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## LINEAR CONTROL SYSTEM

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# CONTENTS

# CHAPTER - 1
# *INTRODUCTION TO CONTROL SYSTEM*

## 1.1 INTRODUCTION

A control System is a combination of elements arranged in a planned manner where in each element causes an effect to produce a desired output. This cause and effect relationship is governed by a mathematical relation.


If the input of system is controlled in desired manner, the system is called control system.
Any system can be characterized mathematically by
1. Transfer function 2. State model

$$\text{Transfer function} = \frac{\text{L.T.of output}}{\text{L.T.of input}} = \frac{L\left[c(t)\right]}{\left[c(s)\right]} = \frac{C(s)}{R(s)}\bigg|_{\text{initial conditions}=0}$$

Transfer function is also called impulse response of the system.

**1. Disturbances**

The signal that has some adverse effect on output of system called disturbances if it is generated inside called internal distributes if it is other called out external disturbances.

**2. Plant**

It is defined as the portion of system when is to be controlled it is also called process.

**3. System**

A system is an arrangement or component such that it gives proper output to given input e.g. classroom example of physical system.

**4. Control System**

It is an arrangement of different physical component such that it gives the desired output for the given input by means of regulate or control either direct or indirect.

**5. Controllers**

It is the element of system it say, may be external to system it controls the plant or process.

**6. Performance Specifications**

Control system are designed to perform specific task. The requirement imposed on control system are usually spelled out as performance specifications. These specifications may be given transact response requirement maximum overshoot settling time is step response.
1. Steady state requirement (steady state error) or may be given in terms of frequency response.
2. Specification of the control system must be given before the design process begins.
3. Most important part of control system design is to sate the performance specification precisely so that they will yield on optional control system for the given purpose.

Mathematical modeling of control system regular must be able to model dynamic system in mathematical terms and analyse their dynamic characteristics.

A mathematical model of dynamic system is defined as a set of equation that represent the dynamics of system.
(a) Principle of causality apples to the system considered.
(b) Current output of system (t = 0) depends an past impact (input t < 0).
(c) But does not depend upon the feature value of impact.

**7. Transfer Function/Impulse Response**

In modern control system engineering transfer function usually used to define input – output relationship.

$a_0\, y^n(t) + y^{n-1}(t) + \ldots\ldots.$

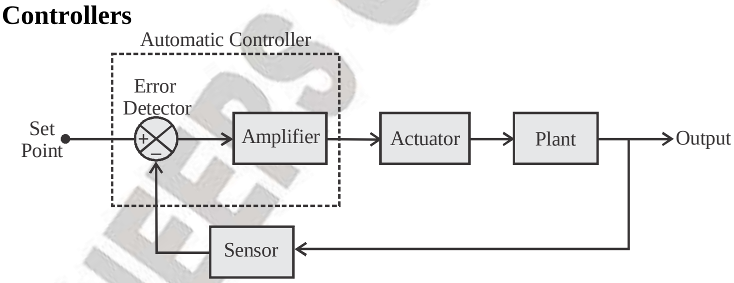
$= b_0 x^m(t) + b_1\, x^{m-1}(t)$

$$\frac{y(s)}{x(s)} = \frac{b_0 s^m + b_1 s^{n-1} + \ldots\ldots..}{a_0 s + a_1 s^{m-1} - \ldots\ldots\ldots.}$$

Applicability of transfer function obeyed only upto linear, time invarient defferential equation system which is extensively used in analysis and design of such a system.
1. If transfer function is know the output or response can be studied for various form of input with a view towards understanding the nature of system.
2. Transfer function is properly of system itself independent of magnitude and nature of input.
3. It does not provide the physical structure of system. The transfer function of many physical statures can be identical.

> The transfer function of a system is a mathematical model in that it is an operational method of expressing the differential equation that relates the output variable and input variable.

**8. Automatic Controllers**




Error detector is actuating the error signal.
Actuator is the power device that produces the input to plant according to control signal.
Sensor is measuring element.
Automatic controller compares actual value of plant output with descried value of plant output measured the deviation and produces a control signal that reduce the domatium to zero.

**9. Control Action**

The manner in which automatic controller generate the control signal is called control action.

**10. Controlled Variable (Control Signal or Manipulated Variable)**

Controlled variable is the quantity or condition that is measured and controlled.

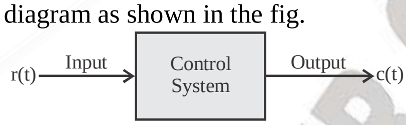
Control Signal or manipulated variable is the quantity or condition that is value of controlled variable normally the controlled variable is the output of the system.

> Control means measuring the value of controlled variable of system and applying control signal to system to correct or limit deviation of measured value from desired value.

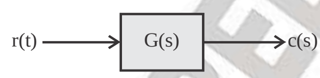
## 1.2 CLASSIFICATION OF CONTROL SYSTEM

### 1.2.1 Open - Loop Control System0

It can be described by a block diagram as shown in the fig.



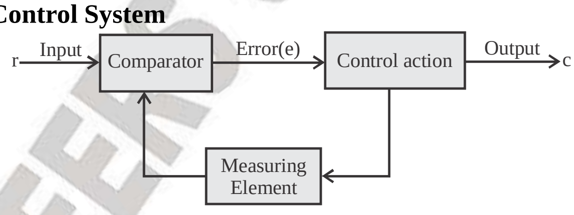

The input 'r' controls the output c through a control action process. In the block diagram shown, it is observed that the output has no effect on the control action. Such a system is termed as open loop control system.




In an open - loop control system, the output is neither measured nor feedback for comparison with the input. Faithfulness of an open - loop control system depends on the accuracy of input calibration.

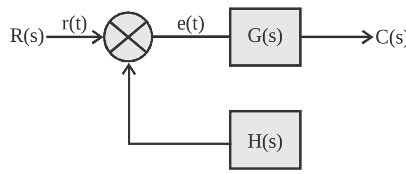
$$\frac{C(s)}{R(s)} = G(s) \text{ or } C(s) = G(s)R(s)$$

### 1.2.2 Closed - Loop Control System




In a closed- loop control system, the output has an effect on control action through a feedback as shown and hence closed – loop control systems are also termed as feedback control systems. The control action is actuated by an error signal 'e' which is the difference between the input single 'r' and the output signal 'c'. This process of comparison between the output and input maintains the output at a desired level through control action process.

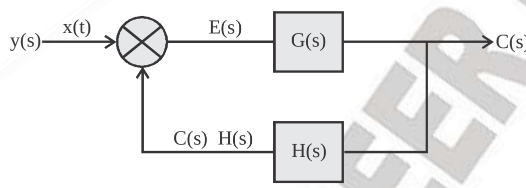
The control system without involving human intervention for normal operation are called automatic control systems. A closed – loop (feedback) control system using a power amplifying device prior to controller and the output of such a system being mechanical i.e., position, velocity , acceleration is called servomechanism.




If error signal e(t) is zero, output is controlled
If error signal is not zero, output is non controlled.
For positive feedback, error signal = x(t) + y(t) or R(s) + y(s)
For negative feedback, error signal = x(t) - y(t) or R(s) – y(s)
The purpose of feedback is to reduce the error between the reference input r(t) and the system output c(t).

Transfer function of closed loop control system will be $\frac{C(s)}{R(s)}$

Now,




$G(s) = \frac{C(s)}{E(s)} \qquad C(s) = G(s)E(s)$

$\text{Now } T(s) = \frac{C(s)}{R(s)}$

Where T (s) is the overall T.F. of a closed loop C.S and H(s) $\frac{B(s)}{C(s)}$

B(s) = H (s) C (s)
For negative feedback, R(s) – y(s)
$\therefore$ G(s) [R(s) – Y(s)] = G(s) [R(s) – H(s) C (s)] C(s) = G(s) R(s) – G(s) H(s) C (s)

$\therefore \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$ , for Positive feedback

$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)}$

G(s) H(s) is called loop transfer function.

### 1.2.3 Comparison of Open – Loop and Closed – Loop Control Systems

| | Open – loop C.S. | | Closed loop C.S. |
|---|---|---|---|
| **1.** | The accuracy of an open – loop system depends on the calibration of the input. Any departure from pre – determined Calibration affects the output | **1.** | As the error between the reference input and the output is continuously measured through feedback, the closed – loop system works more accurately. |

| | | | |
|---|---|---|---|
| **2.** | The open – loop system is simple to construct and cheap. | **2.** | The closed – loop system is complicated to construct and costly |
| **3.** | The open – loop systems are generally stable. | **3.** | The closed – loop systems can become unstable under certain conditions. |
| **4.** | The operation of open – loop system is affected due to presence of non – linearities in its elements. | **4.** | In terms of the performance, the closed – loop systems adjusts to the effects of non – linearities present in its elements. |

| **Positive Feedback** | | **Negative Feedback** | |
|---|---|---|---|
| Unity F/B (H(s) = 1) | Non unity F/B (H(s) ≠ 1) | Unity F/B | Non unity F/B |
| $G(s) = \frac{G(s)}{1 - G(s)}$ | $G(s) = \frac{G(s)}{1 - G(s)H(s)}$ | $G(s) = \frac{G(s)}{1 + G(s)}$ | $G(s) = \frac{G(s)}{1 + G(s)H(s)}$ |

Where G(s) T.F. without feedback (or) T.F. of the forward path
H(s) = T.F. of the feedback path

## 1.3 EFFECT OF FEEDBACK

### 1.3.1 Effect of Feedback on Stability

Stability is a notion that describes whether the system will be able to follow the input command. A system is said to be unstable, if its output is out of control or increases without bound for a bounded input. It can be demonstrated that one of the advantages of incorporating feedback is that it can stabilize an unstable system.

### 1.3.2 Effect of Feedback on Overall Gain

Feedback effects the gain G of a non – feedback system by a factor of 1 ± GH. The general effect of effect of feedback is that it may increase or decrease the gain. In perceptual control system G and H are function of frequency so that 1 + GH >>1 in one range and can be < 1in other range.
Therefore, feedback could increase the gain of the system in one frequency range but decrease it in another.

### 1.3.3 Effect of Feedback on Sensitivity

$$S_G^M = \frac{\partial M / M}{\partial G / G} = \frac{\partial M}{\partial G} . \frac{G}{M} = \frac{1}{1 + G + H}$$

1.In general a good control system should be sensitive to the impact command.
2.Thus sensitivity function can be made arbitrarily small by increasing GH provided that the system remains stable.

### 1.3.4 Effect of Feedback on Sensitivity

***1. Senstivity***

It is a defined as a ratio of variation in a system parameter to the variation in another system parameter.

Mathematically sensitivity $\dfrac{\%\,\text{change in P}}{\%\,\text{change in Parameter K}}$ or $S_k = \dfrac{\partial P/P}{\partial K/K}$

Consider G as a parameter that may vary. The sensitivity of the gain of the overall system T to the variation in G is defined as

$$S_G^T = \frac{\partial T/T}{\partial G/G}$$

Where $\partial$ T denotes the incremental change in M due to the incremental change in G; $\partial T/T$ and $\partial G/G$ denote the percentage change in T and G, respectively.

$$S_G^T = \frac{\partial T}{\partial G}\frac{G}{T} = \frac{1}{1+GH} \qquad S_G^T = \frac{dT\,|\,T}{dG\,|\,G} = \frac{G}{T}\frac{dT}{dG}$$

$$\text{where}\ \frac{dT}{dG} = \frac{d}{dG}\left[\frac{G}{1+GH}\right] = \frac{d}{dG}\left[\frac{(1+GH)\frac{dG}{dG} - G\frac{d}{dG}(1+GH)}{(1+GH)^2}\right]$$

$$= \left[\frac{(1+GH)\times 1 - G\times 0 + H}{(1+GH)^2}\right] = \left[\frac{1+GH-GH}{(1+GH)^2}\right]$$

Now $S_G^T = \dfrac{G}{T}\dfrac{dT}{dG} = \dfrac{G}{\dfrac{G}{1+GH}} \times \dfrac{1}{(1+GH)^2}$ by putting value of $\dfrac{dT}{dG}$ from above equation

$$\Rightarrow S_G^T = \frac{1}{1+GH}$$

***2. Negative Feedback System***

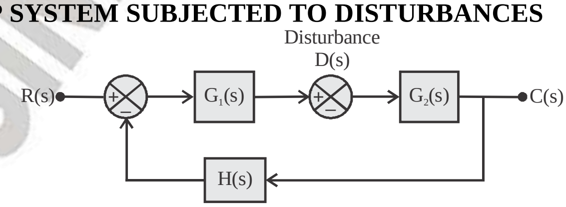
For closed loop system

For an open loop system, $G = T \Rightarrow S_G^T = \dfrac{G}{T}\dfrac{dT}{dG} = 1$

Thus sensitivity of a closed loop is to parameter variations is reduced by a factor of (1+GH)

This relation shows that the sensitivity function can be made arbitrarily small by increasing GH, provided that the system remains stable. In an open – loop system, the gain of the system will respond in a one – to – one fashion to the variation in G. In general, the sensitivity of the system gain of a feedback system to parameter variations depends on where the parameter is located in the control process.

## 1.4. CLOSED LOOP SYSTEM SUBJECTED TO DISTURBANCES




**Superposition Principle**

$C(s) = C(s)\,|_{D(s)=0} + C(s)|_{R(s)=0}$

$$\left.\frac{C_D(s)}{D(s)}\right|_{R(s)=0} = \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}$$

$$\left.\frac{C_R(s)}{R(s)}\right|_{D(s)=0} = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H(s)}$$

$C(s) = C_R(s) + C_D(s)$

$$C(s) = \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}[G_1(s)R(s)+D(s)]$$

If $|G_1(s)H(s)| >> 1$; $G_1(s)G_2(s)H(s)| >> 1$

$\frac{C_D(s)}{D(s)} \simeq 0$ and effect of disturbance is suppressed.

This is an advantage of closed loop control system. On other hand $\frac{C_R(s)}{R(s)} = \frac{1}{H(s)}$

This means if $|G_1(s)G_2(s)H(s)| >> 1$

Thus $\frac{C(s)}{R(s)}$ because independent of $G_1(s)$ and $G_2(s)$ and is inversely proportional to H(s) so that $G_1(s)$ and $G_2(s)$ variation do not affect the closed loop transfer function.

Thus we can conclude that any closed loop system with H(s) = 1 tends to equalize input and output.

Effect of internal disturbances is equalized or vanishes in closed loop control system.

Transfer function = $\frac{C(s)}{R(s)} = \frac{\text{Output}}{\text{Input}}$

# GATE QUESTIONS

**1.** The Nyquist stability criterion and the Routh criterion both are powerful analysis tools for determining the stability of feedback controllers. Identify which of the following statements is FALSE:

**[GATE - 2018]**

(a) Both the criteria provide information relative to the stable gain range of the system.

(b) The general shape of the Nyquist plot is readily obtained from the Bode magnitude plot for all minimum-phase systems.

(c) The Routh criterion is not applicable in the condition of transport lag, which can be readily handled by the Nyquist criterion

(d) The closed-loop frequency response for a unity feedback system cannot be obtained from the Nyquist plot.

**2.** A system is described by the following differential equation:

$$\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} + x(t), \quad x(0) = y(0) = 0$$

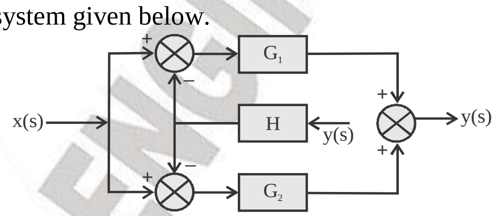
where x(t) and y(t) are the input and output variables respectively. The transfer function of the inverse system is

**[GATE - 2017]**

(a) $\frac{s+1}{s-2}$ (b) $\frac{s+2}{s+1}$

(c) $\frac{s+1}{s+2}$ (d) $\frac{s-1}{s-2}$

**3.** Find the transfer function $\frac{Y(s)}{X(s)}$ of the system given below.




**[GATE - 2015]**

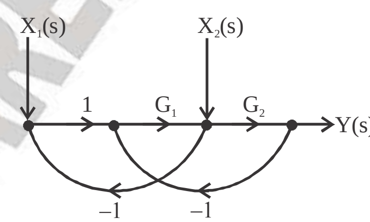
(a) $\frac{G_1}{1+HG_1} + \frac{G_2}{1-HG_2}$

(b) $\frac{G_1}{1+HG_1} + \frac{G_2}{1+HG_2}$

(c) $\frac{G_1+G_2}{1+H(G_1+G_2)}$

(d) $\frac{G_1+G_2}{1-H(G_1+G_2)}$

**4.** For the signal - flow graph shown in the following expressions is equal to the transfer function $\left.\frac{Y(s)}{X_2(s)}\right|_{x_1(s)=0}$ ?

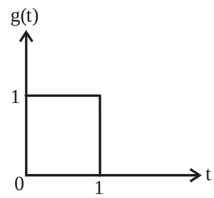



**[GATE - 2015]**

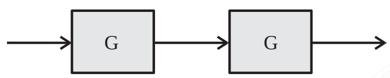
(a) $\frac{G_1}{1+G_2(1+G_1)}$ (b) $\frac{G_2}{1+G_1(1+G_2)}$

(c) $\frac{G_1}{1+G_1G_2}$ (d) $\frac{G_2}{1+G_1G_2}$

**5.** The impulse response g(t) of a system, G, is a shown in Figure (a). What is the maximum value attained by the impulse response of two cascaded blocks of G as shown in Figure (b)?




**(a)**



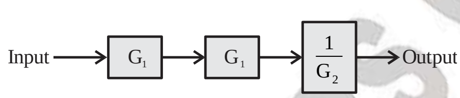

**(b)**

**[GATE - 2015]**

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$

(c) $\frac{4}{5}$ (d) 1

**6.** The measurement system shown in the figure uses three sub-systems in cascade whose gains are specified as $G_1$, $G_2$, $1/G_3$ . The relative small errors associated with each respective subsystem $G_1$, $G_2$ and $G_3$ are $\varepsilon_1$ , $\varepsilon_2$ and $\varepsilon_3$ . The error associated with the output is :




**[GATE - 2009]**

(a) $\varepsilon_1 + \varepsilon_2 + \frac{1}{\varepsilon_3}$ (b) $\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_3}$

(c) $\varepsilon_1 + \varepsilon_2 - \varepsilon_3$ (d) $\varepsilon_1 + \varepsilon_2 + \varepsilon_3$

**7.** Despite the presence of negative feedback, control systems still have problems of instability because the

**[GATE - 2005]**

(a)Components used have non – linearities

(b)Dynamic equations of the subsystem are not known exactly.

(c)Mathematical analysis involves approximations.

(d)System has large negative phase angle at high frequencies.

**8.** Consider the function $F(s) = \frac{5}{s(s^2 + 3s + 2)}$

Where F (s) is the Laplace transform of the function f(t). The initial value of f(t) is equal to

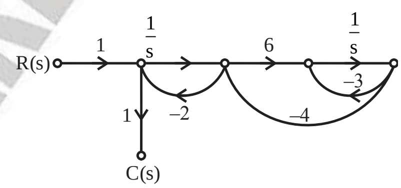
**[GATE - 2004]**

(a) 5 (b) 5/2

(c) 5/3 (d) 0

**9.** The signal flow graph of a system is shown in fig. below. The transfer function C(s)/R(s) of the system is

**[GATE - 2003]**




(a) $\frac{6}{s^2 + 29s + 6}$ (b) $\frac{6s}{s^2 + 29s + 6}$

(c) $\frac{s(s+2)}{s^2 + 29s + 6}$ (d) $\frac{s(s+27)}{s^2 + 29 + 6}$

**10.** The system shown in the figure remains stable when

**[GATE - 2002]**

(a) k < –1 (b) –1 < k < 3

(c) 1 < k < 3 (d) k > 3

# SOLUTIONS

**Sol. 1. (b)**
Bode magnitude plot consists of only magnitude information. But to obtain Nyquist plot we need both magnitude and phase information. Hence statement (b) is false.

**Sol. 2. (b)**

Given $\frac{dy(t)}{dt} + 2y(t) = \frac{dx(t)}{dt} + x(t)$

Apply laplace transform to above equation

SY(s) + 2Y(s) = SX(s) + X(s)

Y(s) [s +2] = X(s) (s +1)

$$H(s) = \frac{Y(s)}{X(s)} = \frac{S+1}{S+2}$$

$$H_{inv}(s) = \frac{S+2}{S+1}$$

**Sol. 3. (c)**

$Y_{(s)} = G_1\,[y(s) - + 1\,y(s)] + G_2\,[x(s) - 4y\,(s)]$

$$\frac{y_{(s)}}{X_{(s)}} = \frac{G_1 + G_2}{1 + H(G_1 + G_2)}$$

**Sol. 4. (b)**

Forward path $P_1 = G_2$

All loops $L_1 = -G_1$

$L_2 = -G_1G_2$

Non touching loops are nil.

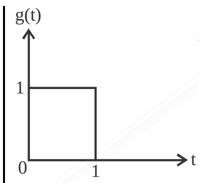
So

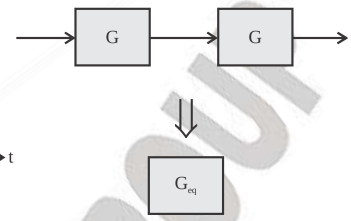
$$\frac{y(s)}{x_2(s)} = \frac{\sum_{n=1}^{k} P_k\Delta_k}{\Delta} = \frac{G_1(1-0)}{1-(-G_1-G_1G_2)}$$

$$\Rightarrow \frac{G_2}{1+G_1(1+G_2)}$$

**Sol. 5. (d)**

Thee given system is




The continuous time signal is defined as

g(t ) = u(t )− u(t − 1)

By convolving *G* with *G* itself, we get

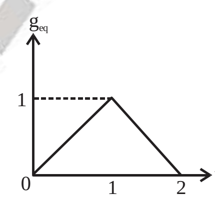
*g*(t) * g(t) = [u(t )− u(t − 1)]x[u(t )− u(t − 1)]

= u(t ) * u(t )− u(t ) * u(t − 1)− u(t − 1) * u(t ) +u(t − 1) * 4(t − 1)

= r(t )− r(t − 1)− r(t − 1)+ r(t − 2)

So, g (t)eq = r(t )− 2r(t − 1)+ r(t − 2)

The continuous time signal is drawn in the figure below.




Hence, maximum value of *geq* is equal to 1

**Sol. 6. (a)**

Overall gain of the system is written as

$$G = G_1G_2\frac{1}{G_3}$$

We know that for a quantity that is product of two or more quantities total percentage error is some of the percentage error in each quantity. So error in overall gain G is

$$\Delta G = \varepsilon_1 + \varepsilon_2 + \frac{1}{\varepsilon_3}$$

**Sol. 7. (a)**

Despite the presence of negative feedback, control systems still have problems of instability because components used have nonlinearity. There are always some variation as compared to ideal characteristics.

**Sol. 8. (d)**
Given transfer function is

$$F(s) = \frac{5}{s(s^2+3s+2)}$$

$$F(s) = \frac{5}{s(s+1)(s+2)}$$

By partial fraction, we get

$$F(s) = \frac{5}{25} - \frac{5}{s+1} + \frac{5}{2(s+2)}$$

Taking inverse Laplace of F(s) we have

$$F(t) = \frac{5}{2}u(t) - 5e^{-t} + \frac{5}{2}e^{-2t}$$

So, the initial value of f(t) is given by

$$\lim_{t\to 0} f(t) = \frac{5}{2} - 5 + \frac{5}{2}(1) = 0$$

**Sol. 9. (d)**
Mason Gain formula

$$T(s) = \frac{\sum p_k \Delta_k}{\Delta}$$

In given SFG there is only forward path and 3 possible loop.
$p_1 = 1$

$$\Delta_1 = 1 + \frac{3}{s} + \frac{24}{s} = \frac{s+27}{s}$$

$$L_1 = \frac{-2}{s}, L_2 = \frac{-24}{s} \text{ and } L_3 = \frac{-3}{s}$$

Where $L_1$ and $L_3$ are non – touching
This

$$\frac{C(s)}{R(s)} = \frac{p_1\Delta_1}{1-(\text{loop gain}) + \text{pair of non} - \text{touching loops}}$$

$$= \frac{\left(\frac{s+27}{s}\right)}{1-\left(\frac{-3}{s} - \frac{24}{s}\right) + \frac{-2}{s}, \frac{-3}{s}} = \frac{\left(\frac{s+27}{s}\right)}{1+\frac{2s}{s}+\frac{s}{s^2}}$$

$$= \frac{s(s+27)}{s^2+29s+6}$$

**Sol. 10. (d)**
From Mason gain formula we can write transfer function as

$$\frac{Y(s)}{R(s)} = \frac{\frac{K}{s}}{1-\left(\frac{3}{5}+\frac{-K}{s}\right)} = \frac{K}{s-3(3-K)}$$

For system to be stable (3 – k) < 0 i.e K > 3

# ESE OBJ QUESTIONS

**1.** The open – loop transfer function of a system is $\frac{10K}{1+10s}$

When the system is converted into a closed – loop with unity feedback, the time constant of the system is reduced by a factor of 20. The value of K is

**[EE ESE - 2018]**

(a) 1.9 (b) 1.6
(c) 1.3 (d) 1.0

**2.** The effects of feedback on stability and sensitivity are

**[EC ESE - 2015]**

(a)Negative feedback improves stability and system response is less sensitive to external inputs and parameter variations.
(b)Feedback does not affect stability but system response is sensitive to disturbances and parameter variations.
(c)Feedback does not affect stability response is sensitive to disturbances and parameter variations
(d)Negative feedback affects stability and system response is more sensitive disturbances and parameter variations.

**3.** The D.C. gain and steady state error for step input for G (s) = $\frac{s+1}{s^2+s+1}$ are:

**[EC ESE - 2013]**

(a) 1 and 1 (b) 0 and 1
(c) 1 and 0 (d) 0 and 0

**4.** In control systems, excessive bandwidth is NOT employed because:

**[EC ESE - 2013]**

(a) Noise is proportional to bandwidth
(b) It leads to low relative stability
(c) It leads to slower response
(d) Noise is proportional to the square of the bandwidth

# SOLUTIONS

**Sol.1. (a)**

$\text{OLTF} = \frac{10k}{1+10s}$

$Z_1 = 10$

$Z_2 = \frac{10}{20} = 0.5$

$\text{CLTF} = \frac{10k}{10k+1+10s}$

$Z_2 = \frac{10}{10k+1} = 0.5$

$\frac{10}{0.5} = 10k+1$

k = 1.9

**Sol.2. (a)**

**Sol.3. (c)**

$G(s) = \frac{s+1}{s^2+s+1}$

$G(s) = |_{s=0} = \frac{0+1}{0+0+1} = 1$

$e_{ss}$ = Steady State Error $= \frac{1}{1+k_p}$

$k_p =^{t}_{s\to 0} G(s)H(s) = 0$

**Sol.4. (a)**

Noise Power = $\eta_0\beta$

Noise Power × Bandwidth × B

# CHAPTER - 2
# *MATHEMATICAL MODES OF PHYSICAL SYSTEMS*

## 2.1 INTRODUCTION
1.A physical system is collection of physical objects connected together to serve an objective.
2.Idealizing assumptions are always made for the purpose of analysis and synthesis of systems. An idealized physical system is called a physical mode.
3.Once a physical model of a physical system is obtained, the next step is to obtain a mathematical model which is the mathematical representation of the physical model through use of appropriate physical laws.

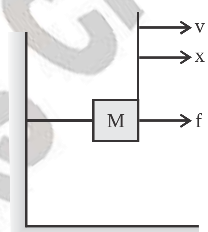
## 2.2 MECHANICAL SYSTEMS
A mechanical system which is modeled using the three ideal elements would yield a mathematical model which is an ordinary differential equation. All mechanical systems are divided into two parts:

### 2.2.1 Mechanical Translational System
In this type of mechanical system input is the forced (F) and the output is linear displacement (x) or linear velocity (v). The three ideal elements are:

***1. Mass Element***

$$F = M\frac{d^2x}{dt^2}$$

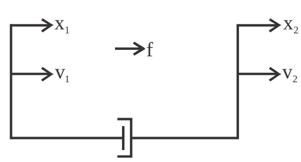
or $F = M\frac{dv}{dt}$




***2. Damper Element***

$$F = f\frac{d}{dt}(x_1 - x_2) = f\frac{dx}{dt}$$

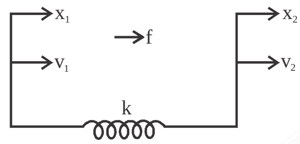
where $x_1 - x_2$ x
or $F = f(v_1 - v_2) = fv$
where $v = v_1 - v_2$




***3. Spring Element***

$F = K(x_1 - x_{2=}) = Kx$

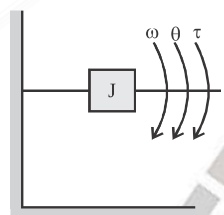
where $x_1 - x_2 = x$
or $F = K \int (v_1 - v_2) = K \int v$
where $v = v_1 - v_2$




### 2.2.2 Mechanical Rotational System

In this type of mechanical system input is the torque ($\tau$) and output is angular displacement ($\theta$) or angular ($\omega$). The three ideal elements are:

***1. Inertial Element***

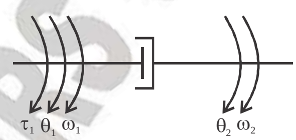



$$\tau = J\frac{d^2\theta}{dt^2} \text{ or } \tau = J\frac{d\omega}{dt}$$

***2. Torsional Damper Element***

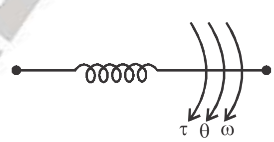
$$\tau = f\frac{d}{dt}(\theta_1 - \theta_2) = f\frac{d\theta}{dt}$$

Where, $\theta = \theta_1 - \theta_2$ or $\tau = \phi(\omega_1 - \omega_2) = f\omega$
where $\omega = \omega_1 - \omega_2$




***3. Torsional Spring Element***

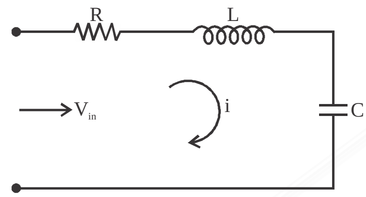
$\tau = K\theta$ Or $\tau = K\int \omega dt$




### 2.3 ELECTRICAL SYSTEM

The resistor, inductor and capacitor are the three basic elements of electrical circuits. These circuits are analysed by the application of Kirchhoff's voltage and current laws.

### 2.3.1 Series RLC Circuits



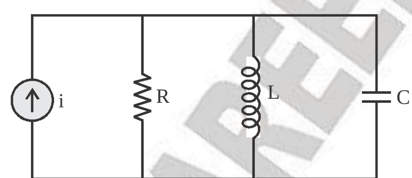

Applying KVL, $iR + L\frac{di}{dt} + \frac{1}{C}\int idt = V_{in}$

but $i = \frac{dq}{dt}$ or $q = \int idt$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V_{in}$$

### 2.3.2 Parallel RLC Circuit

Applying KCL,




$$\frac{V}{R} + \frac{1}{L}\int v\,dt + C\frac{dV}{dt} = i$$

Where, V is node voltage

$$V = \frac{d\phi}{dt}$$

Where $\phi$ is Magnetic flux

$$C\frac{d^2\phi}{dt^2} + \frac{1}{d}\frac{d\phi}{dt} + \frac{\phi}{L} = i$$

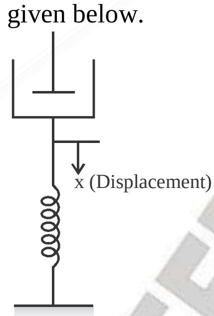
Duality: $C \rightarrow$

$$R \rightarrow \frac{1}{R} \;;\; \phi \rightarrow q$$

### 2.4 NODAL METHOD FOR WRITING DIFFERENTIAL EQUATION OF COMPLEX MECHANICAL SYSTEM

1. Number of nodes = Number of displacements.
2. Take and additional node which is a reference node.
3. Connect the mass and inertial mass elements always between the principle node and reference node.
4. Connect the spring and damping elements either between the principle nodes of between principle nodes and reference depending on their position.
5. Obtain the nodal diagram and write the describing equations at each node.

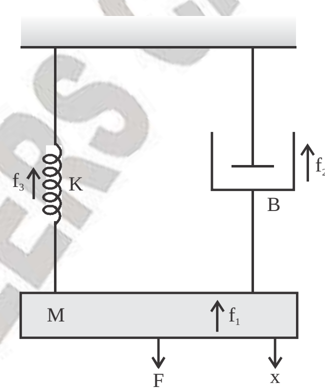
One mass element constitutes order of differential equation = 2
Two mass elements constitute order of differential equation = 4
'n' mass elements constitute order of differential equation = 2n
A point or node between damper and spring must be take as displacement
Each mass must be taken as a displacement
Consider the mechanical systems given below.




### 2.4.1 Mechanical Translations System

$F = F_1 + F_2 + F_3$

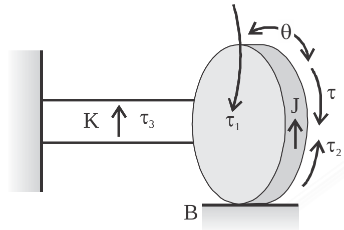
$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx$$




### 2.4.2 Mechanical Rotational System

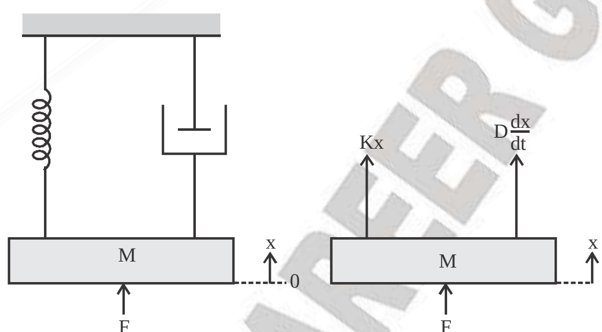
$\tau = \tau_1 + \tau_2 + \tau_3$

$$\tau = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + Kq$$




For anybody, the algebraic sum of externally applied forces and the forces resisting motion in any given direction is zero.

A positive reference direction must first be chosen. Forces acting in the reference direction are considered as positive and those against the reference direction as negative.




The applied force F is balanced by the acceleration of the mass, and resistive forces of spring and damper.

$$F = M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx$$

This is a linear constant coefficient second order differential equation.

**2.5 ANALOGOUS SYSTEMS**

There are two types of analogies:

(i) Force (F) – Torque (τ) – voltage (V) Analogy

(ii) Force (F) – Torque (τ) – Current (i) Analogy

Applied force (F) is analogous to applied voltage $V_{in}$.

Mass M is analogous to inductance L.

Coefficient of viscous friction B is analogy to resistance R.

Spring deflection constant K is analogous to reciprocal of capacitance (1/C).

Displacement is analogous to electric charge q.

As the quantities $L\frac{d^2q}{dt^2}, R\frac{dq}{dt}, \frac{1}{C}q, V_{in}$ are voltages and $M\frac{d^2x}{dt^2}, B\frac{dx}{dt}, Kx, F$ are forces, therefore above said analogy is called force (F) – voltage (V) analogy.

### 2.6.1 Analogy with Various System

| Electrical | Thermal | Mechanical | Liquid |
|---|---|---|---|
| Charge | Heat | Length | Volume |
| Voltage | Temperature | Force | Heat |
| Current | Rate of Heat | Velocity | Rate of Volume |
| Resistance | Resistance | Resistance | Resistance |
| Capacitance | Capacitance | Capacitance | Capacitance |
| Inductance | Not applicable | Mass | Iterance |

### 2.6.2 List of Mechanical Electrical Analogous Variables and Parameter

| Mechanical Translation System | Mechanical Rotational System | Electrical System |
|---|---|---|
| Force: F(t) | Torque: T(t) | Voltage : V(t) |
| Displacement: x(t) | Angular Displacement:θ(t) | Charge: q(t) |
| Velocity: v(t) = x(t) | Angular velocity: ω(t) = θ(t) | Current : L |
| Mass: M | Moment of inertia : J | Inductance : L |
| Friction coefficient : B | Friction coefficient : B | Resistance : R |
| Spring Constant: K | Torsional constant: K | Reciprocal of capacitance:1/c |

# GATE QUESTIONS

**1.** The laplace transform of a causal signal y(t) is $Y(s)=\frac{s+2}{s+6}$ . The value of the signal y(t) at t = 0.1s is _____ unit.

**[GATE - 2017]**

**2.** The transfer function of a system is $\frac{Y(s)}{R(s)}=\frac{s}{s+2}$. The steady state output y(t) is $A\cos(2t+\phi)$ for the input cos(2t). The values of A and $\phi$, respectively are

**[GATE - 2016]**

(a) $\frac{1}{\sqrt{2}},-45°$ (b) $\frac{1}{\sqrt{2}},+45°$

(c) $\sqrt{2},-45°$ (d) $\sqrt{2},+45°$

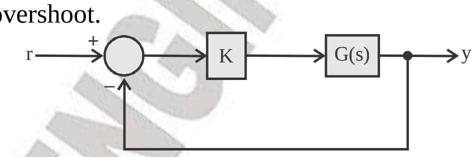
**3.** The particular solution of the initial value problem given below is $\frac{d^2y}{dx^2}+12\frac{dy}{dx}+36y=0$ with y(0) =3 and $\left.\frac{dy}{dx}\right|_{x=0}=-36$

**[GATE - 2016]**

(a) $(3-18x)e^{-6x}$ (b) $(3+25x)e^{-6x}$

(c) $(3+20x)e^{-6x}$ (d) $(3-12x)e^{-6x}$

**4.** In the feedback system shown below $G(s)=\frac{1}{(s^2+2s)}$.The step response of the closed-loop system should have minimum setting time and have no overshoot.




The required value of gain k to achieve this is _______.

**[GATE - 2016]**

**5.** The response of the system $G(s)=\frac{s-2}{(s+1)(s+3)}$ to the unit step input u(t) is y(t).The value of $\frac{dy}{dt}$ at $t=0^+$ is __________.

**[GATE - 2016]**

**6.** The open-loop transfer function of a unity-feedback control system is given by

$$G(s)=\frac{K}{s(s+2)}$$

For the peak overshoot of the closed-loop system to a until step input to be 10%, the value of K is________.

**[GATE - 2016]**

**7.** A first-order low-pass filter of time constant T is excited with different input signals (with zero initial conditions up to t = 0 ). Match the excitation signals X,Y,Z with the corresponding time responses for $t \le 0$

X: Impulse P: $1-e^{-t/T}$

Y: Unit step Q: $t-T(1-e^{-t/T})$

Z: Ramp R: $e^{-t/T}$

**[GATE - 2016]**

(a) X-R, Y-Q, Z-P

(b) X-Q, Y-P, Z-R

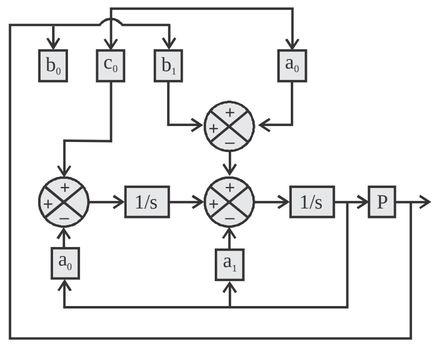
(c) X-R, Y-P, Z-Q

(d) X-P, Y-R, Z-Q

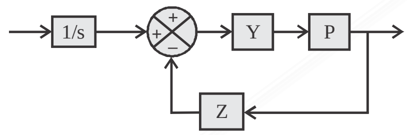
**8.** The closed-loop transfer function of a system is $T(s)=\frac{4}{\left(s^2+0.4s+4\right)}$ . The steady state error due to unit step input is__________.

