conditions.
2.It is only applicable to LTI system and there too it is restricted to single input systems.
3.It reveals only the system O/P for a given i/p and provides no information regarding internal states of the system.
4.Classical design methods (roots locus and freq. domain methods) based on transfer function model are trail and error procedures.

10.3 ADVANTAGES OF STATE VARIABLE METHOD
1. It is applicable for both LTI and LT varying systems.
2. It takes initial conditions into account.
3. All the internal states of the system can be determined.
4. Applicable for multiple input multiple output.
5. Controllability and observability can be determined easily.

10.4 REPRESENTATION OF STAT MODEL
1. Physical variable representation.
2. Phase variable representation
3. Cononical representation.

State model of a system is not unique property. But transfer function of the system is unique.