**[GATE - 2014]**

**9.** The system shown in figure below




Can be reduced to the form




With

**[GATE - 2007]**

(a) $X = c_0s + c_1$, $Y = 1/(s^2 + a_0s + a_{1)},$ $Z = b_0s + b_1$

(b) $X = 1$, $Y = (c_0s + c_1)/(s^2 + a_0s + a_1)$, $Z = b_0s + b_1$

(c) $X = c_1s + c_0$, $Y = (b_1s + b_0)/(s^2 + a_1s + a_0)$, $Z = 1$

(d) $X = c_1s + c_0$, $Y = 1/(s^2 + a_1s + a)$, $Z = b_1s + b_0$

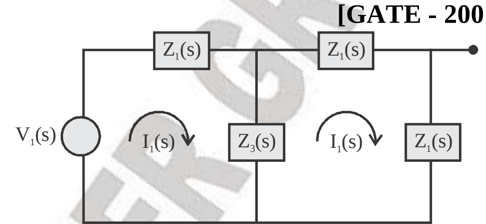
**10.** For a tachometer, if θ(t) is the rotor displacement in radians, e(t) is the output voltage and $K_t$ is the tachometer constant in V/rad/sec, then the transfer function, $\frac{E(s)}{Q(s)}$ will be

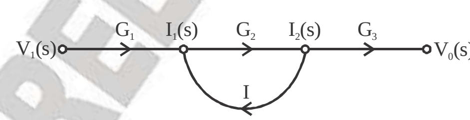
**[GATE - 2004]**

(a) $K_ts^2$
(b) $K_t/s$
(c) $K_ts$
(d) $K_t$

**11.** An electrical system and its signal – flow graph representations are shown the fig (a) and (b) respectively. The values of $C_2$ and H, respectively are

**[GATE - 2001]**




(a) $\frac{Z_3(s)}{Z_1(s)+Z_3(s)+Z_4(s)}, \frac{-Z_3(s)}{Z_1(s)+Z_3(s)}$

(b) $\frac{-Z_3(s)}{Z_2(s)-Z_3(s)+Z_4(s)}, \frac{-Z_3(s)}{Z_1(s)+Z_3(s)}$

(c) $\frac{Z_3(s)}{Z_2(s)+Z_3(s)+Z_4(s)}, \frac{-Z_3(s)}{Z_1(s)+Z_3(s)}$

(d) $\frac{-Z_3(s)}{Z_2(s)-Z_3(s)+Z_4(s)}, \frac{Z_3(s)}{Z_1(s)+Z_3(s)}$

# SOLUTIONS

**Sol. 1. (–2.19 )**

The laplace transform $Y(s)=\frac{s+2}{s+6}$ then y(t) at t = 0.1 is

$$Y(s)=\frac{s+2}{s+6}=\frac{s+6-4}{s+6}$$

$$y(t)=L^{-1}\left[1-\frac{4}{s+6}\right]$$

$Y(t) = [\delta(t) - 4e^{-6t}]$

T = 0.1 $y(0.1) = \delta(0.1) - 4e^{-6(0.1)}$

Y(0.1) = –2.19

**Sol. 2. (b)**

$$A=\left|\frac{j\omega}{j\omega+2}\right|_{\omega=2}=\frac{2}{\sqrt{2^2+2^2}}=\frac{2}{2\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$$\phi=\angle\left.\frac{j\omega}{j\omega+2}\right|_{\omega=2}=90°-\tan^{-1}\frac{2}{2}=45°$$

**Sol. 3. (a)**

$D^2 + 12D + 36 = 0$

$\Rightarrow$ D= -6, -6

The solution is $y = C_1 e^{-6x} + C_2xe^{-6x}$ = (1)

$y(0) = 3 \Rightarrow 3 = C_1$

(1) $\Rightarrow y = 3e^{-6x} + C_2xe^{-6x}$

$$\frac{dy}{dx}=-18e^{-6x}+C_2\{-6xe^{-6x}+e^{-6x}\}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=0}=-18+C_2$$

$\Rightarrow -36 = -18 + C_2$

$C_2 = -18$

$\therefore$ The solution is $y = 3e^{-6x} - 18xe^{-6x}$

**Sol. 4. (1)**

Given $G(s)=\frac{1}{(s^2+2s)}$

From Diagram CE $\Rightarrow$ 1 + KG(s) = 0

$s^2 + 2s + K = 0$

Minimum Settling Time is obtain. For Critical Damped System For Critical Damped System

($\xi$=1) the % $m_p$ = 0%

$2\xi\omega_n=2$

$2\times1\times\omega_n=2$

$\omega_n$ = 1 rad/sec

K = 1

**Sol. 5. (1)**

**Method-I.**

Given $Y(s)=\frac{s-2}{(s+1)(s+3)}u(s)$

$$\Rightarrow Y(s)=\frac{s-2}{(s+1)(s+3)}[\text{Givem } u(s)=\frac{1}{s}]$$

$$L\left[\frac{dy}{dt}\right]=sY(s)$$

$$sY(s)=\frac{s-2}{(s+1)(s+3)}$$

$$\left.\frac{dy}{dt}\right|_{t=0^+}=\lim_{s\to\infty}\left(\frac{s-2}{(s+1)(s+3)}\right)=1$$

**Method-II.**

$$Y(s)=\left(\frac{s-2}{s(s+1)(s+3)}\right)=\frac{-2}{3s}+\frac{3}{2(s+1)}-\frac{5}{6(s+3)}$$

$y(t)=-2/3 + 3/2e^{-t}-5/6e^{-3t}$

$$\frac{dy}{dt}=(t=0)=3/2(-1)e^{-t}-\frac{5}{6}(-3)e^{+3t}$$

= –3/2+5/2=1

**Sol. 6. (2.87)**

Given % $M_p$ = 10%

$M_p$ = 0.1

$\Rightarrow M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$

$0.1 = e^{-\pi\xi/\sqrt{1-\xi^2}}$

$$\ln(0.1)=\frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$$2.3=\frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

$\xi$ = 0.59

Given $G(s) = \frac{K}{s(s+2)}$

**CE:**

$1+G(s) = 0 \Rightarrow s^2 + 2s + K = 0$

$2\varepsilon\omega_n = 2$

$2 \times 0.59 \times \omega_n = 2$

$\omega_n = 1.69$ r/sec

$K = \omega_n^2 = 2.87$

**Sol. 7. (c)**

$$H(s) = \frac{1}{1+s\tau}$$

$V_0(s) = H(s).V_1(s)$

**(i)** if $v_i(t) = \delta(t)$

$V_1(s) = 1$

$$V_0(s) = H(s).V_1(s) = \frac{1}{1+s\tau}$$

$$V_0(t) = \frac{1}{\tau}e^{-\frac{t}{\tau}}$$

**(ii)** if $v_i(t) = u(t)$

$V_1(s) = 1/s$

$$V_0(s)\frac{1}{s(1+s\tau)} = \frac{1}{s} - \frac{1}{s+\frac{1}{\tau}}$$

$v_0(t) = (1 - e^{-t/\tau})$

**(iii)** $v_i(t) = r(t)$

$$\Rightarrow V_1(s) = \frac{1}{s^2}$$

$$V_0(s) = H(s).\ V_1(s) = \frac{1}{s^2(1+s\tau)}$$

$$= \frac{1}{s^2} - \frac{\tau}{s} + \frac{\tau}{s+\frac{1}{\tau}}$$

$V_0(t) = t - \tau(1 - e^{-t/\tau})$

**Sol. 8. (0)**

Closed loop T.F.

$$T(s) = \frac{4}{s^2+0.4s+4}$$

$$\frac{G(s)}{1+G(s)} = \frac{4}{s^2+0.4s+4}$$

$$\frac{1+G(s)}{G(s)} = \frac{s^2+0.45s+4}{4}$$

$$1+\frac{1}{G(s)} = \frac{s^2+0.4s+4}{4}$$

$$\frac{1}{G(s)} = \frac{s^2+0.4+4-4}{4}$$

Open loop. T.F. $G(s) = \frac{4}{s(s+0.4)}$

Error constant, $K_P = \lim_{s\to 0} G(s)$

$$= \lim_{s\to 0}\frac{4}{s(s+0.4)} = \infty$$

Steady state error, $e_{ss} = \frac{1}{1+K_P} = 0$

**Sol. 9. (d)**

From the given block diagram we can obtain signal flow graph of the system. Transfer function from the signal flow graph is written as

$$\text{T.F.} = \frac{\frac{c_0P}{s^2}+\frac{c_1P}{s}}{1+\frac{a_1}{s}+\frac{a_0}{s^2}-\frac{Pb_0}{s^2}-\frac{Pb_1}{s}}$$

$$= \frac{(c_0+c_1s)P}{(s^2+a_1s+a_0)-P(b_0+sb_1)}$$

$$= \frac{\frac{(c_0+c_1s)P}{(s^2+a_1s+a_0)}}{1-\frac{P(b_0+sb_1)}{s^2+a_1s+a_0}}$$

from the given reduced from transfer function is given by

$$\text{T.F} = \frac{XYP}{1-YPZ}$$

By comparing above two we have

$X = (c_0 + c_1s)$

$$Y = \frac{1}{s^2+a_1s+a_0}$$

$Z = (b_0 + sb_1)$

**Sol. 10. (c)**

In A.C tachometer output voltage is directly proportional to differentiation of rotor displacement

$$e(t) \propto \frac{d}{dt}[\theta(t)] \quad e(t) = K_t \frac{d\theta(t)}{dt}$$

Taking Laplace transformation on both sides of above equation

$E(s) = K_t s\theta(s)$

So transfer function

$$T.F = \frac{E(s)}{\theta(s)} = (K_t)s$$

**Sol. 11. (c)**

From SFG we have

$I_1(s) = G_1V_1(s) + HI_2(s)$ …(i)

$I_2(s) = G_2I_1(s)$ …(ii)

$V_0(s) = G_3I_2(s)$ …(iii)

Now applying KVL in given block diagram we have

$V_1(s) = I_1(s)\, Z_1(s) + [I_1(s) - I_2(s)]Z_3(s)$ …(iv)

$0 = [I_2(s) - I_1(s)]Z_3(s) + I_2(s)\, Z_2(s) + I_2(s)\, Z_4(s)$ …(v)

From(iv) we have

Or $V_1(s) = I_1(s)\,[Z_1(s) + Z_3(S)] - I_2(s)Z_3(S)$

$$\text{Or } I_1(s) = V_1 \frac{1}{Z_1(s) + Z_3(s)} + I_2 \frac{Z_3(s)}{Z_1(s) + Z_3(s)} \quad \text{…(vi)}$$

From (v) we have

$I_1(s)\, Z_3(S) = I_2(s)[Z_2(s) + Z_3(s) + Z_4(s)]$ …(vii)

$$\text{Or } I_s(s) = \frac{I_1(s)\, Z_3(s)}{Z_3(s) + Z_2(s) + Z_4(s)}$$

Comparing (ii) and (vii) we have

$$G_2 = \frac{Z_3(s)}{Z_3(s) + Z_2(s) + Z_4(s)}$$

Comparing (i) and (vi) we have

$$H = \frac{Z_3(s)}{Z_1(s) + Z_3(s)}$$

# ESE OBJ QUESTIONS

**1.** In a system, the damping coefficient is –2. The system, response will be

**[EE ESE - 2017]**

(a) Undamped
(b) Oscillations with decreasing magnitude
(c) Oscillations with increasing magnitude
(d) Critically damped

**2.** A dominant pole is determined as

**[EC ESE - 2017]**

(a)The highest frequency pole among all poles
(b)The lowest frequency pole at least two octaves lower than other poles
(c)The lowest frequency pole among all poles
(d)The highest frequency pole at least two octaves higher than other poles

**3.** The desirable features of a servomotor are

**[EE ESE - 2016]**

(a) Low rotor inertia and low bearing friction
(b) High rotor inertia and high bearing friction
(c) Low rotor inertia and low bearing friction
(d) High rotor inertia and low bearing friction.

**4. Statement (I)**: Open-loop system is inaccurate and unreliable due to internal disturbances and lack of adequate calibration.
**Statement (II)**: Closed-loop system is inaccurate as it cannot account environmental or parametric changes and may become unstable.

**[EE ESE - 2016]**

(a)Both Statement (I) and Statement (II) are individually true and statement (II) is the correct explanation of Statement (I).
(b)Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
(c)Statement (I) is true but Statement (II) is false
(d)Statement (I) is false but Statement (II) is true.

**5.** In a closed loop system for which the output in the speed of motor, the output rate control can be used to

**[EC ESE - 2015]**

(a) Limit the speed of the motor
(b) Limit the torque output of the motor
(c) Reduce the damping of the system
(d) Limit the acceleration of the motor

**6.** In a servo-system, the device used for providing derivative feedback is known as

**[EC ESE - 2015]**

(a) Synchro (b) Servometer
(c) Poteniometer (d) Tachogenerator

**7.** The z – transform X(z) of the signal

$$x[n] = \alpha^n u(n)$$

where u(n) is sequence of unit pulses, is

**[EE ESE - 2015]**

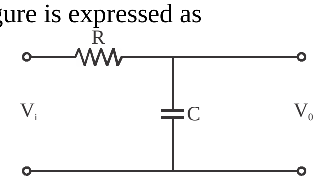
(a) $\frac{\alpha}{z-1}$ (b) $\frac{z}{z-1}$
(c) $\frac{z}{z-\alpha}$ (d) $\frac{1}{z-\alpha}$

**8.** The servomotor differs from the standard motors principally, in that, it has

**[EE ESE - 2015]**

(a) Entirely different construction
(b) High inertia and hence high torque
(c) Low inertia and low torque
(d) Low inertia and higher starting torque

**9.** The transfer function of the circuit as shown in the figure is expressed as




**[EE ESE - 2015]**

(a) $\frac{R}{1+sRC}$ (b) $\frac{S}{1+sCR}$

(c) $\frac{1}{1+sRC}$ (d) $1+sCR$

**10.** The transfer function of a low-pass RC network is

**[EE ESE - 2014]**

(a) RCs (1 + RCs) (b) $\frac{1}{(1+RCs)}$

(c) $\frac{RC}{(1+RCs)}$ (d) $\frac{s}{(1+RCs)}$

**11.** The transfer function of a zero order hold is given by

**[EE ESE - 2014]**

(a) $\frac{1}{s}$ (b) $1 - e^{Ts}$

(c) $s(1 - T^{Ts})$ (d) $\frac{1-e^{-Ts}}{s}$

**12.** When deriving the transfer function of a linear element

**[EE ESE - 2013]**

(a)Both initial conditions and loading are taken into account.
(b)Initial conditions are taken into account but the element is assumed to be not loaded.
(c)Initial conditions are assumed to be zero but loading is taken into account.
(d)Initial conditions are assumed to be zero and the element is assumed to be not loaded.

**13.** In control system, excessive bandwidth is not employed because

**[EE ESE - 2013]**

(a)Noise its proportional to bandwidth
(b)It leads to low relative stability
(c)It leads to slower time response
(d)Noise is proportional to the square of the bandwidth

**14.** Consider the following statements regarding advantages of close loop negative feedback control systems over open loop systems:
(i)The overall reliability of the closed loop system is more than at open loop system.
(ii)The transient response in a closed loop system decays more quickly than in open loop system.
(iii)In an open loop system, closing of the loop increases the overall gain of the system.
(iv)In the closed loop system, the effect of variation of component parameters on its performance is reduced.
Which of these statements are correct?

**[EE ESE - 2013]**

(a) i and ii (b) i and iii
(c) ii and iv (d) iii and iv

**15.** The open-loop transfer function of a unity feedback control system is

$G(s) = \frac{1}{(s+2)^2}$. The closed-loop transfer function will have poles at

**[EE ESE - 2012]**

(a) –2, –2 (b) –2, –1
(c) –2, +2 (d) $-2 \pm j1$

**16.** Match List-I (Mechanical translation system) with List-II (Electrical element for analogues) and select the correct answer using the code given below the lists:

**List-I**
A. Mass
B. Damper
C. Spring
D. Displacement

**List -II**
(i) Resistor
(ii) Inductor
(iii) Capacitor
(iv) Charge

**[EE ESE - 2012]**

**Codes:**
(a) A-iv, B-iii, C-i, D-ii
(b) A-ii, B-iii, C-i, D-iv
(c) A-iv, B-i, C-iii, D-ii
(d) A-ii, B-i, C-iii, D-iv

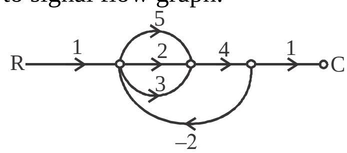
**17.** The law/principle in mechanical systems, analogous to Kirchhoff's laws in electrical systems, is

**[EE ESE - 2012]**

(a) First law of motion

(b) Second law of motion
(c) Third law of motion
(d) d'Alembert's principle

**18.** Consider the following statements with regards to signal flow graph:

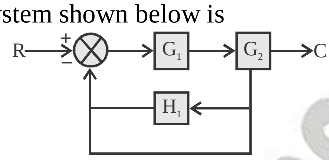



(i) The number of loops are 3.
(ii)The number of loops are 2.
(iii)The number of forward paths are 3.
(iv) $\frac{C}{R}$ ratio is $\frac{40}{81}$
(v) $\frac{C}{R}$ ratio is $\frac{28}{21}$
Which of these statements are correct?

**[EE ESE - 2012]**

(a) i, iii, iv and v
(b) i, iii and iv
(c) ii, iii and iv
(d) iii, iv and v

**19.** The resulting equivalent transfer function of the system shown below is




**[EE ESE - 2013]**

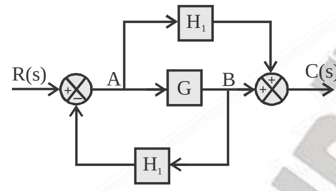
(a) $\frac{G_1G_2}{1+G_1G_2+G_1G_2H_1}$

(b) $\frac{G_1G_2}{1+G_1G_2+G_1H_1}$

(c) $\frac{G_1G_2}{1+H_1G_1G_2}$

(d) $\frac{G_1G_2}{1+G_1G_2+H_1}$

**20.** The transfer function $\frac{C(s)}{R(s)}$ for the system shown below is

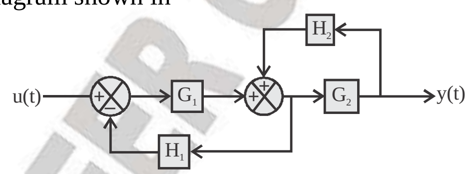



**[EE ESE - 2011]**

(a) $\frac{G+H}{1+GH_2}$
(b) $\frac{G+H_2}{1+GH_1}$

(c) $\frac{H_2}{1+GH_1}$
(d) $\frac{GH_2}{1+GH_1}$

**21.** The system transfer function for the block diagram shown in

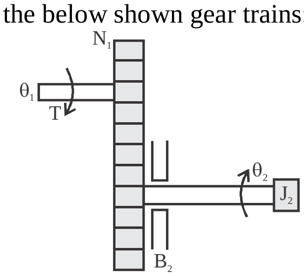



**[EE ESE - 2011]**

(a) $\frac{G_1G_2}{1-G_2H_2+G_1H_1}$
(b) $\frac{G_1G_2}{1-H_1G_1+G_2H_1}$

(c) $\frac{G_1G_2H_1}{1+G_2H_1+G_1H_1}$
(d) $\frac{G_1G_2H_1}{1+G_2H_2+G_1H_1}$

**22.** Consider the following relations with regard to the below shown gear trains:




(i) $\frac{\theta_1}{\theta_2}=\frac{N_2}{N_1}$

(ii) $T_2=J_2\frac{d^2\theta_2}{dt^2}+B_2\frac{d\theta_2}{dt}$

(ii) $T_1 = J_2\left(\frac{N_1}{N_2}\right)^2 \frac{d^2\theta_1}{dt^2} + B_2\left(\frac{N_1}{N_2}\right)^2 \frac{d\theta_1}{dt}$

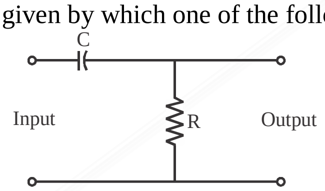
$$T_1 = J_2\left(\frac{N_1}{N_2}\right)^2 \frac{d^2\theta_1}{dt^2} + B_2\left(\frac{N_1}{N_2}\right)^2 \frac{d\theta_1}{dt}$$

Which of these relations are correct?

**[EE ESE - 2011]**

(a) i, ii and iii (b) i and ii only
(c) ii and iii only (d) i and iii only

**23.** The transfer function for the diagram shown below is given by which one of the following ?




**[EE ESE - 2008]**

(a) $\frac{1}{(1+sRC)}$ (b) $\frac{sRC}{(1+sRC)}$

(c) $\frac{sRC}{(1-sRC)}$ (d) 1 + sRC

**24.** Which one of the following statements is correct of phase – shift type and Wein bridge type R-C oscillators ?

**[EE ESE - 2007]**

(a) Both use positive feedback
(b) The former uses positive feedback while the latter uses both positive and negative feedback
(c) The former uses both positive and negative feedback while the latter uses positive feedback only
(d) Both use negative feedback

**25. Assertion (A):** For a prototype second order system, the larger the bandwidth, the faster the system will respond.
**Reason (R):** Bandwidth and rise time are inversely proportional.

**[EE ESE - 2007]**

(a)Both A and R are true and R is the correct explanation of A.
(b)Both a and R are true but R is not the correct explanation of A.
(c)A is true but R is false.
(d)A is false but R is true.

**26.** Consider the following statements in connection with feedback in control system:
(i)With an increase in forward gain, the output value approaches the input value in the case of negative feedback closed – loop system.
(ii)A negative feedback closed – loop system when subjected to an input of 5 V with forward gain of 1 and a feedback gain of 1 gives output 4.999 V.
(iii)The transfer function is dependent only upon its internal structure and components, and is independent of the input applied to the system.
(iv)The overall gain of the block diagram shown is 10.


Which of the statements given above are correct?

**[EE ESE - 2006]**

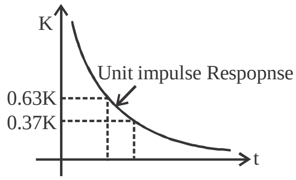
(a) Only i and ii (b) Only ii and iii
(c) Only iii and iv (d) Only i and iii

**27.** Consider the following statements with regard to the bandwidth of a closed-loop system:
(i)In systems where the low frequency magnitude is 0 dB on the Bode diagram, the bandwidth is measured at the-3 dB frequency.
(ii)The bandwidth of the closed loop control system is a measurement of the range of fidelity of response of the system.
(iii)The speed of response to a step input is proportional to the bandwidth.
(iv)The system with the larger bandwidth provides slower step response and lower fidelity ramp response.
Which of the statements given are correct ?

**[EE ESE - 2005]**

(a) i, ii and iii (b) i, ii and iv
(c) i, iii and iv (d) ii, iii and iv

**28.** The unit impulse response of a system having transfer function $K/(s + \alpha)$ is shown below. The value of $\alpha$ is



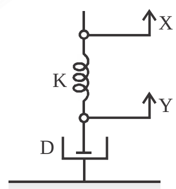

**[EC ESE - 2003]**

(a) $t_1$ (b) $1/t_1$
(c) $t_2$ (d) $1/t_2$

**29.** Which one of the following effects in the system is NOT caused by negative feedback ?
**[EE ESE - 2003]**

(a) Reduction in gain
(ii) Increase in bandwidth
(c) Increase in distortion
(d) Reduction in output impedance

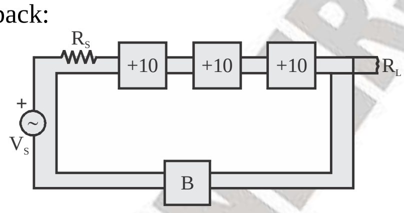
**30.** The mechanical system shown below has its poles(s) at




**[EC ESE - 2002]**

(a) -K/D (b) -D/K
(c) -DK (d) O, -K/D

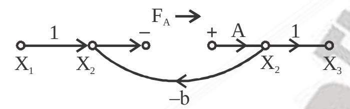
**31.** Consider the following amplifier with –ve feedback:




If the closed loop gain of the above amplifier is +100, the value B will be
**[EC ESE - 2002]**

(a) $-9 \times 10^{-3}$ (b) $+9 \times 10^{-3}$
(c) $-11 \times 10^{-3}$ (d) $+1.1 \times 10^{-3}$

**32.** Consider the following signal-loop feedback structure illustrating the return difference:




The return difference for A is
**[EC ESE - 2002]**

(a) 1Aβ (b) 1 + Aβ
(c) $\frac{AB}{1+A\beta}$ (d) $\frac{A\beta}{1-A\beta}$

**33.** Match List-I (Property) with List-II (Specification) and select the correct answer:

**List-I**
A. Relative stability
B. Speed of response
C. Accuracy
D. Sensitivity

**List-II**
(i) Rise time
(ii) Velocity error constant
(iii) Return difference
(iv) M-peak

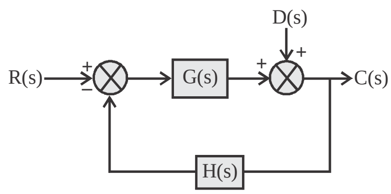
**[EE ESE - 2002]**

**Codes:**
(a) A-iv, B-iii, C-ii, D-i
(b) A-ii, B-i, C-iv, D-iii
(c) A-iv, B-i, C-ii, D-iii
(d) A-ii, B-iii, C-iv, D-i

**34.** Which of the following are the characteristics of closed loop systems ?
(i)It does not compensate for disturbances.
(ii)It reduces the sensitivity of plant – parameter variations.
(iii)It does not involve output measurements.
(iv)It has the ability to control the system transient response.
Select the correct answer using the codes given below:
**[EE ESE - 2002]**

(a) i and iv (b) ii and iv
(c) i and iii (d) ii and iii

**35.** In the feedback system C(s), R(s) and D(s) are the system output, input and disturbance, respectively




**Assertion (A):** For system

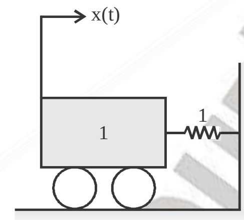
$$\frac{C(s)\{R(s)+D(s)\}}{R(s)D(s)} + \frac{1+G(s)}{1+G(s)H(s)}$$

**Reason (R):** Transfer function of a system is defined as the ratio of output Laplace transform and input Laplace transform setting other inputs and the initial conditions to zero.

**[EE ESE - 2002]**

(a)Both A and R are true and R is the correct explanation of A
(b)Both A and R are true but R is NOT the correct explanation of A
(c)A is true but R is false
(d)A is false but R is true

**36.** Consider the mechanical system shown in the given figure. If the system is set into motion by unit impulse force, the equation of the resulting oscillation will be




**[EC ESE - 2001]**

(a) x(t) = sin t    (b) $x(t) = \sqrt{2}\sin t$
(c) x(t) = 1/2 sin 2t    (d) $x(t) = \sin\sqrt{2}t$

**37.** Open loop transfer function of a system having one zero with a positive real value is called:

**[EC ESE - 2001]**

(a) Zero phase function
(b) Negative phase function
(c) Positive phase function
(d) Non-minimum phase function

**38.** Consider the following operations in respect of a Wheatstone bridge:
(Key "$K_b$" is used for the supply battery and Key "$K_g$" is used for the galvanometer)
1. Open $K_b$    2. Close $K_g$
3. Close $K_b$    4. Open $K_g$
The correct sequence of these operations is:

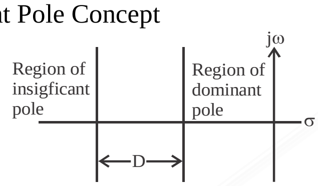
**[EC ESE - 1999]**

(a) 1, 2, 3, 4    (b) 3, 1, 2, 4
(c) 4, 3, 2, 1    (d) 3, 2, 4, 1

# SOLUTIONS

**Sol.1. (c)**
A system with negative damping coefficient is dynamically unstable. So, the system response will be oscillations with increasing magnitude.

**Sol.2. (b)**
Dominant Pole Concept




The pole which are nearer to jω axis is dominant pole and the pole which are away from the jω axis is known as insignificant pole.
The distance D between dominant pole and insignificant pole is 5 to 10 times of the magnitude of dominant pole or pair of complex dominant pole.

**Sol.3. (a)**

**Sol.4. (c)**
Closed loop system has feedback to account environment changes and became stable.

**Sol.5. (*)**

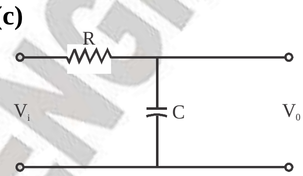
**Sol.6. (d)**

**Sol.7. (c)**
$x[n] = \alpha^n . i(n)$

$$x(z) = Z[\alpha^n.u(n)] = \frac{z}{z-\alpha}$$

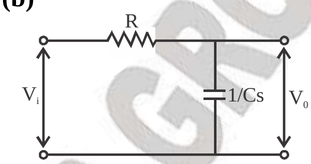
**Sol.8. (d)**

**Sol.9. (c)**




$$\frac{V_0(s)}{V_i(s)} = \frac{\frac{1}{sC}}{r+\frac{1}{sC}} = \frac{1}{1+sRC}$$

**Sol.10. (b)**




$$\frac{V_0}{V_i} = \frac{1/Cs}{R+\frac{1}{Cs}} = \frac{1}{RCs+1}$$

**Sol.11. (d)**
The transfer function of a Zero Order Hold (ZOH).

**Sol.12. (d)**
While deriving the transfer function of a linear element only initial conditions are assumed to be zero, whereas it is independent of loading condition.

**Sol.13. (a)**
Higher the bandwidth means lower the selectivity and hence higher the noise.

**Sol.14. (c)**
Statement 4 is correct because sensitivity of close loop negative feedback control system is less than sensitivity of open loop control system.

**Sol.15. (d)**
Closed – loop transfer function

$$M(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{1}{s^2+4s+5}$$

Closed – loop poles = $-2 \pm \sqrt{4-5}$
$= -2 \pm j1$

**Sol.16. (d)**
By comparing displacement with charge, we came to know that it is force voltage analogy, mass is analogous to inductor, damper to register, spring to capacitor and displacement to charge.

**Sol.17. (d)**
D'Alembert's principle for the translational mechanical system is as follows:
The aldebraic sum of the externally applied forces on a given body and the force resisting the motion of the body in a given direction is zero.

**Sol.18. (b)**

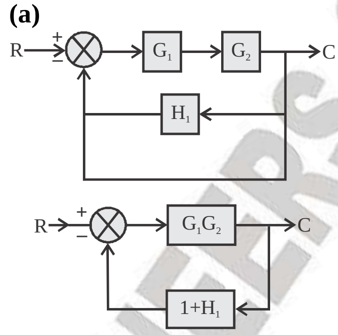
$$\frac{C}{R}=\frac{P_1\Delta_1+P_2\Delta_2+P_3\Delta_3}{\Delta}$$

$$=\frac{5\times4+2\times4+3\times4}{1-\{-5\times4\times2-2\times4\times2-3\times4\times2\}}$$

$$\Rightarrow\frac{C}{R}=\frac{40}{81}$$

Hence, option (b) is correct

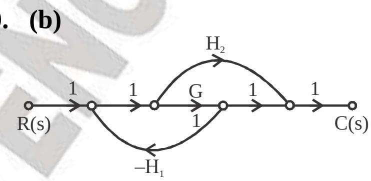
**Sol.19. (a)**




$$\text{Transfer function }=\frac{G_1G_2}{1+G_1G_2(1+H_1)}$$

Hence, option (a) is correct.

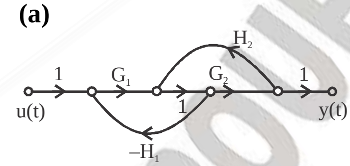
**Sol.20. (b)**




$$\frac{C(s)}{R(s)}=\frac{P_1\Delta_1+P_2\Delta_2}{\Delta}=\frac{G+H_2}{1-(-GH_1)}$$

Hence, option (b) is correct.

**Sol.21. (a)**




$$\text{Transfer function}=\frac{P_1\Delta_1}{\Delta}$$

$$=\frac{G_1G_2}{1-\{-G_1H_1+G_2H_2\}}$$

Hence, option (a) is correct

**Sol.22. (a)**
Number of teeth is proportional to the radius

$$\frac{r_1}{r_2}=\frac{N_1}{N_2}$$

Distance travelled on the surface of the gear is the same for both

$$r_1\theta_1=r_2\theta_2\Rightarrow\frac{r_1}{r_2}=\frac{\theta_2}{\theta_1}$$

Work done by one gear is equal to the other

$$T_1\theta_1=T_2\theta_2\Rightarrow\frac{T_1}{T_2}=\frac{\theta_2}{\theta_1}$$

Combining,

$$\frac{T_1}{T_2}=\frac{\theta_2}{\theta_1}=\frac{N_1}{N_2}=\frac{r_1}{r_2}=\frac{\omega_2}{\omega_1}$$

Torque on one gear can be transferred to other gear similar to transformer's transferred impedance with ration $N_1/N_2$.
Hence option (a) is correct.

**Sol.23. (b)**

$$\text{T.F.}=\frac{V_0}{V_i}=\frac{R}{R+\frac{1}{Cs}}=\frac{RCs}{1+RCs}$$

**Sol.24. (b)**
Branch made by $R_1$ and $R_2$ provide negative feedback in wein bridge oscillator.

**Sol.25. (a)**

$$\text{B.W.} = \frac{0.35}{\text{Rise time}}$$

**Sol.26. (d)**

**Sol.27. (a)**
Speed of response is proportional to the bandwidth of the system. Higher the bandwidth faster the response.

**Sol.28. (d)**

$$G(s) = \frac{C(s)}{R(s)} = \frac{K}{s+\alpha}$$

$\Rightarrow C(s) = \dfrac{K}{s+\alpha}$ since R(s) = 1

$\Rightarrow c(t) = Ke^{-\alpha t}$

Time constant $\tau = 1/\alpha$

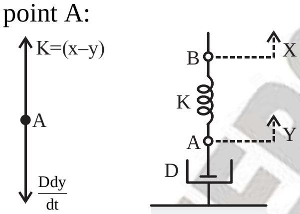
Time constant is the time at which

$c(t) = Ke^{-1} = 0.37\ K$

So, $\tau = t_2 = \dfrac{1}{\alpha} \Rightarrow \alpha = \dfrac{1}{t_2}$

**Sol.29. (c)**

**Sol.30. (a)**
F.B.D. of point A:




so, $K(x-y) = \dfrac{Ddy}{dt}$

Taking Laplace transform

$K\{X(s) - Y(s)\} = DsY(s)$

$\Rightarrow (Ds + K)\, y(s) = KX(s)$

$$\Rightarrow \frac{Y(s)}{X(s)} = \frac{K}{D\left(s+\frac{K}{D}\right)}$$

Thus the pole of the system is at $s = -\dfrac{K}{D}$.

**Sol.31. (b)**
Open loop gain, $A = 10^3$

$$\frac{A}{1+AB} = 100 \text{ or } \frac{10^3}{1+1000\,dB} = 100$$

$1 + 1000B = 10 \Rightarrow B = +9 \times 10^{-3}$

**Sol.32. (b)**

**Sol.33. (c)**

**Sol.34. (b)**

**Sol.35. (a)**
Transfer function is defined for linear systems, so superposition principle is applicable.

**Sol.36. (a)**
Force equation is

$$\frac{Md^2x}{dt^2} + f\frac{dx}{dt} + kx = F$$

Taking Laplace transform

$(Ms^2 + fs + k) \times (s) = F(s)$

Given:

M = 1, k = 1, F(s) = 1

Let f = 0 (Assume)

$\Rightarrow (s^2 + 1) \times (s) = 1$

$$\Rightarrow X(s) = \frac{1}{s^2+1}$$

$\Rightarrow x(t) = \sin t$

**Sol.37. (d)**
(i) When transfer function has at least one pole or zero in the RHS of s-plane, it is called non–minimum phase transfer function.
(ii) When transfer function has no pole or zero in the RHS of s-plane. It is called minimum phase transfer function.

**Sol.38. (d)**
The steps in the operation of a Wheatstone bridge are as follows:

(i) Close $K_b$ (ii) Close $K_g$

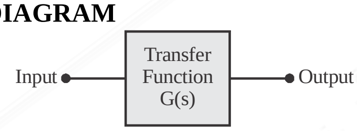
(iii) Open $K_g$ (iv) Open $K_b$

# CHAPTER - 3
# *BLOCK DIAGRAM ALGEBRA*

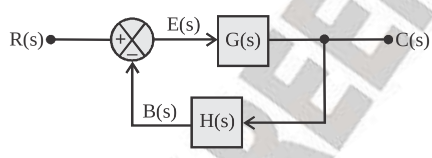
## 3.1 BLOCK DIAGRAM

It is a pictorial representation of function performed by each component and of flow of signals. Such a diagram depicts the inter-relationship that exists among various components differing from a purely abstract mathematical model. The block diagram has the advantage of indicating more realistically the signal flows of actual system.

## 3.2 ELEMENT OF BLOCK DIAGRAM




This represents the elements of a block diagram. The arrow heads pointing towards the block diagram indicate the input and the arrowheads leaving the block represent output. Such arrows are represented to as signal.




**Open Loop Transfer Function**

**1. Open Loop Transfer Function**

B(s) = C(s) H(s)

$$\frac{B(s)}{E(s)} = G(s)\,H(s)$$

**2. Feed Forward Transfer Function**

$$\frac{C(s)}{E(s)} = G(s)$$

If H(s) = 0 then

G(s) = G(s) H(s) $\therefore$ H(s) = 1

**3. Closed Loop Transfer Function**

C(s) = G(s) E(s)

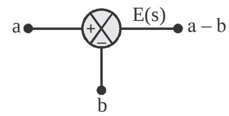
E(s) = R(s) – B(s)

E(s) = R(s) – H(s) C(s)

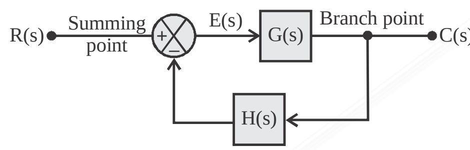
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)}$$

**4. Branch Point**

A branch point is a point from which the signal from the block goes concurrently to other block or summing points.




**Summing point**




**General Block Diagram**

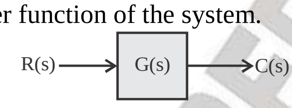
## 3.3 BLOCK DIAGRAM REDUCTION

Some of the important rules for block diagram reduction are given below:

**1. The block diagram** shown below relates the output and input as per the transfer function relations given below;

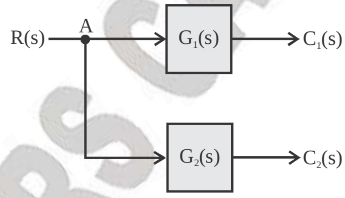
$$G(s) = \frac{C(s)}{R(s)} \quad \text{or} \quad C(s) = R(s).\, G(s)$$

Where G(s) is known as the transfer function of the system.



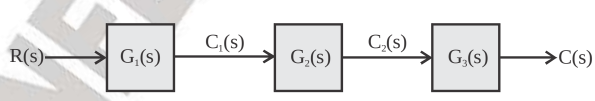

**2. Take off point**

Application of one input source to two or more systems is represented by a take off point as shown at point A in the below figure.




**3. Blocks in cascade**

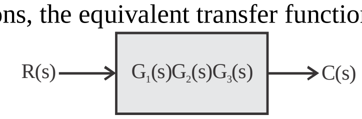
When several blocks are connected in cascade, the overall equivalent transfer function is determined below.




$$\frac{C_1(s)}{R(s)} = G_1(s) \frac{C_2(s)}{C_1(s)} = G_2(s)$$

$$\frac{C(s)}{C_2(s)} = G_3(s)$$

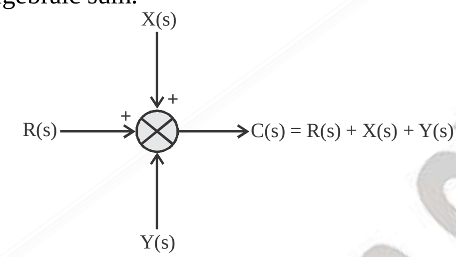
Multiplying above three equations, the equivalent transfer function is



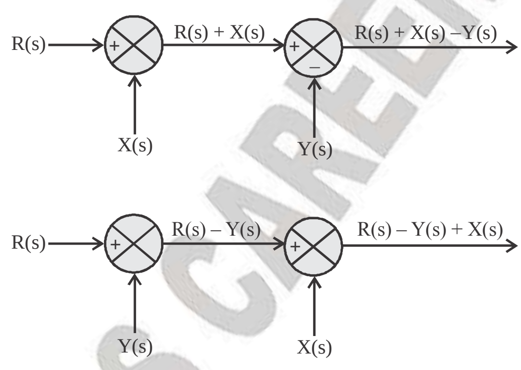
$$\frac{C(s)}{R(s)} = G_1(s)G_2(s)G_3(s)$$

**4. Summing Point**

Summing point represents summation of two or more signal entering in a system. The output of a summing point being the algebraic sum.

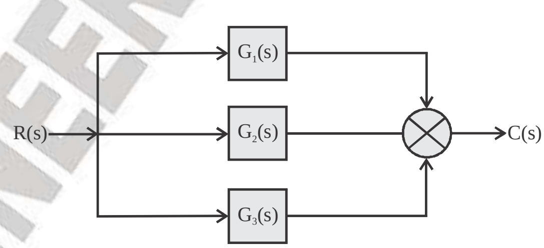



**5. Consecutive summing points can be interchanged, as this interchange does not alter the output signal**




**6. Blocks in Parallel**

When one or more blocks are connected in parallel, the overall equivalent transfer function is determined below



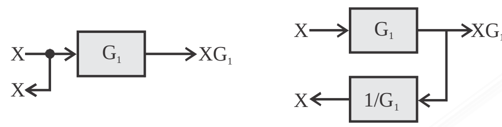

$C(s) = R(s)\,G_1(s) + R(s)\,G_2\,(s) + R(s)\,G_3(s)$

Or $C(s)\ \ R(s)\,[G_1\,(s) + G_2\,(s) + G_3(s)\,]$

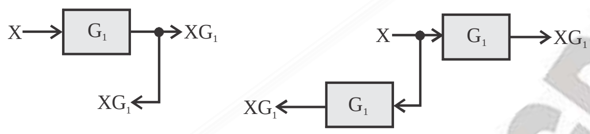
Therefore, the overall equivalent transfer function is,

$$\frac{C(s)}{R(s)} = [G_1(s) + G_2(s) + G_3(s)]$$

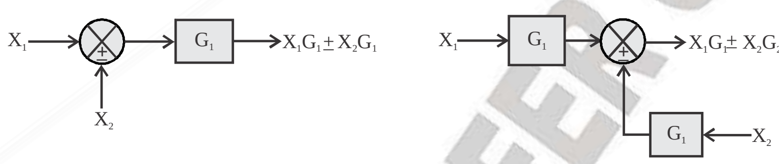
**7. Shifting of a take off point from a position before a block to a position after the block is shown below**



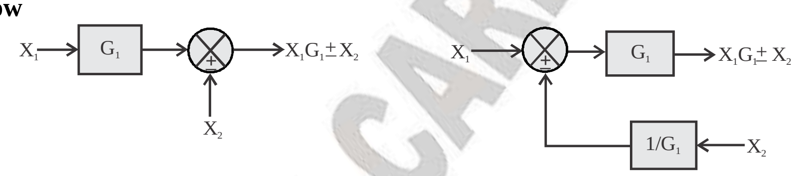

**8. Shifting of a take off point from a position after a block to a position before the block is shown below**



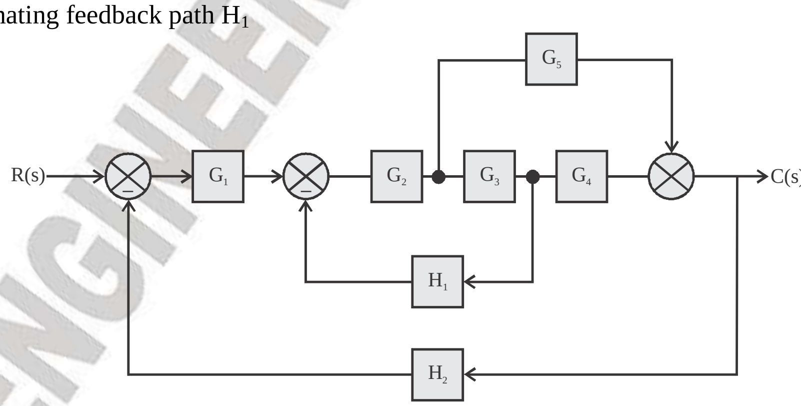

**9. Shifting of a summing point from a position before a block to a position after the block is shown**

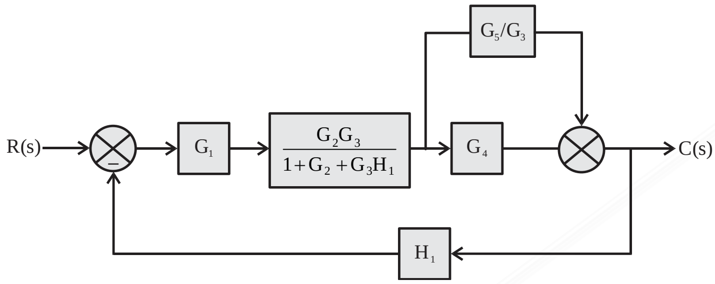


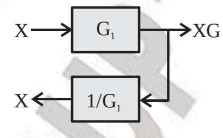

**10. Shifting of a summing point from a position after a block to a position before the block is shown below**

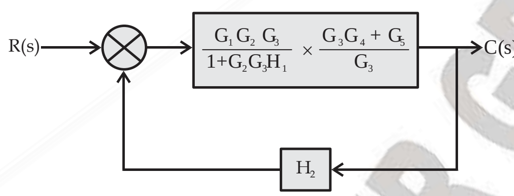


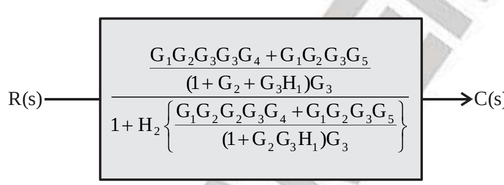

**Example 1.** Reduce the block diagram to its canonical form and obtain C(s) / R(s).
**Solution.**
Shifting take off point towards right as shown, we get
Now eliminating feedback path $H_1$





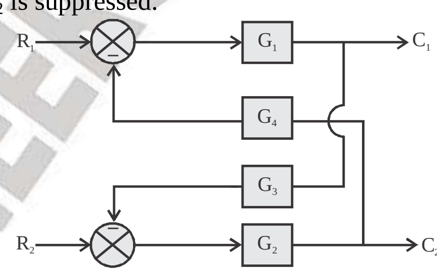



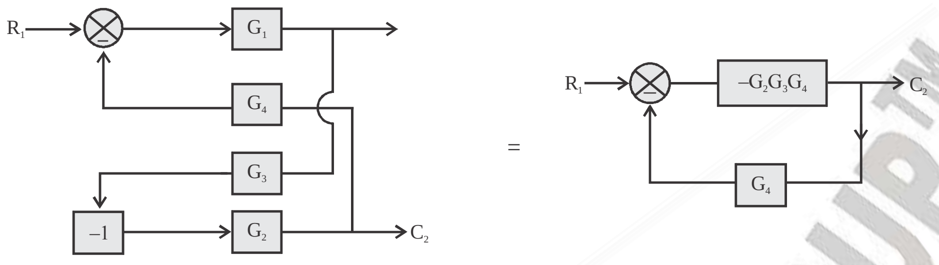

After simplification, $\dfrac{C(s)}{R(s)} = \dfrac{G_1G_2G_3[G_3G_4+G_5]}{1+G_2G_3H_1G_3+G_1G_2G_3H_2[G_3G_4+G_5]}$

**Example 2.** Obtain the expression for $C_1$ and $C_2$ for the given multiple input multiple output system.

**Solution.**

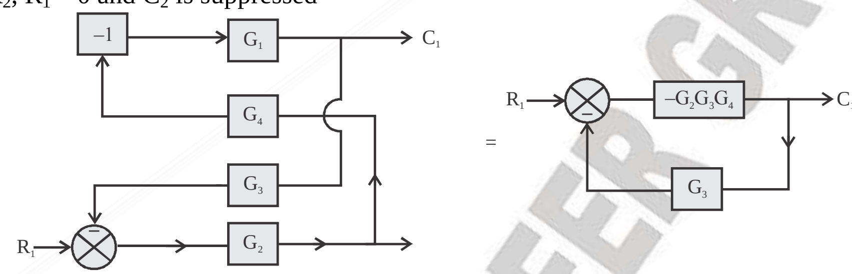
Consider $R_1$ is acting and $R_2$ is suppressed.





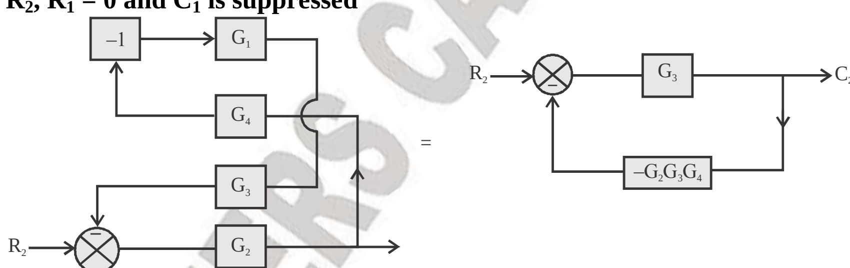

$$\frac{C_2}{R_1}=\frac{-G_2G_1G_3}{1+(1-G_1G_2G_3G_4)}=\frac{-G_1G_2G_3}{1-G_1G_2G_3G_4}\therefore\frac{C_1}{R_2}=\frac{G_1G_2G_3}{1-G_1G_2G_3G_4}$$

For $C_1/R_2$, $R_1 = 0$ and $C_2$ is suppressed




$$\therefore\frac{C_1}{R_2}=\frac{G_1G_2G_4}{[+G_3]-[G_1G_2G_4]}=\frac{G_1G_2G_4}{1-G_1G_2G_3G_4}$$

**For $C_2 / R_2$, $R_1 = 0$ and $C_1$ is suppressed**




$$\therefore\frac{C_2}{R_2}=\frac{G_2}{1+G_2[-G_1G_3G_4]}=\frac{G_2}{1-G_1G_2G_3G_4}$$

## 3.4 SIGNAL FLOW GRAPH METHOD

A signal flow graph may be defined as a graphical means of portraying the input – output relationships between the variables of a set of linear algebraic equations.

### 3.4.1 Basic Properties of Signal Flow Graphs

1. A signal flow graphs applies only to linear systems.
2. The equations based on which a signal flow graph is drawn must be algebraic equations in the form of effects as functions of causes:

3. Nodes are used to represent variable. Normally, the nodes are arranged from left to right, following a successor of causes and effects through the system.
4. Signals travel along branches only in the direction described by the arrows of the branches.

### 3.4.2 Definitions for Signal Flow Graphs

***1. Input Node (Source)***
An input node is a node that has only outgoing branches.

***2. Output Node (Sink)***
An output node is a node which has only incoming branches. For feedback output node is extended by a unity gain signal.

***3. Path***
A path is any collection of a continuous succession of branches traversed in the same direction.

***4. Forward Path***
A forward path is a path that starts at an input node and ends at an output node and along which no node is traversed more than once.

***5. Loop***
A loop is path that originates and terminates on the same node and along which no other node is encountered more than once.

***6. Path Gain***
The product of the branch gains encountered in traversing a path is called the path given.

***7. Forward Path Gain***
Forward path gain is defined as the path gain of a forward path.

***8. Loop gain***
Loop gain is defined as the path gain of a loop.

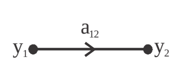
### 3.4.3 Mason Gain Formula

The general gain formula is

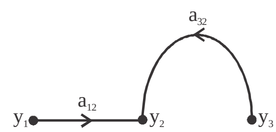
$$T.F = \sum_{k=1}^{N} \frac{P_k \Delta_k}{\Delta}$$

N = total number of forward paths
$P_k$ = gain of the $k^{th}$ forward path
$\Delta$ = 1 – (sum of all individual loop gains) + (sum of gain products of all possible combinations of two non – touching loops) – (sum of the gain products of all possible combinations of three non – touching loops) + ….
$\Delta_k$ = the $\Delta$ for the part of the signal flow graph which is non – touching with the $k^{th}$ forward path.

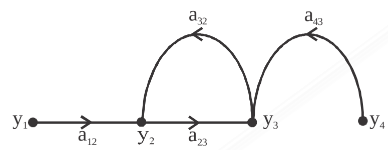
### 3.4.4 Signal Flow Graphical (SFG)




Where $y_1$ is input and $y_2$ is output.
$a_{12}$ is gain of system.

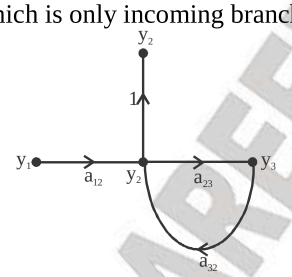



$y_2 = a_{12}y_1 + a_{32}y_2$

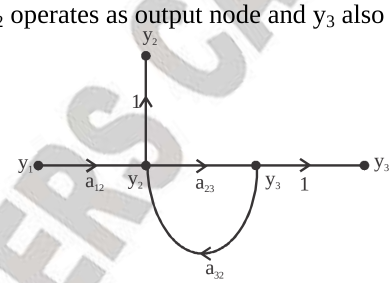


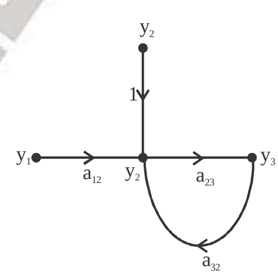

$y_2 = a_{12}y_1 + a_{32}y_3, y_3 = a_{32}y_2 + a_{43}y_4$

1. Signal flow graph is only applicable to linear system
2. Equation for which an SFG is drawn must be algebraic in form of cause and effect relationship.
3. Node used to represent variables. Normally nodes are arranged from left to right succession of cause and effect relationship. Where input node is source which is a node that has only outgoing branches and output node is sink which is only incoming branches.




Modification in SFG so that $y_2$ operates as output node and $y_3$ also as shown below.

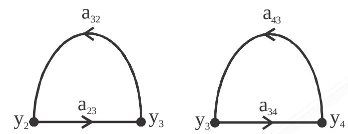


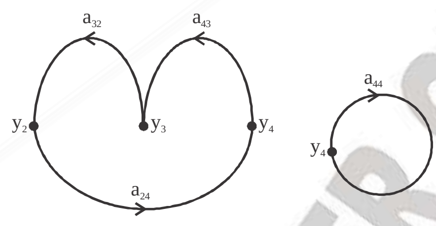

X Erroneous way to make $y_2$ as input
Because $y_2 = a_{12}y_1 + a_{12}y_1 + a_{32}\ y_3 + y_2$ X

***1. Loop***

A loop is a path that originates and terminate on the same node and along which no other node is encountered more than once.

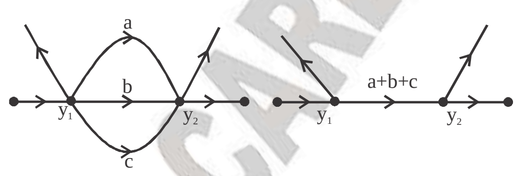



**Loop 1** **Loop 2**



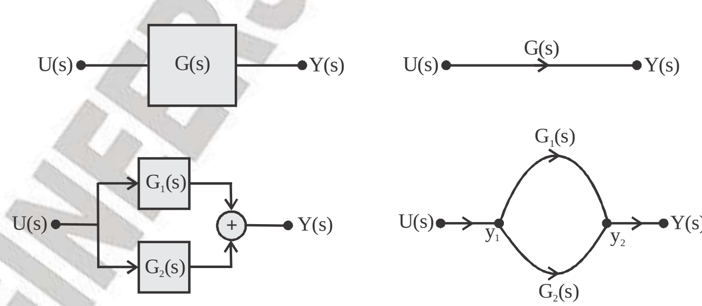

**Loop 3** **Loop 4**

1. Value of variable represented by a node is equal to sum of the entire signal entering the node.
2. Value of variable represented by a node is transmitted through all the branches leaving the nodes.




**Parallel Branches Connected**

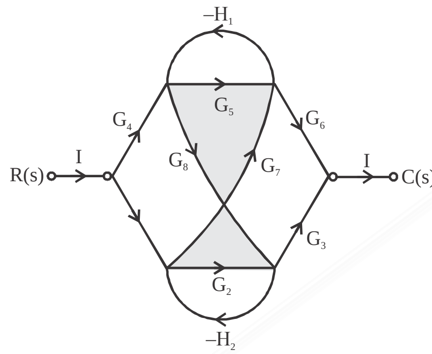
**3.4.5 Simple Transfer Function between two nodes can be added as**




**Example 3.** Using mason's gain formula, find the gain of the following system is figure below:

**Solution.**

The forward path gains are



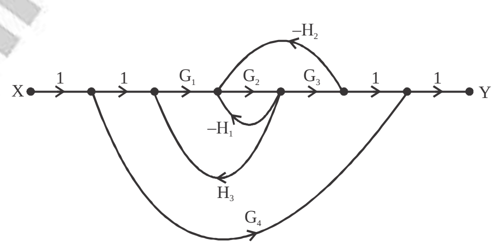

$P_1 = G_1G_2G_3$,
$P_2 = G_4G_5G_6$
$P_3 = G_1G_7G_6$,
$P_4 = G_4G_8G_3$
$P_5 = G_4G_8\,(-H_2)\,G_7G_6$,
$P_6 = G_1G_7\,(-H_1)\,G_8G_3$
The feedback loop gains are,
$L_1 = -G_5H_1$ $\quad L_2 = -G_2H_2$ $\quad L_3 = +\,G_7\,H_1\,G_8\,H_2$
The two non – touching loops are $L_1L_2$
$\therefore\ L_1L_2 = +\,G_5H_1G_2H_2$
$\therefore\ \Delta = 1 - [L_1 + L_2 + L_3] + [L_1\,L_2]$
$1 + G_5H_1 + G_2H_2 - G_7H_8\,H_1H_2 + G_2G_5\,H_1H_2$
For $P_1$, $L_1$ is non – touching
$\Delta_1 - 1\,L_1 = 1 + G_5H_1$
For $P_2$, $L_2$ is non – touching
$\Delta_2 = 1 - L_2 = 1 + G_2H_2$
For $P_3$ to $P_6$ all loops are touching to all forward paths,
$\therefore\ \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$

$$\therefore \text{Gain} = \frac{\sum P_k\Delta_k}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

$G_1G_2G_3\,(1 + G_5H_1) + G_4G_5G_6\,(1 + G_2H_2) +$

$$\text{Gain} = \frac{G_1G_7G_6 + G_4G_8G_3 - G_4G_8G_7G_6H_2 - G_1G_3G_7G_8H_1}{1 + G_5H_1 + G_2H_2 - G_7H_8H_1H_2 + G_2G_5H_1H_2}$$

**Example 4.** For the signal flow graph given below, find the transfer function.
**Forward Paths** $P_1 = G_1G_2G_3$
$P_2 = G_4$
**Loops** $L_1 = -\,G_2\,H_1$



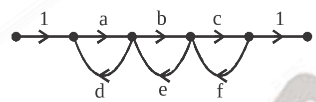
$L_2 = -G_2 G_3 H_2$
$L_3 = G_1 G_2 H_2$
$\Delta = 1 - (L_1 + L_2 + L_3)$
$= 1 + G_2 H_1 + G_2 G_3 H_2 - G_1 G_2 H_3$

$$\therefore T = \frac{Y}{X} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$T = \frac{Y}{X} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

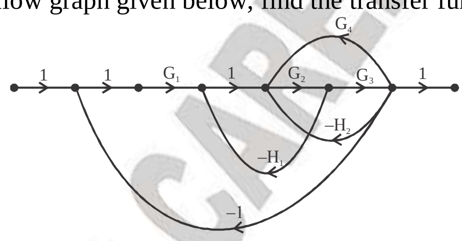
Where $\Delta_1 = 1$ , $\Delta_2 = 1 + G_2 H_1 + G_2 G_3 H_2 - G_2 G_3 H_2 - G_1 G_2 H_3$

**Example 5.** For the signal flow graph given below, find the transfer function.



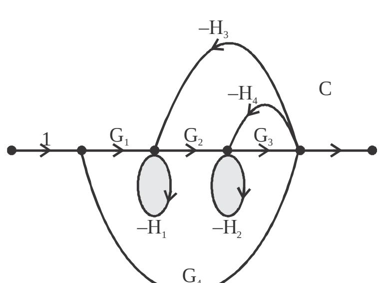

$$T = \frac{abc}{1-(ad+bc+cf)+(adcf)}$$

**Example 6.** For the signal flow graph given below, find the transfer function.




$P_1 = G_1 G_2 G_3$
$P_2 = G_1 G_4$
$L_1 = -G_1 G_2 H_1$
$L_2 = -G_1 G_2 G_3$
$L_3 = -G_2 G_3 H_2$
$L_4 = -G_4 H_2$
$L_5 = -G_1 G_4$
$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5)$
$\Delta_1 = \Delta_2 = 1$

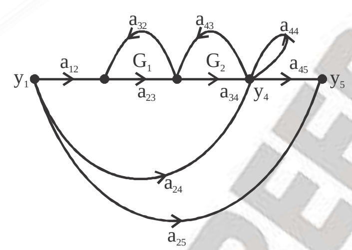
**Example 7.** For the signal flow graph given below, find the transfer function.



$P_1 = G_1 G_2 G_3 \quad L_1 = -H_1$
$P_2 = G_4 \quad L_2 = -H_2$
$L_3 = -G_3 H_4$
$L_4 = -G_2 G_3 H_3$
$N_1 = H_1 H_2$
$N_2 = H_1 \times (H_4 G_3)$
$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (N_1 + N_2)$
$\Delta_1 = 1$
$\Delta_2 = 1 + H_1 + H_2 + H_1 H_2$

**Example 8.** Draw the signal flow graph for the following equations. $Y_2$ is the input node and $y_5$ is the output node and find the transfer function.

$y_2 = a_{42} y_1 + a_{32} y_3 \quad y_3 = a_{23} y_2 + a_{43} y_4$
$y_4 = a_{24} y_3 + a_{34} y_3 + a_{44} y_4 \quad y_5 = a_{25} y_2 + a_{45} y_4$
$P_1 = a_{12} a_{23} a_{34} a_{45}$

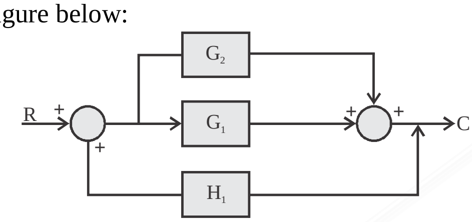



$P_2 = a_4 a_{45}$
$P_3 = a_{25}$
$L_1 = a_{23} a_{32}$
$L_2 = a_{34} a_{43}$
$L_3 = a_{44}$
$N_1 = a_{23} a_{32} a_{44}$
$\Delta = 1 - (L_1 + L_2 + L_3) + N_1$
$\Delta_1 = 1; \quad \Delta_2 = 1 - L_1$
$\Delta_3 = \Delta$

$$\therefore TF = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$$

# ASSIGNMENT

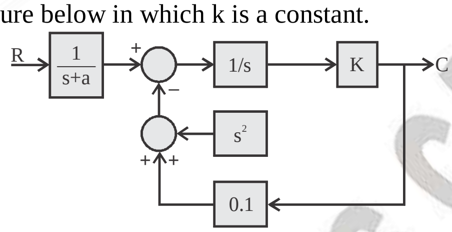
**1.** Determine C/R from the system shown in figure below:




(a) $\dfrac{G_1+G_2}{1-G_1H_1+G_2H_1}$ (b) $\dfrac{G_1}{1-G_1H_1}$

(c) $\dfrac{G_1+G_2}{1-G_1H_1-G_2H_1}$ (d) None

**2.** Find the transfer C/R for the system shown in figure below in which k is a constant.



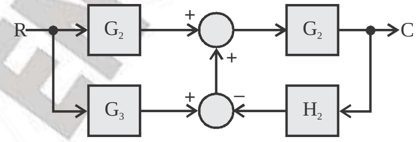

(a) $\dfrac{1}{(s+a)(s^2+s+0.1k)}$

(b) $\dfrac{k}{(s+a)(s^2+s+0.1k)}$

(c) $\dfrac{k}{(s+a)(s^2+s-0.1k)}$

(d) None of these

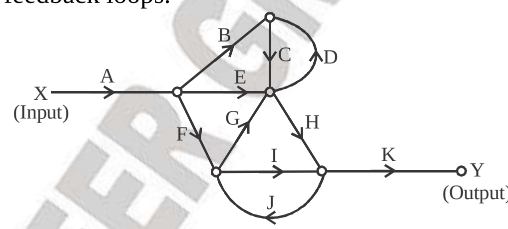
**3**. Determine C/R for the system given in figure below. Then put $G_3 = G_1\,G_2\,H_2$. Now the new transfer function will be:




(a) $\dfrac{G_1G_2+G_2G_3}{1+G_2H_2}$ (b) $\dfrac{G_1G_2+G_2G_3}{1-G_2H_2}$

(c) $G_1\,G_3$ (d) $G_1\,G_2$

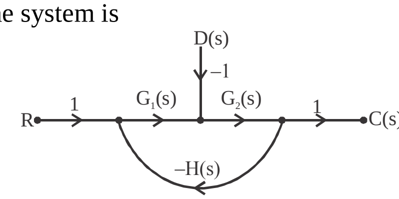
**4.** The signal flow graph of figure shown below has ________ forward paths and ________ feedback loops.




(a) (4, 4) (b) (4, 5)

(c) (4, 3) (d) (3, 3)

**5.** The signal flow graph of the system is shown in the given figure. The transfer function $\dfrac{C(s)}{D(s)}$ of the system is



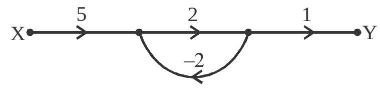

(a) $\dfrac{G_1(s)G_2(s)}{1+G_1(s)H(s)}$

(b) $\dfrac{G_1(s)G_2(s)}{1-G_1(s)G_2(s)H(s)}$

(c) $\dfrac{G_2(s)}{1+G_1(s)G_2(s)H(s)}$

(d) $\dfrac{-G_2(s)}{1-G_1(s)G_2(s)H(s)}$

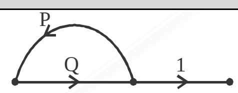
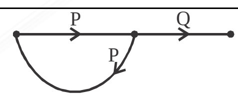
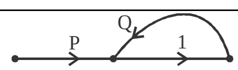
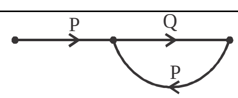
**6.** In the signal flow graph of figure y/x equal.




(a) 3 (b) 5/2
(c) 2 (d) None

**7.** Signal flow graph is used to find:
(a) Stability of the system
(b) Controllability of the system
(c) Poles of the system
(d) Transfer function of the system
(e) All of above

**8**. Match List-I (SFG) with List-II (Transfer function) and select the correct answer using the codes given below the lists:

| List-I | | List-II | |
|---|---|---|---|
| A. |  | (i) | $\frac{P}{1-Q}$ |
| B. |  | (ii) | $\frac{Q}{1-PQ}$ |
| C. |  | (iii) | $\frac{PQ}{1-PQ}$ |
| D. |  | (iv) | $\frac{PQ}{1-P^2}$ |

**9**. With a negative feedback, the system gain and stability:
(a) Decrease, increase
(b) Increase, decrease
(c) Increase, increase
(d) Decrease, decrease

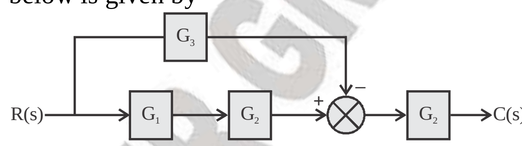
**10.** A positive feedback signal improves the performance of automatic control system.
(a) False
(b) True
(c) Can't be determined
(d) Data insufficient

**11** The transfer function $\frac{C(s)}{R(s)}$ of a system in regenerative feedback is given by

(a) $\frac{G(s)}{1+G(s)H(s)}$ (b) $\frac{G(s)H(s)}{1+G(s)H(s)}$

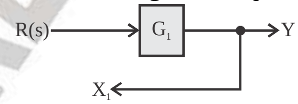
(c) $\frac{G(s)}{1-G(s)H(s)}$ (d) $\frac{G(s)H(s)}{1-G(s)}$

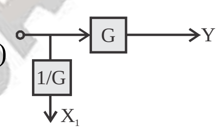
**12.** The total gain $\frac{C(s)}{R(s)}$ of the system shown below is given by

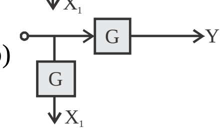


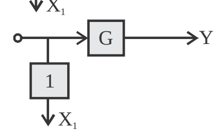

(a) $G_1 G_2 G_3 + G_4$ (b) $G_1 G_2 + G_3 + G_4$
(c) $(G_1 G_2 – G_3) G_4$ (d) $G_1 G_2 G_4 + G_3$

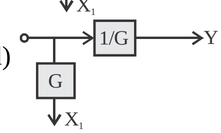
**13**. The given block diagram is equivalent to



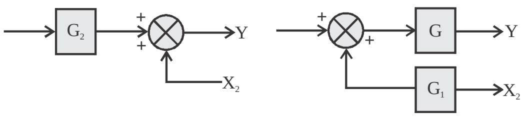

(a)



(b)



(c)



(d)



**14.** Two equivalent block diagrams are shown below $G_1$ is equal to




(a) 1/G (b) G
(c) 1 (d) None

**15.** Feedback control system is basically.
(a) High pas filter (b) Band pass filter
(c) Low pass filter (d) Band stop filter

**16.** The block diagram shown in figure (a) and (b) are equivalent if 'x' in figure (b) is equal to:

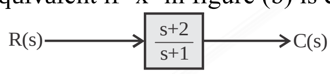



**Figure (A)**

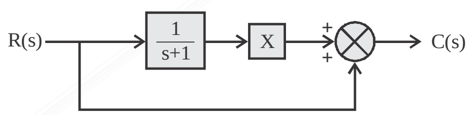
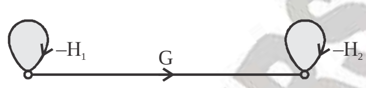



**Figure (B)**

(a) 1 (b) 2
(c) s + 1 (d) s + 2

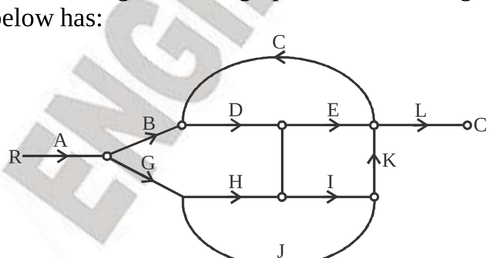
**17.** For the signal flow graph shown in figure below, the overall transfer function of the system will be:




(a) G (b) $\dfrac{G}{1+H_2}$

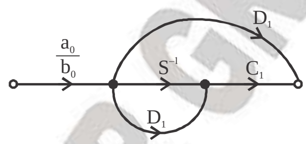
(c) $\dfrac{G}{1+H_1+H_2}$ (d) $\dfrac{G}{1+H_1+H_2}$

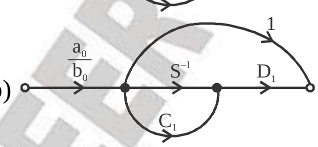
**18**. The signal-flow graph shown in figure below has:

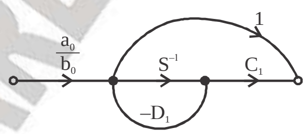



(a) Three forward paths and two non-touching loops
(b) Three forward paths and three loops
(c) Two forward paths and two non-touching loops
(d) Two forward paths and three loops

**19.** The represents of $\dfrac{C(s)}{R(s)} = \dfrac{a_0(s+c_1)}{b_0(s+d_1)}$ in state diagram will be:

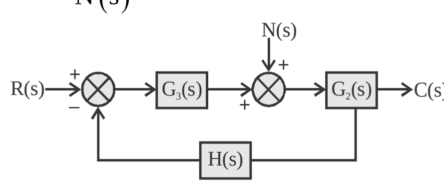
(a) 


(b) 


(c) 


(d) None

**20.** The closed-loop system shown in the figure is subjected to a disturbance N(s). the transfer function $\dfrac{C(s)}{N(s)}$ is



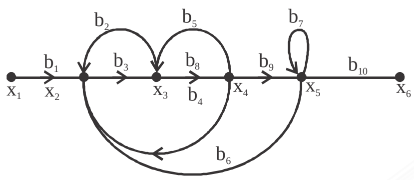

(a) $\dfrac{G_1(s)+G_2(s)}{1+G_1(s)G_2(s)H(s)}$

(b) $\dfrac{G_1(s)}{1+G_1(s)H(s)}$

(c) $\dfrac{G_2(s)}{1+G_2(s)H(s)}$

(d) $\dfrac{G_2(s)}{1+G_1(s)G_2(s)H(s)}$

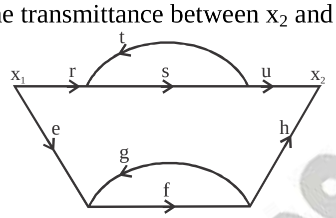
**21.** A signal flow graph is shown in the following figure: Consider the following statements regarding the signal flow graph:




1. There are three forward paths.
2. There are three individual loops.
3. There are two non-touching loops.

Of these statements
(a) 1, 2 and 3 are correct
(b) 1 and 2 are correct
(c) 2 and 3 are correct
(d) 1 and 3 are correct

**22.** For the signal flow diagram shown in the given, the transmittance between $x_2$ and $x_1$ is

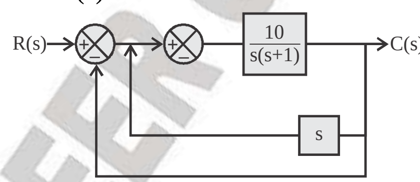



(a) $\dfrac{rsu}{1-st}+\dfrac{efh}{1-fg}$ (b) $\dfrac{rsu}{1-fg}+\dfrac{efh}{1-st}$

(c) $\dfrac{efh}{1-ru}+\dfrac{rsu}{1-eh}$ (d) $\dfrac{rst}{1-eh}+\dfrac{rsu}{1-st}$

**23.** Signal flow graph is used to find.
(a) Stability of the system.
(b) Controllability of the system.
(c) Transfer function of the system
(d) Poles of the system.

**24.** For the system shown in fig. the transfer function $\dfrac{C(s)}{R(s)}$ is equal to :

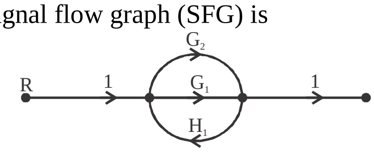



(a) $\dfrac{10}{s^2+s+10}$ (b) $\dfrac{10}{s^2+11s+10}$

(c) $\dfrac{10}{s^2+9s+10}$ (d) $\dfrac{10}{s^2+2s+10}$

# SOLUTIONS

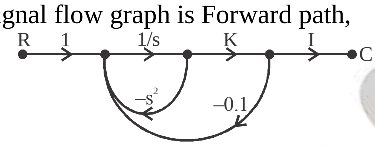
**Sol. 1.**
The signal flow graph (SFG) is




The two forward path gains are
$P_1 = G_1$ and $P_2 = G_2$
The two feedback loop gains are
$L_1 = G_1H_1$ and $L_2 = G_2H_1$
There are no non-touching loops
$\Delta = 1 - G_1H_1 - G_2H_1$
$\Delta_1 = 1$ and $\Delta_2 = 1$

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1 + G_2}{1 - G_1H_1 - G_2H_2}$$

**Sol. 2.**
The signal flow graph is Forward path,




$$P_1\left(\frac{1}{s+a}\right).\left(\frac{1}{s}\right)k = \frac{k}{s(s+a)}$$

The two feedback loop gains are

$$L_1 = -s \text{ and } L_2 - \frac{0.1k}{s}$$

There are no non touching loops.

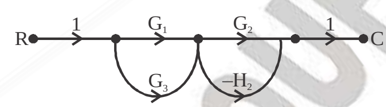
$$\Delta = 1 + s + \frac{0.1k}{s} = \frac{s^2 + s + 0.1k}{s}$$

$\Delta_1 = 1$

$$\frac{C}{R} = \frac{P_1\Delta_1}{\Delta} = \frac{k.s}{s(s+a)(s^2+s+0.1k)}$$

$$= \frac{k}{(s+a)(s^2+s+0.1k)}$$

**Sol. 3.**
The signal flow graph is




Forward path are:
$P_1 = G_1G_2$ and $P_2 = G_2G_3$
Individual loop is,
$L_1 = -G_2H_2$
$\Delta = 1 + G_2H_2$
$\Delta_1 = 1$ and $\Delta_2 = 1$

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2 + G_2G_3}{1 + G_2H_2}$$

$G_3 = G_1G_2H_2$

$$\left.\frac{C}{R}\right| = \frac{G_1G_2 + G_2G_1G_2H_2}{1 + G_2H_2}$$

$$= \frac{G_1G_2(1+G_2H_2)}{(1+G_2H_2)} = G_1G_2$$

**Sol. 4.**
The forward paths are:
$P_1 = a\ f\ i\ k$ $\quad P_3 = a\ b\ c\ h\ k$
$P_2 = a\ e\ h\ k$ $\quad P_4 = a\ f\ g\ h\ k$
So, total number of forward path = 4
Individual loops are:
$L_1 = cd$
$L_2 = ij$
$L_3 = ghj$
So, total number of individual loops = 3.

**Sol. 5.**
To find $\frac{C(s)}{D(s)}$ put another input R(s) = 0
Forward path, $P_1 = -G_2(s)$
Individual path,
$L_1 = -G_1(s)\ G_2(s)\ H(s)$
$\Delta = 1 + G_1(s)\ G_2(s)\ H(s)$
$\Delta_1 = 1$

$$\frac{C(s)}{D(s)} = \frac{P_1\Delta_1}{\Delta} = \frac{-G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

**Sol. 6.**

Forward path, $P_1 = 5 \times 2 = 10$

$L_1 = -2 \times 2 = -4$

$\Delta = 1 + 4 = 5, \Delta_1 = 1$

$$\frac{y}{x} = \frac{P_1\Delta_1}{\Delta} = \frac{10}{5} = 2$$

**Sol. 7. (e)**

**Sol. 8 . (*)**

A-ii, B-iv, C-i, D-iii

**A.** $\frac{Q}{1-PQ} = \text{T.F.}$

Mason Gain Formula

$$\text{T.F.} = \frac{\Delta_k P_k}{\Delta}$$

$P_k$ = No. if forward path in case (A)

$P_1 = Q$

$\Delta = 1-[L_1] = 1 - PQ$

$\Delta_1 = 1$

$$\text{T.F.} = \frac{Q}{1-PQ}$$

**B.** Similarly using Mason Gain

$\text{T.F.} = \frac{PQ}{1-P^2}$; $P_1$ = Forward Path = PQ

$\Delta = 1 - [L_1] = 1-P^2$

**C.** Similarly using Mason Gain

$\text{T.F.} = \frac{P}{1-Q}$; $P_1$ = Forward path = P

$\Delta = 1-[L_1] = 1-Q$

**D.** Similary using Mason Gain

$$\text{T.F.} = \frac{P}{1-PQ}$$

$P_1$ = Forward path = PQ

$\Delta = 1 - [L_1] = 1 - PQ$

**Sol. 9 . (a)**

$$A_{CL} = \frac{A_{OL}}{1+AB}$$

$A_{CL} < A_{OL}$

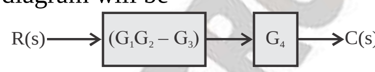
**Sol. 10 . (c)**

**Sol. 11.**

The transfer function $\frac{C(s)}{R(s)}$ of a system in regenerative feedback $= \frac{G(s)}{1-G(s)|-|(s)}$

**Sol. 12.**

G1 and G2 are series cascade andG3is in parallel with negative sign, then the reduced block diagram will be




$$\frac{C(s)}{R(s)} = (G_1G_2 - G_3)G_4$$

**Sol. 13.**

It is the rule to move take-off point before a block.

**Sol. 14. (a)**

$y = xG + x_2$

$y=(x+x_2G_1)G = G_x + G_1G_{x2}$

$G_1 = 1/G$

**Sol. 15.**

Feedback control system is basically low-pass filter.

**Sol. 16.**

In figure

(a) $\frac{C(s)}{R(s)} - \frac{s+2}{s+1}$ and in figure

(b) $\frac{C(s)}{R(s)} = \frac{x}{s+1} + 1$

Both are equivalent so, $\frac{x}{s+1} + 1 = \frac{s+2}{s+1}$

$$\frac{x}{s+1} = \frac{s+2}{s+1} - 1 \quad \therefore \frac{x}{s+1} = \frac{s+2}{s+1} - 1 \quad \therefore X = 1$$

**Sol. 17.**

Forward path, $P_1 = G$

$L_1 = -H_1$ $L_2 = -H_2$

Non-touching loop = $(-H_1)(-H_2) = H_1 H_2$

$\Delta = 1 + H_1 + H_2 + H_1 H_2 = (1 + H_1)(1 + H_2)$

$\Delta_1 = 1$

$$= \frac{C}{R} = \frac{G}{(1+H_1)(1+H_2)}$$

**Sol.18.**

Forward paths are:

$P_1 = a\ b\ d\ e\ l$

$P_2 = a\ g\ h\ I\ k\ l$

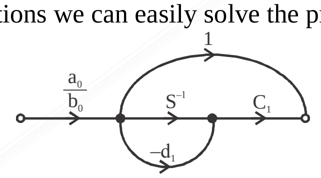
$P_3 = a\ b\ d\ f\ I\ k\ l$

$L_1 = cde$ and $L_2 = h\ i\ j$

$L_3 = fikcd$

**Sol.19.**

From options we can easily solve the problem




Forward paths are:

$$P = \frac{a_0}{b_0}, P_2 = \frac{a_0}{b_0}\frac{c_1}{s}$$ Individual loop,

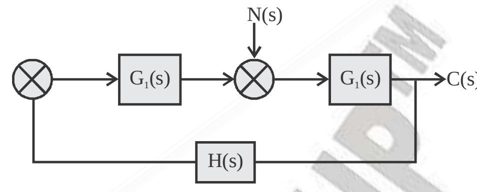
$$L_1 - \frac{d_1}{s}$$

$$\Delta = 1 + \frac{d_1}{s}\ \Delta_1 = 1, \Delta_2 = 1$$

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{\frac{a_0}{b_0}\left(1+\frac{c_1}{s}\right)}{\left(1+\frac{d_1}{s}\right)} = \frac{a_0(s+c_1)}{b_0(s+d_1)}$$

**Sol. 20.**

To find $\frac{C(s)}{N(s)}$ put $R(s) = 0$




Forward path gain = $G_2(s)$

Feedback path gain = $-G_1(s)\ H(s)$

$$\frac{C(s)}{N(s)} = \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)}$$

**Sol. 21. (d)**

**Sol. 22. (a)**

$$\frac{X_2}{X_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$P_1 = rsu,\ \Delta_1 = 1 - fg$

$P_2 = efh,\ \Delta_s = 1 - st,\ \Delta = 1 - fg - st + fgst$

$$\frac{X_2}{X_1} = \frac{rsu(1-fg)+efh(1-st)}{1-fg-st+fgst}$$

$$= \frac{rsu(1-fg)+efh(1-st)}{(1-fg)(1-st)}$$

$$\frac{X_2}{X_1} = \frac{rsu}{1-st} + \frac{efh}{1-fg}$$

**Sol. 23. (c)**

Signal flow graph is used to find the transfer function of the system.

**Sol. 24. (b)**

By using Mason's gain formula

$$\frac{Y(s)}{R(s)} = \frac{\frac{10}{s(s+1)}}{1-\left(\frac{10}{s(s+1)} - \frac{10s}{s(s-1)}\right)}$$

$$\frac{Y(s)}{R(s)} = \frac{10}{s^2+11s+10}$$

# GATE QUESTIONS

**1.** Let a causal LTI system be characterized by the following differential equation, with initial rest condition

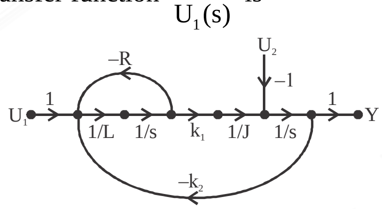
$$\frac{d^2y}{dt^2}+7\frac{dy}{dt}+10y(t)=4x(t)+5\frac{dx(t)}{dt}$$

Where, x(t) and y(t) are the input and output respectively. The impulse response of the system is (u(t) is the unit step function)

**[GATE - 2017]**

(a) $2e^{-2t}\,u(t)\;-7e^{-5t}\,u(t)$
(b) $-2e^{-2t}\,u(t)+7e^{-5t}\,u(t)$
(c) $7e^{-2t}u(t)-2e^{-5t}\,u(t)$
(d) $-7e^{-2t}\,u(t)+2e^{-5t}\,u(t)$

**2.** In the system whose signal flow graph is shown in the figure. $U_1(s)$ and $U_2(s)$ are inputs. The transfer function $\frac{Y(s)}{U_1(s)}$ is




**[GATE - 2017]**

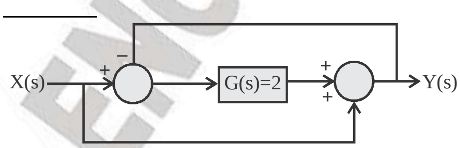
(a) $\dfrac{k_1}{JLs^2+JRs+k_1k_2}$

(b) $\dfrac{k_1}{JLs^2-JRs-k_1k_2}$

(c) $\dfrac{k_1-U_2(R+sL)}{JLs^2+(JR-U_2L)s+k_1k_2-U_2R}$

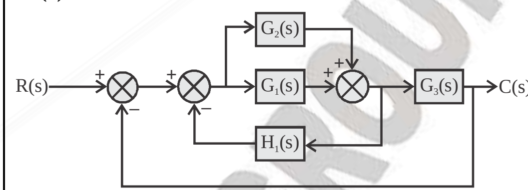
(d) $\dfrac{k_1-U_2(sL-R)}{JLs^2-(JR+U_2L)s-k_1k_2+U_2R}$

**3.** For the system shown in the figure, Y(s)/X(s) = ________




**[GATE - 2017]**

**4.** The overall closed loop transfer function $\frac{C(s)}{R(s)}$, represented ion the figure, will be




**[GATE - 2017]**

(a) $\dfrac{(G_1(s)+G_2(s))G_3(s)}{1+(G_1(s)+G_2(s))(H_1(s)+G_3(s))}$

(b) $\dfrac{(G_1(s)+G_3(s))}{1+(G_1(s)H_1(s)+G_2(s)G_3(s)}$

(c) $\dfrac{(G_1(s)-G_2(s))H_1(s)}{1+(G_1(s)+G_3(s))(H_1(s)+G_1(s))}$

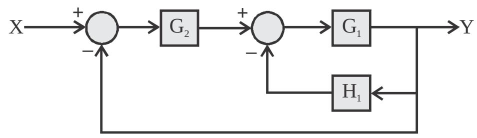
(d) $\dfrac{G_1(s)G_2(s)H_1(s)}{1+G_1(s)H_1(s)+G_1(s)G_3(s)}$

**5.** Match the inferences X, Y, and Z, about a system, to the corresponding properties of the elements of first column in Routh's Table of the system characteristic equation.
X: The system is stable
Y: The system is unstable
Z: The test breaks down
P: When all elements are positive
Q: When any on element is zero
R:When there is a change in sign of coefficients

**[GATE - 2016]**

(a) X→P, Y→Q, Z→R
(b) X→Q,Y→P, Z→R
(c) X→R, Y→Q, Z→P
(d) X→P, Y→R, Z→Q

**6.** The block diagram of a feedback control system is shown in the figure. The overall closed-loop gain G of the system is




**[GATE - 2016]**

(a) $G = \dfrac{G_1G_2}{1+G_1H_1}$

(b) $G = \dfrac{G_1G_2}{1+G_1G_2+G_1H_1}$

(c) $G = \dfrac{G_1G_2}{1+G_1G_2H_1}$

(d) $G = \dfrac{G_1G_2}{1+G_1G_2+G_1G_2H_1}$

**7.** By performing cascading and/or summing/differencing operations using transfer function blocks $G_1(s)$ and $G_2(s)$, one CANNOT realize a transfer function of the form

**[GATE - 2015]**

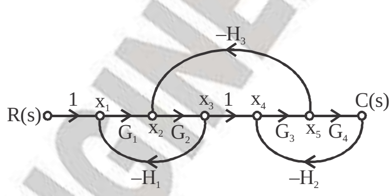
(a) $G_1(s)G_2(s)$

(b) $\dfrac{G_1(s)}{G_2(s)}$

(c) $G_1(s)\left(\dfrac{1}{G_1(s)}+G_2(s)\right)$

(d) $G_1(s)\left(\dfrac{1}{G_1(s)}-G_2(s)\right)$

**8.** For the signal flow graph shown in the figure, the value of $\dfrac{C(s)}{R(s)}$ is




**[GATE - 2015]**

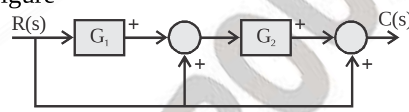
(a) $\dfrac{G_1G_2G_3G_4}{1-G_1G_2H_1-G_2G_4H_2-G_2G_3H_3+G_1G_2G_3G_4H_1H_2}$

(b) $\dfrac{G_1G_2G_3G_4}{1+G_1G_2H_1+G_3G_4H_2+G_2G_3H_3+G_1G_2G_3G_4H_1H_2}$

(c) $\dfrac{1}{1+G_1G_2H_1+G_3G_4H_2+G_2G_3H_3+G_1G_2G_3G_4H_1H_2}$

(d) $\dfrac{1}{1-G_1G_2H_1-G_3G_4H_2-G_2G_3H_3+G_1G_2G_3G_4H_1H_2}$

**9.** Consider the following block diagram in the figure



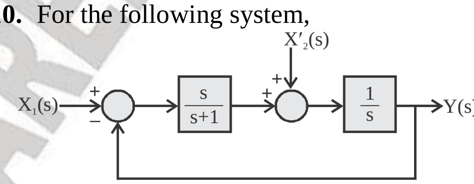

**[GATE - 2014]**

The transfer function $\dfrac{C(s)}{R(s)}$ is

(a) $\dfrac{G_1G_2}{1+G_1G_2}$ (b) $G_1G_2 + G_1 + 1$

(c) $G_1G_2 + G_2 + 1$ (d) $\dfrac{G_1}{1+G_1G_2}$

**10.** For the following system,




When $X_1(s) = 0$, the transfer function $\dfrac{Y(s)}{X_2(s)}$ is

**[GATE - 2014]**

(a) $\dfrac{s+1}{s^2}$ (b) $\dfrac{1}{s+1}$

(c) $\dfrac{s+2}{s(s+1)}$ (d) $\dfrac{s+1}{s(s+2)}$

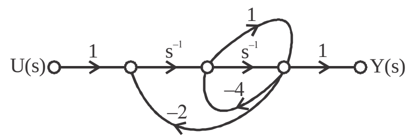
**11.** A system with the open loop transfer function

$$G(s) = \frac{K}{s(s+2)(s^2+2s+2)}$$

is connected in a negative feedback configuration with a feedback gain of unity. For the closed loop system to be marginally stable, the value of K is _______________.

**[GATE - 2014]**

**12.** The signal flow graph for a system is given below. The transfer function $\frac{Y(s)}{U(s)}$ for this system is

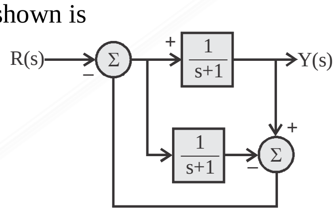



**[GATE - 2013]**

(a) $\frac{s+1}{5s^2+6s+2}$ (b) $\frac{s+1}{s^2+6s+2}$

(c) $\frac{s+1}{s^2+4s+2}$ (d) $\frac{1}{5s^2+6s+2}$

**13.** The transfer function Y(s)/R(s) of the system shown is

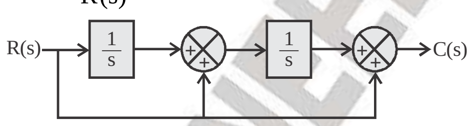



**[GATE - 2010]**

(a) 0 (b) $\frac{1}{s+1}$

(c) $\frac{2}{s+1}$ (d) $\frac{2}{s+3}$

**14.** For the block diagram shown, the transfer function $\frac{C(s)}{R(s)}$ is equal to

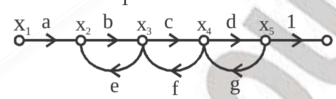



**[GATE - 2004]**

(a) $\frac{s^2+1}{s^2}$ (b) $\frac{s^2+s+1}{s^2}$

(c) $\frac{s^2+s+1}{s}$ (d) $\frac{1}{s^2+s+1}$

**15.** Consider the signal flow graph shown in figure. The gain $\frac{X_5}{X_1}$ is




**[GATE - 2004]**

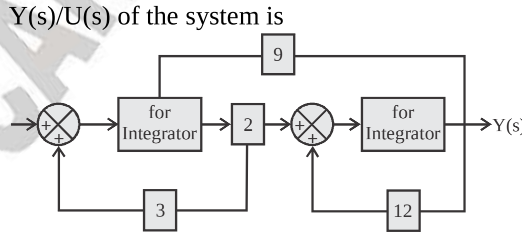
(a) $\frac{1-(be+cf+dg)}{abcd}$

(b) $\frac{bedg}{1-(be+cf+dg)}$

(c) $\frac{abcd}{1-(be+cf+dg)+bedg}$

(d) $\frac{1-(be+cf+dg)+bedg}{abcd}$

**16.** The block diagram of a control system is shown in fig. The transfer function G(s) = Y(s)/U(s) of the system is




**[GATE - 2003]**

(a) $\frac{1}{18\left(1+\frac{s}{6}\right)\left(1+\frac{s}{9}\right)}$ (b) $\frac{1}{27\left(1+\frac{s}{6}\right)\left(1+\frac{s}{9}\right)}$

(c) $\frac{1}{27\left(1+\frac{s}{12}\right)\left(1+\frac{s}{9}\right)}$ (d) $\frac{1}{27\left(1+\frac{s}{9}\right)\left(1+\frac{s}{3}\right)}$

# SOLUTIONS

**Sol. 1. (b)**
Taking the laplace transform

$$TF = \frac{Y(s)}{X(s)} = \frac{5s+4}{s^2+7s+10}$$

$$= \frac{5s+4}{(s+2)(s+5)} = \frac{-2}{s+2} + \frac{7}{s+5}$$

IR = $L^{-1}$ [TF] = $-2e^{-2t}$ u(t) + $7e^{-5t}$ u(t)

**Sol. 2. (d)**

$$\angle\left(\frac{1-j\omega}{1+j\omega}\right) = \angle(-\tan^{-1}\omega - \tan^{-1}\omega)$$

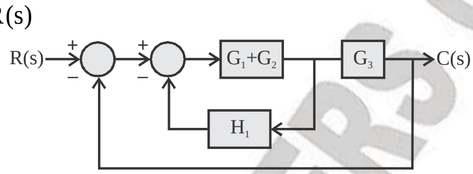
At ω = 0, = 0° (Maximum)
At ω = ∞, ϕ = –180° (Minimum)

**Sol. 3. (1)**

$$\frac{Y(s)}{X(s)} = \frac{2+1}{1+2} = 1$$

**Sol. 4. (a)**

$$\frac{C(s)}{R(s)}$$

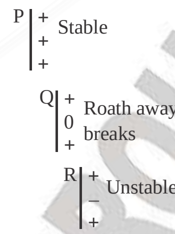



$$\frac{C(s)}{R(s)} = \frac{G_3(G_1+G_2)}{[1-(-H_1(G_1+G_2)-G_3(G_1+G_2))]}$$

$$= \frac{G_3(G_1+G_2)}{1+H_1(G_1+G_2)+G_3(G_1+G_2)}$$

$$= \frac{G_3(G_1+G_2)}{1+(G_1+G_2)(1+1+G_3)}$$

**Sol. 5. (d)**
X-P, Y-R, Z-Q




**Sol. 6. (b)**
From block diagram

$$\frac{Y(s)}{X(s)} = G(s) = \frac{G_1G_2}{1+G_1H_1+G_1G_2}$$

**Sol. 7. (b)**

$$\frac{G_1(s)}{G_2(s)}$$

**Sol. 8. (b)**

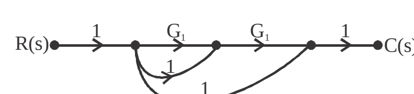
$$\frac{P(s)}{R(s)} = \sum_{k=1}^{n} \frac{P_k\Delta}{\Delta}$$

$P_K$ = Gain of forward path, $P_1 = G_1G_2G_3G_4$
$D_K$ = 1 (sum of all individual loop gain not touching to $k^{th}$ forward parts
Loop 1 ⇒ $L_1 = -G_1G_2H_1$ Non Touching loops+ ……
$L_2 = -G_2G_3H_3$ ↳$L_1L_3$
$= G_1G_2G_3G_4H_1H_2$, $L_3 = -G_3G_4H_2$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4(1-0)}{1-(-G_1G_2H_1 = G_2G_3H_3 - G_3G_4H_2) + G_1G_2G_3G_4H_1H_2}$$

$$\Rightarrow \frac{G_1G_2G_3G_4}{1+G_1G_2H_1+G_1G_3H_3+G_3G_4H_2+G_1G_2G_3G_4H_1H_2}$$

**Sol. 9. (c)**
By drawing the signal flow graph for the given block diagram



Number of parallel paths are three

Gains $P_1G_1G_2$, $P_2 = G_2$ , $P_3 = 1$

By mason's gain formula,

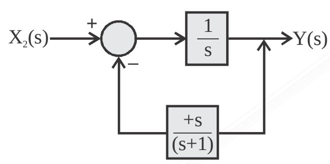
$$\frac{C(s)}{R(s)} = P_1 + P_2 + P_3$$

$$\Rightarrow G_1G_2 + G_2 + 1$$

**Sol. 10. (d)**

If $X_1(s) = 0$

$\frac{Y(s)}{X_2(s)}$; The block diagram becomes




$$\frac{Y(s)}{X_2(s)} = \frac{\frac{1}{s}}{1+\frac{1}{s}.\frac{s}{9s+1)}} = \frac{\frac{1}{s}}{(s+2)/s+1}$$

$$\Rightarrow \frac{(s+1)}{s(s+2)}$$

**Sol. 11. (5)**

$$G(s) = \frac{k}{s(s+2)(s^2+2s+2)}$$

$$\text{Closed loop T.F.} = \frac{G(s)}{1+G(s)}$$

$$= \frac{\frac{K}{s(s+2)(s^2+2s+2)}}{\frac{K}{s(s+2)(s^2+2s+2)}}$$

$$\text{Closed loop} = \frac{K}{s(s+2)(s^2+2s+2)+K}$$

Characteristic equation

$s(s + 2) (s^2 + 2s + 2) + K = 0$

$(s^2 + 2s) (s^2 + 2s + 2) + K = 0$

$S^4 + 4s^3 + 4s^2 + 4s + K = 0$

Routh array

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 6 | k |
| $s^3$ | 4 | 4 | 0 |
| $s^2$ | 5 | K | 0 |
| $s^1$ | $\frac{20+4k}{5}$ | 0 | |
| $s^2$ | K | | |

For marginally stable

$20 - 4k = 0$ or $k = 5$

**Sol. 12. (a)**

For the given SFG. We have two forward paths

$P_{k1} = (1)(s^{-1})(s^{-1})(1) = s^{-2}$

$P_{k2} = (1)(s^{-1})(1)(1) = s^{-1}$

Since, all the loops are touching to both the paths $P_{k1}$ and $P_{k2}$ so,

$\Delta k_1 = \Delta k_2 = 1$

Now, we have $\Delta$ = 1 – (sum of individual loops) + (sum of product of nontouching loops)

Here, the loops are

$L_1 = (-4)(1) = -4$

$L_2 = (-4)(s^{-1}) = 4s^{-1}$

$L_3 = (-2)(s^{-1})(s^{-1}) = -2s^{-2}$

$L_4 = (-2)(s^{-1})(1) = -2s^{-1}$

As all the loop $L_1$, $L_2$, $L_3$ and $L_4$ are touching to each other so,

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4)$

$= 1 - (-4 - 4s^{-1} - 2s^{-2} - 2s^{-1})$

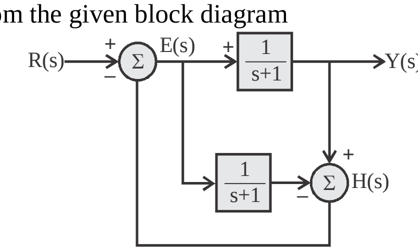
$= 5 + 6s^{-1} + 2s^{-2}$

From Mason's gain formulae

$$\frac{Y(s)}{U(s)} = \frac{\sum_i P_k\Delta_k}{\Delta} = \frac{s^{-2}}{5+6s^{-1}+2s^{-2}} = \frac{s+1}{5s^2+6s+2}$$

**Sol. 13. (b)**

From the given block diagram




$$H(s) = Y(s) - E(s).\frac{1}{s+1}$$

$$E(s) = R(s) - H(s) = R(s) - Y(s) + \frac{E(s)}{(s+1)}$$

$$E(s)\left[1+\frac{1}{s+1}\right] = R(s) - Y(s)$$

$$\frac{sE(s)}{(s+1)} = R(s) - y(s) \qquad ...(1)$$

$$Y(s) = \frac{E(s)}{s+1} \qquad ...(2)$$

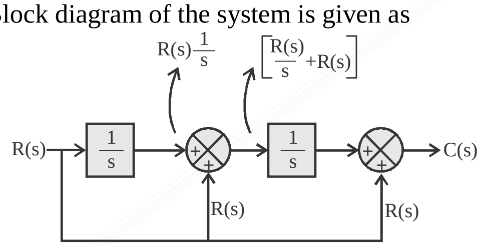
From (1) and (2) sY(s) = R(s) – Y(s)

(s +1) Y(s) = R(s)

Transfer function $\frac{Y(s)}{R(s)} = \frac{1}{s+1}$

**Sol. 14. (b)**

Block diagram of the system is given as




From the figure we can see that

$$C(s) = \left[R(s)\frac{1}{s} + R(s)\right]\frac{1}{s} + R(s)$$

$$C(s) = R(s)\left[\frac{1}{s^2} + \frac{1}{s} + 1\right]$$

$$\frac{C(s)}{R(s)} = \frac{1+s+s^2}{s^2}$$

**Sol. 15. (c)**

Mason Gain formula, $T(s) = \frac{\sum P_k \Delta_k}{\Delta}$

In Given SFG there is only one forward path and 3 possible loop. $p_1$ = abcd ; $\Delta_1 = 1$

Δ = 1 – (sum of individual loops) – (sum of two non touching loops)

= 1 – ($L_1 + L_2 + L_3$) + ($L_1L_3$)

Non touching loop are $L_1$ and $L_3$ where

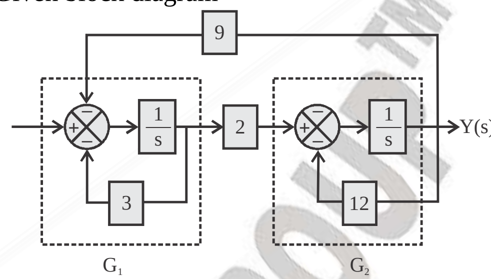
$L_1 L_2$ = bedg

Thus $\frac{C(s)}{R(s)} = \frac{P_1\Delta_1}{1-(be+cf+dg)+bedg}$

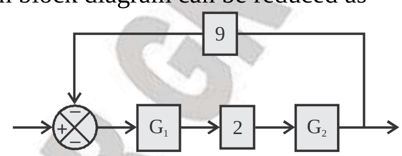
$$= \frac{abcd}{1-(be+cf+dg)+bedg}$$

**Sol. 16. (b)**

Given block diagram



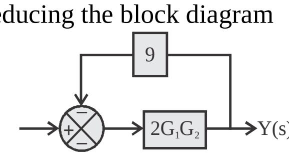

Given block diagram can be reduced as




$$\text{Where } G_1 \frac{\left(\frac{1}{s}\right)}{1+\left(\frac{1}{s}\right)^3} = \frac{1}{s+3}$$

$$G_2 = \frac{\left(\frac{1}{s}\right)}{1+\left(\frac{1}{s}\right)12} = \frac{1}{s+12}$$

Further reducing the block diagram




$$Y(s) = \frac{2G_1G_2}{1+2(G_1G_2)9}$$

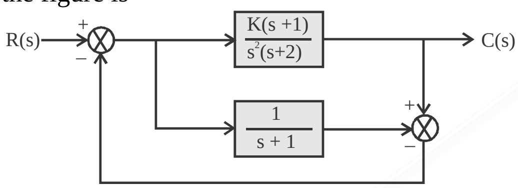
$$= \frac{(2)\left(\frac{1}{s+3}\right)\left(\frac{1}{s+12}\right)}{1+(2)\left(\frac{1}{s+3}\right)\left(\frac{1}{s+12}\right)(9)}$$

$$= \frac{2}{(s+3)(s+12)+18} = \frac{2}{s^2+15s+54}$$

$$= \frac{2}{(s+9)(s+6)} = \frac{1}{27\left(1+\frac{s}{9}\right)\left(1+\frac{s}{6}\right)}$$

# ESE OBJ QUESTIONS

**1.** The closed-loop transfer function $\frac{C(s)}{R(s)}$ of the system represented by the block diagram in the figure is




**[EE ESE - 2018]**

(a) $\frac{1}{(s+1)^2}$ (b) $\frac{1}{s+1}$

(c) s + 1 (d) 1

**2.** Consider the following statements for signal flow graph.
1. It represents linear as well as non-linear systems.
2. It is not unique for a given system
Which of the above statements is/are correct?

**[EE ESE - 2018]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

**3.** In Force-Voltage Analogy

**[EC ESE - 2016]**

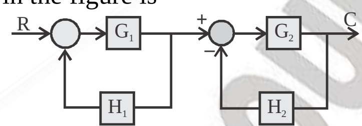
(a)Force is analogous to current
(b)Mass is analogous to capacitance
(c)Velocity is analogous to current
(d)Displacement is analogous to magnetic flux linkage

**4.** In position control systems, the Tacho-generator feedback is used to

**[EC ESE - 2016]**

(a)Increase the effective damping in the system
(b)Decrease the effective damping in the system
(c)Decrease the steady state error
(d)Increase the steady state error

**5.** The transfer function C/R of the system shown in the figure is




**[EE ESE - 2015]**

(a) $\frac{G_1G_2}{1+G_1H_1+G_2H_2}$

(b) $\frac{G_1H_1G_2H_2}{(1+G_1H_1)(1+G_2+H_2)}$

(c) $\frac{G_1G_2}{1-G_1-G_2+G_1G_2H_1H_2}$

(d) $\frac{G_1G_2}{1+G_1G_1+G_2H_2+G_1G_2H_1H_2}$

**6. Statement (I):** Servo motors have small diameter and large axial length.
**Statement (II):** Servo motors must have low inertia and high starting torque.

**[EE ESE - 2014]**

(a)Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b)Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c)Statement (I) is true but Statement (II) is false.
(d)Statement (I) is false but Statement (II) is true.

**7.** With negative feedback, the system stability and system gain respectively

**[EE ESE - 2014]**

(a) Increase and increases
(b) Increases and decreases
(c) Decreases and increases
(d) Decreases and decreases

**8.** A second order system is described by the equation $\frac{d^2x}{dt^2}+5\frac{dx}{dt}+7x=7y$ The frequency and damping ratio respectively are

**[EC ESE - 2013]**

(a) 1 rad/sec and 5
(b) 5 rad/sec and 7
(c) 1 rad/sec and $\sqrt{7}$
(d) $\sqrt{7}$ rad/sec and 0.94

**9.** The sensitivity of an overall transfer function M(s) of a closed – loop control system with respect to the forward path transfer function G(s) is:

**[EE ESE - 2012]**

(a) $\frac{G}{1+GH}$
(b) $\frac{G}{1-GH}$
(c) $\frac{1}{1-GH}$
(d) $\frac{1}{1+GH}$

**10.** Match List-I and List-II and select the correct answer using the code given below the lists:

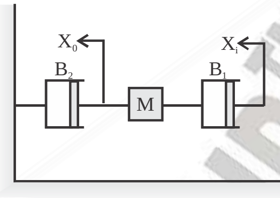
**List-I**
A. Mass
B. Damper
C. Spring
D. Force

**List-II**
(i) Capacitor
(ii) Voltage
(iii) Resistor
(iv) Inductor

**[EE ESE - 2011]**

**Codes:**
(a) A-ii, B-i, C-iii, D-iv
(b) A-iv, B-i, C-iii, D-ii
(c) A-ii, B-iii, C-i, D-iv
(d) A-iv, B-iii, C-i, D-ii

**11.** For the mechanical system with mass and viscous friction components, shown in figure, $\frac{X_0(s)}{X_i(s)}$ is

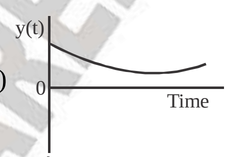


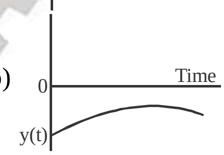

**[EE ESE - 2011]**

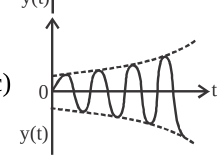
(a) $\frac{B_2}{MS+B_1+B_2}$
(b) $\frac{1}{Ms+(B_1+B_2)}$
(c) $\frac{B_1}{Ms+B_1+B_2}$
(d) None of these

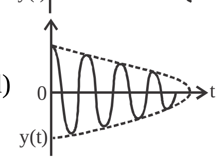
**12.** Which of the following is the response of a spring-mass damper with under-damping?

**[EC ESE - 2010]**

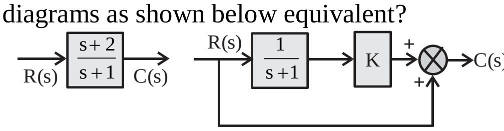
(a)



(b)



(c)



(d)


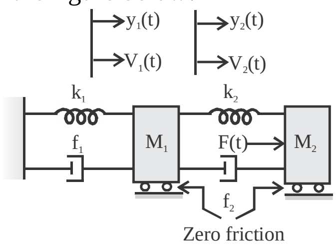

**13.** For what value of K, are the two block diagrams as shown below equivalent?




**[EE ESE - 2009]**

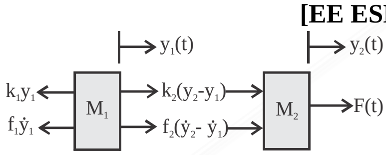
(a) 1
(b) 2
(c) (s + 1)
(d) (s + 2)

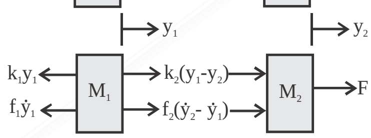
**14.** Which one of the following is the correct free body diagram for the physical system as shown in the figure below?

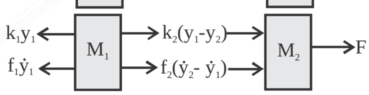


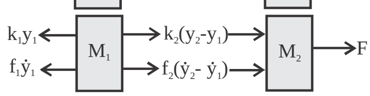

$y_1(t)$ and $y_2(t)$ are displacements
$v_1(t)$ and $v_2(t)$ are velocities

**[EE ESE - 2009]**

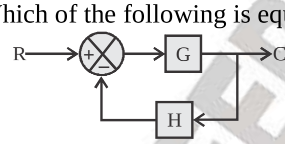
(a) 


(b) 

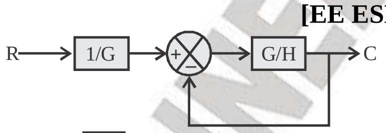

(c) 

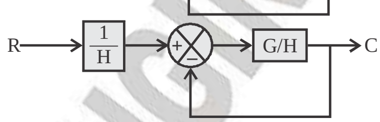

(d) 

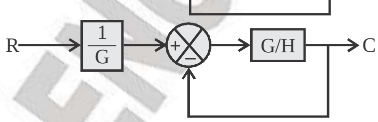

**15.** The below shown feedback control system has to be reduced to equivalent unity feedback system. Which of the following is equivalent?

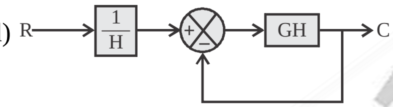



**[EE ESE - 2009]**

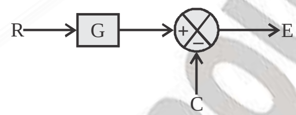
(a) 


(b) 

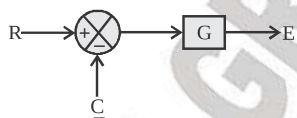

(c) 

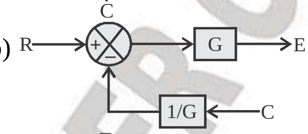

(d) 

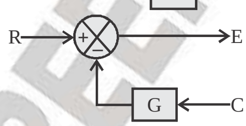

**16.** Which one of the following block diagrams is equivalent to the below shown block diagram?

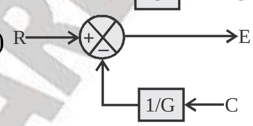



**[EE ESE - 2009]**

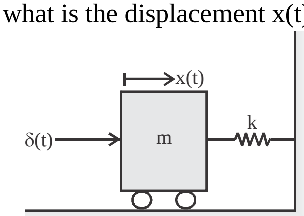
(a) 


(b) 


(c) 


(d) 


**17.** A mechanical system is as shown in the figure below. The system is set into motion by applying a unit impulse force δ(t). Assuming that the system is initially at rest and ignoring friction, what is the displacement x(t) of mass?




**[EE ESE - 2009]**

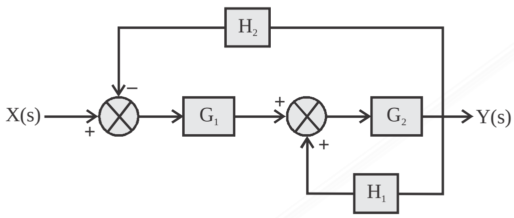
(a) $\frac{1}{\sqrt{x}}\exp(-m.t)$

(b) $\frac{1}{\sqrt{mk}}\sin(t)$

(c) $\frac{1}{\sqrt{mk}}\sin\left(\sqrt{\frac{k}{m}}.t\right)$

(d) $\frac{1}{\sqrt{mk}}\left(\sqrt{\frac{k}{m}}.t\right)$

**18.** Which one of the following is the transfer function $\frac{Y(s)}{X(s)}$ for the block diagram given below?




**[EE ESE - 2007]**

(a) $\frac{G_1G_2}{1+H_2G_1G_2+H_1G_2}$

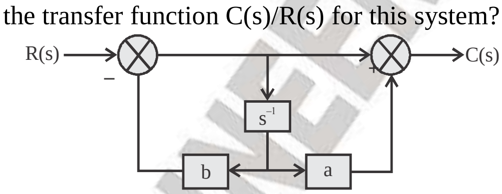
(b) $\frac{G_1G_2}{1+H_2G_1G_2+H_1G_2}$

(c) $\frac{H_1G_1G_2}{1-H_2G_1G_2+H_1G_2}$

(d) $\frac{H_1G_1G_2}{1-H_2G_1G_2+H_1G_2}$

$\frac{H_1G_1G_2}{1+H_2G_1G_2-H_1G_2}$

**19.** The block diagram for a particular control system is shown in the below figure. What is the transfer function C(s)/R(s) for this system?

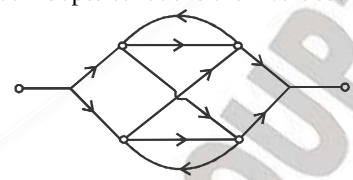



**[EE ESE - 2007]**

(a) $\frac{s+a}{s-b}$ (b) $\frac{s+a}{s+b}$

(c) $\frac{s-a}{s+b}$ (d) $\frac{s-a}{s-b}$

**20.** The signal flow graph shown below has M number of forward paths and P number of individual loops. What are their values?

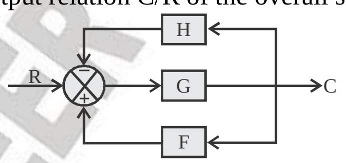


**[EE ESE - 2006]**

(a) M = 4 and P = 2 (b) M = 6 and P = 3

(c) M = 4 and P = 3 (d) M = 6 and P = 2

**21.** For the feedback system shown in the figure below, which one of the following expresses the input output relation C/R of the overall system?




**[EE ESE - 2006]**

(a) $\frac{G}{1-FG+GH}$ (b) $\frac{G}{1+FG-GH}$

(c) $\frac{FG}{1+FGH}$ (d) $\frac{GH}{1-FGH}$

**22.** Consider the following statements with respect to feedback control systems:

1. Accuracy cannot be obtained by adjusting loop gain.
2. Feedback decreases overall gain.
3. Introduction of noise due to sensor reduces overall accuracy.
4. Introduction of feedback may lead to the possibility of instability of closed loop system.

Which of the statements given above are correct?

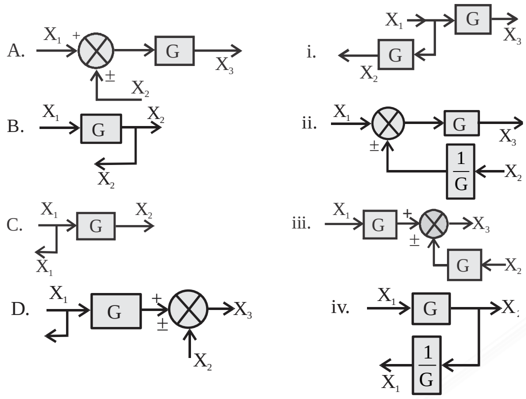
**[EC ESE - 2006]**

(a) 1, 2, 3 and 4 (b) Only 1, 2 and 4

(c) Only 1 and 3 (d) Only 2, 3 and 4

**23.** Match List-I (Original Diagram) with List-II (Equivalent Diagram) and select the correct answer using the code given below the Lists:

**List-I** **List-II**




**[EE ESE - 2005]**

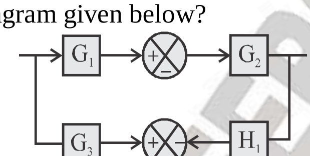
**Codes:**
(a) A-iii, B-i, C-iv, D-ii
(b) A-ii, B-iv, C-i, D-iii
(c) A-iii, B-iv, C-i, D-ii
(d) A-ii, B-i, C-iv, D-iii

**24.** The maximum temperature rise of a transformer is $50^0$C. It attains a temperature $31.6^0$C in 1/2 hour. What its thermal time constant?

**[EE ESE - 2005]**

(a) 2 hours (b) 1/2 hours
(c) 1 hours (d) 1/4 hours

**25.** What is the overall transfer function of the block diagram given below?

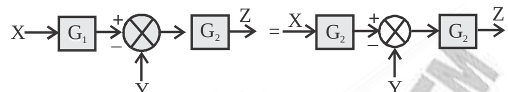



**[EE ESE - 2005]**

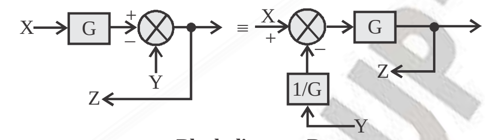
(a) $\dfrac{G_1G_2+G_2G_3}{1+G_2H_1}$ (b) $\dfrac{G_1G_3+G_2G_3}{1+G_3H_1}$

(c) $G_1G_2+G_2G_3$ (d) $\dfrac{G_1G_3+G_2G_3}{1+G_2G_3H_1}$

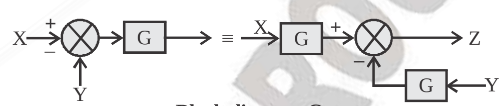
**26.** Consider the following three cases of block diagram algebra A, B and C.



**Block diagram A**



**Block diagram B**



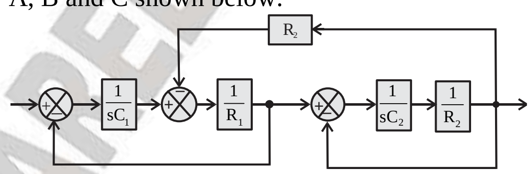
**Block diagram C**

Which of the above relations are correct?

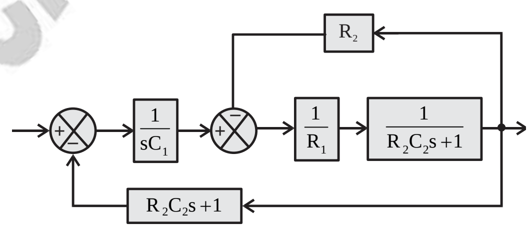
**[EE ESE - 2004]**

(a) A and B (b) B and C
(c) A and C (d) A, B and C

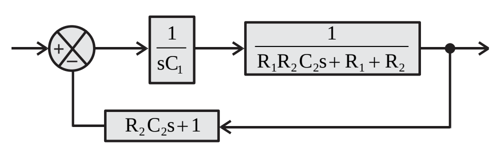
**27.** Consider the following three block diagram A, B and C shown below:



**Block diagram A**



**Block diagram B**



**Block diagram C**

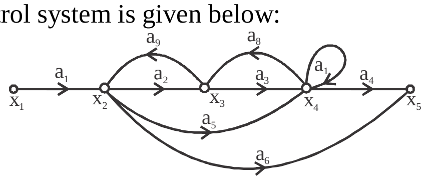
Which one of the following statements is correct in respect of the above blocks diagrams?

**[EE ESE - 2004]**

(a) Only A and B are equivalent
(b) Only A and C are equivalent

(c) Only B and C are equivalent
(d) A, B and C are equivalent

**28.** The signal flow graph for a certain feedback control system is given below:




Now consider the following set of equations for the nodes:

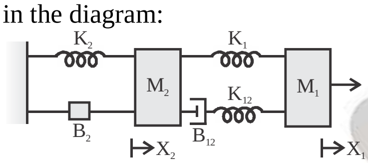
(i) $x_2 = a_1x_1 + a_9x_3$
(ii) $x_3 = a_2x_2 + a_8x_4$
(iii) $x_4 = a_3x_3 + a_5x_2$
(iv) $x_5 = a_4x_4 + a_6x_2$

Which of the above equations are correct?

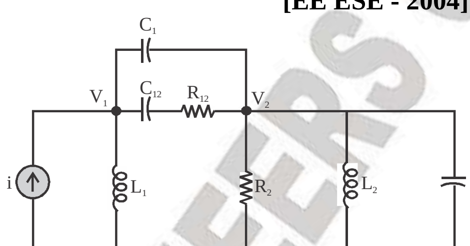
**[EE ESE - 2004]**

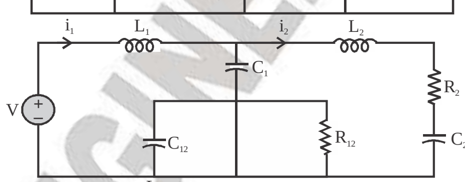
(a) i, ii and iii (b) i, iii and iv
(c) ii, iii and iv (d) i, ii and iv

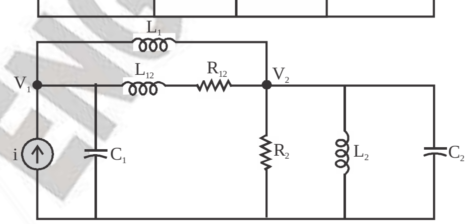
**29.** Consider the following mechanical system shown in the diagram:

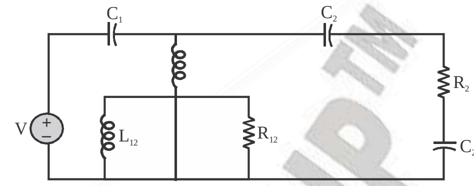



**[EE ESE - 2004]**

(a)


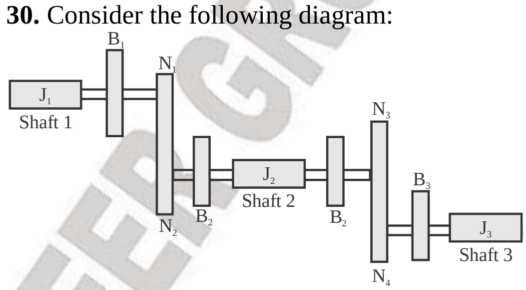

(b)



(c)



(d)



Which one of the following circuits shows the correct force-current analogous electrical circuit for the mechanical diagram shown above?

**30.** Consider the following diagram:




For the multiple gear system shown above, which one of the following gives the equivalent inertia referred to shaft 1?

**[EE ESE - 2004]**

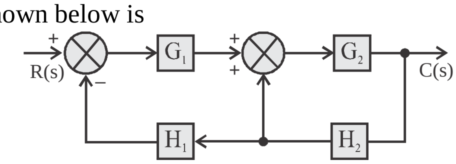
(a) $J_1 + J_2\left(\frac{N_1}{N_2}\right)^2 + J_3\left(\frac{N_1N_3}{N_2N_4}\right)^2$

(b) $J_1 + J_2\left(\frac{N_2}{N_1}\right)^2 + J_3\left(\frac{N_2N_4}{N_1N_3}\right)^2$

(c) $J_1 + J_2\left(\frac{N_1}{N_2}\right)^2 + J_3\left(\frac{N_1N_2}{N_3N_4}\right)^2$

(d) $J_1 + J_2\left(\frac{N_2}{N_1}\right)^2 + J_3\left(\frac{N_1N_2}{N_3N_4}\right)^2$

**31.** The overall gain $\frac{C(s)}{R(s)}$ of the block diagram shown below is




**[EE ESE - 2003]**

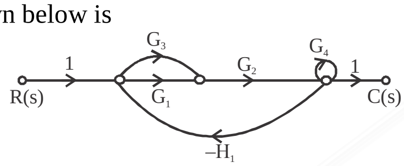
(a) $\frac{G_1G_2}{1-G_1G_2H_1H_2}$

(b) $\dfrac{G_1G_2}{1-G_2H_2-G_1G_2H_1}$

(c) $\dfrac{G_1G_2}{1-G_2H_2+G_1G_2H_1H_2}$

(d) $\dfrac{G_1G_2}{1-G_1G_2H_1-G_1G_2H_2}$

**32.** The gain C(s)/R(s) of the signal flow graph shown below is




**[EE ESE - 2003]**

(a) $\dfrac{G_1G_2+G_2G_3}{1+G_1G_2H_1+G_2G_3H_1+G_4}$

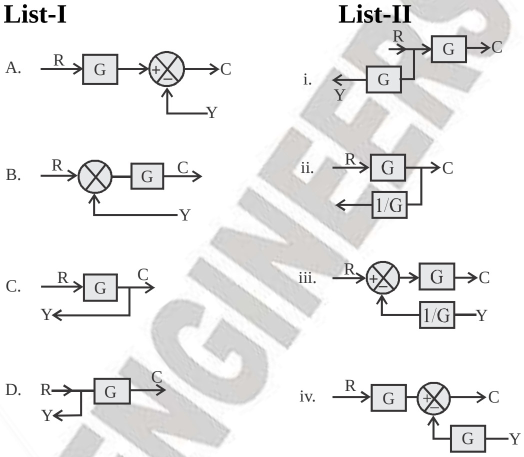
(b) $\dfrac{G_1G_2+G_2G_3}{1+G_1G_2H_1+G_2G_3H_1-G_4}$

(c) $\dfrac{G_1G_3+G_2G_3}{1+G_1G_3H_1+G_2G_3H_1+G_4}$

(d) $\dfrac{G_1G_3+G_2G_3}{1+G_1G_3H_1+G_2G_3H_1+G_4}$

**33.** Match List- I (Block Diagram) with List- II (Transformed Block Diagram) and select the correct answer.

**List-I** **List-II**




**[EE ESE - 2003]**

**Codes:**

(a) A-iii, B-iv, C-ii, D-i

(b) A-iv, B-iii, C-i, D-ii

(c) A-iii, B-iv, C-i, D-ii

(d) A-iv, B-iii, C-ii, D-i

**34.** Which one of the following statements is NOT correct?

**[EE ESE - 2003]**

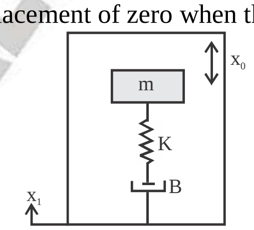
(a)The action of bellows in pneumatic control system is similar to that of a spring.

(b)The flapper valve converts large changes in the position of the flapper into small changes in the black pressure.

(c)The common name of pneumatic amplifier is pneumatic relay.

(d)The transfer function of a pneumatic actuator is of the form $\dfrac{A}{Ms^2+fs+K}$

**35.** A seismic transducer using a spring- mass-damper system as shown below will have an output displacement of zero when the input $x_1$ is




**[EE ESE - 2003]**

(a) Constant displacement

(b) Constant velocity

(c) Constant acceleration

(d) Sinusoidal displacement

**36. Assertion (A)**: A linear, negative feedback control system is invariable stable if its open loop configuration is stable.

**Reason (R)**: The negative feedback reduces the overall gain of the system.

**[EC ESE - 2003]**

(a)Both A and R are true and r is the correct explanation of A

(b)Both A and R are true but R is NOT the correct explanation of A

(c)A is true but R is false

(d)A is false but R is true.

**37. Assertion (A)**: in a shunt regulator, the control element is connected in shunt with the load to achieve constant output voltage.
**Reason (R)**: The impedance of the control element varies to keep the total current flowing through the load and the control element constant.

**[EC ESE - 2003]**

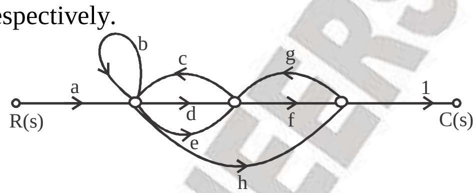
(a)Both A and R are true and r is the correct explanation of A
(b)Both A and R are true but R is NOT the correct explanation of A
(c)A is true but R is false
(d)A is false but R is true.

**38.** Which one of the following statements is INCORRECT with reference to pneumatic system?

**[EE ESE - 2002]**

(a)Operating pressure is low compared to hydraulic system
(b)Leaks can create problems as well as fire hazards
(c)They are insensitive to temperature changes
(d)High compressibility of air results in longer time delays

**39.** The number of forward paths and the number of non-touching loop pairs for the signal flow graph given in the figure below are, respectively.




**[EE ESE - 2002]**

(a) 1, 3 (b) 3, 2
(c) 3, 1 (d) 2, 4

**40.** Match List-I (Physical action or activity) with List-II (Category of system) and select the correct answer:

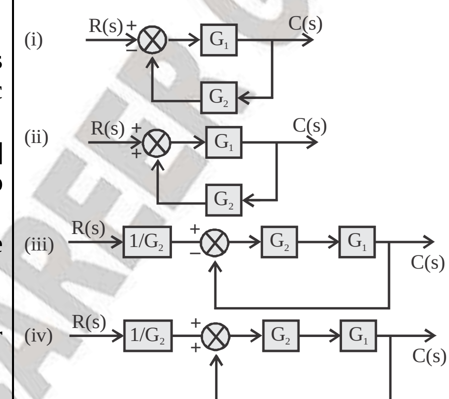
**List-I**
A. Human respiration system
B. Pointing of an object with a finger
C. A man driving a car
D. A thermostatically controlled room heater

**List-II**
(i) Man-made control system
(ii) Natural including biological control system
(iii) Control system whose components are both man – made and natural

**[EE ESE - 2001]**

**Codes:**
(a) A- ii, B-ii, C-iii, D-i
(b) A-iii, B-i, C-ii, D-i
(c) A-iii, B-ii, C-ii, D-iii
(d) A-ii, B-i, C-iii, D-iii

**41.** Consider the following block diagrams:

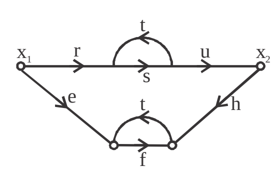



Which one of these block diagrams can be reduced to transfer function $\frac{C(s)}{R(S)} = \frac{G_1}{1-G_1G_2}$ ?

**[EE ESE - 2001]**

(a) i and iii (b) ii and iv
(c) i and iv (d) ii and iii

**42.** For the signal flow diagram shown in the given figure, the transmittance between $x_2$ and $x_1$ is

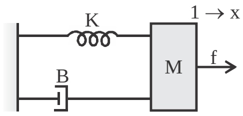


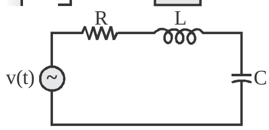

**[EC ESE - 2001]**

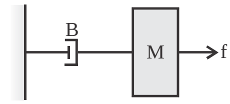
(a) $\frac{rsu}{1+st} + \frac{efh}{1-fg}$ (b) $\frac{rsu}{1-fg} + \frac{efh}{1-st}$

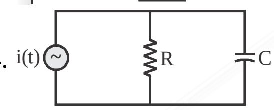
(c) $\frac{efh}{1-ru}+\frac{rsu}{1-eh}$ (d) $\frac{rst}{1-eh}+\frac{rsu}{1-st}$

**43.** Consider the following system:

1.



2.



3.



4.



Which of these systems can be modeled by the differential equation.

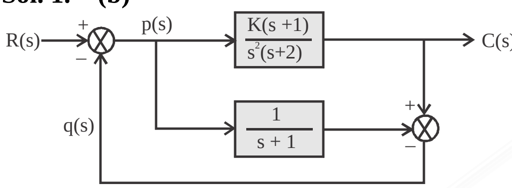
$$a_2\frac{d^2y(t)}{dt^2}+a_1\frac{dy(t)}{dt}+a_0y(t)=x(t)?$$

**[EC ESE - 1999]**

(a) 1 and 2 (b) 1 and 3
(c) 2 and 4 (d) 1, 2 and 4

# SOLUTIONS

**Sol. 1. (b)**




$$q(s) = \left[C(s) - \frac{P(s)}{S+1}\right] \text{ and } C(s) = \frac{P(s)}{S+1}$$

Hence q(s) = 0

So

P(s) = R(s)

$$C(s) = \frac{P(s)}{s+1}$$

$$\frac{C(s)}{R(s)} = \frac{1}{s+1}$$

**Sol. 2. (b)**

Signal flow graph is valid only for LTI system and SISO system and signal flow graph is not unique for any electric N/w it depend upon solution of variable.

**Sol. 3. (c)**

As per theory of analogy velocity is analogous to current.

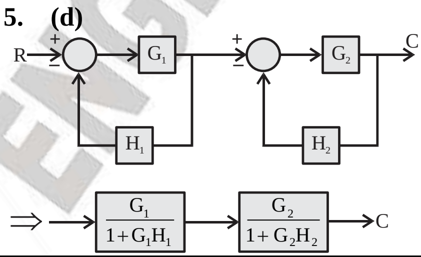
**Sol. 4. (a)**

Tacho generator (or) derivative controller mainly used to increase system damping

$$\xi_{new} = \xi_{old} = \frac{K_D\omega_n}{2}$$

$K_D$ = tachometer constant

**Sol. 5. (d)**



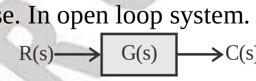

$$\frac{C}{R} = \frac{G_1G_2}{(1+G_1H_1)(1+G_2H_2)}$$

$$= \frac{G_1G_2}{1+G_1H_1+G_2H_2+G_1G_2H_1H_2}$$

**Sol. 6. (a)**

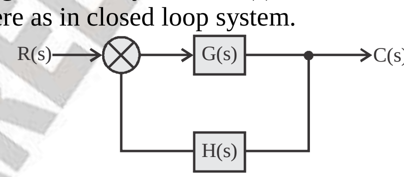
**Sol. 7. (b)**

With negative feedback, the system stability will increase. In open loop system.




The gain of the system is G(s).

Where as in closed loop system.




The closed loop gain of the systems $\frac{G(s)H(s)}{1+G(s)H(s)}$.

hence it is divided by 1 + G(s)H(s), in closed loop system with negative feedback gain decreases.

**Sol. 8. (a)**

**Sol. 9. (d)**

$$M(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{G}{1+GH} \qquad ...(i)$$

$$S_G^M = \frac{\frac{\partial M}{M}}{\frac{\partial G}{G}} = \frac{G}{M}\cdot\frac{\partial M}{\partial M} \qquad ...(ii)$$

Differentiating equation (i) w.r.t. G

$$\frac{\partial M}{\partial G} = \frac{1+GH-GH}{(1+GH)^2} = \frac{1}{(1+GH)^2}$$

Equation (ii) becomes

$$S_G^M = \frac{G}{\frac{G}{1+GH}} = \frac{1}{(1+GH)^2} = \frac{1}{1+GH}$$

**Sol. 10. (d)**

| **Voltage analogy** | | **Current analogy** |
|---|---|---|
| Force | Voltage | Current |
| Mass | Inductor | Capacitor |
| Spring | 1/C | 1/L |
| Damper | R | 1/R |

Hence, option (d) is correct.

**Sol. 11. (c)**

Dynamic equation,

$$M\frac{d^2x_0}{dt^2} + B_2\frac{dx_0}{dt} + B_t\frac{d}{dt}(x_0 - x_i) = 0$$

$MX_0(s)s^2 + B_2X_0(s)s + B_1(X_0(s) - X_i(s))\, s = 0$

$$\frac{X_0(s)}{X_i(s)} = \frac{B_1 s}{Ms^2 + (B_1 + B_2)s} = \frac{B_1}{Ms + B_1 + B_2}$$

**Sol. 12. (d)**

**Sol. 13. (a)**

$$\frac{C(s)}{R(s)} = \frac{k}{s+1} + 1 = \frac{k+s+1}{s+1}$$

Comparing with $\frac{C(s)}{R(s)} = \frac{s+2}{s+1}$

$\therefore k = 1$

**Sol. 14. (a)**

**Sol. 15. (d)**

$$C = R\left(\frac{G}{1+GH}\right)$$

Which is satisfied by (d) option

$$C = \frac{R}{H}\times\left[\frac{GH}{1+GH}\right] = R\left[\frac{G}{1+GH}\right]$$

**Sol. 16. (b)**

E = RG – C

Which is satisfied by (b) option.

**Sol. 17. (c)**

$$\delta(t) = \frac{md^2x(t)}{dt^2} + k\ x(t)$$

Taking laplace transform

$1 = ms^2X(s) + k[X(s)]$

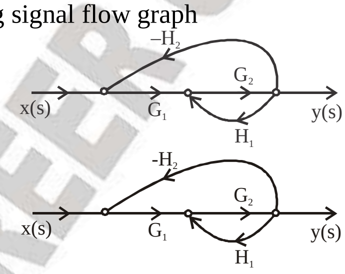
$$\therefore X(s) = \frac{1}{ms^2 + k}$$

$$X(s) = \frac{1}{m\left[s^2 + \frac{k}{m}\right]}$$

$$X(t) = \frac{1}{\sqrt{mK}}\sin\left(\sqrt{\frac{k}{m}}.t\right)$$

**Sol. 18. (a)**

Making signal flow graph




$$\frac{y|s|}{x|s|} = \frac{G_1G_2}{1-(G_2H_1 - G_1G_2H_2)}$$

$$= \frac{G_1G_2}{1 - G_2H_1 + G_1G_2H_2}$$

**Sol. 19. (b)**

Only one loop and two path so using Mason's gain formulae

$$\frac{C(s)}{R(s)} = \frac{1+s^{-1}a}{1-(-bs^{-1})} = \frac{1+\frac{a}{s}}{1+\frac{b}{s}} = \frac{s+a}{s+b}$$

**Sol. 20. (b)**

**Sol. 21. (a)**

Solving positive feedback

$$T.F. = \frac{G}{1-GF}$$

Now solving negative feedback path

$$T.F_1 = \frac{\frac{G}{1-GF}}{1+\frac{GH}{1-GF}}$$

$$T.F_1 = \frac{G}{1-GF+GH}$$

**Sol. 22. (d)**

**Sol. 23. (a)**

**Sol. 24. (b)**

$31.6 = (1 - e^{-t/\tau}) \Rightarrow \tau = \frac{1}{2}$ hour

**Sol. 25. (a)**

$$T(s) = (G_1 + G_3)\frac{G_2}{1+G_2H_1} = \frac{G_1G_2 + G_2G_3}{1+G_2H_1}$$

**Sol. 26. (b)**

**Sol. 27. (d)**

Block diagram 'B' can be obtained from 'A' and 'C' can be obtained from 'B'.

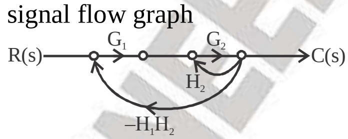
**Sol. 28. (d)**

**Sol. 29. (c)**

**Sol. 30. (a)**

**Sol. 31. (c)**

Making signal flow graph




$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_2}{1-(G_2H_2 - G_1G_2H_1H_2)}$$

$$= \frac{G_1G_2}{1-G_2H_2 + G_1G_2H_1H_2}$$

**Sol. 32. (b)**

$P = G_1G_2$; $P_2 = G_2G_3$; $\Delta_1 = \Delta_2 = 1$

$L_1 = -G_1G_2H_1$; $L_2 = G_4$; $L_3 = -G_3G_2H_1$

$$\therefore T(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{1-(L_1+L_2+L_3)}$$

$$= \frac{G_1G_2 + G_2G_3}{1+G_1G_2H_1 + G_2G_3H_1 - G_4}$$

**Sol. 33. (c)**

**Sol. 34. (b)**

Pneumatic Flapper valve converts small changes in the position of the flapper into large changes in the back pressure.

**Sol. 35. (a, b)**

**Sol. 36. (b)**

**Sol. 37. (c)**

**Sol. 38. (b)**

**Sol. 39. (c)**

Forward path = adfI, aefI, ahI

Non touching loop pairs = (fg and b) one pair only

**Sol. 40. (a)**

**Sol. 41. (b)**

**Sol. 42. (a)**

$$\frac{X_2}{X_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$P_1$ = rsu, $\Delta_1 = 1 - fg$

$P_2$ = efh, $\Delta_2 = 1 - st$, $\Delta = 1 - fg - st + fgst$

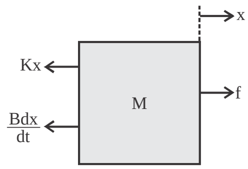
$$\frac{X_2}{X_1} = \frac{rsu(1-fg) + efh(1-st)}{1-fg-st+fgst}$$

$$= \frac{rsu(1-fg) + efh(1-st)}{(1-fg)(1-st)}$$

$$\frac{X_2}{X_1} = \frac{rsu}{1-st} + \frac{efh}{1-fg}$$

**Sol. 43. (a)**

This is a second order differential equation which means that there must be present all the three components in the system, i.e. either R, L and C or K, B and M.




**Free body diagram of M in Fig. 1**

In Figure 1,

$$f - kX - B\frac{dx}{dt} = M\frac{d^2x}{dt^2}$$

Or

$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = f \quad ...(i)$$

In figure 2,

$$V(t) = Ri + \frac{Ldi}{dt} + \frac{q}{c}$$

Or

$$V(t) = \frac{Ld^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} \quad ...(ii)$$

Both the equations are symmetric to the given equation.

# CHAPTER - 4
# *TIME RESPONSE ANALYSIS OF CONTROL SYSTEM*

## 4.1 INTRODUCTION

### 4.1.1 Types of System
(No. of open loop poles of the system at origin)

**Example.**

(i) $G(s)=\frac{K}{(s+1)(s+2)}$, No pole at origin. So it is type 0.

(ii) $G(s)=\frac{K}{s(s+1)(s+1)}$, 1 pole at origin. So type 1.

(iii) $G(s)=\frac{K}{s^2(s+1)(s+2)}$, 2 poles at origin. So type 2

Order is the highest coefficient of s in the denominator of closed loop transfer function.

**Example.** Consider a unity feedback system whose open loop transfer function is

$G(s)=\frac{K}{(s+1)(s+2)}$ What is the type and order of the system?

**Solution.**
The closed loop transfer function is

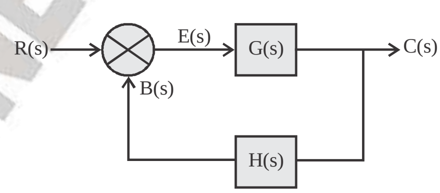
$$=\frac{K}{s^2+3s+2}$$

So it is a type 0 and order 2 system.

## 4.2 ERROR ANALYSIS

### 4.2.1 Steady State Error
A desirable feature of a control system is the faithful following of its input by the output. However, if the actual output of a control system during steady state deviates from the reference input (i.e. desired output, the system is said to possess a steady state error.

As the steady state performance of a control system is assessed by the magnitude of the steady state error possessed by the system and the system input specified as either step or ramp or parabolic.




The magnitude of the steady state error in a closed-loop control system depends on its open-loop transfer function, i.e. G(s) H(s) of the system. The classification of open loop transfer function of a control system is explained below:

GH is loop transfer function

G is open loop transfer function

E(s) = R(s) – B(s) …(i)
B(s) = H(s) C(s) …(ii)
C(s) = G(s) E(s) …(iii)
From (i), (ii) and (iii)
E(s) = R(s) – H(s) G (s) E(s)

$$\therefore E(s) = \frac{1}{1+G(s)H(s)}R(s)$$

**(a) Type '0'**: If there are no poles at origin, this is type '0' system
(I) unit step input

$$R(s) = \frac{1}{s}$$

$$E(s) = \frac{1}{S+SG(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0}(SE(s))$$

$$= s\left\{\frac{1}{S+SG(s)H(s)}\right\}$$

$$= \frac{1}{1+\lim\limits_{s\to 0}}(G(s)H(s)) = \frac{1}{1+K_p}$$

Where $K_p = \lim\limits_{s\to 0}(G(s)H(s))$ position error constant
Whatever may be the system, for the step input we have exp.

$$e_{ss} = \frac{1}{1+K_p}$$

**Case-I.** For Type 'o'
$e_{ss}$ = constant
**Case-II.** For Type '1'
$K_p \to \infty$

$$e_{ss} = \frac{1}{1+\infty} = 0$$

**Case-III.** For Type '2'
$K_p \to \infty$

$$e_{ss} = \frac{1}{1+\infty} = 0$$

For the same type of input. As the system type increases the steady state error decreases.
II Ramp input, r(t) = tu(t)

$$R(s) = \frac{1}{s^2}$$

$$E(s) = \frac{1}{S^2 + S^2 G(s)\,H(s)}$$

$$e_{ss} = \lim_{s\to 0}(SE(s)) = s\left\{\frac{1}{S^2 + S^2\,G(s)\,H(s)}\right\} = \underset{s\to 0}{\text{Lt}}\,\frac{1}{S + SG(s)\,H(s)} = \frac{1}{\underset{s\to 0}{\text{Lt}}\,SG(s)\,H(s)}$$

$$e_{ss} = \frac{1}{K_v}$$

Where $K_v$ is velocity error constant

$$K_V = \underset{s\to 0}{\text{Lt}}\,SG(s)\,H(s)$$

**Case-I.** Type '0'

$$K_V = \underset{s\to 0}{\text{Lt}}\,SG(s)\,H(s) = 0$$

$$\therefore e_{ss} = \frac{1}{K_v} = 0$$

**Case-II.** Type "1"

$$K_V = \underset{s\to 0}{\text{Lt}}(SG(s)\,H(s)) = \text{constant}$$

Here s is in denominator of G(s) H(s)

$$e_{ss} = \frac{1}{K_v} = \text{constant}$$

**Case-III.** Type '2'

$$K_V = \underset{s\to 0}{\text{Lt}}(SG(s)\,H(s)) = \infty$$

Here $s^2$ is in denominator of G(s) H(s)

$e_{ss}$ = '0'

As the unit input changes from unit step to ramp and ramp to parabola the steady state error increases for the same type.

**III parabolic input, r(t) =** $\frac{t^2}{2}$

$$R(s) = \frac{1}{S^3}$$

$$E(s) = \frac{1}{S^3 + S^3 G(s)\,H(s)}$$

$$e_{ss} = \underset{s\to 0}{\text{Lt}}\,SE(s) = \frac{1}{S^2 + S^2 G(s)\,H(s)}$$

$$e_{ss} = \frac{1}{0 + \lim_{s\to 0} S^2 G(s)\,H(s)}$$

$$K_a = \lim_{s\to 0} S^2 G(s)\,H(s)$$

**(a) Type-0**

$K_a = 0$

$$e_{ss} = \frac{1}{0} = \infty$$

**(b) Type-1**

$K_4 = 0$

$$e_{ss} = \frac{1}{0} = \infty$$

**(c) Type-2**

$K_a$ is constant

$e_{ss}$ is constant

| | Unit Step | Remp | Parabola |
|---|---|---|---|
| Type-0 | $\frac{1}{1+K_p}$ | $\infty$ | $\infty$ |
| Type-1 | 0 | $\frac{1}{K_v}$ | $\infty$ |
| Type-2 | 0 | 0 | $\frac{1}{K_a}$ |
| | $K_p = \lim_{s\to 0} G(s)H(s)$ | $K_V = \mathrm{Lt}_{s\to 0}\, sG(s)H(s)$ | $\lim_{s\to 0} S^2G(s)H(s)$ |

**Example 2.** A unity feedback control system has $G(s) = \frac{20(s+1)}{s^2(s+2)(s+4)}$. Find the static error constant and steady state error if the i/p is :- r(t)= (40t + 20t + $5t^2$) 4(t).

**Solution.**

Position error constant, $K_p = \mathrm{Lt}\, sG(s) = \frac{s20(s+1)}{s^2(s+2)(s+4)} = \infty$

Acc. Error constant, $K_a$ Lt $s^2$ G(s)

$$= \frac{s^2 20(s+1)}{s^2(s+2)(s+4)} = \frac{20}{2\times 4} = 2.5$$

Now $e_{ss} = \frac{40}{1+Kp} + \frac{20}{K_v} + \frac{5\times 2}{K_s}$

[due to 3 basic inputs]

$$e_{ss} = \frac{40}{\infty} + \frac{20}{\infty} + \frac{5\times 2}{2.5} = 4$$

**Example 3.** A system has position error constant, $K_p$ = 3, Find the steady state error if the i/p is 8tu(t) [i.e. unit ramp input]

**Solution.**

$K_p$ is defined for type-0 system. So for the type-0 system, $K_v$ = 0

$$\therefore e_{ss} = \frac{1}{K_v} = \infty$$

**Example 4.** For the system represented by the following block diagram, find steady state error.

$$\frac{C(s)}{R(s)} = \frac{K(s+1)(s+3)}{s^4 + 5s^3 + 5s^2 + Ks + K}$$

**Solution.**

So it is a type o system & for type 0 system, $K_p$ is defined

$$K_p = \underset{s\to 0}{Lt}\, G(s)$$

$$= 3 \Rightarrow e_s = \frac{1}{1+K_p} = \frac{1}{1+3} = \frac{1}{4}$$

## 4.3 SECOND ORDER CONTROL SYSTEM

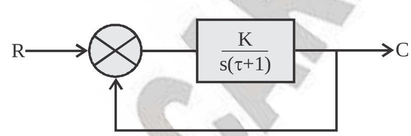
$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + \frac{1}{\tau}s + \frac{K}{\tau}}$$

τ is Time constant

K is gain

Characteristic equation

$$s^2 + \frac{1}{\tau}s + \frac{k}{\tau} = 0$$



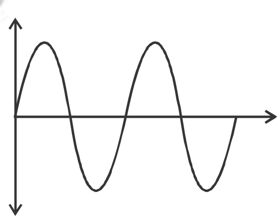

Compare with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$\omega_n = \sqrt{\frac{k}{\tau}} \quad , \ \xi = \frac{1}{2\sqrt{k\tau}}$$

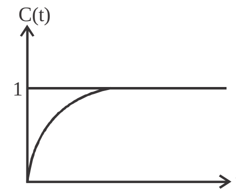
### 4.3.1 Consider the following cases of ε(Damping Ratio)

**(i) When ε = 0,** the o/p response to a unit step input is:

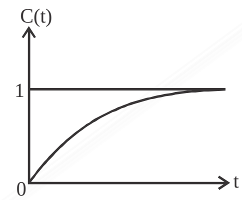
i.e the output response is not damped but oscillatory in nature with frequency of $\omega_n$ rad/sec. Where $\omega_n$ is undamped natural frequency.



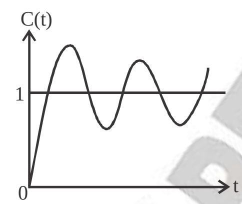
**(ii) When ε = 1**, the output response is critically damped and exhibits no overshoots.




**(iii) When ε > 1**, the output response is over damped i.e. the response takes longer time to reach its final value.




**(iv) When ε < 1,** it is an under damped system i.e damped sinusoidal. Where slope of sinusoidal is exponential decreasing.




## 4.4 TIME RESPONSE SPECIFICATION

### 4.4.1 Definition of Transient Response Specification

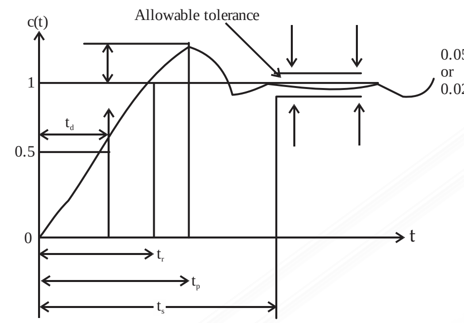
In specifying the transient – response characteristics of a second order control system for ε = 1 to a unit – step input, it is common to specify the following;

1. Delay time , $t_d$
2. Rise time $t_r$
3. Peak time , $t_p$
4. Maximum overshoot, $M_p$
5. Settling time, $t_s$

These specifications are defined in what follows and are shown graphically in fig.

***1. Delay Time ($t_d$)***

The delay time is the time required for the response to reach half the final value of very first time.




**Second order system**

***2. Rise Time ($t_r$)***

The rise time is the time required for the response to rise from 10% to 90%, 5% to 95%, or 0% to 100% rise time is normally used.

***3. Peak Time ($t_p$)***

The peak time is the time required for the response to reach the first peak of the overshoot.

***4. Maximum (Percent) Overshoot ($M_p$)***

The maximum overshoot is the maximum peak value of the response curve measured from unity. If the final steady state value of the response differs from unity, then it is common to use the maximum percent overshoot. It is defined by Maximum percent overshoot $=\dfrac{c(t_p)-c(\infty)}{c(\infty)}\times 100\%$.

The amount of the maximum (percent) overshoot directly indicates the relative stability of the system.

***5. Settling Time ($t_s$)***

The settling time is the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2% or 5%). The settling time is related to the largest time constant of the control system. Which percentage error criterion to use may be determined from the objectives of the system design in equation.

The time - domain specification just given are quite important since most control systems are time - domain systems that is, they must exhibit acceptable time responses. (This means that the control system must be modified until the transient response is satisfactory). Note that if we specify the values of $t_d$, $t_r$, $t_p$ ,$t_s$ and $M_p$ , then the shape of the response curve is virtually determined. This may be seen clearly from figure.

Note that not all these specifications necessarily apply to any given case. For example, for an over damped system, the terms peak time and maximum, overshoot do not apply. (for systems that yield steady-state errors for step inputs, this error must be kept within a specified percentage level. Detailed discussions of steady – state errors are there in sections to follow).

**4.4.2 Second Order Systems And Transient Response Specifications**

***1. The Rise Time***

$$t_r=\frac{\pi-\phi}{\omega_n\sqrt{1-\varsigma^2}} \text{ or } t_r=\frac{\pi-\phi}{\omega_d} \qquad \ldots(i)$$

Where $\omega_d$ is damped natural frequency in rad/sec.

$$\omega_n\sqrt{1-\varepsilon^2}=\omega_d$$

Where $\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$

(Where $\phi$ is in radians)

***2. Maximum Overshoot $M_p$ and Peak Time $t_p$***

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \text{ or } t_p = \frac{\pi}{\omega_d}$$

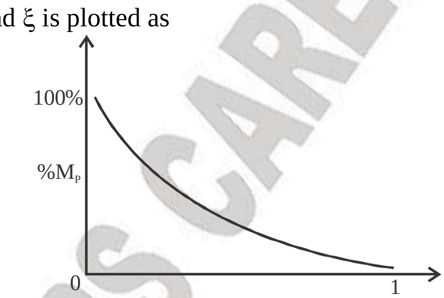
$c(t)_{max}$ is determined by putting $t = t_p$ in the time response expression. Therefore finally we get

$$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \text{ or } c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi)$$

$$\%M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$

The instant which first undershoot occurs can be determined by putting n = 2 in equation (i)

graph relating $M_p$ and $\xi$ is plotted as




i.e. as $M_P$ increase, $\varepsilon$ decreases.

***3 The Settling Time ($t_s$)***

For an under damped system the magnitude of the oscillations present in the output time response decay exponentially with a time constant $1/\xi\ \omega_n$. The time needed to settle down aforesaid oscillations with 2% of the desired value of the output is known as setting time and denoted as $t_s$. The settling time for a second order control system is approximately four times the time constant of the system, hence, $t_s = \frac{4}{\zeta\omega_n}$

On 5% basis, the settling time for second order control system is approximately three times the time constant, i.e.

$$t_s = \frac{3}{\xi\omega_n}$$

An exponentially decaying function will come to its 5% value in 3 times constant ($e^{-t/\tau} = e^{-t/3t} = e^{-3} \cong 0.5$) or 2% value in 4 times constant ($e^{-4} \cong 0.2$)

### 4.5 A FEW COMMENTS ON TRANSIENT – RESPONSE SPECIFICATIONS

Except for certain applications where oscillations cannot be tolerated, it is desirable that the transient response be sufficiently fast and be sufficiently damped. Thus, for a desirable transient response of a second – order system, the damping ratio must be between 0.4 and 0.8 small values of $\xi(\xi < 0.4)$ yield excessive overshoot in the transient response, and a system with a large value of $(\xi > 0.8)$ responds sluggishly.

We shall see later that the maximum overshoot and the rise time conflict with each other. In other words, both the maximum overshoot and the rise time cannot be made smaller simultaneously. If one of them is made smaller, the other necessarily becomes larger.

**Example 1.** A unity feedback system is characterized by an open loop transfer function, $\left[G(s) = \frac{K}{s(s+10)}\right]$. Determine K such that $\varepsilon = 0.5$. Find $t_s$, $t_p$, $M_P$

**Solution.**

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{K}{s^2+10s+K}$$

Comparing this characteristic equation with $s^2 + 2\xi w_n s + w_n^2$

We get : $2\xi w_n = 10$

$$\therefore \frac{C(s)}{R(s)} = \frac{100}{s^2+10s+100}$$

$\Rightarrow w_n = 10$rad/sec

$$t_s = \frac{4}{\xi w_n} = 0.8\,\text{sec}, t_p \frac{\pi}{w_n\sqrt{1-\xi^2}} = 0.362\,\text{sec}$$

$K = w_n^2 = 100$

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

$$= e^{-0.5\pi/\sqrt{1-0.5^2}}\ 16.3\%$$

### 4.6 SOME PRACTICAL SECOND ORDER SYSTEMS

#### 4.6.1 RLC Series Circuit

Characteristic equation $s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$

Compare with standard Equation

$S^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$\omega_n = \frac{1}{\sqrt{LC}}$$ [Undamped natural frequency]

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

Where, $\xi$ is the damping ratio.

***1. For Under Damped System***

$0 < \xi\ 1$

$$\frac{R}{2}\sqrt{\frac{C}{L}} < 1$$

$$R < 2\sqrt{\frac{L}{C}}$$

***2. Critically Damped (For Critically Damped System, $\xi$= 1)***

$$\frac{R}{2}\sqrt{\frac{C}{L}} = 1$$

$$R = 2\sqrt{\frac{L}{C}}$$

***3. Over Damped ($\xi$ > 1)***

$$R > 2\sqrt{\frac{L}{C}}$$

***4. For Undamped System ($\xi$ = 0)***

R = 0

**4.6.2 RLC Parallel Circuit**

Characteristic equation

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

Compare with $s^2 + 2\xi\,\omega_n s + \omega_n^2 = 0$

$$\omega_n = \frac{1}{\sqrt{LC}}$$

$$\xi = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

***1. For Under Damped System***

$$\frac{1}{2} \times \sqrt{\frac{L}{C}} < 1$$

$$R > \frac{1}{2}\sqrt{\frac{L}{C}}$$

***2. Critically Damped***

$$R = \frac{1}{2}\sqrt{\frac{L}{C}}$$

***3. Over Damped***

$$R < \frac{1}{2}\sqrt{\frac{L}{C}}$$

### 4.6.3 Translatory System

$$s^2 + \frac{f}{M}s + \frac{k}{M} = 0$$

Where M is mass
k is spring constant
f is damping coefficient
Compare with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$\omega_n = \sqrt{\frac{k}{M}},$$

$$\xi = \frac{f}{2M\omega_n} = \frac{f}{2M}\sqrt{\frac{M}{k}}, \ \xi = \frac{f}{2\sqrt{kM}}$$

### 4.6.4 Rotational System

$$s^2 + \frac{f}{J}s + \frac{k}{J} = 0$$

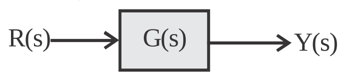
Where J is Moment of Inertia
Replace M by J

$$\omega_n = \sqrt{\frac{k}{J}}$$

$$\xi = \frac{f}{2\sqrt{kJ}}$$

# ASSIGNMENT

**1.** Consider the response of the system shown in figure below:




For an unit step input when $G(s) = \frac{4}{s+5}$ the steady-state error will be

(a) 0.4 unit (b) 0.2 unit
(c) 0.5 unit (d) 1.0 unit

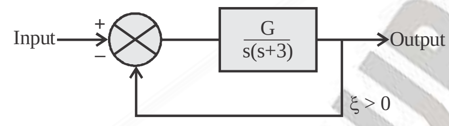
**2.** Consider the following statement:
1.For the positive value of feedback the time constant of closed loop system is less than the time constant of open loop system.
2.Less time constant means response is faster. Therefore feedback improves the time response of the system.
Which of these statements are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) None

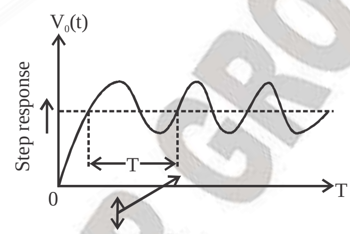
**3.** A second order system has a transfer function given by $G(s) = \frac{25}{s^2 + 8s + 25}$. If the system initially at rest is subjected to a unit step input at t = 0, the second peak in the response will occur at:

(a) $\frac{\pi}{2}$ sec. (b) $\frac{2\pi}{3}$ sec
(c) $\frac{\pi}{3}$ sec (d) $\pi$ sec.

**4.** The block diagram of feedback system is shown in figure (a). Find the minimum value of G for which the step response of the system would exhibit an overshoot as shown in figure (b).




**(A)**




**(B)**

(a) 9 (b) 2.25
(c) 5.25 (d) 6.25

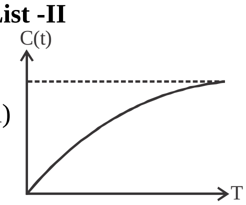
**5.** For G equal to twice this minimum value, find the time period 't' indicated in figure (b).

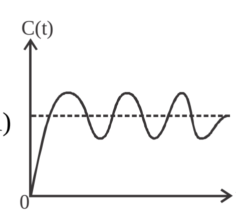
(a) 1.56 sec. (b) 1.96 sec.
(c) 1.45 sec. (d) 2.96 sec.

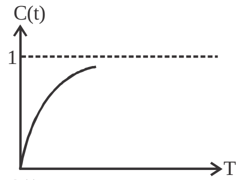
**6.** Match List-I with List-II and select the correct answer using the code given below the lists:

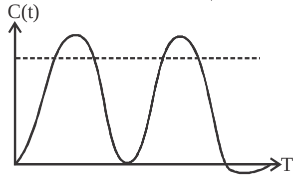
**List-I**
A. Overdamped
B. Underdamped
C. Critical damped
D. Undamped

**List -II**

(i)



(ii)


(iii)



(iv)



**Codes:**
(a) A-i, B-ii, C-iii, D-iv
(b) A-iii, B-ii, C-i, D-iv
(c) A-i, B-iv, C-iii, D-ii
(d) A-iii, B-iv, C-i, D-iii

**7.** The roots of the characteristics equal of the second order system in which real part and imaginary part represents.
(a) Damped frequency and damping
(b) Damping and damped frequency
(c) Natural frequency and damping ratio
(d) Damping ratio and natural frequency

**8.** Consider the following statements:
**1.** The delay time ($t_d$) is the time required for the response to reach 50% of the final value in first time.
**2.** The rise time ($t_T$) is the time required for the response to rise from 10% to 90% of its final value for under damped systems.
**3.** The rise time ($t_T$) is the time required for the response to rise from 0 to 100% for under damped system
Which of these statements are correct?
(a) 1, 2 and 3 only (b) 1 and 2 only
(c) 1 and 3 only (d) 2 and 3 only

**9.** The unit impulse response of a system is h(t) = $e^{-2t}$, $t \geq 0$.
For this system, the steady-state value of the output for unit step input is equal to
(a) 0.6 (b) 0.7
(c) 1 (d) 0.5

**10.** The unit step response of a system starting from rest is given by C(t) $1 - e^{-3t}$ for $t \geq 0$
The transfer function of the system is
(a) $\frac{3}{s(s+3)}$ (b) $\frac{3}{s+3}$
(c) $\frac{3s}{s+3}$ (d) $\frac{1}{s+3}$

**11.** A control system has input r(t) and output c(t). if the input is first passed through a block whose transfer function is $e^{-2s}$ and then applied to the system, the modified output will be
(a) $c(t-2)\,u(t-2)$ (b) $c(t-2)\,u(t)$
(c) $c(t)\,u(t-2)$ (d) None

**12.** The impulse response of the system is c(t)= $-te^{-t} + 2e^{-t}$, its open loop transfer function will be
(a) $\frac{2s+1}{(s+1)^2}$ (b) $\frac{2s+1}{s}$
(c) $\frac{2s+1}{s^2}$ (d) $\frac{2s+1}{s+1}$

**13.** The unit step response of the system is c(t) = $1 - 10e^{-t}$. Its transfer function will be
(a) $\frac{10}{s+1}$ (b) $\frac{1-9s}{s+1}$
(c) $\frac{1+9s}{s+1}$ (d) $\frac{1}{s+1}$

**14.** A ramp input applied to an unity feedback system results in 4% steady state error. The type number and zero frequency gain of the system are respectively
(a) 1 and $\frac{1}{25}$ (b) 1 and 25
(c) 0 and 25 (d) 0 and $\frac{1}{25}$

**15.** A parabolic input applied to an unity feedback system result in 5% steady state error. The type of number and zero frequency gain of the system are respectively|.
(a) 1 and 20 (b) 2 and 20
(c) 2 and $\frac{1}{20}$ (d) 1 and $\frac{1}{20}$

**16.** Consider the following statements regarding time constant of the system:
1.Time constant of a system is related to the speed of the response.
2.Smaller the time constant slower is the system response.
3.It is define as time taken by the system response to reach 98% of the final value.
Which of these statements are correct?
(a) Only 1 (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3

**17.** Find the initial value and final values of the following function

$$F(s)=\frac{12(s+1)}{s(s+2)^2(s+3)}$$

(a) 1, 0 (b) 0, ∞
(c) ∞, 1 (d) 0, 1

**18.** A unity feedback system is characterized by open loop transfer function $G(s)=\frac{k}{s(s+10)}$
Determine the gain k so that the system will have a damping ratio of 0.5
(a) 10 (c) 50
(c) 100 (d) None

**19.** Also find peak overshoot and time to peak overshoot for a unit step input.
(a) 0.326 sec, 16.3% (b) 16.3%, 0.326 sec
(c) 16.3%, 32.6 sec (d) None

**20.** The following transfer function of a unity feedback type 1, second order system has a pole at –2. The nature of gain 'k' is so adjusted that damping ratio is 0.4.
(a) 2.5 (b) 62.5
(c) 6.25 (d) None

**21.** The above equation is subjected to input r(t) = 1 + 4t. Find the steady state error.

(a) ∞ (b) 3.125
(c) 0 (d) 1.28

**22.** Consider the system with the transfer function $P(s)=\frac{1}{(s+1)(s+2)}$
The magnitude and angle of the transfer function for ω = 1.
(a) 3.16 and 75° (b) 0.316 and 71.6°
(c) 31.6 and 71.6° (d) None

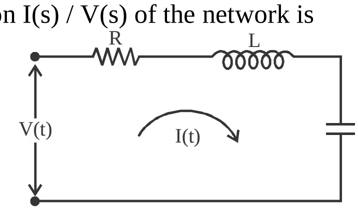
**23.** The transfer function of a system whose input and output are related by the following differential equation?

$$\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=u+\frac{du}{dt}$$

(Ignoring terms due to initial condition).
(a) $\frac{s+1}{s^2+3s+2}$ (b) $\frac{s^2+3s+2}{s+1}$
(c) $\frac{s+1}{s^2+5s+2}$ (d) None

**24.** A particular system containing a time delay has the differential equation $\frac{d}{dt}y(t)+y(t)=u(t-T)$. Find the transfer function of this system (Ignoring term due to initial condition).
(a) $\frac{1}{s+1}$ (b) $\frac{1}{s+2}$
(c) $\frac{e^{-sT}}{s+1}$ (d) $\frac{e^{-sT}}{s+2}$

**25.** For the network shown in figure below v(t) is the input and i(t) is the output. The transfer function I(s) / V(s) of the network is




(a) $\frac{LCs^2+RCs+1}{Cs}$ (b) $\frac{C}{LCs^2+RCs+1}$f

(c) $\frac{C}{RC+LCs+1}$ (d) $\frac{Cs}{LCs^2+RCs+1}$

**26.** In a circuit the current i(t) has the Laplace transform $l(s)=\frac{3(s+10)}{(s+12)}$. The final value of i(f) is

(a) 0.25 (b) 2.5

(c) 3 (d) Infinity

**27.** If the unit step response of a system is a unit impulse function, then the transfer function of such a system will be

(a) 1 (b) $\frac{1}{s}$

(c) S (d) $\frac{1}{s^2}$

**28.** The unit-impulse response of unity-feedback control system is given by the open-loop transfer function is equal to

(a) $\frac{2s+1}{(s+1)^2}$ (b) $\frac{s+2}{(s+1)^2}$

(c) $\frac{2s+1}{s^2}$ (d) $\frac{s+1}{s^2}$

**29.** For a second order system, damping ratio ($\xi$) is $0 < \xi < 1$, then the roots of the characteristic polynomial are:

(a) Real but not equal

(b) Real and equal

(c) Complex conjugates

(d) Imaginary

**30.** Backlash in a stable control system may cause

(a) Underdamping

(b) Overdamping

(c) High level oscillations

(d) Low level oscillation

**31.** Which one of the following is the correct statement about a stable system?

poles in the right half of s-plane

zeroes in the right half of s-plane

poles in the left half of s-plane and zeros in the right half of s-plane

no poles or zeroes in the right half of s-plane or on the j$\omega$-axis excluding the origin.

**32.** The transfer function H(s) of a system is given by $H(s)=\frac{Y(s)}{X(s)}=\frac{s+2}{s^2+s+4}$. Given that under steady-state condition, the sinusoidal input and output are respectively X(t) = cos2t and y(t) = cos(2t + θ). Then the angle θ will be

(a) 45° (a) Zero

(b) –45° (d) –90°

**33.** The forward path transfer function of a unity feedback system is

$G(s)=\frac{k}{(s+a)}$

The system has 10% overshoot and velocity error constant $k_v$ = 100. The value of k is

(a) $237 \times 10^3$ (b) 144

(c) $14.4 \times 10^3$ (d) 237

**34.** The value of a is

(a) $23.7 \times 10^3$ (b) 237

(c) $14.4 \times 10^3$ (d) 144

**35.** A system has $k_P$ = 4, the steady state error for input of 10 u(t) and 10t u(t) are respectively

(a) 2, ∞ (b) 0.4, ∞

(c) 0.4, 0 (d) 2, 0

**36.** In second order control system the value of the resonant peak will be unity when the damping ratio has a value of

(a) Zero (b) Unit

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

**37.** Octave frequency range is specified by

(a) $\frac{\omega_2}{\omega_1}=2$ (b) $\frac{\omega_2}{\omega_1}=10$

(c) $\frac{\omega_2}{\omega_1}=8$ (d) None

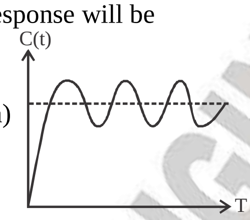
**38.** A decade frequency range in given by $\frac{\omega_2}{\omega_1}$ equal to
(a) 2 (b) 4
(c) 8 (d) 10

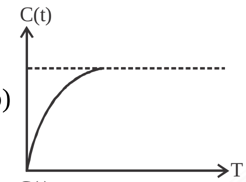
**39.** The steady state error of a type 1 second order system to unit ramp input is
(a) $2\,\xi\,\omega_n$ (b) $\frac{2\xi}{\omega_n}$
(c) $\frac{4\xi}{\omega_n}$ (d) None of these

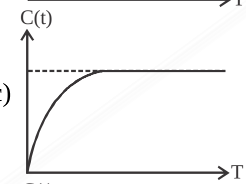
**40.** The response C(t) of a system is given by the differential equation
$$\frac{d^2C(t)}{dt^2}+\frac{4dC(t)}{dt}+5C(t)=0 \text{ is}$$
(a) Undamped
(b) Underdamped
(c) Critically damped
(d) Oscillatory

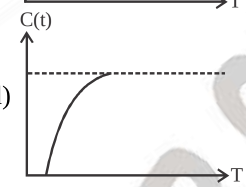
**41.** If two identical first order low-pass filters are cascaded non-interactively, then the unit step response of the composite filter will be
(a) Critically damped
(b) Underdamped
(c) Overdamped
(d) Oscillatory

**42.** A step input is applied to a system will the transfer function $G(s)=\frac{e^{-s}}{1+0.5s}$. The output response will be

(a)


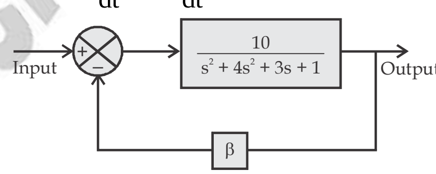

(b)



(c)



(d)



**43.** If a second order system has poles ate $-1 \pm j$, then step response of the system will exhibit a peak value at
(a) 4.5 s (b) 3.5 s
(c) 3.14 s (d) 1 s

**44.** The dynamic equation of a second-order system is $2\frac{d^2y}{dt^2}+4\frac{dy}{dt}+8y=8$




The damping coefficient is
(a) 0.1 (b) 0.25
(c) 0.5 (d) 1.0

**45.** Match List-I with List-II and select the correct answer using the codes given below the lists:

**List-I (Transfer function)**

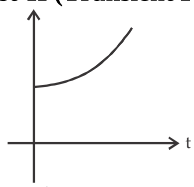
A. $\frac{1}{(s+\alpha)(s+\beta)}$

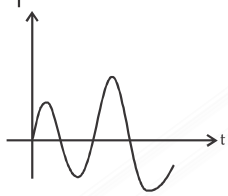
B. $\frac{1}{(s-\alpha)(s-\beta)}$

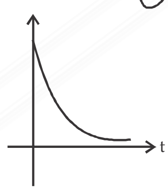
C. $\dfrac{1}{(s-\alpha+j\beta)(s-a-j\beta)}$

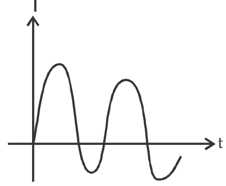
D. $\dfrac{1}{(s+\alpha+j\beta)(s+\alpha-j\beta)}$

**List-II (Transient Function)**

(i)



(ii)



(iii)



(iv)



Assume $\alpha > 0$ , $\beta > 0$

**Codes:**

(a) A-i, B-iii, C-ii, D-iv
(b) A-iii, B-i, C-ii, D-iv
(c) A-iii, B-iv, C-ii, D-i
(d) A-iv, B-ii, C-i, D-iii

**46.** For a unity feedback system, the open-loop transfer function is

$$G(s) = \frac{16(s+2)}{s^2(s+1)(s+4)}$$

What is the steady-state error if the input is r(t) = $(2 + 3t + 4t^2)$ u(t) ?

(a) 0 (b) 1
(c) 4 (d) 5

**47.** A casual system having the transfer function $H(s) = \dfrac{1}{s+2}$ is excited with 10u(t). The time at which the output reaches 99% of its steady state value is

(a) 2.7 sec (b) 2.5 sec
(c) 2.3 sec (d) 2.1 sec

**48.** The open-loop transfer function G(s) of a unity feedback control system is $\dfrac{1}{s(s+1)}$. The system is subjected to an input r(t) = sin t. The steady state error will be

(a) Zero (b) 1
(c) $\sqrt{2}\sin\left(1-\dfrac{\pi}{4}\right)$ (d) $\sqrt{2}\sin\left(1+\dfrac{\pi}{4}\right)$

**49.** Which one of the following is the response y(t) of a casual LTI system described by

$$H(s) = \frac{(s+1)}{s^2+2s+2}$$

For a given input $x(t) = e^{-t}u(t)$ ?

(a) $Y(t) = e^{-t}\sin tu(t)$
(b) $Y(t) = e^{-(t-1)}\sin(t-1)\,u(t-1)$
(c) $Y(t) = \sin(t-1)\,u(t-1)$
(d) $Y(t) = e^{-t}\cos t\,u(t)$

**50.** The unit step response of a particular control system is given by $c(t) = 1 - 10e^{-t}$. then its transfer function is

(a) $\dfrac{10}{s+1}$

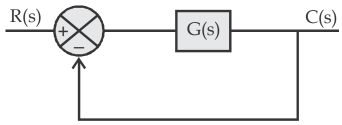
(b)

| | | |
|---|---|---|
| $S^3$ | 1 | 4 |
| $S^2$ | K | 3 |
| $S^1$ | $\frac{4k-3}{k}$ | |
| $S^0$ | 3 | |

(c) $\dfrac{1-9s}{s+1}$ (d) $\dfrac{1-9s}{s(s+1)}$

**51.** Assuming unit ramp input, match List-I (system type) with List-II (Steady state error) and select the correct answer using the codes given below lists:

| **List-I** | **List-II** |
|---|---|
| A. 0 | (i) K |
| B. 1 | (ii) ∞ |
| C. 2 | (iii) 0 |
| D. 3 | (iv) 1/K |




**Codes:**
(a) A-ii, B-iv, C-iii, D-i
(b) A-i, B-ii, C-iii, D-iv
(c) A-ii, B-i, C-iv, D-iii
(d) A-i, B-ii, C-iv, D-iii

**52.** Match List-I (System G(s)) with List-II (Nature of response) and select the correct answer using the code given below the Lists:

**List-I**

A. $\frac{400}{s^2+12s+400}$

B. $\frac{900}{s^2+90s+900}$

C. $\frac{225}{s^2+30s+225}$

D. $\frac{625}{s^2+625}$

**List-II**
(i) Undamped
(ii) Critically damped
(iii) Underdamped
(iv) Overdamped

**Codes:**
(a) A-iii, B-i, C-ii, D-iv
(b) A-ii, B-iv, C-iii, D-i
(c) A-iii, B-iv, C-ii, D-i
(d) A-ii, B-i, C-iii, D-iv

**53.** The unit step response of a second order system is = $1 - e^{-5t} - 5t\, e^{-5t}$
Consider the following statements:
The undamped natural frequency is 5 rad/s
The damping ratio is 1
The impulse response is $25t\, e^{-5t}$
Which of the statements given above are correct?
(a) Only 1 and 2 (b) Only 2 and 3
(c) Only 1 and 3 (d) 1, 2 and 3

**54.** The steady state error due to a ramp input for a type two system is equal to
(a) Zero
(b) Infinite
(c) Constant
(d) Data is insufficient

**55.** Given the transfer function, $G(s) = \frac{121}{s^2+13.2s+121}$ of a system. Which of the following characteristics does it have ?
(a) Overdamped and setting time 1.1s
(b) Underdamped and settling time 0.6s
(c) Critically damped and setting time 0.8s
(d) Underdamped and setting time 0.707 s.

**56.** The impulse response of an initially relaxed linear system is $e^{-2t}$ u(t). to produce a response of $te^{-2t}$ u(t). the input must be equal to
(a) 2e–t u(t) (b) $\frac{1}{2}e^{-2t}u(t)$
(c) e–2t u(t) (d) e–tu(t)

**57.** Consider a system with the transfer function $G(s) = \frac{s+6}{ks^2+s+6}$. Its damping ratio will be 0.5 when the value of k is
(a) 2/6 (b) 3
(c) 1/6 (d) 6

**58.** The unit impulse response of a system is given by c(t) = $0.5\, e^{-t/2}$ its transfer function is
(a) $\frac{1}{(s+2)}$ (b) $\frac{1}{(1+2s)}$
(c) $\frac{2}{(1+2s)}$ (d) $\frac{2}{(s+2)}$

**59.** Which one of the following is the steady state error of a step input applied to a unity feedback system will the open loop transfer function
$G(s) = \frac{10}{s^2+14s+50}$ ?
(a) $e_{ss} = 0$ (b) $e_{ss} = 0.83$
(c) $e_{ss} = 1$ (d) $e_{ss} = \infty$

**60.** The steady state error of a stable type 0 unity feedback system for a unit step function is

(a) 0 (b) $\frac{1}{1+K_p}$

(c) ∞ (d) $\frac{1}{K_p}$

**61.** What is the steady state error for a unity feedback control system having $G(s)=\frac{1}{s(s+1)}$ due to unit ramp input?

(a) 1 (b) 0.5

(c) 0.25 (d) $\sqrt{0.5}$

**62.** If the closed-loop transfer functions T(s) of a unity negative feedback system is given by

$$T(s)=\frac{a_{n-1}s+a_n}{s^n+a_1s^{n-1}+.......+a_{n-1}s+a_n}$$

Then the steady state error for a unity ramp input is

(a) $\frac{a_n}{a_n-1}$ (b) $\frac{a_n}{a_{n-2}}$

(c) $\frac{a_n}{a_n-2}$ (d) zero

**63.** If the characteristic equation of a closed-loop system is $s^2 + 2s + 2 = 0$, then the system is

(a) Overdamped

(b) Critically damped

(c) Underdamped

(d) Undamped

**64.** The OPEN-loop DC gain of a unity negative feedback system with closed-loop transfer Function $\frac{s-4}{s^2+7s+13}$ is

(a) $\frac{4}{13}$ (b) $\frac{4}{9}$

(c) 4 (d) 13

**65.** Consider a system with the transfer function

$G(s)=\frac{s+6}{Ks^2+s+6}$. Its damping ratio will be 0.5 when the value of K is

(a) 2/6 (b) 3

(c) 1/6 (d) 6

**66.** The transfer function Y(s)/U(s) of a system described by the state equations x(t) = – 2u(t) and y(t) = 0.5x(t) is

(a) 0.5/(s – 2) (b) 1/(s – 2)

(c) 0.5/(s + 2) (d) 1/(s + 2)

**67.** The transfer function of a system is $G(s)=\frac{s+6}{(s+1)(s+100)}$ for a unit-step input to the system the approximate setting time for 2% criterion is

(a) 100 sec (b) 4 sec

(c) 1 sec (d) 0.01 sec

**68.** If the system, initially at rest, is subjected to a unit step input at t = 0, the second peak in the response will occur at.

(a) π sec. (b) π/3sec.

(c) 2π/3sec. (d) π/2sec.

## ANSWER KEY

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | b | **2.** | c | **3.** | d | **4.** | d | **5.** | b | **6.** | a | **7.** | b | **8.** | c | **9.** | d | **10.** | b |
| **11.** | a | **12.** | c | **13.** | b | **14.** | b | **15.** | b | **16.** | a | **17.** | d | **18.** | c | **19.** | b | **20.** | c |
| **21.** | d | **22.** | b | **23.** | a | **24.** | c | **25.** | d | **26.** | b | **27.** | c | **28.** | c | **29.** | c | **30.** | d |
| **31.** | d | **32.** | c | **33.** | c | **34.** | d | **35.** | a | **36.** | c | **37.** | a | **38.** | d | **39.** | b | **40.** | b |
| **41.** | a | **42.** | d | **43.** | c | **44.** | c | **45.** | b | **46.** | b | **47.** | c | **48.** | a | **49.** | a | **50.** | c |
| **51.** | a | **52.** | c | **53.** | d | **54.** | a | **55.** | b | **56.** | c | **57.** | c | **58.** | b | **59.** | b | **60.** | b |
| **61.** | a | **62.** | d | **63.** | c | **64.** | b | **65.** | c | **66.** | d | **67.** | b | **68.** | a | | | | |

# SOLUTIONS

**Sol. 1.**

$Y(s) = G(s) \times R(s) = \frac{4}{s+5}.\frac{1}{s} = \frac{4}{s(s+5)}$

$\therefore$ y(t) = response during the transient period steady-state response = $y(t)|_{t\to\infty} = y(\infty) = 0.8$

$\therefore$ stead-state error = $e_{ss} = \underset{s\to 0}{Lt}\ sE(s) = 1.0 - 0.8f$

= 0.2 unit

**Sol. 3.**

$R(s) = \frac{1}{s}$

$C(s) = R(s)\ .\ G(s) = \frac{5}{s(s^2+8s+25)}$

Compare equation (i) with

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$2\xi\omega_n = 8$ and $\omega_n = \sqrt{25} = 5 rad/sec$

$\therefore$ Required time = $\frac{3\pi}{\omega_n\sqrt{1-\xi^2}}$ ($3\pi$ because of second peak)

**Sol. 4.**

Closed loop transfer function:

$\frac{C(s)}{R(s)} = \frac{G}{s^2+3s+G}$

Characteristic equation $s^2 + 3s + G = 0$

Compare with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$2\xi\omega_n = 3$ and $\omega_n = \sqrt{G}$

For minimum value of G 'ξ' should be 0.6.

$\therefore 2 \times 0.6\ \sqrt{G} = 3$

G = 6.25

$= \frac{3\pi}{5\sqrt{1-(0.8)^2}} = \pi sec.$

**Sol. 5.**

G' = 2 G = 2 × 6.25 = 12.5

$\omega_n = \sqrt{G'} = \sqrt{12.5} = 3.53 rad/sec.fg$

$2\xi\omega_n = 3$

$\xi = \frac{3}{2\omega_n} = \frac{3}{2\times 3.53} = 0.424$

$\omega_d = \omega_n\sqrt{1-\xi^2} = 3.53\sqrt{1-(0.424)^2}$

= 3.197 rad.

$\omega_d = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega_d} = \frac{2\pi}{3.197}$ = 1.96 sec.

**Sol. 7.**

Characteristics equation of 2nd order system is

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

The root of the equation are

$s_1 = -\xi\omega_n + j\omega_n\sqrt{1-\xi^2}$

and $s_2 = -\xi\omega_n - j\omega_n\sqrt{1-\xi^2}$

Real part of the roots ($+\ \xi\omega_n$) represents damping and imaginary part $\left(\omega_n\sqrt{1-\xi^2}\right)$ represents the damped frequency ($\omega_d$), or conditional frequency

**Sol.8.**

Rise time ($t_r$): It is the time required for the response to rise 10% to 90% of its final value for over damped system and 0 to 100% for under damped system.

**Sol. 9.**

$h(t) = e^{-2t}$

$H(s) = \frac{1}{s+2}$ and $R(s) = \frac{1}{s(s+2)}$

$\therefore$ Output $C(s) = H(s).R(s) = \frac{1}{s(s+2)}$

Steady-state value,

$e_{ss} = \underset{s\to 0}{Lt}\ sC(s) = \underset{s\to 0}{Lt}\ s;\frac{1}{s(s+2)}$ = 0.5

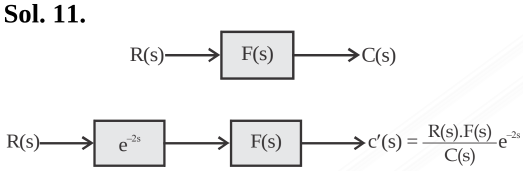
**Sol. 10.**

$C(t) = 1 - e^{-3t}$

$$= \frac{1}{s} - \frac{1}{s+3} = \frac{3}{s(s+3)} \text{ and } R(s) = \frac{1}{s}$$

$$\therefore \text{ Transfer function } H(s) = \frac{C(s)}{R(s)} = \frac{3}{s+3}$$

**Sol. 11.**




$L^{-1}c(s).e^{-2s} = c(t-2)\,u(t-2)$
Using above formula,
$L^{-1}\,c(s)\,.\,e^{-2s} = c(t-2)\,u(t-2)$

**Sol. 12.**

We know that,

L [Impulse response] = Transfer function = $\frac{C(s)}{R(s)}$

$$c(t) = -\frac{1}{(s+1)^2} + \frac{2}{s+1}$$

$$\therefore = \frac{2s+1}{(s+1)^2} + \frac{2}{s+1} = \text{closed loop T.F.}$$

$$\frac{G(s)}{1+G(s)H(s)} = \frac{2s+1}{(s+1)^2}$$

$$\frac{G(s)}{1+G(s)} = \frac{2s+1}{(s+1)^2}$$

$$\underset{\text{Open loop T.F.}}{\underset{\Downarrow}{G(s)}} = \frac{2S+1}{s^2}$$

**Sol. 13.**

$$H(s) = \frac{C(s)}{R(s)}$$

$$H(s)\frac{\frac{1}{s} - \frac{10}{s+10}}{1/s} = \frac{1-9s}{(s+1)}$$

**Sol. 14.**

If ramp input is applied and steady-state error ($e_{ss}$) is the finite then the type of system is 1.

and $e_{ss} = \frac{1}{k}$

$$\frac{4}{100} = \frac{1}{k}$$

$\therefore$ k = 25

**Sol. 15.**

If parabolic input applied and steady-state error is finite then the type of system is 2.

and $e_{ss} = \frac{1}{k}$

$\frac{5}{100} = \frac{1}{k}$, therefore: k = 20

Above problem can be solve with the help of table:

| | **Step input** | **Ramp input** | **Parabolic input** |
|---|---|---|---|
| **Type-0** | $\frac{A}{1+k}$ | $\infty$ | $\infty$ |
| Type-1 | 0 | $\frac{A}{k}$ | $\infty$ |
| Type-2 | 0 | 0 | $\frac{A}{k}$ |

**Sol. 16.**

Time constant is defined as time taken by the system response to reach 63% of the final value. Smaller the time constant faster is the system response and larger its value, slower is the response.

**Sol. 17.**

$$F(s) = \frac{12(s+1)}{s(s+2)^2\,(s+3)}$$

Initial value = $\lim_{s\to\infty} \frac{12\left(1+\frac{1}{s}\right)}{s^3\left(1+\frac{4}{s^2}+\frac{4}{s}\right)\left(1+\frac{3}{s}\right)} = 0$

$= \lim_{s\to\infty} \frac{12s\left(1+\frac{1}{s}\right)}{s^3\left(1+\frac{4}{s^2}+\frac{4}{s}\right)\left(1+\frac{3}{s}\right)} = 0$

And Find value

$= \lim_{s\to\infty} s.F(s) = \lim_{s\to\infty} \frac{12(s+1)}{(s+2)^2\,(s+3)} \quad \frac{12}{4\times 3} = 1$

**Sol. 18.**

The characteristics equation is 1 + G(s) H(s) = 0

$1+\frac{k}{s(s+10)} = 0$

Compare with standard second order transfer function.

$2\xi\omega_n = 10$

$\omega_n = \frac{10}{2\xi} = \frac{10}{2\times 0.5} = 10$ and $\omega_n = k$

$\therefore$ $K = 10^2 = 100$

**Sol.19.**

$M_P = \frac{-\pi\xi}{e\sqrt{1-\xi^2}} = e\frac{-0.5\pi}{\sqrt{1-(0.5)^2}} = 16.3\%$

$T_P = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{10\sqrt{1-(0.5)^2}} = 0.326\,\text{sec}$

**Sol. 20.**

$G(s) = \frac{k}{s(s+2)}$

The characteristic equation,

1 + G(s) H(s) = 0

$s^2 + 2s + k = 0$

$2\xi\omega_n = 2$

$\omega_n = \frac{2}{2\times 0.4} = 2.5$ and $\omega_n^2 = k$

$k = (2.5)^2 = 6.25$

**Sol. 21.**

$G(s) = \frac{k}{s(s+2)} = \frac{6.25}{s(s+2)}$

r(t) = 1 + 4t

$e_{ss} = \frac{k}{1+k_p} + \frac{4}{k_v}$

$k_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{6.25}{s(s+2)} = \infty$

$k_v = \lim_{s\to 0} S.G(s) = \lim_{s\to 0} \frac{6.25}{s+2} = \frac{6.25}{2}$

$e_{ss} = \frac{1}{1+\infty} + \frac{4}{3.125} = 1.28$

**Sol. 22.**

$P(j\omega) = \frac{1}{(1+j\omega)(2+j\omega)}$ (Put, $s = j\omega$)

for $\xi = 1$, $|P(j)| = \frac{1}{\sqrt{2}.\sqrt{5}} = 0.316$

$\angle P(j) = -\tan^{-1}(1) - \tan^{-1}(0.5)$

$= -45° - 26.6°$

$= 71.6°$

**Sol. 23.**

Taking Laplace transform of the equation:

$S^2Y(s) + 3\,sY(s) + 2\,Y(s) = u(s) + s\,u(s)$

$\frac{Y(s)}{u(s)} = \frac{s+1}{s^2+3s+2}$

$\therefore$ Transfer function $\frac{Y(s)}{u(s)} = \frac{s+1}{s^2+3s+2}$

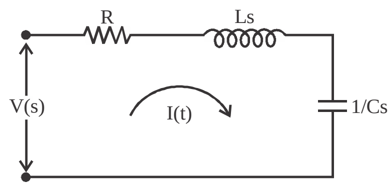
**Sol. 24.**

The Laplace transform of the differential equation:

$sY(s) + Y(s) = e^{-sT}\,u(s)$

$\frac{Y(s)}{u(s)} = \frac{e^{-sT}}{s+1}$

**Sol. 25.**




Now apply KVL,

$$V(s) = \left(R + Ls + \frac{1}{Cs}\right) I(s)$$

$$\frac{V(s)}{I(s)} = \frac{RCs + LCs^2 + 1}{Cs}$$

$$\frac{I(s)}{V(s)} = \frac{Cs}{LCs^2 + RCs + 1}$$

**Sol. 26.**

Find value in s-domain

$$= \underset{s\to 0}{Lt} \frac{3(s+10)}{(s+12)} = \frac{3\times 10}{12} = 2.5$$

**Sol. 27.**

$$H(s) = \frac{Y(s)}{R(s)} = \frac{1}{1/s} = s$$

**Sol. 28.**

Closed loop transfer function = L [impulse response]

= L [$-t\,e^{-t} + 2\,e^{-t}$]

$$= \frac{-1}{(s+1)^2} + \frac{2}{s+1} = \frac{2s+1}{(s+1)^2}$$

$$\therefore \quad \frac{G(s)}{1+G(s)H(s)} = \frac{2s+1}{(s+1)^2}$$

$$\text{or} \quad \frac{G(s)}{1+G(s)} = \frac{2s+1}{(s+1)^2}$$

$G(s)[s^2 + 1 + 2s - 2s - 1] = 2s + 1$

$$\therefore \quad \underset{\text{openloop transfer function}}{\underset{\Downarrow}{G(s)}} = \frac{2s+1}{s^2}$$

**Sol. 30.**

In a servo system, the gear backlash may cause sustained oscillations or chattering phenomenon, and the system may even turn unstable for large backlash.

**Sol. 32.**

$$H(j\omega) = \frac{2 + j\omega}{(j\omega)^2 + j\omega + 4} = \frac{2 + j\omega}{-\omega^2 + j\omega + 4}$$

$\omega = 2$

$$H(2j) = \frac{2 + 2j}{2j}$$

$\angle H(2j) = \tan^{-1}(1) - \tan^{-1}(\infty)$

$= 45° - 90° = -45°$

$Y(t) = \cos(2t - 45°)$

**Sol. 33.**

Velocity error constant, $k_v = 100$ i.e. finite so.

This indicate type-1 system

n = 1

$$\therefore \quad G(s) = \frac{k}{s(s+a)}$$

$$K_v = \underset{s\to 0}{Lt}\, sG(s) = \frac{k}{a}$$

$$\therefore \quad \frac{k}{a} = 100 \Rightarrow a = \frac{k}{100}$$

For 10% overshoot,

$$0.1 = \frac{e - \pi\xi}{\sqrt{1-\xi^2}}$$

$\therefore\ \xi = 0.6$

$\therefore$ Transfer function

$$= \frac{G(s)}{1+G(s)} = \frac{kl}{s^2 + as + k}$$

Compare with standard equation,

$2\xi\omega_n = a$ and $\omega_n = \sqrt{k}$

$2\xi\sqrt{k} = a$

$$\frac{2\times 6\times \sqrt{k}}{10} = \frac{k}{100}$$

$120\sqrt{k} = k$

$K^2 - 14400\,k = 0$

K = 14400

**Sol.35.**

For 10 u(t) i.e. (step input)

$$e_{ss} = \frac{10}{1+k_p} = \frac{10}{5} = 2$$

and for 10t u(t), i.e. (ramp input)

$e_{ss} = \infty$

so the system is type '0'.

**Sol. 40.**

In s-domain the equation becomes:

$s_2 + 4_s + 5 = 0$

Compare this equation with standard equation, we get,

$$\omega_n = \sqrt{5} \text{ and } 2\times\xi\times\omega_n = 4$$

$$\xi = \frac{4}{2\times\sqrt{5}} = 0.89$$

$\xi = 0.89$ which is less than 1. So the response of the system is underdamped.

**Sol. 41.**

Let the first order system $= \frac{1}{s+5}$.

If two identical first order L.P.F. are cascaded then,

$$\frac{1}{(s+5)^2} = \frac{1}{s^2+10s+25} f$$

Characteristics equation = $s^2 + 10s + 25$

$\omega_n = 5$ and $2 \times \xi \times \omega_n = 10$

$$k_p = \frac{1}{k-1} \quad \xi = \frac{10}{2\times 5} = 1$$

$\Rightarrow \xi = 1$. So the composite filter will be critically damped.

**Sol. 43. (c)**

$$\frac{C(s)}{R(s)} = \frac{1}{(s+1+j)(s+1-j)}$$

$$= \frac{1}{(s+1)^2+1} = \frac{1}{s^2+2s+2}$$

$$\omega_n = \sqrt{2}, 2\xi\omega_n = 2$$

$$\xi = \frac{1}{\sqrt{2}}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\varepsilon^2}}$$

$$= \frac{\pi}{\sqrt{2}\sqrt{1-\frac{1}{2}}} = \frac{\pi}{\sqrt{2}\times\frac{1}{\sqrt{2}}} = \pi \sec$$

**Sol. 44. (c)**

$$2\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 8$$

$$y(s) = \frac{8}{2s^2+4s+8}$$

$\omega_n = 2$,

$2\xi\omega_n = 2$

$$\xi = \frac{2}{2\omega_n}$$

$$= \frac{2}{2\times 2} = \frac{1}{2} = 0.5$$

**Sol. 45. (b)**

**Sol. 46. (b)**

Steady state error due to 3 basic inputs is:

$$e_{ss} = \frac{1}{1+Kp} + \frac{1}{Kv} + \frac{1}{Ka}$$

**Sol. 47. (c)**

$$\frac{C(s)}{R(s)} = \frac{10}{s(s+2)} \Bigg| \begin{matrix} R(t) = 10\,u(t) \\ R(s) = \frac{10}{s} \end{matrix}$$

$$C(s) = \frac{10}{s^2+2s}$$

$$C(s) = \frac{10}{s(s+2)} = 5\left(\frac{1}{s} - \frac{1}{s+2}\right)$$

$C(t) = 5(1 - e^{-2t})$ …(i)

Steady state value is when t = s i.e. 5

$$\frac{99}{100}\times 5 = 5(1-e^{-2t})$$

$$e^{-2t} = 1 - \frac{99}{100} = \frac{1}{100} = 10^{-2}$$

t = 2.3

**Sol. 48. (a)**

Steady state error $e_{ss} = \lim_{s\to0} \frac{sR(s)}{1+G(s)}$

$R(s) = \frac{1}{s^2+1}$

$e_{ss} = \lim_{s\to0} \frac{s\frac{1}{s^2+1}}{1+\frac{1}{s(s+1)}}$

$e_{ss} = \lim_{s\to0} \frac{s^2(s+1)}{(s^2+1)\{s(s+1)+1\}}$

$e_{ss} = 0$

**Sol. 49. (a)**

$X(s) = \frac{1}{s+1}$

Y(s) = X(s) H (s)

$= \frac{1}{(s+1)} \cdot \frac{(s+1)}{\{(s+1)^2+1\}} = \frac{1}{(s+1)^2+1}$

$\Rightarrow$ y(t) = $e^{-t}$ sin t u (t)

**Sol. 50. (c)**

$R(s) = \frac{1}{s} \quad C(s) = \frac{1}{s} - \frac{10}{s+1}$

$= \frac{s+1-10s}{s(s+1)} = \frac{1-9s}{s(s+1)}$

$T(s) = \frac{C(s)}{R(s)} = \frac{1-9s}{s+1}$

**Sol. 51. (a)**

Table for steady error

| Type | Unit step | Unit Ramp | Unit Parabola |
|---|---|---|---|
| Type 0 | $\frac{1}{1+K_p}$ | $\infty$ | $\infty$ |
| Type 1 | 0 | $\frac{1}{K_v}$ | $\infty$ |
| Type 2 | 0 | 0 | $\frac{1}{K_a}$ |

where

$K_p = \lim_{s\to0} G(s)\ H(s)$

$K_v = \lim_{s\to0} sG(s)\ H(s)$

$K_a = \lim_{s\to0} s^2G(s)H(s)$

**Sol. 52. (c)**

$S^2 + 12s + 400 = 0$

$\Rightarrow \xi = \frac{12}{2\sqrt{400}} = \frac{12}{40} < 1 \Rightarrow$ underdamped

$S^2 + 90s + 900 = 0$

$\Rightarrow \xi = \frac{900}{2\sqrt{900}} = \frac{90}{2\times30} > 1 \Rightarrow$ overdamped

$S^2 + 30s + 225 = 0$

$\Rightarrow \xi = \frac{30}{2\sqrt{225}} = \frac{30}{2\times15} = 1$

$\Rightarrow$ critically damped

$S^2 + 625 = 0$

$\Rightarrow \xi = 0 \Rightarrow$ undamped

**Sol. 53. (d)**

$C(s) = \frac{1}{s} - \frac{1}{s+5} - \frac{5}{(s+5)^2}$

$= \frac{(s+5)^2-(s+5)s-5s}{s(s+5)^2} = \frac{25}{s(s+5)^2}$

$C(s) = \frac{25}{s(s^2+10s+25)}$

$R(s) = \frac{1}{s}$

$G(s) = \frac{C(s)}{R(s)} = \frac{25}{s^2+10s+25}$

$\omega_n = \sqrt{25} \Rightarrow \omega_n = 5 rad/s$

$\xi = \frac{10}{2\times5} = 1$

Impulse response = $\frac{d}{dt}(1-e^{-5t}-5te^{-5t})$

= $5e^{-5t} - 5e^{-5t} + 25\ te^{-5t} = 25\ te^{-5t}$

**Sol. 54. (a)**

**Sol. 55. (b)**

Characteristic equation:

$S^2 + 13.2s + 121$

Comparing it with $s^2 + 2\xi w_n s + w_n^2$

$w_n = \sqrt{121} = 11 rad/sec$

$2\xi w_n = 13.2$

Since $\xi < 1$, so the system is under damped and setting time,

$$T_s = \frac{2}{\xi\omega_n} = \frac{4}{0.6\times 11} = 0.6 sec$$

**Sol. 56. (c)**

$C(t) = t.e^{-2t}\, u(t)$

$$C(s) = \frac{1}{(s+2)^2}$$

$$G(s) = \frac{1}{(s+2)}$$

$$\therefore\ \frac{C(s)}{R(s)} = G(s)$$

$$R(s) = \frac{C(s)}{G(s)} = \frac{1}{(s+2)^2}\times(s+2)$$

$$= \frac{1}{s+2}$$

$\therefore\ r(t) = e^{-t}\, u(t)$

**Sol. 57. (c)**

$$G(s) = \frac{s+6}{ks^2+s+6} = \frac{k(s+6)}{s^2+\frac{s}{k}+\frac{6}{k}}$$

Comparing it with $s^2 + 2\xi\omega_n^s + \omega_n^2$

$$\omega_n = \sqrt{\frac{6}{k}}$$

$$2\xi\omega_n = \frac{1}{k} \Rightarrow 2\times 0.5\times\sqrt{\frac{6}{k}} = \frac{1}{k}$$

$$\Rightarrow \frac{6}{k} = \frac{1}{k^2} \Rightarrow k = \frac{1}{6}$$

**Sol. 58. (b)**

$$Cs = \frac{0.5}{\left(s+\frac{1}{2}\right)}$$

$\&R(s) = 1[\because r(t) = s(t)]$

$$\therefore G(s) = \frac{C(s)}{R(s)} = \frac{0.5}{\left(s+\frac{1}{2}\right)} = \frac{1}{1+2s}$$

**Sol.59. (b)**

Since it is a step input. So position error constant $k_p$ is defined for it.

$$\text{i.e. } k_p = \underset{s\to 0}{Lt}\frac{10}{s^2+14s+50} = \frac{1}{5}$$

$\therefore$ steady state error

$$e_{ss} = \frac{1}{1+k_p}$$

$$e_{ss} = \frac{1}{1+\frac{1}{5}} = 0.83$$

**Sol. 60. (b)**

**Sol. 61. (a)**

For a unit ramp i/p, velocity error constant is defined i.e. $K_V$

$$K_v = \underset{s\to 0}{Lt}\, sG(s)$$

$$= \underset{s\to 0}{Lt}\frac{s}{s(1+s)}$$

$\therefore$ steady state error,

$$e_{ss} = \frac{1}{kv} = 1$$

**Sol. 62. (d)**

$$T(s) = \frac{\dfrac{a_{n-1}S+a_n}{s^n+a_n s^{n-1}+......a_{n-2}s^2}}{1+\dfrac{a_{n-1}s+a_n}{s^n+a_1 s^{n-1}+.....a_{n-2}s^2}}$$

$$G(s) = \frac{a_{n-1}s+a_n}{s^n+a_1 s^{n-1}+......a_{n-s}s^2}$$

For type 2, Ramp input

$$K_v = \lim_{s\to 0} sG(s)$$

$K_v = \infty$

$$e_{ss} = \frac{1}{K_v} = 0.$$

**Sol. 63. (c)**

$$s^2 + 2\xi\omega_n + \omega_n^2 = 0$$

$$2\xi\omega_n = 2, \xi = \frac{1}{\omega_n}$$

$$\omega_2 = \sqrt{2}$$

$$= \frac{1}{\sqrt{2}} \xi < 1 \text{ (under damped)}$$

**Sol. 64. (b)**

$$\text{CLTF} = \frac{G(s)}{1 + G(s)H(s)}$$

$$= \frac{s+4}{s^2 + 7s + 13}$$

$$\frac{1 + G(s)H(s)}{G(s)} = \frac{s^2 + 7s + 13}{s+4}$$

H(s) = 1 for unity feedback.

$$\frac{1}{G(s)} = \frac{s^2 + 7s + 13}{s+4} - 1$$

$$\frac{1}{G(s)} = \frac{s^2 + 6s + 9}{s+4}$$

$$\therefore\ G(s) = \frac{s+4}{s^2 + 6s + 9}$$

For D.C. s = 0

$\therefore$ Open Loop Gain, G(s) $= \frac{4}{9}$.

**Sol. 65. (c)**

$$\frac{s+6}{K\left(s^2 \frac{s}{K} + \frac{6}{K}\right)} f$$

Comparing with $s^2 + 2\xi\omega_n + \omega_n^2$

$$\omega_n = \sqrt{\frac{6}{K}}$$

$$2\xi\omega_n = \frac{1}{K}$$

$$2 \times 0.5 \times \sqrt{\frac{6}{K}} = \frac{1}{K}$$

$$\Rightarrow \frac{6}{K} = \frac{1}{K^2}$$

$$K = \frac{1}{6}$$

**Sol. 66. (d)**

x(t) = –2 × (t) + 2u(t) …(i)

y(t) = 0.5 ×(t) …(ii)

From (i), Taking Laplace transform of (i)

sX(s) = – 2X(s) + 2U(s)

X(s)[2 + 2] = 2U(s)

$$\Rightarrow X(s) \frac{2U(s)}{(s+2)}$$

Taking Laplace transform of (ii)

Y(s) = 0.5X (s)

$$Y(s) = \frac{0.5 \times 2U(s)}{s+2}$$

$$\therefore\ \frac{Y(s)}{U(s)} = \frac{1}{2}.$$

**Sol. 67. (b)**

$$G(s) \frac{100}{(s+1)(s+100)}$$

Taking dominant pole consideration, s = –100 pole is not taken.

$$\therefore\ G(s) = \frac{100}{s+1}$$

Now it is 1st order system

$t_s$ = 4T = 4× 1 = 4s.

**Sol. 68. (a)**

Compare given transfer function.

With 2nd order equation.

$\omega_n = 5$, $\zeta = 0.8$

$$\omega_d = \omega_n\sqrt{1 - \xi_2} = 3$$

For 2nd peak is

$$(t_p) = \frac{3\pi}{\omega_d} = \frac{3\pi}{3} = \pi$$

# GATE QUESTIONS

**1.** A discrete-time all-pass system has two of its poles at $0.25\angle 0^0$ and $2\angle 30°$. Which one of the following statements about the system is TRUE?

**[GATE - 2018]**

(a) It has two more poles at $0.5\angle 30^0$ and $4\angle 0°$
(b) It is stable only when the impulse response is two-sided.
(c) It has constant phase response over all frequencies.
(d) It has constant phase response over the entire z-plane.

**2.** Which of the following systems has maximum peak overshoot due to a unit step input?

**[GATE - 2017]**

(a) $\frac{100}{s^2+10s+100}$  (b) $\frac{100}{s^2+15s+160}$
(c) $\frac{100}{s^2+5s+100}$  (d) $\frac{100}{s^2+20s+100}$

**3.** What a unit ramp input is applied to the unity feedback system having closed loop transfer function $\frac{C(s)}{R(s)} = \frac{Ks+b}{s^2+as+b}, (a>0, b>0, K>0)$, the steady error will be

**[GATE - 2017]**

(a) 0  (b) a/b
(c) $\frac{a+K}{b}$  (d) $\frac{a-K}{b}$

**4.** A unity feedback control system is characterized by the open loop transfer function

$$G(s) = \frac{2(s+1)}{s^3+ks^2+2s+1}$$

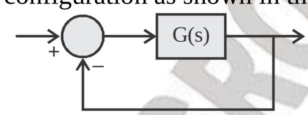
The value of k for which the system oscillates at 2 rad/s is ______

**[GATE - 2017]**

**5.** The open loop transfer function

$$G(s) = \frac{(s+1)}{s^p(s+2)(s+3)}$$

Where p is an integer, is connected in unity feedback configuration as shown in the figure.

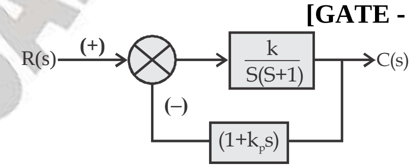



Given that the steady state error is zero for unit step input and is 6 for unit ramp input, the value of the parameter p is _____.

**[GATE - 2017]**

**6.** The block diagram of a closed - loop control system is shown in the figure. The values of k and $k_p$ are such that the system has a damping ration of 0.8 and an undamped natural frequency $\omega_n$ of 4 rad/s respectively. The value of $k_p$ will be _____

**[GATE - 2017]**




**7.** A second-order real system has the following properties:
(a) The damping ratio ζ= 0.5 and Undamped natural frequency $\omega_n$ = 10 rad/s,
(b) The steady state value of the output, to a unit step input, is 1.02.
The transfer function of the system is

**[GATE - 2016]**

(a) $\frac{1.02}{s^2+5s+100}$  (b) $\frac{102}{s^2+10s+100}$
(c) $\frac{100}{s^2+10s+100}$  (d) $\frac{102}{s^2+5s+100}$

**8.** Consider a linear time-invariant system with transfer function $H(s) = \dfrac{1}{1+s}$

If the input is cos(t) and the steady state output is Acos(t $+\alpha$), then the value of A is __________.

**[GATE - 2016]**

**9.** Consider a causal LTI system characterized by differential equation $\dfrac{dy(t)}{dt} + \dfrac{1}{6}y(t) = 3x(t).$ The response of the system to the input x(t) = $x(t) = 3e^{-\frac{t}{3}}u(t)$, where u(t) denotes the unit step function, is

**[GATE - 2016]**

(a) $9e^{-\frac{t}{3}}u(t)$

(b) $9e^{-\frac{t}{6}}u(t)$

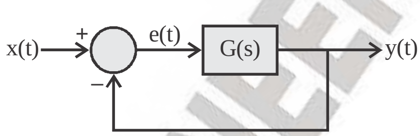
(c) $9e^{-\frac{t}{3}}u(t) - 6e^{-\frac{1}{6}}u(t)$

(d) $54e^{-\frac{t}{6}}u(t) - 54e^{-\frac{t}{3}}u(t)$

**10.** For the unity feedback control system shown in the figure, the open-loop transfer function G(s) is given as

$$G(s) = \frac{2}{s(s+1)}$$

The steady state error $e_{ss}$ due to a unit step input is



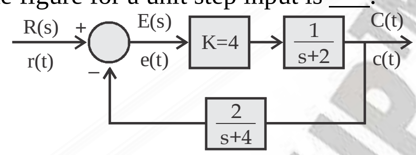

**[GATE - 2016]**

(a) 0 (b) 0.5
(c) 1.0 (d) ∞

**11.** The natural frequency of an undamped second – order system is 40 rad/s. If the system is damped with a damping ratio 0.3, the damped natural frequency in radius is __

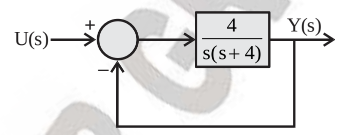
**[GATE - 2014]**

**12.** The steady state error of the system shown in the figure for a unit step input is ___.




**[GATE - 2014]**

**13.** For the second order closed – loop system shown in the figure, the natural frequency (in rad/s) is




**[GATE - 2014]**

(a) 16 (b) 4
(c) 2 (d) 1

**14.** The forward path transfer function of a unity negative feedback system is given by

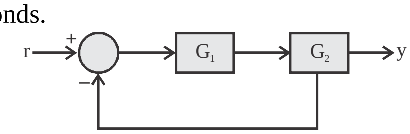
$$G(s) = \frac{K}{(s+2)(s-1)}$$

The value of K which will place both the poles of the closed – loop system at the same location is____.

**[GATE - 2014]**

**15.** For the following feedback system

$G(s) = \dfrac{1}{(s+1)(s+2)}$. The 2% settling time of the step response is required to be less than 2 seconds.




Which one of the following compensators C(s) achieves this

**[GATE - 2014]**

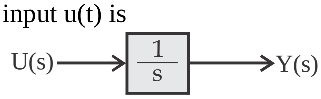
(a) $3\left(\dfrac{1}{s+5}\right)$ (b) $5\left(\dfrac{0.03}{s}+1\right)$

(c) $2(s+4)$ (d) $4\left(\dfrac{s+8}{s+3}\right)$

**16.** The input $-3e^{2t}u(t)$, where u(t) is the unit step function, is applied to a system with transfer function $\frac{s-2}{s+3}$. If the initial value of the output is –2, then the value of the output at steady state is ____.

**[GATE - 2014]**

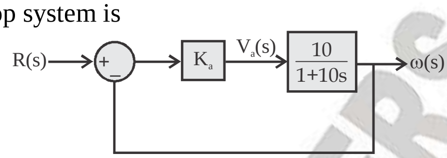
**17.** Assuming zero initial condition, the response y(t) of the system given below to a unit step input u(t) is




**[GATE - 2013]**

(a) u(t) (b) tu(t)
(c) $\frac{t^2}{2}u(t)$ (d) $e^{-1}u(t)$

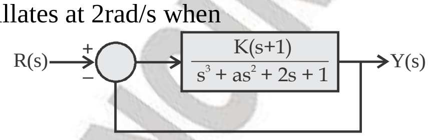
**18.** The open - loop transfer function of a dc motor is given as $\frac{\omega s}{V_a(s)} = \frac{10}{1+10s}$. When connected in feedback as shown below, the approximate value of $K_a$ that will reduce the time constant of the closed loop system by one hundred times as compared to that of the open – loop system is




**[GATE - 2013]**

(a) 1 (b) 5
(c) 10 (d) 100

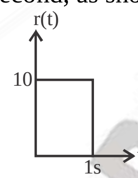
**19.** The feedback system shown below oscillates at 2rad/s when




**[GATE - 2012]**

(a) K = 2 and a = 0.75
(b) K = 3 and a = 0.75
(c) K = 4 and a = 0.5
(d) K = 2 and a = 0.5

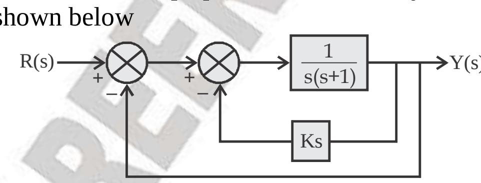
**20.** The steady state error of a unity feedback linear system for a unit step input is 0.1. The steady state error of the same system, for a pulse input r(t) having a magnitude of 10 and a duration of one second, as shown in the figure .




**[GATE - 2011]**

(a) 0 (b) 0.1
(c) 1 (d) 10

**21.** A two loop position control system is shown below




The gain K of the Tacho – generator influences mainly the

**[GATE - 2011]**

(a) Peak overshoot
(b) Natural frequency of oscillation
(c) Phase shift of the closed loop transfer function at very low frequencies ($\omega \to 0$)
(d) Phase shift of the closed loop transfer function at very high frequencies ($\omega \to \infty$)

**22.** A system with transfer function $\frac{Y(s)}{X(s)} = \frac{s}{s+p}$ has an output $y(t) = \cos\left(2t - \frac{\pi}{3}\right)$ for the input signal $x(t) = p\cos\left(2t - \frac{\pi}{2}\right)$. Then, the system parameter p is

**[GATE - 2010]**

(a) $\sqrt{3}$ (b) $2/\sqrt{3}$
(c) 1 (d) $\sqrt{3}/2$

**23.** A unity negative feedback closed loop system has a plant with the transfer function

$G(s) = \frac{1}{s^2 + 2s + 2}$ and a controller $G_C(s)$ in the feed forward path. For a unit set input, the transfer function of the controller that gives minimum steady state error is

**[GATE - 2010]**

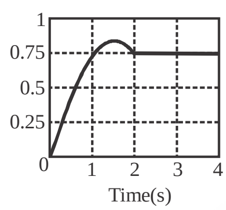
(a) $G_c(s) = \frac{s+1}{s+2}$

(b) $G_c(s) = \frac{s+2}{s+1}$

(c) $G_c(s) = \frac{(s+1)(s+4)}{(s+2)(s+3)}$

(d) $G_c(s) = 1 + \frac{2}{s} + 3s$

**24.** The unit - step response of a unity feedback system with open loop transfer function G(s) = K/((s+1)(s+2)) is shown in the figure. The value of K is




**[GATE - 2009]**

(a) 0.5 (b) 2
(c) 4 (d) 6

**25.** A function y(t) satisfies the following differential equation:

$$\frac{dy(t)}{dt} + y(t) = \delta(t)$$

Where δ(t) is the delta function. Assuming zero initial condition, and denoting the unit step function by u(t), y(t) can be of the form

**[GATE - 2008]**

(a) $e^t$ (b) $e^{-t}$
(c) $e^t u(t)$ (d) $e^{-t}u(t)$

**26.** The transfer function of a linear time invariant system is given as

$$G(s) = \frac{1}{s^2 + 3s + 2}$$

The steady state value of the output of the system for a unit impulse input applied at time instant t = 1 will be

**[GATE - 2008]**

(a) 0 (b) 0.5
(c) 1 (d) 2

**27.** The transfer function of a system is given as $\frac{100}{s^2 + 20s + 100}$

The system is

**[GATE - 2008]**

(a) An over damped system
(b) An under damped system
(c) A critically damped system
(d) An unstable system

**28.** Group I lists a set of four transfer functions. Group II gives a list of possible step response y(t). Match the step responses with the corresponding transfer functions.

**[GATE - 2008]**

**Group-I**

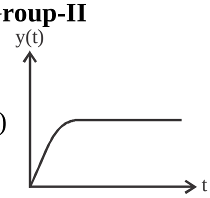
$P = \frac{25}{s^2 + 25}$

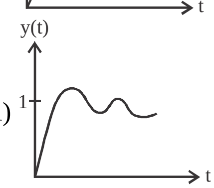
$Q = \frac{36}{s^2 + 20s + 36}$

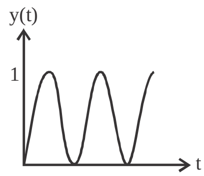
$R = \frac{36}{s^2 + 2s + 36}$

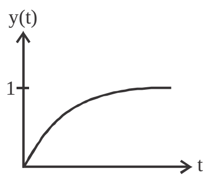
$S = \frac{49}{s^2 + 7s + 49}$

**Group-II**

(i)



(ii)


(iii)


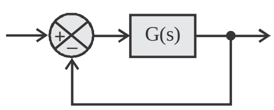

(iv)



**Codes:**
(a) P-iii, Q-i, R-iv, S-ii
(b) P-iii, Q-ii, R-iv, S-i
(c) P-ii, Q-i, R-iv, S-ii
(d) P-3, Q-4, R-i, S-ii

**29.** A certain system has transfer function

$$G(s)=\frac{s+8}{s^2+\alpha s-4}$$

Where $\alpha$ is a parameter. Consider the standard negative unity feedback configuration as shown below.




Which of the following statements is true?

**[GATE - 2008]**

(a) The closed loop systems is never stable for any value of $\alpha$.
(b) For some positive value of $\alpha$, the closed loop system is stable, but not for all positive values.
(c) For all positive values of $\alpha$, the closed loop system is stable.
(d) The closed loop system stable for all values of $\alpha$, both positive and negative.

**30.** The number of open right half plane of

$$G(s)=\frac{10}{s^5+2s^4+3s^3+6s^2+5\delta+3}$$ is

**[GATE - 2008]**

(a) 0 (b) 1
(c) 2 (d) 3

**31.** The magnitude of frequency responses of an underdamped second order system is 5 at 0 rad/sec and peaks to $\frac{10}{\sqrt{3}}$ at $5\sqrt{2}$ rad/sec. The transfer function of the system is

**[GATE - 2008]**

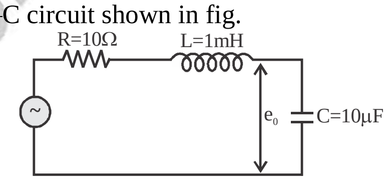
(a) $\frac{500}{s^2+10s+100}$ (b) $\frac{375}{s^2+5s+75}$
(c) $\frac{720}{s^2+12s+144}$ (d) $\frac{1125}{s^2+25s+225}$

**32.** If the loop gain K of a negative feedback system having a loop transfer function $K(s+3)/(s+8)^2$ is to be adjusted to induced a sustained oscillation then

**[GATE - 2007]**

(a) The frequency of this oscillation must be $4\sqrt{3}\,\text{rad/s}$
(b) The frequency of the oscillation must be 4 rad/s
(c) The frequency of this oscillation must be 4 or $4\sqrt{3}\,\text{rad/s}$
(d) Such a K does not exist

**Common Data for Q. 33 & Q.34**
R–L–C circuit shown in fig.




**33.** For a step – input $e_f$, the overshoot in the output $e_0$ will be

**[GATE - 2007]**

(a) 0, since the system is not under damped
(b) 5%
(c) 16%
(d) 48%

**34.** If the closed – loop transfer function of a control system is given as $T(s)=\frac{s-5}{(s+2)(s+3)}$ then it is

**[GATE - 2007]**

(a) An unstable system
(b) An uncontrollable system
(c) A minimum phase system
(d) A non – minimum phase system

**35.** The transfer function of a plant is $T(s)=\frac{5}{(s+5)(s^2+s+1)}$.
The second order approximation of T(s) using dominant pole concept is

**[GATE - 2007]**

(a) $\frac{1}{(s+5)(s+1)}$
(b) $\frac{5}{(s+5)(s+1)}$
(c) $\frac{5}{s^2+s+1}$
(d) $\frac{1}{s^2+s+1}$

**36.** Consider two transfer function $G_1(s)=\frac{1}{s^2+as+b}$ and $G_2 s=\frac{s}{s^2+as+b}$. The 3-dB bandwidths of their frequency responses are respectively.

**[GATE - 2006]**

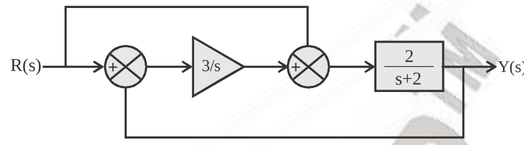
(a) $\sqrt{a^2-4b}, \sqrt{a^2+4b}$
(b) $\sqrt{a^2+4b}, \sqrt{a^2-4b}$
(c) $\sqrt{a^2-4b}, \sqrt{a^2-4b}$
(d) $\sqrt{a^2+4b}, \sqrt{a^2+4b}$

**37.** A system with zero initial conditions has the closed loop transfer function
$T(s)=\frac{5}{(s+5)(s^2+s+1)}$.
The system output is zero at the frequency

**[GATE - 2005]**

(a) 0.5 rad/sec
(b) 1 rad/sec
(c) 2 rad/sec
(d) 4 rad/sec

**38.** When subject to a unit step input, the closed loop control system shown in the figure will have a steady state error of




**[GATE - 2005]**

(a) –1.0
(b) –0.5
(c) 0
(d) 0.5

**39.** In the derivation of expression for peak percent overshoot

$$M_p = \exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right)\times 100\%$$

Which one of the following conditions is NOT required?

**[GATE - 2005]**

(a) System is linear and time invariant
(b) The system transfer function has a pair of complex conjugate poles and no zeroes.
(c) There is no transportation delay in the system.
(d) The system has zero initial conditions.

**40.** A ramp input applied to an unity feedback system results in 5% steady state error. The type number and zero frequency gain of the system are respectively

**[GATE - 2005]**

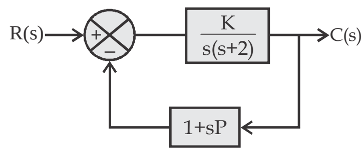
(a) 1 and 20
(b) 0 and 20
(c) 0 and $\frac{1}{20}$
(d) 1 and $\frac{1}{20}$

**41.** For the equation , $s^3 - 4s^2 + s + 6 = 0$ the number of roots in the left half of s plane will be

**[GATE - 2004]**

(a) Zero
(b) One
(c) Two
(d) Three

**42.** The block diagram of a closed loop control system is given by figure. The values of K and P such that the system has a damping ratio of 0.7 and an undamped natural frequency $\omega_n$ of 5 rad/sec, are respectively equal to




**[GATE - 2004]**

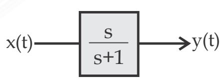
(a) 20 and 0.3 (b) 20 and 0.2
(c) 25 and 0.3 (d) 25 and 0.2

**43.** The unit impulse response of a second order underdamped system starting from rest is given by $c(t) = 12.5e^{-6t}\sin 8t,\ t \geq 0$. The steady – state value of the unit step response of the system is equal to

**[GATE - 2004]**

(a) 0 (b) 0.25
(c) 0.5 (d) 1.0

**44.** In the system shown in figure, the input x(t) = sin t. In the steady – state, the response y(t) will be




**[GATE - 2004]**

(a) $\frac{1}{\sqrt{2}}\sin(t-45°)$ (b) $\frac{1}{\sqrt{2}}\sin(t+45°)$
(c) $\sin(t-45^o)$ (d) $\sin(t+45^0)$

**45.** A causal system having the transfer function H(s) = 1/(s+2) is excited with 10u(t). The time at which the output reaches 99% of its steady state value is

**[GATE - 2004]**

(a) 2.7 sec (b) 2.5 sec
(c) 2.3 sec (d) 2.1 sec

**46.** A control system is defined by the following mathematical relationship

$$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+5x=12(I-e^{-2t})$$

**[GATE - 2003]**

The response of the system as $t \to \infty$ is
(a) x = 6 (b) x = 2
(c) x = 2.4 (d) x = –2

**47.** A control system with certain excitation is governed by the following mathematical equation

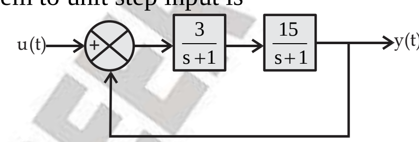
$$\frac{d^2x}{dt^2}+\frac{1}{2}\frac{dx}{dt}+\frac{1}{18}x=10+5e^{-4t}+2e^{-5t}$$

The natural time constant of the response of the system are

**[GATE - 2003]**

(a) 2 sec and 5 sec
(b) 3 sec and 6 sec
(c) 4 sec and 5 sec
(d) 1/3 sec and 1/6 sec

**48.** The block diagram shown in figure gives a unit feedback closed loop control system. The steady state error in the response of the above system to unit step input is



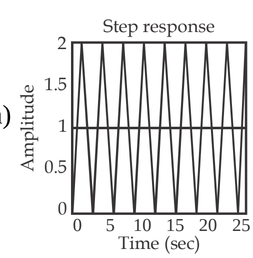

**[GATE- 2003]**

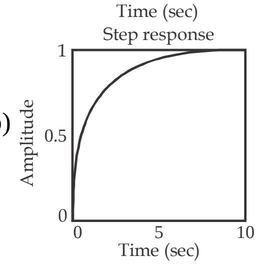
(a) 25% (b) 0.75%
(c) 6% (d) 33%

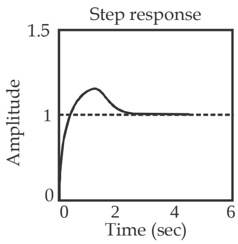
**49.** A second order system has the transfer function $\frac{C(s)}{R(s)}=\frac{4}{s^2+4s+4}$

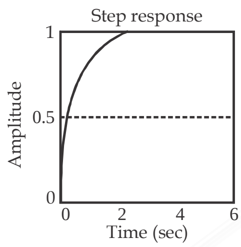
With r(t) as the unit-step function, the response c(t) of the system is represented by

**[GATE - 2003]**

(a)



(b)


(c) 


(d) 


**50.** Consider a system with transfer function $G(s) = \frac{s+6}{ks^2+s+6}$. Its damping ratio will be 0.5 when the value of k is

**[GATE - 2002]**

(a) $\frac{2}{6}$ (b) 3

(c) $\frac{1}{6}$ (d) 6

**51.** The characteristic polynomial of a system is $q(s) = 2s^5 + s^4 + 4s^3 + 2s^2 + 2s + 1$
The system is

**[GATE - 2002]**

(a) Stable (b) Marginally stable
(c) Unstable (d) Oscillatory

**52.** The transfer function of a system is $G(s) = \frac{100}{(s+1)(s+100)}$. For a unit – step input to the system the approximate settling time for 2% criterion is

**[GATE - 2002]**

(a) 100 sec (b) 4 sec
(c) 1 sec (d) 0.01 sec

**53.** If the characteristic equation of a closed – loop system is $s^2 + 2s + 2 = 0$, then the system is

**[GATE - 2001]**

(a) Over damped (b) Critically damped
(c) Under damped (d) Undamped

**54.** The open - loop DC gain of a unity negative feedback system with closed – loop transfer function $\frac{s+4}{s^2+7s+13}$ is

**[GATE - 2001]**

(a) $\frac{4}{13}$ (b) $\frac{4}{9}$
(c) 4 (d) 13

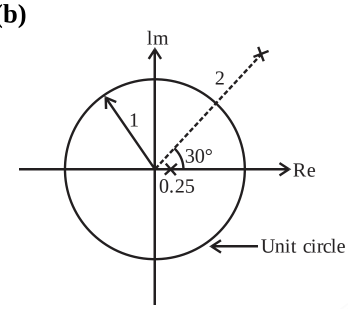
**55.** An amplifier with resistive negative feedback has two left half plane poles in its open - loop transfer function. The amplifier

**[GATE - 2000]**

(a) Will always be unstable at high frequency
(b) Will be stable for all frequency
(c) May be unstable, depending on the feedback factor
(d) Will oscillate at low frequency.

# SOLUTIONS

**Sol.1. (b)**




The ROC should include unit circle to make the system stable. From the given pole pattern it is clear that, to make the system stable, the ROC should be two-sided and hence the impulse response of the system should be also two-sided.

**Sol.2. (c)**

(a) $\frac{100}{s^2+10s+100}$

$\omega_n = 10, \xi = \frac{10}{2\omega_n} = 0.5$

(b) $\frac{100}{s^2+15s+160}$

$\omega_n = 10, \xi = \frac{15}{2\omega_n} = 0.75$

(c) $\frac{100}{s^2+5s+100}$

$\omega_n = 10, \xi = \frac{5}{2\omega_n} = 0.25$

This has maximum peak over shoot.

(d) $\frac{100}{s^2+20s+100}$

$\omega_n = 10, \xi = \frac{20}{2\omega_n} = 1$

**Sol.3. (d)**

$$\text{OLTF} = \frac{\text{CLTF}}{1-\text{CLTF}} = \frac{\frac{ks+b}{s^2+as+b}}{1-\frac{ks+b}{s^2+as+b}}$$

$$G(s) = \frac{ks+b}{s^2+(a-k)s}$$

$$k_v = \lim_{s\to 0} s.G(s) = \frac{b}{a-k}$$

$$\text{Error} = \frac{1}{k_v} = \frac{a-k}{b}$$

**Sol.4. (0.75)**

$$G(s) = \frac{2(s+1)}{s^3+ks^2+2s+1}$$

Given ω = 2 rad/sec

CE ⇒ $S^3 + kS^2 + 4S + 3 = 0$

| | | |
|---|---|---|
| $S^3$ | 1 | 4 |
| $S^2$ | K | 3 |
| $S^1$ | $\frac{4k-3}{k}$ | |
| $S^0$ | 3 | |

For marginal stable $\frac{4k-3}{k} = 0$

$$\Rightarrow K = \frac{3}{4} = 0.75$$

**Sol.5. (1)**

To get steady state error zero for unit step input and 6 for unit ramp input, the type of the system is one.

**Sol.6. (0.3375)**

$$\frac{C(s)}{R(s)} = \frac{\frac{k}{s(s+1)}}{\frac{1+k(1+k_p s)}{s(s+1)}}$$

$$\frac{C(s)}{R(s)} = \frac{k}{s^2 + s + kk_p s + k}$$

By comparing with standard second order system

$k = \omega_n^2 = 16$

$(1 + kk_p) = 2\xi\omega_n$

$1 + 16(k_p) = 2\,(0.8)4$

$k_p = 0.3375$

**Sol.7. (b)**

$$TF = \frac{102}{s^2 + 10s + 100}$$

$\omega_n = 10$ rad/s, $\zeta = 0.5$

DC gain = $\frac{102}{100} = 1.02$

**Sol.8. (0.707)**

$$A = \left|\frac{1}{j\omega + 1}\right|_{\omega=1} = \frac{1}{\sqrt{2}} = 0.707$$

**Sol.9. (d)**

$$TF = \frac{Y(s)}{X(s)} = \frac{3}{s + \frac{1}{6}}$$

$$X(s) = \frac{3}{s + \frac{1}{3}}$$

$$Y(s) = \frac{3}{s + \frac{1}{6}} \times \frac{3}{s + \frac{1}{3}} = \frac{8}{\left(s + \frac{1}{6}\right)\left(s + \frac{1}{6}\right)}$$

$Y(t) = L^{-1}[Y(s)] = 54\,e^{-\frac{1}{6}}u(t) - 54\,e^{-\frac{1}{3}}u(t)$

**Sol.10. (a)**

Given $G(s) = \frac{2}{s(s+1)}, H(s) = 1$

Type -1 System, to the unit step input the $e_{ss} = 0$

**Sol.11. (38.15)**

Given $\omega_n = 40$r/sec $\omega_n$

$\xi = 0.3$

$\omega_d = \omega_n\sqrt{1-\xi^2}\,\omega_d = 40\sqrt{1-(0.3)^2}$

$\omega_d = 38.15$ r/sec

**Sol.12. (0.5)**

Given $G(s) = \frac{4}{s+2}; H(s) = \frac{2}{s+4}$

For unit step input,

$k_p = \lim_{s\to 0} g(s)$

$$k_p = \lim_{s\to 0}\left(\frac{4}{s+1}\right)\left(\frac{2}{s+4}\right)$$

$k_p = 1$

Steady state error $e_{88} = \frac{A}{1+k_p}$

$$e_{88} = \frac{1}{1+1} e_{88} = \frac{1}{2} \Rightarrow 0.50$$

**Sol.13. (c)**

Transfer function $\frac{Y(s)}{U(s)} = \frac{4}{s^2 + 4s + 4}$

If we compare with standard 2nd order system transfer function

i.e. $\frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$

$w_n^2 = 4 \Rightarrow w_n = 2$rad/sec

**Sol.14. (2.25)**

Given $G(s) = \frac{K}{(s+1)(s-1)}$

$H(s) = 1$

Characteristic equation: $1 + G(s)H(s) = 0$

$$1 + \frac{K}{(s+2)(s-1)} = 0$$

The poles are $s_{1,2} = -1 \pm \sqrt{\frac{9}{4} - 4K}$

If $\frac{9}{4} - K = 0$ then both poles of the closed loop system at the same location

So,

$K = \frac{9}{4} \Rightarrow 2.25$

**Sol.15. (c)**

By observing the option, if we place other options, characteristic equation will have 3rd order one, where we can describe the settling time .

If C(s) = 2(s+4) is considered

The characteristic equation, is

$s^2 + 3s + 2 + 2s + 8 = 0$

$\Rightarrow s^2 + 5s + 10 = 0$

Standard character equation

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\omega_n^2 = \sqrt{10}; \xi\omega_n = 2.5$

Given,

2% settling time,

$$\frac{4}{\xi w_n} < 2 \Rightarrow \xi w_n > 2$$

**Sol.16. (0)**

**Exp-I.**

$$\frac{Y(s)}{X(s)} = \frac{S-2}{S+3}$$

$\Rightarrow$ SY (s) + 3Y(s) = S × (s) – 2X (s)

Due to initial condition, we can write above equation as

$Sy(s) - y(0) + 3y(s) = sx(s) - x(0^-) - 2x(s)$

$y(0^-) = -2,\ x(0^-) = 0\ [x(t) = 3e^{2t}u(t)]$

$$\Rightarrow Sy(s) + 2 + 3y(s) = (s-2)\left(\frac{-3}{s-2}\right)$$

$$(s+3)y(s) = -3-2 \Rightarrow y(s) = \frac{-5}{5+3}$$

$\Rightarrow y(t) = -5e^{-3t}u(t)$

$y(\infty)$ (steady state) = 0

**Exp-II.**

$$H(s) = \frac{s-2}{s+3}; X(t) = -3e^{2t}.u(t)$$

$$\therefore\ X(s) = \frac{-3}{s-2} \Rightarrow Y(s) = \frac{-3}{s+3}$$

$$y(t)|_{at\ t=x} \Rightarrow y(\infty) = \lim_{s\to 0} S.y(s) = \lim_{s\to 0}\frac{-3s}{s+3}$$

$Y(\pi) = 0$

**Sol.17. (b)**

The Laplace transform of unit step function is

$$U(s) = \frac{1}{2}$$

So, the O/P of the system is given as

$$Y(s) = \left(\frac{1}{s}\right)\left(\frac{1}{s}\right) = \frac{1}{s^2}$$

For zero initial condition, we check

$$U(t) = \frac{dy(t)}{dt}$$

$\Rightarrow U(s) = SY(s) - y(0)$

$$\Rightarrow U(s) = s\left(\frac{1}{s^2}\right) - y(0)$$

$$\text{or}\quad U(s) = \frac{1}{2} \qquad (y(0) = 0)$$

Hence, the output is correct which is

$$Y(s) = \frac{1}{s^2}$$

Its inverse Laplace transform is given by

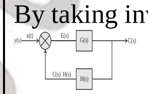
$Y(t) = tu(t)$

**Sol.18. (c)**

Given, open loop transfer function

$$G(s) = \frac{10K_a}{1+10s} = \frac{K_a}{s+\frac{1}{10}}$$

By taking inverse Laplace transform, we have



Comparing with standard form of transfer function, $Ae^{-t/\tau}$ , we get the open loop time constant.

$T_{ol} = 10$

Now, we obtain the closed loop transfer function for the given system as

$$H(s) = \frac{G(s)}{1+G(s)} = \frac{10K_a}{1+10s+10K_a}$$

$$= \frac{K_a}{s+\left(K_a + \frac{1}{10}\right)}$$

By taking inverse laplace transform, we get

$$h(t) = k_a .\ e^{\left(k_1+\frac{1}{10}\right)t}$$

So, the time constant of closed loop system is obtained as

$$T_{ol} = \frac{1}{K_a + \frac{1}{10}}$$

Or, $T_{ol} = \frac{1}{K_a}$ (approximately)

Now, given that $k_a$ reduces open loop time constant by a factor of 100. i.e.,

$$T_{ol} = \frac{T_{ol}}{100}$$

or, $\frac{1}{K_a} = \frac{10}{100}$

or, $k_a = 10$

**Sol.19. (a)**

$$Y(s) = \frac{K(s+1)}{s^3 + as^2 + 2s + 1}[R(s) - Y(s)]$$

$$Y(s)\left[1 + \frac{K(s+1)}{s^3 + as^2 + 2s + 1}\right]$$

$$= \frac{K(s+1)}{s^3 + as^2 + 2s + 1}R(s)$$

$Y(s)\ [s^3 + as^2 + s(2+k) + (1+k) = K(s+1)R(s)$

Transfer function,

$$H(s) = \frac{Y(s)}{R(s)} = \frac{K(s+1)}{s^3 + as^2 + s(2+k) + (1+k)}$$

Routh Table:

| | | |
|---|---|---|
| $s^3$ | 1 | 2+K |
| $s^2$ | A | 1+K |
| $s^1$ | $\frac{a(2+K) - (1+K)}{a}$ | 0 |

For oscillation,

$$\frac{a(2+K) - 1(1+K)}{a} = 0$$

$$a = \frac{K+1}{K+2}$$

Auxiliary equation

$A(s) = as^2 + (k+1) = 0$

$$s^2 = -\frac{k+1}{a}$$

$$s^2 = \frac{-k+1}{a}$$

$$s^2 = \frac{-k+1}{(k+1)}(k+2) = -(k+2)$$

$$s = j\sqrt{k+2}$$

$$j\omega = j\sqrt{k+2}$$

$\omega = \sqrt{k+2} = 2$ (Oscillation frequency)

K = 2

and $a = \frac{2+1}{2+2} = \frac{3}{4} = 0.75$

**Sol.20. (a)**

We know that steady state error is given by

$$e_{88} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

Where $R(s) \to$ input

$G(s) \to$ open loop transfer function

For unit step input

$$R(s) = \frac{1}{s}$$

So $e_{ss} = \lim_{s\to 0} \frac{s\left(\frac{1}{s}\right)}{1+G(s)} = 0.1$

1+ G(0) = 10

G(0) = 9

Given input $r(t) = 10[\mu(t) - \mu(t-1)]$

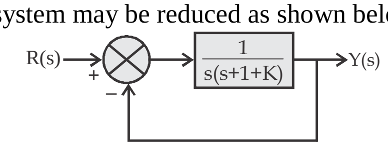
Or $R(s) = 10\left[\frac{1}{s} - \frac{1}{s}e^{-s}\right] = 10\left[\frac{1-e^{-s}}{s}\right]$

So steady state error

$$e_{ss} = \lim_{s\to 0} \frac{s \times 10 \frac{(1-e^{-s})}{s}}{1+G(s)} = \frac{10(1-e^{0})}{1+9} = 0$$

**Sol.21. (a)**

The system may be reduced as shown below



$$\frac{Y(s)}{R(s)}=\frac{\frac{1}{s(s+1+K)}}{1+\frac{1}{s(s+1+K)}}=\frac{1}{s^2+s(1+K)+1}$$

This is a second order system transfer function, characteristic equation is

$s^2 + s(1 + K) + 1 = 0$

Comparing with standard form

$$s^2+2\xi\omega_n s+\omega_n^2=0$$

We get $\xi=\frac{1+K}{2}$

Peak overshoot

$$M_p=e^{-\pi\xi/\sqrt{1-\xi^2}}$$

So the Peak overshoot is effected by k.

**Sol.22. (b)**

Transfer function is given as

$$H(s)=\frac{Y(s)}{X(s)}=\frac{s}{s+p}$$

$$H(j\omega)=\frac{j\omega}{j\omega+p}$$

Amplitude Response

$$|H(j\omega)|=\frac{\omega}{\sqrt{\omega^2+p^2}}$$

Phase Response $\theta_h(\omega)=90^o-\tan^{-1}\left(\frac{\omega}{p}\right)$

Input $x(t)=p\cos\left(2t-\frac{\pi}{2}\right)$

Output $y(t) = H(j\omega)|x(t-\theta_h) = \cos\left(2t-\frac{\pi}{3}\right)$

$$|H(j\omega)|=p=\frac{\omega}{\sqrt{\omega^2+p^2}}$$

$$\frac{1}{p}=\frac{2}{\sqrt{4+p^2}},\ (\omega = 2 \text{ rad/sec})$$

Or $4p^2 = 4 + p^2 \Rightarrow 3p^2 = 4$

Or $p=2/\sqrt{3}$

Alternative

$$\theta_h=\left[-\frac{\pi}{3}-\left(-\frac{\pi}{2}\right)\right]=\frac{\pi}{6}$$

So, $\frac{\pi}{6}=\frac{\pi}{2}-\tan^{-1}\left(\frac{\omega}{p}\right)$

$$\tan^{-1}\left(\frac{\omega}{p}\right)=\frac{\pi}{2}-\frac{\pi}{6}=\frac{\pi}{3}$$

$$\frac{\omega}{p}=\tan\left(\frac{\pi}{3}\right)=\sqrt{3}$$

$$\frac{2}{p}=\sqrt{3},\ (\omega = 2 \text{ rad/sec})$$

Or $p = 2/\sqrt{3}$

**Sol.23. (d)**

Steady state error is given as

$$e_{ss}=\lim_{s\to 0}\frac{sR(s)}{1+G(s)G_c(s)}$$

$R(s) = \frac{1}{2}$ (unit step unit)

$$e_{88}=\lim_{s\to 0}\frac{1}{1+G(s)G_c(s)}$$

$$=\lim_{s\to 0}\frac{1}{1+\frac{G_c(s)}{s^2+2s+2}}$$

$e_{ss}$ will be minimum if $\lim_{s\to 0}G_c(s)$ is maximum

In option (d)

$$\lim_{s\to 0}G_c(s)=\lim_{s\to 0}1+\frac{2}{s}+3s=\infty$$

So, $e_{ss}=\lim_{s\to 0}\frac{1}{\infty}=0$ (minimum)

**Sol.24. (d)**

From the figure we can see that steady state error for given system is

$$e_{ss}=1-0.75=0.25$$

Steady state error for unity feedback system is given by

$$e_{ss}=\lim_{s\to 0}\left[\frac{sR(s)}{1+G(s)}\right]$$

$$= \lim_{s\to 0}\left[\frac{s\left(\frac{1}{s}\right)}{1+\frac{K}{(s+1)(s+2)}}\right]; R(s)=\frac{1}{s}(\text{unit step input})$$

$$= \frac{1}{1+\frac{K}{2}} = \frac{2}{2+K}$$

So, $e_{ss} = \frac{2}{2+K} = 0.25$

2 + 0.5 + 0.25 K

$$K = \frac{1.5}{0.25} = 6$$

**Sol.25. (d)**

Given differential equation for the function

$$\frac{dy(t)}{dt} + y(t) = \delta(t)$$

Taking Laplace on both the sides we have,

sY(s)+Y(s)

(s +1) Y(s) = 1

$$Y(s) = \frac{1}{s+1}$$

Taking inverse Laplace of Y(s)

$Y(t) = e^{-t}u(t), t > 0$

**Sol.26. (a)**

Given transfer function

$$G(s) = \frac{1}{s^2+3s+2}$$

Input r(t) = δ(t –1)

$R(s) = L[\delta(t-1)] = e^{-s}$

Output is given by

$$Y(s) = R(s)G(s) = \frac{e^{-s}}{s^2+3s+2}$$

Steady state value of output

$$\lim_{t\to y} y(t) = \lim_{s\to 0} Y(s) = \lim_{s\to 0}\frac{Se^{-s}}{s^2+3s+2} = 0$$

**Sol.27. (c)**

Given transfer function is

$$H(s) = \frac{100}{s^2+20s+100}$$

Characteristic equation of the system is given by

$S^2 + 20s + 100 = 0$

$\omega_n^2 = 100 \Rightarrow \omega_n = 10\text{rad/sec}$

$2\xi\omega_n = 20$

Or $\xi = \frac{20}{2\times 10} = 1$

(ξ = 1) so system is critically damped.

**Sol.28. (d)**

| | | |
|---|---|---|
| $P = \frac{25}{s^2+25}$ | $2\xi\omega_n = 0$, $\xi = 0 \rightarrow$ Undamped | Graph 3 |
| $Q = \frac{6^2}{s^2+20s+6^2}$ | $2\xi\omega n = 20$, $\xi > 1 \rightarrow$ Overdamped | Graph 4 |
| $R = \frac{6^2}{s^2+12s+6^2}$ | $2\xi\omega_n = 12$, $\xi = 1 \rightarrow$ Critically | Graph 1 |
| $S = \frac{7^2}{s^2+7s+7^2}$ | $2\xi\omega_n = 7$, $\xi < 1 \rightarrow$ underdamped | Graph 2 |

**Sol.29. (c)**

The characteristic equation of closed loop transfer function is

1+G(s)H(s) = 0

$$1 + \frac{s+8}{s^2+\alpha s-4} = 0$$

Or $s^2 + \alpha s - 4 + s + 8 = 0$

Or $s^2 + (\alpha + 1)s + 4 = 0$

This will be stable if $(\alpha + 1) > 0 \rightarrow \alpha > -1$. Thus system is stable for all positive value of α.

**Sol.30. (c)**

The characteristic equation is

1 + G(s) = 0

Or $s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3 = 0$

Substituting $s = \frac{1}{z}$ we have

$3z^5 + 5z^4 + 6z^3 + 3z^2 + 2z + 1 = 0$

The routh table is shown below. As thee are two sign change in first column, there are two RHS poles.

| | | | |
|---|---|---|---|
| $z^5$ | 3 | 6 | 2 |
| $z^4$ | 5 | 3 | 1 |

| | | | |
|---|---|---|---|
| $z^3$ | $\frac{21}{5}$ | $\frac{7}{5}$ | |
| $z^2$ | $\frac{4}{3}$ | 3 | |
| $z^1$ | $-\frac{7}{4}$ | | |
| $z^0$ | 1 | | |

**Sol.31. (a)**

For underdamped second order system the transfer function is

$$T(s) = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

It peaks at resonant frequency. Therefore

Resonant frequency $\omega_r = \omega_n\sqrt{1-2\xi^2}$

And peak at this frequency

$$\mu_r = \frac{5}{2\xi\sqrt{1-2\xi^2}}$$

We have $\omega_r = 5\sqrt{2}$, and $\mu_r = \frac{10}{\sqrt{3}}$. Only options (a) satisfy these values.

$$\omega_n = 10, \xi = \frac{1}{2}$$

Where $\omega_r = 10\sqrt{1-2\left(\frac{1}{4}\right)} = 5\sqrt{2}$

And $\mu_r = \frac{5}{2\frac{1}{2}\sqrt{1-\frac{1}{4}}} = \frac{10}{\sqrt{3}}$ Hence satisfied.

**Sol.32. (b)**

Characteristic equation for the given system

$$1 + \frac{K(s+3)}{(s+8)^2} = 0$$

$(s+8)^2 + K(s+3) = 0$

$s^2 + (16 + K)s + (64 + 3K) = 0$

by applying Routh's criteria.

| | | |
|---|---|---|
| $s^2$ | 1 | 64+3K |
| $s^1$ | 16 + K | 0 |
| $s^0$ | 64 + 3K | |

For system to be oscillatory

$16 + K = 0 \Rightarrow K = -16$

Auxiliary equation $A(s) = s^2 + (64 + 3K) = 0$

$\Rightarrow s^2 + 64 + 3\times(-16) = 0$

$s^2 + 64 - 48 = 0$

$s^2 = -16 \Rightarrow j\omega = 4j$

$\omega = 4$ rad/sec

**Sol.33. (c)**

System response of the given circuit can be obtained as

$$H(s) = \frac{e_0(s)}{e_i(s)} = \frac{\left(\frac{1}{Cs}\right)}{\left(R + Ls + \frac{1}{Cs}\right)}$$

$$H(s) = \frac{1}{LCs^2 + RCs + 1} = \frac{\left(\frac{1}{LC}\right)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Characteristic equation is given by

$$s^2 + \frac{R}{L}s + \frac{1}{Lc} = 0$$

Here natural frequency $\omega_n = \frac{1}{\sqrt{LC}}$

$$2\xi\omega_n = \frac{R}{L},$$

Damping ratio $\xi = \frac{R}{2L}\sqrt{LC} = \frac{R}{2}\sqrt{\frac{C}{L}}$

$$\xi = \frac{10}{2}\sqrt{\frac{1\times10^{-3}}{10\times10^{-6}}} = 0.5 \text{ (under damped)}$$

So peak overshoot is given by

% peak overshoot

$$= e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}\times100 = e^{\frac{-\pi\times0.5}{\sqrt{1-(0.5)^2}}}\times100 = 16\%$$

**Sol.34. (d)**

In a minimum phase system, all the poles as well as zeros are on the left half of the s-plane. In given system as there is right half zero (s = 5), the system is a non-minimum phase system.

**Sol.35. (d)**

We have $T(s) = \frac{1}{(s+5)(s^2+s+1)}$

$$=\frac{5}{5\left(1+\frac{s}{5}\right)\left(s^2+s+1\right)}$$

In given transfer function denominator is $(s+5)\left[(s+0.5)^2+\frac{3}{4}\right]$. We can see easily that pole at $s=-0.5\pm j\frac{\sqrt{3}}{2}$ is dominant then pole at s = -5. Thus we have approximated it.

**Sol.36. (d)**

**Sol.37. (c)**
Closed loop transfer function of the given system is,

$$T(s)=\frac{s^2+4}{(s+1)(s+4)}$$

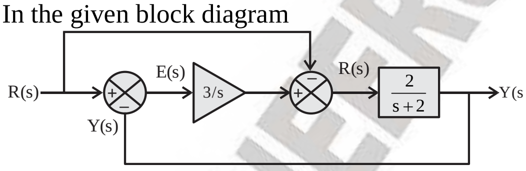
$$T(j\omega)=\frac{(j\omega)^2+4}{(j\omega+1)(j\omega+4)}$$

If system output is zero

$$|T(j\omega)| = \frac{|4-\omega^2|}{|(j\omega+1)(j\omega+4)|}=0$$

$4-\omega^2 = 0$
$\omega^2 = 4$
$\Rightarrow \omega = 2$ rad/sec

**Sol.38. (c)**
In the given block diagram




Steady state error is given as

$$e_{88} = \lim_{s\to 0} sE(s)$$

E(s) = R(s) – Y(s)
Y(s) can be written as

$$Y(s)=\left[\{R(s)-Y(s)\}\frac{3}{5}-R(s)\right]\frac{2}{s+2}$$

$$=R(s)\left[\frac{6}{s(s+2)}-\frac{2}{s+2}\right]-Y(s)\left[\frac{6}{s(s+2)}\right]$$

$$Y(s)\left[1+\frac{6}{s(s+2)}\right]=R(s)\left[\frac{6-2s}{s(s+2)}\right]$$

$$Y(s)=R(s)\frac{(6-2s)}{(s^2+2s+6)}$$

So, $E(s)=R(s)-\frac{(6-2s)}{(s^2+2s+6)}R(s)$

$$=R(s)\left[\frac{s^2+4s}{s^2+2s+6}\right]$$

For unit step input $R(s) = \frac{1}{s}$

Steady state error $e_{ss}=\lim_{s\to 0} sE(s)$

$$e_{88} = \lim_{s\to 0}\left[s\cdot\frac{1}{s}\frac{(s^2+4s)}{(s^2+2s+6)}\right]=0$$

**Sol.39. (c)**
The peak percent overshoot is determined for LTI second order closed loop system with zero initial condition. It's transfer function is

$$T(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

Transfer function has a pair of complex conjugate poles and zeroes.

**Sol.40. (a)**

For ramp input we have $R(s)=\frac{1}{s^2}$

Now $e_{ss}=\lim_{s\to 0} sE(s)$

$$=\lim_{s\to 0} s\frac{R(s)}{1+G(s)}=\lim_{s\to 0}\frac{1}{s+sG(s)}$$

Or $e_{ss} = \lim_{s\to 0}\frac{1}{sG(s)}=5\%=\frac{1}{20}$ Finite

But $k_v=\frac{1}{e_{ss}}=\lim_{s\to 0} sG(s)=20$

$k_v$ is finite for type 1 system having ramp input.

**Sol.41. (b)**
Given characteristic equation
$s^3-4s^2+s+6=0$
Applying Routh's method,

| | | |
|---|---|---|
| $s^3$ | 1 | 1 |
| $s^2$ | –4 | 6 |
| $s^1$ | $\frac{-4-6}{-4}=2.5$ | 0 |
| $s^0$ | 6 | |

Three are two sign changes in the first column, so no. of right half poles is 2.
No. of roots in left half of s-plane
= (3 –2) = 1

**Sol.42. (d)**
For the given system, characteristic equation can be written as,

$$1+\frac{K}{s(s+2)}(1+sP)=0$$

s(s +2) + K( 1 + sP) = 0
$s^2 + s(2 + KP) + K = 0$
From the equation

$$\omega_n=\sqrt{K}=5\text{rad/sec (given)}$$

so, K = 25
and $2\xi\omega_n = 2 + KP$
$2 \times 0.7 \times 5 = 2 + 25P$
Or P = 0.2
So K = 25, P = 0.2

**Sol.43. (d)**
Unit-impulse response of the system is given as,
$c(t) = 12.5e^{-6t}\sin 8t,\ t \geq 0$
So transfer function of the system.

$$H(s)=L\,|\,c(t)]=\frac{12.5\times 8}{(s+6)^2+(8)^2}$$

$$H(s)=\frac{100}{s^2+12s+100}$$

Steady state value of output for unit step input.

$$\lim_{t\to\infty}y(t)=\lim_{s\to 0}y(s)=\lim_{s\to 0}sH(s)\,R(s)$$

$$=\lim_{s\to 0}s\left[\frac{100}{s^2+12s+100}\right]\frac{1}{s}=1.0$$

**Sol.44. (a)**
System response is

$$H(s)=\frac{s}{s+1};\quad H(j\omega)=\frac{j\omega}{j\omega+1}$$

Amplitude response $|H(j\omega)|=\frac{\omega}{\sqrt{\omega+1}}$

Given input frequency ω = 1 rad/sec

So $|H(j\omega)|_{\omega=1\text{ rad/sec}}=\frac{1}{\sqrt{1+1}}=\frac{1}{\sqrt{2}}$

Phase response
$\theta_h(\omega) = 90^o - \tan^{-1}(\omega)$
$\theta_h(\omega)|_{\omega=1} = 90^o - \tan^{-1}(1) = 45^o$
So the output of the system is

$$y(t)=|H(j\omega)|\,x(t-\theta_h)=\frac{1}{\sqrt{2}}\sin(t-t-45°)$$

**Sol.45. (c)**
We have r(t) = 10u(t)

Or $R(s)=\frac{10}{s}$

Now $H(s)=\frac{1}{s+2}$

$$C(s)=H(s).R(s)=\frac{1}{s+2}.\frac{10}{s}\frac{10}{s(s+2)}$$

or $C(s)=\frac{5}{s}-\frac{5}{s+2}$

$c(t) = 5[1 - e^{-2t}]$
The steady state value of c(t) is 5. It will reach 99% of steady state value reaches at t, where
$5[1 - e^{-2t}] = 0.99 \times 5$
or $1-e^{-2t} = 0.99$
$e^{-2t} = 0.1$
or –2t = in 0.1 or t = 2.3 sec

**Sol.46. (c)**
Given system equation is

$$\frac{d^2x}{dt^2}+6\frac{dx}{dt}+5x=12(1-e^{-2t})$$

Taking Laplace transform on both side

$$S^2X(s)+6sX(s)+5X(s)=12\left[\frac{1}{s}-\frac{1}{s+2}\right]$$

$S^2X(s)+6sX(s)5$

$$(s^2+6s+5)\,X(s)=12\left[\frac{2}{s(s+2)}\right]$$

System transfer function is

$$X(s)=\frac{24}{s(s+2)(s+5)+(s+1)}$$

Response of the system as $t\to\infty$ is given by

$\lim_{t\to\infty} f(t)=\lim_{x\to 0} sF(s)$ (Final value theorem)

$$=\lim_{s\to 0}\left[\frac{24}{s(s+2)(s+5)(s+1)}\right]=\frac{24}{2\times 5}=2.4$$

**Sol.47. (b)**

Given equation

$$\frac{d^2x}{dt^2}+\frac{1}{2}\frac{dx}{dt}+\frac{1}{18}x=10+5e^{-4t}+2e^{-5t}$$

Taking Laplace on both sides we have

$$s^2X(s)+\frac{1}{2}sX(s)+\frac{1}{18}X(s)+\frac{1}{18}X(s)=\frac{10}{s}+\frac{5}{s+4}+\frac{2}{s+5}$$

$$(s^2+\frac{1}{2}s+\frac{1}{18})X(s)$$

$$=\frac{10(s+4)(s+5)+5s(s+5)+2s(s+4)}{s(s+4)(s+5)}$$

System response is,

$$X(s)=\frac{10(s+4)(s+5)+5s(s+5)+2s(s+4)}{s(s+4)(s+5)\left(s^2+\frac{1}{2}s+\frac{1}{18}\right)}$$

$$=\frac{10(s+4)(s+5)+5s(s+5)+2s(s+4)}{s(s+4)(s+5)\left(s+\frac{1}{3}\right)\left(s+\frac{1}{6}\right)}$$

We know that for a system having many poles, nearness of the poles towards imaginary axis in s-plane dominates the nature of time response. So here time constant given by two poles which are nearest to imaginary axis. Poles nearest to imaginary axis.

$$s_1=-\frac{1}{3}, s_2=-\frac{1}{6}$$

So, time constants $\begin{cases}\tau_1=3\text{sec}\\ \tau_2=6\text{sec}\end{cases}$

**Sol.48. (a)**

Steady state error for a system is given by

$$e_{ss}=\lim_{s\to 0}\frac{sR(s)}{1+G(s)H(s)}$$

Where input $R(s)=\frac{1}{s}$ (unit step)

$$G(s)=\left(\frac{3}{s+15}\right)\left(\frac{15}{s+1}\right)$$

H(s) = 1 (unity feedback)

So $e_{ss}=\lim_{s\to 0}\frac{s\left(\frac{1}{s}\right)}{1+\frac{45}{(s+15)(s+1)}}$

$$=\frac{15}{15+45}=\frac{15}{60}$$

$$\%e_{ss}=\frac{15}{60}\times 100=25\%$$

**Sol.49. (b)**

The characteristics equation is

$s^2+4s+4=0$

Comparing with

$s^2+2\xi\omega_n+\omega_n^2=0$

We get $2\xi\omega_n=4$ and $\omega_n^2=4$

Thus $\xi=1$ Critically damped

$$t_s=\frac{4}{\xi\omega_n}=\frac{4}{1\times 2}=2$$

**Sol.50. (c)**

The characteristics equation is

$Ks^2+s+6=0$

or $\quad s^2+\frac{1}{K}s+\frac{6}{K}=0$

comparing with $s^2+2\xi\omega_n s+\omega_n^2=0$ we get

we get $2\xi\omega_n=\frac{1}{K}$ and $\omega_n^2=\frac{6}{K}$

or $2\times 0.5\times\sqrt{6}K\omega=\frac{1}{K}$

Given $\xi=0.5$

or

$$\frac{6}{K}=\frac{1}{K^2}\Rightarrow K=\frac{1}{6}$$

**Sol.51. (b)**

Routh table is shown below. Here all element in 3rd row are zero, so system is marginal stable.

| | | | |
|---|---|---|---|
| $s^5$ | 2 | 4 | 2 |
| $s^4$ | 1 | 2 | 1 |
| $s^3$ | 0 | 0 | 0 |
| $s^2$ | | | |
| $s^1$ | | | |
| $s^0$ | | | |

**Sol.52. (c)**
The characteristics equation is
$s^2 + 2s + 2 = 0$
Comparing $s^2 + 2\xi\omega_n + \omega_n^2 = 0$ we get
$2\xi\omega_n = 2$ and $\omega_n^2 = 2$
$\omega = \sqrt{2}$
and $\xi = \frac{1}{\sqrt{2}}$
Since $\xi < 1$ thus system is under damped.

**Sol.53. (b)**
The characteristics equation is
$(s+1)(s + 100) = 0$
$s^2 + 101s + 100 = 0$
Comparing with
$s^2 + 2\xi\omega_n + \omega_n^2 = 0$ we get
$2\xi\omega_n = 101$ and $\omega_n^2 = 100$
Thus
$\xi = \frac{101}{20}$ Overdamped

For overdamped system settling time can be determined by the dominate pole of the closed loop system. In given system dominant pole consideration is at s = –1. Thus
$\frac{1}{T} = 1$ and $T = \frac{4}{T} = 4$ sec

**Sol.54. (b)**
For unity negative feedback system the closed loop transfer function is
$$\text{CLTF} = \frac{G(s)}{1+G(s)} = \frac{s+4}{s^2+7s+13}$$
$G(s) \to$ OL Gain
$$\text{Or } \frac{1+G(s)}{G(s)} = \frac{s^2+7s+13}{s+4}$$
$$\text{Or } \frac{1}{G(s)} = \frac{s^2+7s+13}{s+4} - 1 = \frac{s^2+6s+9}{s+4}$$
$$\text{Or } G(s) = \frac{s+4}{s^2+6s+9}$$
For DC gain s = 0, thus
Thus $G(0) = \frac{4}{9}$

**Sol.55. (b)**
It is stable at all frequencies because for resistive network feedback factor is always less than unity. Hence overall gain decreases.

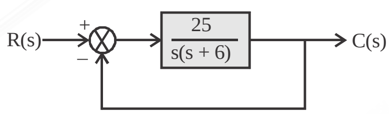
# ESE OBJ QUESTIONS

**1.** The steady state error for a Type 0 system for unit – step input is 0.2. In a certain instance, this error possibility was removed by insertion of a unity gain block. Thereafter, a unit ramp was applied. The nature of the block and new steady – state error in this changed configuration will, respectively, be

**[EE ESE - 2018]**

(a) Integrator; 0.25
(b) Differentiator; 0.25
(c) Integrator; 0.20
(d) Differentiator; 0.20

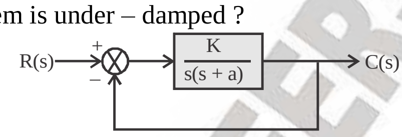
**2.** For a closed loop system shown in the figure, what is the settling time for ±2% settling of the steady state condition, assuming unit–step input?




**[EE ESE - 2018]**

(a) 0.33 s
(b) 1.33 s
(c) 2.33 s
(d) 3.33 s

**3.** A unity feedback system is shown in the figure. What is the magnitude of K so that the system is under – damped ?




**[EE ESE - 2018]**

(a) K = 0
(b) $K = \frac{a^2}{4}$
(c) $K < \frac{a^2}{4}$
(d) $K > \frac{a^2}{4}$

**4.** Settling time is the time required for the system response to settle within a certain percentage of

**[EE ESE - 2018]**

(a) Maximum value
(b) Final value
(c) Input amplitude value
(d) Transient error value

**5.** In a unity feedback control system, the open – loop transfer function is

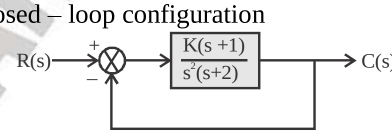
$$G(s) = \frac{K(s+2)}{s^2(s^2+7s+12)}$$

Then the error constants $K_p$, $K_v$ and $K_a$, respectively, are

**[EE ESE - 2018]**

(a) ∞, ∞ and $\frac{K}{6}$
(b) 0, 0 and $\frac{K}{6}$
(c) $\frac{K}{6}$, 0 and 0
(d) $\frac{K}{6}$, ∞ and ∞

**6.** Consider the stability of the system shown in the figure when analyzed with a positive real value of gain K in
1. open – loop configuration
2. closed – loop configuration




Which of the following statements is correct?

**[EE ESE - 2018]**

(a) Both 1 and 2 are stable
(b) 1 is stable and 2 is unstable
(c) 1 is unstable and 2 is stable
(d) Both 1 and 2 are unstable

**7.** What is the effect on the natural frequency ($\omega_n$) and damping factor ($\delta$) in the control systems when derivative compensation is used?

**[EE ESE - 2017]**

(a) $\omega_n$ increases and $\delta$ decreases
(b) $\omega_n$ remains unchanged and $\delta$ increases
(c) $\omega_n$ remains unchanged and $\delta$ decreases
(d) $\omega_n$ decreases and $\delta$ increases

**8.** Consider the following statements:
For a type – 1 and a unity feedback system, having unity gain in the forward parth
1.Positional error constant $K_p$ is equal to zero

2.acceleration error constant $K_a$ is equal to zero
3.Steady state error $e_{ss}$ per unit – step displacement intpu is equal to 1
Which of the above statements are correct?
**[EC ESE - 2017]**
(a) 1, 2 and 3 (b) 1 and 2 Only
(c) 2 and 3 (d) 1 and 3 only

**9.** The largest error between reference input and output during the transient period is called:
**[EC ESE - 2017]**
(a) Peak error
(b) Transient overshoot
(c) Peak overshoot
(d) Transient deviation

**10.** If the characteristic equation of a closed-loop system is $2s^2 + 6s + 6 = 0$, then the system is
**[EC ESE - 2017]**
(a) Overdamped (b) Critically damped
(c) Underdamped (d) Undamped

**11.** What is the time required to reach 2% of steady – state value, for the closed-loop transfer function $\frac{2}{(s+10)(s+100)}$, when the input is u(t)?
**[EC ESE - 2017]**
(a) 20 s (b) 2s
(c) 0.2s (d) 0.02s

**12.** A control system has $G(s) = \frac{10}{s(s+5)}$ and H(s) = K. What is the value of K for which the steady state error for unit-step input is less than 5%?
**[EC ESE - 2017]**
(a) 0.913 (b) 0.927
(c) 0.953 (d) 1.050

**13.** A system has a transfer function

$$\frac{C(s)}{R(s)} = \frac{4}{s^2 + 1.6s + 4}$$

For a unit-step response and 2% tolerance band, the settling time will be
**[EE ESE - 2016]**
(a) 5 seconds (b) 4 seconds
(c) 3 seconds (d) 2 seconds

**14.** The open-loop transfer function of a unity feedback system is $\frac{K}{s(s+4)}$ For a damping factor of 0.5, the value of the gain K must be set to
**[EE ESE - 2016]**
(a) 1 (b) 2
(c) 4 (d) 16

**15.** For a unity feedback control system the forward path transfer function is given by

$$G(s) = \frac{40}{s(s+2)(s^2+2s+30)}$$

The steady-state error of the system for the input $\frac{5t^2}{2}$ is
**[EE ESE - 2016]**
(a) 0 (b) $\infty$
(c) $20t^2$ (d) $30t^2$

**16.** Consider the following statements:
1. Adding a zero to the G(s) H(s) tends to push root locus to the left.
2. Adding a pole to the G(s) H(s) tends to push root locus to the right.
3. Complementary root locus (CRL) refers to root loci with positive K.
4. Adding a zero to the forward path transfer function reduces the maximum overshoot of the system.
Which of the above statement are correct?
**[EE ESE - 2016]**
(a) 1, 2 and 3 only (b) 3 and 4 only
(c) 1, 2 and 4 only (d) 1, 2 3 and 4

**17.** For a critically damped system, the closed-loop poles are
**[EE ESE - 2016]**
(a) Purely imaginary
(b) Real, equal and negative
(c) Complex conjugate with negative real part
(d) Real, unequal and negative

**18.** A second-order position control system has an open-loop transfer function.

$$G(s) = \frac{57.3K}{s(s+10)}$$

What value of K will result in a steady-state error of 1°, when the input shaft rotates at 10 r.p.m.?

**[EE ESE-2016]**

(a) 21.74 (b) 10.47
(c) 5.23 (d) 0.523

**19. Statement (I)**: In type-0 and type-I systems, stable operation is possible if gain is suitably reduced.
**Statement (II)**: Any one of the compensators lag, lead, lag-lead may be used to improve the performance.

**[EE ESE - 2016]**

(a) Both Statement (I) and Statement (II) are individually true and statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
(c) Statement (I) is true but Statement (II) is false
(d) Statement (I) is false but Statement (II) is true.

**20.** The transfer function $\frac{1}{2s+1}$ will have

**[EC ESE - 2016]**

(a) DC gain 1 and high frequency gain 1
(b) DC gain 0 and high frequency gain ∞
(c) DC gain 1 and high frequency gain 0
(d) DC gain 0 and high frequency gain 1

**21.** The closed-loop transfer function of a certain control system is given by $\frac{C}{R}(s) = \frac{100}{s^2+10s+100}$. Then the settling time for a 2% tolerance band is given by

**[EC ESE - 2016]**

(a) 0.8 s (b) 1.2 s
(c) 1.5 s (d) 2.1 s

**22.** The unit step input response of a certain control system is given by c(t) = 1 + 0.2 $e^{-60t}$ – 1.2 $e^{-10t}$. Then the undamped natural frequency $\omega_n$ and damping ratio $\xi$ are, respectively.

**[EC ESE - 2016]**

(a) 24.5 and 1.27 (b) 33.5 and 1.27
(c) 24.5 and 1.43 (d) 33.5 and 1.43

**23.** For a unity feedback control system having an open-loop transfer function $G(s) = \frac{25}{s(s+6)}$, what is the time $t_p$ at which of the step input response occurs?

**[EC ESE - 2016]**

(a) 0.52 s (b) 2.75 s
(c) 0.79 s (d) 1.57 s

**24.** The closed-loop transfer function of a unity feedback control system is, $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n+\omega_n^2}$. The velocity error constant of the system is

**[EC ESE - 2016]**

(a) $\frac{\omega_n}{2\xi}$ (b) $\frac{\omega_n}{\xi}$

(c) $\frac{2\omega_n}{\xi}$ (d) $\frac{3\omega_n}{2\xi}$

**25.** A proportional controller with transfer function, $K_p$ is used with a first-order system having its transfer function as $G_C(s) = \frac{K}{(1+S\tau)}$, in unity feedback structure. For step inputs, an increase in $K_p$ will

**[EC ESE - 2016]**

(a) Increase the time constant and decrease the steady state error
(b) Decrease the time constant and decrease the steady state error.
(c) Decrease the time constant and increase the steady state error.
(d) Increase the time constant and increase the steady state error.

**26.** For a second-order differential equation, if the damping ratio $\xi$, is unity, then

**[EC ESE - 2016]**

(a)The poles are imaginary and complex conjugate
(b)The poles are in the right half of s-plane
(c)The poles are equal, negative and real
(d)Both the poles are unequal, negative and real.

**27.** When the unit impulse response of a second order system is $\frac{1}{6}e^{-8t}\sin 0.6t$, the natural frequency and damping ratio of the system are respectively.

**[EC ESE - 2015]**

(a) 1 rad/s and 0.8
(b) 0.64 rad/s and 0.8
(c) 1 rad/s and 1
(d) 0.64 rad/s and 1

**28.** Given that the transfer function $G(s)=\frac{k}{s^2(1+sT)}$, the type and order of this system are respectively.

**[EC ESE - 2015]**

(a) 5 and 2
(b) 2 and 2
(c) 2 and 3
(d) 3 and 3

**29.** The closed loop transfer function of a unity negative feedback system is $\frac{100}{s^2+8s+100}$. Its open loop transfer function is

**[EC ESE - 2015]**

(a) $\frac{100}{s+8}$
(b) $\frac{1}{s^2+8s}$
(c) $\frac{100}{s^2-8s}$
(d) $\frac{100}{s^2+8s}$

**30.** The roots of the characteristic equation 1 + G(s) H(s) = 0 are the same as the

**[EC ESE - 2015]**

(a) Poles of the closed loop transfer function
(b) Poles of the open loop transfer function
(c) Zeros of the closed loop transfer function
(d) Zeros of the open loop transfer function

**31.** The derivative of a parabolic function becomes

**[EE ESE - 2015]**

(a) A unit-impulse function
(b) A ramp function
(c) A gate function
(d) A triangular function

**32.** A unit impulse function is defined as
(i) A pulse of area 1
(ii) A pulse compressed along horizontal axis and stretched along vertical axis keeping the area unity
(iii) $\frac{du}{dt}$
(iv) $\delta(t) = 0, \neq 0$
Which of the above statements are correct?

**[EE ESE - 2015]**

(i) i,ii and iii only
(b) i, iii and iv only
(c) ii, iii and iv only
(d) i, ii, iii and iv

**33.** Phase lead compensation

**[EE ESE - 2015]**

(a)Increase bandwidth and increases steady – state error.
(b)Decreases bandwidth and decreases steady state error
(c)Will not affect bandwidth but decreases steady – state error
(d)Increases bandwidth but will not affect steady – state error.

**34.** In time domain specification, decay ratio is the ratio of the

**[EE ESE - 2015]**

(a) Amplitude of the first peak and the steady – state value
(b) Amplitudes of the first two successive peaks
(c) Peak value to the steady-state value
(d) None of the above

**35.** Consider the time response of a second – order system with damping coefficient less than 1 to a unit step input:
(i) It is overdamped.
(ii) It is a periodic function.
(iii) Time duration between any two consecutive values of 1 is the same.
Which of the above statements is/are correct?

**[EE ESE - 2015]**

(a) i , ii and iii
(b) i only
(c) ii only
(d) iii only

**36.** A sensor requires 30 s to indicate 90% of the response to a step input. If the sensor is a first – order system, the time constant is
[given, $\log_e(0.1) = -2.3$ ]

**[EE ESE - 2015]**

(a) 15 s (b) 13 s
(c) 21 s (d) 28 s

**37.** Consider the following input and system types:

| **Input Type** | **System Type** |
|---|---|
| Unit step | Type '0' |
| Unit ramp | Type '1' |
| Unit parabolic | Type '2' |

Which of the following statements are correct ?
(i)Unit step input is acceptable to all the three types of system.
(ii)Type '0' system cannot accept unit parabolic input.
(iii)Unit ramp input is acceptable to Type '2' system only.

**[EE ESE - 2015]**

(a) i and ii only (b) i and iii only
(c) ii and iii only (d) i, ii and iii

**38.** The characteristic equation of a closed loop system is $s^2 + 4s + 16 = 0$. The natural frequency of oscillation and damping constant respectively are

**[EE ESE - 2015]**

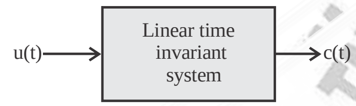
(a) $2\,\text{rad/s}$ and $\frac{1}{2}$

(b) $2\sqrt{3}\,\text{rad/s}$ and $\frac{1}{\sqrt{3}}$

(c) 4rad/s and $\frac{1}{2}$

(d) $4\,\text{rad/s}$ and $\frac{1}{\sqrt{2}}$

**39.** A quiescent linear time – invariant system subjected to a unit step input u(t) has the response $c(t) = te^{-t}, t \geq 0$. Then $\frac{C(s)}{R(s)}$ would be




**[EE ESE - 2015]**

(a) $\frac{1}{s(s+1)}$ (b) $\frac{1}{s+1}$

(c) $\frac{1}{(s+1)^2}$ (d) None of above

**40.** The unit impulse response of a system is given as $c(t) = -4e^{-t} + 6e^{-2t}$. The step response of the same system for $t \geq 0$ is equal to

**[EE ESE - 2014]**

(a) $3e^{-2t} - 4e^{-t} + 1$ (b) $-3e^{-2t} + 4e^{-t} + 1$
(c) $-3e^{-2t} - 4e^{-t} - 1$ (d) $3e^{-2t} + 4e^{-t} + 1$

**41.** A unity feedback second order control system is characterized by the open loop transfer function

$$(s) = \frac{K}{s(Js+B)}$$

J = moment of inertia, B = damping constant and K = system gain.
The transient response specification which is not affected by system gain variation is

**[EE ESE - 2014]**

(a) Peak overshoot
(b) Rise time
(c) Settling time
(d) Time to peak overshoot

**42. Statement (I):** Transfer function approach is inadequate, when time domain in solution is required.
**Statement (II):** All initial conditions of the system are neglected in derivation of transfer function.

**[EE ESE - 2014]**

(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).

(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

**43.** For a unit step input, a system with forward path transfer function $G(s)=\frac{20}{s^2}$ and feedback path transfer function H(s) = (s + 5) has a steady – state output of

**[EE ESE - 2014]**

(a) 2 (b) 0.5
(c) 1 (d) 0.2

**44.** Consider the open – loop transfer function :

$$G(s)H(s)=\frac{5(s+1)}{s^2(s+5)(s+12)}$$

The steady state error due to ramp input is

**[EE ESE - 2014]**

(a) 0 (b) 5
(c) 12 (d) $\infty$

**45.** The position and velocity error coefficient for the system of transfer function,

$G(s)=\frac{50}{(1+0.1s)(1+2s)}$ are respectively.

**[EE ESE - 2014]**

(a) Zero and zero (b) Zero and infinity
(c) 50 and zero (d) 50 and infinity

**46.** The overall transfer function of a second order control system is given by,

$$\frac{C(s)}{R(s)}=\frac{2}{s^2+3s+2}$$

The time response of this system, when subjected to a unit step response is

**[EE ESE - 2014]**

(a) $1 - e^{-2t} + 2e^{-t}$ (b) $1 + e^{-2t} + 2e^{-t}$
(c) $1 - 2e^{-t} + e^{-2t}$ (d) $1 + e^{-2t}$

**47.** For a unity feedback control with $G(s)=\frac{9}{s(s+3)}$, the damping ratio is

**[EE ESE - 2014]**

(a) 0.5 (b) 1
(c) 0.707 (d) 0.33

**48.** The dominant poles of a servo – system are located at s = (-2 ± j2). The damping ratio of the system is

**[EE ESE - 2014]**

(a) 1 (b) 0.8
(c) 0.707 (d) 0.6

**49.** What damping ratio is equal to zero, the damping frequency of a system is

**[EC ESE - 2014]**

(a) Equal to natural frequency
(b) Zero
(c) More than natural frequency
(d) Less than

**50.** A unity feedback system has $G(s)=\frac{K(s+12)}{(s+14)(s+18)}$. What is the value of K to yield 10% error in steady state?

**[EC ESE - 2014]**

(a) 672 (b) 189
(c) 100 (d) 21

**51.** A unity feedback system has an open-loop transfer function $G(s)=\frac{K}{s(s+10)}$

If the damping ratio is 0.5, then what is the value of K?

**[EC ESE - 2014]**

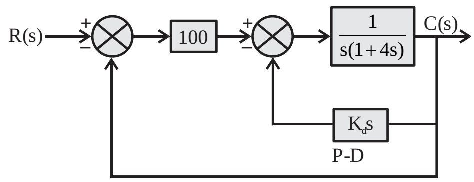
(a) 150 (b) 100
(c) 50 (d) 10

**52.** The loop transfer function of a system is $\frac{K}{s(s+1)(s+5)}$. The loop gain K is adjusted for inducing sustained of K for this objective?

**[EC ESE - 2014]**

(a) 15 (b) 25
(c) 30 (d) 45

**53.** Derivative feedback is employed in the control system shown in the figure, to improve damping. It the required damping factor of the system is 0.5, the value of $K_d$ must be adjusted to




**[EC ESE - 2013]**

(a) 4
(b) 19
(c) 0.25
(d) 6

**54.** The transfer function, of s system is $G(s)=\dfrac{100}{s^2+10s+100}$. The unit step response of the system will settle in approximately

**[EC ESE - 2013]**

(a) 2 sec
(b) 1 sec
(c) 0.8 sec
(d) 1.5

**55.** The open-loop transfer function of a unity feedback control system is $G(s)=\dfrac{1}{(s+2)^2}$. The closed loop transfer function poles are located at:

**[EC ESE - 2013]**

(a) –2, –2
(b) –2, –1
(c) –2, +2
(d) –2, ± j1

**56.** Which has one of the following transfer functions the greatest overshoot?

**[EC ESE - 2013]**

(a) $\dfrac{9}{s^2+2s+9}$
(b) $\dfrac{16}{s^2+2s+16}$
(c) $\dfrac{25}{s^2+2s+25}$
(d) $\dfrac{36}{s^2+2s+36}$

**57.** If the overshoot of the unit-step response of a second of a second order system is 30%, then the time all which peak overshoot occurs (assuming $\omega_n$ = 10 rad/sec):

**[EC ESE - 2013]**

(a) 0.36 sec
(b) 0.363 sec
(c) 0.336 sec
(d) 0.633 sec

**58.** A first order linear system is initially relaxed for a unit step signal u(t), the response is $V(t) = (1 - e^{-3t})$, for t > 0. If a signal $3u(t) + \delta(t)$ is applied to the same system, the response is

**[EE ESE - 2013]**

(a) $(3 - 6e^{-3t})\,u(t)$
(b) $(3 - 3e^{-3t})\,u(t)$
(c) 3 u(t)
(d) $(3 + 3e^{3t})\,u(t)$

**59.** Unit impulse response of a given system is $C(t) = -4e^{-t} + 6e^{-2t}$. The step response for t≥0 is

**[EE ESE - 2013]**

(a) $-3e^{-2t} - 4e^{-t} + 1$
(b) $3e^{+2t} + 4e^{-t} + 1$
(c) $-3e^{-2t} - 4e^{-t} + 1$
(d) $3e^{-2t} + 4e^{-t} + 1$

**60.** The working of a PMMC (Permanent magnet moving coil) meter is described by a second order differential equation

$$J\frac{d^2\theta}{dt^2}+D\frac{d\theta}{dt}+S\theta=T$$

Where,
J is Moment of inertia of the system
D is Damping coefficient
S is Spring constant
θ is Angular deflection and
T is Activating torque

Assuming D = 0, an undamped natural angular frequency is

**[EE ESE - 2013]**

(a) $\sqrt{\dfrac{S}{J}}$
(b) $\sqrt{\dfrac{J}{S}}$
(c) $\dfrac{1}{\sqrt{JS}}$
(d) $\dfrac{1}{2\mu\sqrt{JS}}$

**61.** A unit impulse response of a second order system is $\dfrac{1}{6}e^{-0.8}\sin(0.6t)$. Then natural frequency and damping ratio of the system are respectively.

**[EE ESE - 2013]**

(a) 1 and 0.6
(b) 1 and 0.8
(c) 2 and 0.4
(d) 2 and 0.3

**62.** For a critically damped second order system, if gain constant (K) is increased, the system behavior

**[EE ESE - 2013]**

(a) Becomes oscillatory
(b) Becomes under damped
(c) Becomes over damped
(d) Shows no change

**63.** The transfer function of a system is $\frac{1}{1+sT}$. The input to this system is the ramp function, tu(t). The output would track this system with an error given by

**[EE ESE - 2013]**

(a) Zero (b) $\frac{T}{2}$

(c) T (d) $\frac{T^2}{2}$

**64.** Damping ratio $\xi$ and peak overhsoot $M_p$ are measures of

**[EE ESE - 2013]**

(a) Relative stability
(b) Absolute stability
(c) Speed of response
(d) Steady state error

**65.** A forcing function $(t^2 - 2t)\, u(t - 1)$ is applied to a linear system. The $\mathcal{L}$ - transform of the forcing function is

**[EE ESE - 2013]**

(a) $\frac{2-s}{s^3}e^{-2s}$ (b) $\left(\frac{1-s^2}{s}\right)e^{-s}$

(c) $\frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-2s}$ (d) $\left(\frac{2-s^2}{s^3}\right)e^{-s}$

**66.** A second order system is described by

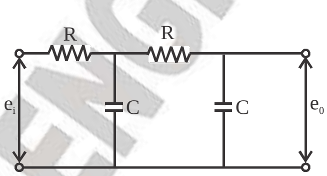
$$2\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = 8x$$

The damping ratio of the system is

**[EE ESE - 2013]**

(a) 0.1 (b) 0.25
(c) 0.333 (d) 0.5

**67.** The transfer function of the network shown below is




**[EE ESE - 2013]**

(a) $\frac{1}{s^2T^2 + 2sT + 1}$ (b) $\frac{1}{s^2T^2 + 3sT + 1}$

(c) $\frac{1}{s^2T^2 + sT + 1}$ (d) $\frac{1}{s^2T^2 + 1}$

**68.** A transfer function has its zero in the right half of the s-plate. The function

**[EE ESE - 2013]**

(a) Is positive real
(b) Is minimum phase
(c) Will give stable impulse response
(d) Is non- minimum phase

**69.** An open loop T.F. of a unity feedback system is given by

$$G(s) = \frac{1}{(s+2)^2}G(s)$$

The closed loop transfer function, will have poles at

**[EE ESE - 2013]**

(a) – 2, – 2 (b) –2, –1
(c) –2, + j, –2 – j (d) –2, 2

**70.** A unity feedback control system has

$$G(s) = \frac{K}{s^2(s+sT)}$$

The order and type of the closed- loop system will be

**[EE ESE - 2012]**

(a) 3 and 1 (b) 2 and 3
(c) 3 and 2 (d) 3 and 3

**71.** The open-loop transfer function of a control system is $\frac{10}{s+1}$ The steady-state error due to unit step input signal when operated as a unity feedback system is

**[EE ESE - 2012]**

(a) 10 (b) 0

(c) $\frac{1}{11}$ (d) $\infty$

**72.** The impulse response of a linear system is $e^{-t}$m t>0. The corresponding transfer function is

**[EE ESE - 2012]**

(a) $\frac{1}{s(s+1)}$ (b) $\frac{1}{s+1}$

(c) $\frac{1}{s}$ (d) $\frac{s}{s+1}$

**73.** A unity feedback system has a forward path transfer function

$G(s) = \frac{K}{s(s+8)}$

Where K is the gain of the system. The value of K, for making this system critically damped, should be

**[EE ESE - 2012]**

(a) 4 (b) 8
(c) 16 (d) 32

**74.** Match List-I (Conditions) with List-II (Damping constant ξ) and select the correct answer using the code given below the lists:

**List-I**
A. Undamped
B. Underdamped
C. Critically damped
D. Overdamped

**List - II**
(i) 0.5
(ii) 2.0
(iii) 0.0
(iv) 1.0

**[EE ESE-2012]**

**Codes:**
(a) A-iii, B-iv, C-i, D-ii
(b) A- ii, B-iv, C-i, D-iii
(c) A-iii, B-i, C-iv, D-ii
(d) A-ii, B-i, C-iv, D-iii

**75.** Match List-I and List-II and select the correct answer using the code given below the lists:

**List-I**
A. $s^2 + 18s + 64$
B. $s^2 + 25$
C. $s^2 + 12s + 36$
D. $s^2 + 8s + 25$

**List-II**
(i) Underdamped
(ii) Critically damped
(iii) Undamped
(iv) Overdamped

**[EE ESE - 2012]**

**Codes:**
(a) A-i, B-ii, C-iii, D-iv
(b) A-iv, B-ii, C-iii, D-i
(c) A-iii, B-i, C-iv, D-ii
(d) A-ii, B-i, C-iv, D=iii

**76.** A system has the following transfer function:

$G(s) = \frac{1}{s^2 + 0.1s + 1}$

If step input is applied to this system, then its setting time with 5% tolerance band will be

**[EE ESE - 2012]**

(a) 60 sec (b) 40 sec
(c) 20 sec (d) 10 sec

**77.** A second-order control system exhibits 100% overshoot. Its damping coefficient is

**[EE ESE - 2012]**

(a) Greater than 1 (b) Less than 1
(c) Equal to 0 (d) Equal to 1

**78.** By using feedback in control system, the sensitivity to parameter variation is improved. This is achieved at rate the cost of

**[EE ESE - 2012]**

(a) Stability
(b) Loss of system gain
(c) Transient response
(d) Reliability

**79.** The characteristic equation of a particular system is given by $s^3 + 2s^2 + 6s + 12 = 0$. The damping ratio δ will be

**[EC ESE - 2012]**

(a) $\delta = 0$ (b) $0 < \delta < 1$
(c) $\delta = 1$ (d) $\delta > 1$

**80.** A third system is approximated to an equivalent second order system. The rise time of this approximated system will be

**[EC ESE - 2012]**

(a)Same as the original system for any input

(b)Smaller than the original system for any input
(c)Larger than the original system for any input
(d)Smaller or larger depending on the type of input.

**81.** The effect of integral controller on the steady state error ($e_{ss}$) and the relative stability ($R_s$) of the system are

**[EC ESE - 2012]**

(a) Both are increased
(b) $e_{ss}$ is increased but $R_s$ is reduced
(c) $e_{ss}$ is reduced but $R_s$ is increased
(d) Both are reduced

**82.** For a second order dynamic system, if the damping ratio is 1 then the poles are

**[EC ESE - 2012]**

(a) Imaginary and complex conjugate
(b) In the right-half of s*plane
(c) Equal, negative and real
(d) Negative and real

**83.** In a feedback control system, if $G(s) = \frac{4}{s(s+3)}$ and $H(s) = \frac{1}{s}$, then the closed-loop system will be of type

**[EC ESE - 2012]**

(a) 3 (b) 2
(c) 1 (d) 0

**84.** The following quantities give a measure of the transient characteristics of a control system, when subjected to unit step excitation:
1. Maximum overshoot.
2. Maximum undershoot
3. Overall gain
4. Delay time
5. Rise time
6. Fall time

**[EC ESE - 2012]**

(a) 1, 3 and 5 (b) 2, 4 and 5
(c) 2, 4 and 6 (d) 1, 4 and 5

**85.** The time taken for the output to settle within ±2% of step input for the control system represented by $\frac{25}{s^2 + 5s + 25}$ is given by

**[EC ESE - 2012]**

(a) 1.2 s (b) 1.6 s
(c) 2.0 s (d) 0.4 s

**86.** The type of system which is used for determination of static error constants is determined from the number of

**[EC ESE - 2012]**

(a)Zeros at origin for open loop transfer function
(b)Poles at origin for open loop transfer function.
(c)Zeros at origin for closed loop transfer function.
(d)Poles at origin for closed loop transfer function.

**87.** Given a unity feedback system with $G(s) = \frac{K}{s(s+6)}$, the value of K for damping ratio of 0.75 is

**[EC ESE - 2011]**

(a) 1 (b) 4
(c) 16 (d) 64

**88.** Consider a second order all-pole function model, if the desired settling time (5%) is 0.60 sec and the desired damping ratio 0.707, where should the poles be located in s-plane?

**[EC ESE - 2011]**

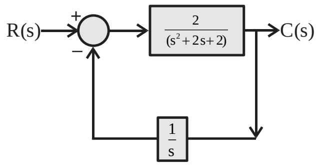
(a) $-5 \pm j4\sqrt{2}$ (b) $-5 \pm j5$
(c) $-4 \pm j5\sqrt{2}$ (d) $-4 \pm j7$

**89.** Given the differential equation model of a physical system, determine the time constant of the system $40\frac{dx}{dt} + 2x = f(t)$

**[EC ESE - 2011]**

(a) 10 (b) 20
(c) 1/10 (d) 4

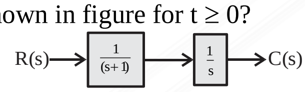
**90.** The block diagram of a closed-loop control system is given in figure. What is the type of this system?




**[EC ESE - 2011]**

(a) Zero (b) One
(c) Two (d) Three

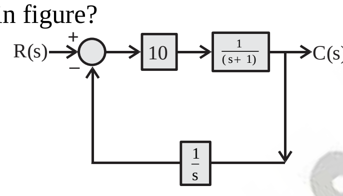
**91.** What is the unit impulse response of the system shown in figure for $t \geq 0$?




**[EC ESE - 2011]**

(a) $1 + e^{-1}$ (b) $1 - e^{-t}$
(c) $e^{-t}$ (d) $-e^{-t}$

**92.** What is the steady-state value of the unit-step response of a closed-loop control system shown in figure?

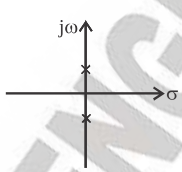


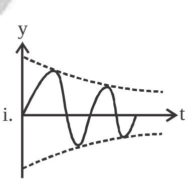

**[EC ESE - 2011]**

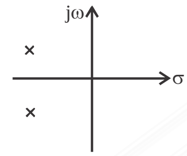
(a) –0.5 (b) 0
(c) 2 (d) ∞

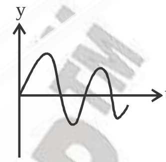
**93.** Match List-I with List-II and select the correct answer using the code given below the lists:

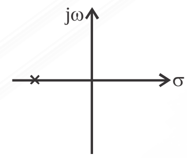
**List-I** **List-II**

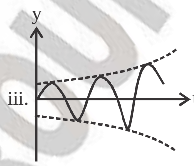
A.



i.



B.


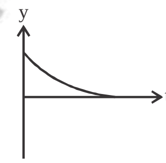

ii.



C.



iii.



D.

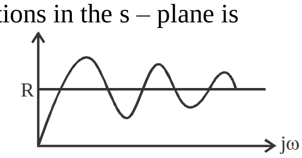


iv.


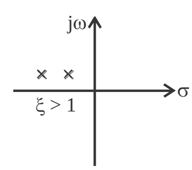

**[EE ESE - 2011]**

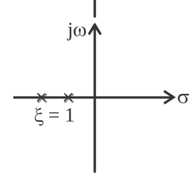
**Codes:**
(a) A-iii, B-i, C-iv, D-ii
(b) A-ii, B-i, C-iv, D-iii
(c) A-iii, B-iv, C-i, D-ii
(d) A-ii, B-iv, C-i, D-iii

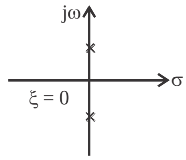
**94.** For the response show below, the correct root locations in the s – plane is

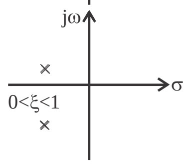



**[EE ESE - 2011]**

(a)


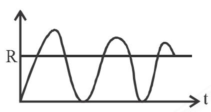

(b)


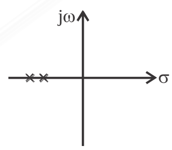
(c)


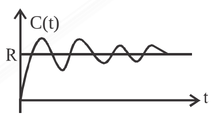

(d)


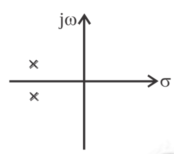

**95.** Match List-I with List-II and select the correct answer using the code given below the lists:

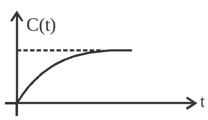
**List-I** | **List-II**

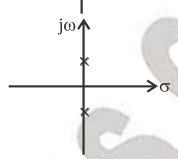
A.


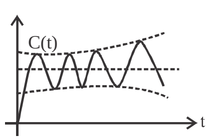

i.


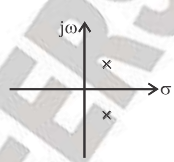

B.



ii.



C.



iii.



D.



iii.


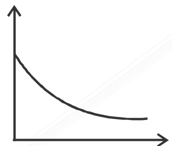

**[EE ESE - 2011]**

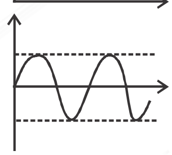
**Codes:**
(a) A-iv, B-i, C-ii, D-iii
(b) A-iii, B-i, C-ii, D-iv
(c) A-iv, B-ii, C-i, D-iii
(d) A-iii, B-ii, C-i, D-iv

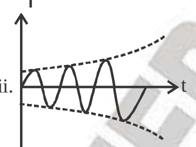
**96.** Match List-I with List-II and select the correct answer using the code given blow the lists:

**List-I**
A. Two imaginary roots
B. Two complex roots in RHS of s-plane
C. A single root on negative real axis
D. a single root at the origin

**List-II**

i.


ii.


iii.



iv.


**[EE ESE - 2011]**

**Codes:**
(a) A-iv, B-i, C-iii, D-ii
(b) A-ii, B-i, C-iii, D-iv
(c) A-iv, B-iii, C-i, D-ii
(d) A-ii, B-iii, C-i, D-iv

**97. Assertion (A):** Process industry applications should ideally be tuned for critical damping.

**Reason (R):** Critically damped response has no oscillations in the output.

**[EE ESE - 2010]**

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is NOT the correct explanation of A.
(c) A is true but R is false
(d) A is false but R is true

**98.** Given a unity feedback system with $G(s) = \dfrac{K}{s(s+4)}$, the value of K for damping ratio of 0.5 is

**[EE ESE - 2010]**

(a) 1 (b) 16
(c) 4 (d) 2

**99.** Consider a unity feedback control system with open – loop transfer function

$$G(s) = \frac{K(s+1)}{s(s+2)(s+3)}$$

The steady – state error of the system due to a unit step input is

**[EE ESE - 2010]**

(a) Zero (b) $\dfrac{K}{6}$

(c) $\dfrac{6}{K}$ (d) Infinite

**100.** A transfer function has a zero at s = –1 and poles at s = –1 ± j1. The multiplier being unity, if the input is unit step function, the steady state response is given by

**[EE ESE - 2010]**

(a) $0.5\angle 0°$ (b) $1.0\angle 0°$
(c) $2.0\angle 0°$ (d) $2.0\angle 90°$

**101.** Consider the following statements in connection with the feedback of control systems:

(i) Feedback can improve stability or be harmful to stability if it is not properly applied.
(ii) Feedback can always improve stability.
(iii) In many situations the feedback can reduce the effect of noise and disturbance on system performance.
(iv) In general the sensitivity of the system gain of a feedback system to a parameter variation depends on where the parameter is located.

Which of these statements are correct?

**[EE ESE - 2010]**

(a) i, ii and iii only (b) i, iii and iv only
(c) i, ii and iv only (d) i, ii, iii and iv

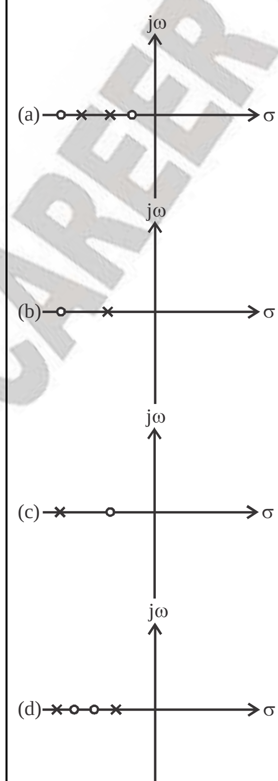
**102. Assertion (A)**: Steady state error can be reduced by increasing integral gain.
**Reason (R)**: Overshoot can be reduced by increasing derivative gain.

**[EC ESE - 2010]**

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

**103.** Which one of the following represents the pole-zero location in the s-plane for lead-compensator?

**[EC ESE - 2010]**




**104.** The transfer function $G(s) = \dfrac{1}{(3s+1)}$ has a corner frequency is

**[EC ESE - 2010]**

(a) 3 rad/s (b) 0.33 rad/s

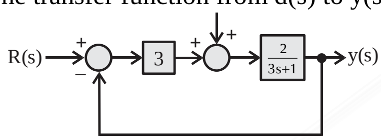
(c) 1 rad/s (d) 30 rad/s

**105.** The magnitude and phase of the transfer function $G(s) = \frac{1}{s+1}$ at ω = 1 is

**[EC ESE - 2010]**

(a) 0.707 and 45°
(b) –3 dB and 0.78 rad
(c) 0.707 and –45°
(d) 3 dB and –90°

**106.** The transfer function from d(s) to y(s) is




**[EC ESE - 2010]**

(a) $\frac{2}{3s+7}$ (b) $\frac{2}{3s+1}$

(c) $\frac{6}{3s+7}$ (d) $\frac{2}{3s+6}$

**107.** In a unity feedback control system with $G(s) = \frac{4}{s^2+0.4s}$ when subjected that to unit step unit, it is required that system response should be settled within 2% tolerance band; the system settling time is

**[EC ESE - 2010]**

(a) 1 sec (b) 2 sec
(c) 10 sec (d) 20 sec

**108.** Consider the function $F(s) = \frac{5}{s(s^2+3s+2)}$ where F(s) is Laplace transform of function f(t). The initial value of f(t) is:

**[EE ESE - 2010]**

(a) 5 (b) 5/2
(c) 5/3 (d) 0

**109.** A linear time-invariant system initially at rest, when subjected to a unit-step input, gives a response y |t| = t $e^{-t}$, t > 0. The transfer function of the system is:

**[EC ESE - 2010]**

(a) $\frac{1}{(s+1)^2}$ (b) $\frac{1}{s(s+1)^2}$

(c) $\frac{s}{(s+1)^2}$ (d) $\frac{1}{s+1}$

**110.** In closed loop control system, what is the sensitivity of the gain of the overall system, m to the variation in G?

**[EC ESE - 2009]**

(a) $\frac{1}{1+G(s)H(s)}$ (b) $\frac{1}{1+G(s)}$

(c) $\frac{G(s)}{1+G(s)H(s)}$ (d) $\frac{G(s)}{1+G(s)}$

**111.** A negative-feedback closed-loop system is supplied to an input of 5V. The system has a forward gain of 1 and a feedback gain of 1. What is the output voltage?

**[EC ESE - 2009]**

(a) 1.0 V (b) 1.5 V
(c) 2.0 V (d) 2.5 V

**112.** Which of the following may result in instability problem?

**[EC ESE - 2009]**

(a) Large error (b) High selectivity
(c) High gain (d) Noise

**113.** What is the characteristic of a good control system?

**[EC ESE - 2009]**

(a)Sensitive to parameter variation
(b)Insensitive to input command
(c)Neither sensitive to parameter variation nor sensitive to input commands
(d)Insensitive to parameter variation but sensation to input commands.

**114.** The transfer function of a linear-time-invariant system is given as $\frac{1}{(s+1)}$. What is the steady-state value of the unit-impulse response?

**[EC ESE - 2009]**

(a) Zero (b) One
(c) Two (d) Infinite

**115.** Consider the function $F(s) = \frac{\omega}{s^2 + \omega^2}$ where F(s) is the Laplace transform of f (t). What is the steady-state value of f(t)?

**[EC ESE - 2009]**

(a) Zero
(b) One
(c) Two
(d) A value between –1 and + 1

**116.** What will be the type of the system, if the steady performance of control system yields a non-zero finite value of the velocity error constant?

**[EE ESE - 2009]**

(a) Type – 0 (b) Type – 1
(c) Type – 2 (d) Type – 3

**117.** The impulse response of a second-order under-damped system started from rest is given by: $C(t) = 12.5\, e^{-6t} \sin 8t,\ t \geq 0$
What are the natural frequency and the damping factor of the system respectively?

**[EE ESE - 2009]**

(a) 10 and 0.6 (b) 10 and 0.8
(c) 8 and 0.6 (d) 8 and 0.8

**118.** A unity feedback system with open loop transfer function of $\frac{20}{s(s+5)}$ is excited by a unit step input. How much time will be required for the response to settle within 2% of final desired value?

**[EE ESE - 2009]**

(a) 0.25 sec (b) 1.60 sec
(c) 2.40 sec (d) 4.00 sec

**119.** Consider the following:
(i) Rise time (ii) Settling time
(iii) Delay time (iv) Peak time
What is the correct sequence of the time domain specifications of a second order system in the ascending order of the values.

**[EC ESE - 2009]**

(a) ii-iv-i-iii (b) iii-iv-i-ii
(c) ii-i-iv-iii (d) iii-i-iv-iii

**120.** In a fluid flow system two fluids are mixed in appropriate proportion. The concentration at the mixing point is y(t) andi s reproduced without change, $T_d$ seconds later at the monitoring point as b(t). What is transfer function between b(t) and y(t) ? (Where S is distance between monitoring point and mixing point)

**[EE ESE - 2009]**

(a) $e^{-Td}$ (b) $e^{+Tds}$
(c) $e^{-Tds}$ (d) $e^{+Td}$

**121. Assertion (A):** Addition of a pole to the forward path transfer function of unity feedback system increases the rise time of step response.
**Reason (R):** The additional pole has the effect of increasing the bandwidth of the system.

**[EE ESE - 2009]**

(a) Both A and R are true and R is the correct explanation of A.
(b) Both a and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

**122.** The response of an initially relaxed, linear constant parameter network to a unit impulse applied at t = 0 is $4e^{-21}$ u(t). What is the response of this network to unit step function ?

**[EE ESE - 2009]**

(a) $2(1 - e^{-2t}) - u(t)$ (b) $4(e^{-t} - e^{-2t})\, u(t)$
(c) sin 2t (d) $(1 - 4e^{-4t})\, u(t)$

**123.** A second order system has a natural frequency of oscillations of 3 rad/sec and damping ratio of 0.5. What are the values of resonant frequency and resonant peak of the system?

**[EE ESE - 2009]**

(a) 1.5 rad/sec and 1.16
(b) 1.16 rad/sec and 1.5
(c) 1.16 rad/sec and 2.1
(d) 2.1 rad/sec and 1.16

**124.** A control system has a transfer function

$$\frac{K(1+0.5s)(1+2s+5s^2)}{s^2(1+s)(1+5s+10s^2)(1+100s+500s^2)}$$

What is the type of the system?

**[EE ESE - 2008]**

(a) 0 (b) 1
(c) 11 (d) 111

**125.** What is the Laplace transform of a function $\delta(t-2)$?

**[EE ESE - 2008]**

(a) 2 (b) 0
(c) $e^{-2s}$ (d) 2s

**126.** Which one of the following is correct ?
Final value theorem is not applicable for the system when the input is

**[EE ESE - 2008]**

(a) Step (b) Ramp
(c) Parabolic (d) Exponential

**127.** Which one of the following statements regarding steady state errors in control system is not correct?

**[EE ESE - 2008]**

(a)Steady state error analysis relies on the use of initial value theorem
(b)Steady state error is a measure of system accuracy when a specific type of input is applied to a control system.
(c)The error constants do not give information regarding steady state error when inputs are other than step, ramp and parabolic
(d)Steady state error does not provide information on how the error varies with time

**128.** Which one of the following is the most likely reason for large overshoot in a control system ?

**[EE ESE - 2008]**

(a) High gain in a system
(b) Presence of dead time delay in a system
(c) High positive correcting torque
(d) High retarding torque

**129.** The input-output relationship of a system is given by

$$r(t)=\frac{d^2c(t)}{dt^2}+3\frac{dc(t)}{dt}+2c(t)$$

where r(t) and c(t) are input and output respectively. The transfer function of the system is equal to

**[EE ESE - 2008]**

(a) $\frac{1}{(s^2+s+2)}$ (b) $\frac{1}{(s^2+3s+2)}$
(c) $\frac{2}{s^2+3s+2}$ (d) $\frac{1}{(s^2+5s+3)}$

**130.** Consider the function

$$F(s)=\frac{\omega}{s^2+\omega^2}$$

Where F(s) = Laplace transform of f(t). The final value of f(t) is equal to

**[EE ESE - 2008]**

(a) Infinite
(b) Zero
(c) Finite constant
(d) A value in between –1 and +1

**131.** Give the Laplace transform f(t) = F9s), the Laplace transform of [f(t)$e^{-at}$ ] is equal to

**[EE ESE - 2008]**

(a) F(s +a) (b) $\frac{F(s)}{(s+a)}$
(c) $e^{as}$ F(s) (d) $e^{-as}$ F(s)

**132.** The type number of the control system with $G(s)H(s)=\frac{K(s+2)}{s(s^2+2s+3)}$

**[EE ESE - 2008]**

(a) One (b) Two
(c) Three (d) Four

**133.** For type 2 system, the steady-state error due to ramp input is equal to

**[EE ESE - 2008]**

(a) Zero (b) Finite constant
(c) Infinite (d) Indeterminate

**134.** Given a unity feedback system with

$$G(s)=\frac{K}{s(s+4)}$$

The value of K for damping ratio of 0.5 is

**[EE ESE - 2008]**

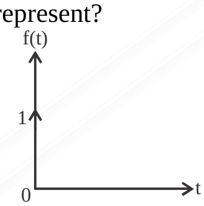
(a) 1 (b) 4
(c) 16 (d) 64

**135.** The impulse response of a second–order under-damped system starting from rest is given by $c(t)=12.5e^{-6t}\sin 8t,\ t \geq 0$.
The natural frequency and the damping factor of the system are respectively

**[EE ESE - 2008]**

(a) 10 and 0.6 (b) 10 and 0.8
(c) 8 and 0.6 (d) 8 and 0.8

**136.** What does the function f(t) plotted in the below figure represent?




**[EE ESE - 2008]**

(a) Unit step function
(b) Unit impulse function
(c) Unit ramp function
(d) Unit parabolic function

**137.** Consider the following statements for pneumatic and hydraulic control systems:
1.The normal operating pressure of pneumatic control is very much higher than that of hydraulic control.
2.In pneumatic control, external leakage is permissible to a certain extent, but there should be no leakage in a hydraulic control.
Which of the statements given above is/are correct?

**[EC ESE - 2008]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

**138.** The closed loop transfer function of a control system has the following poles and zeros

| **Poles** | **Zeros** |
|---|---|
| $p_1 = -0.5$ | $z_1 = -6$ |
| $p_2 = -1.0$ | $z_2 = -8$ |
| $p_3 = -5$ | |
| $p_4 = -10$ | |

The closed loop response can be closely approximated by considering which of the following?

**[EC ESE - 2008]**

(a) $p_1$ and $p_2$ (b) $p_3$ and $p_4$
(c) $p_3$ and $z_1$ (d) $p_4$ and $z_2$

**139.** The closed loop transfer function of a control system is $\dfrac{K}{s(s+1)(s+5)+K}$

What is the frequency of the sustained oscillations for marginally stable conditions?

**[EC ESE - 2008]**

(a) $\sqrt{5}$ rad/s (b) $\sqrt{6}$ rad/s
(c) 5 rad/s (d) 6 rad/s

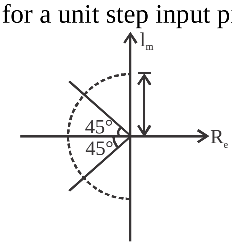
**140.** A second order control system has a transfer function $\dfrac{16}{s^2+4s+16}$. What is the time for the first overshoot?

**[EC ESE - 2008]**

(a) $\dfrac{2\pi}{\sqrt{3}}$ s (b) $\dfrac{\pi}{\sqrt{3}}$ s

(c) $\dfrac{\pi}{2\sqrt{3}}$ s (d) $\dfrac{\pi}{4\sqrt{3}}$ s

**141.** A diaphragm type pressure sensor has two poles as shown in the figure below. The zeros are at infinity. What is its steady state deformation for a unit step input pressure?




**[EC ESE - 2008]**

(a) 0.25 (b) 0.5
(b) 0.707 (d) 1

**142.** The impulse response of a linear time invariant system is given as
$g(t) = e^{-t},\ t > 0$
The transfer function of the system is equal to

**[EC ESE - 2008]**

(a) 1/s (b) 1/[s(s + 1)]
(c) 1/(s + 1) (d) s/(s + 1)

**143. Assertion (A)**: The system having characteristic equation $4s^2 + 6s + 1 = 0$ gives rise to under-damped oscillations for a step input.
**Reason (R)**: The un-damped natural frequency of oscillation of the system is $\omega_n = 0.5$ rad/s.

**[EC ESE - 2008]**

(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

**144.** For the unity feedback system with $G(s) = G(s) = \frac{10}{s^2(s+4)}$, what is the steady state error resulting from an input 10t?

**[EC ESE - 2007]**

(a) 10 (b) 4
(c) Zero (d) 1

**145.** For a second-order system, $\xi$ is equal to zero in the transfer function given by

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Which one of the following is correct?

**[EC ESE - 2007]**

(a) Closed-loop poles are complex conjugate with negative real part.
(b) Closed-loop poles are purely imaginary
(c) Closed-loop poles are real, equal and negative
(d) Closed-loop poles are real, unequal and negative.

**146.** If the initial conditions for a system are inherently zero, what does it physically mean?

**[EC ESE - 2007]**

(a) The system is at rest but stores energy
(b) The system is working but does not store energy
(c) The system is at rest or no energy is stored in any of its parts
(d) The system is working with zero reference input.

**147.** A control system whose step response is $-0.5(1 + e^{-2t})$ is cascaded to another control block whose impulse response is $e^{-t}$. What is the transfer function of the cascaded combination?

**[EC ESE - 2007]**

(a) $\frac{1}{(s+1)(s+2)}$ (b) $\frac{1}{s(s+1)}$
(c) $\frac{1}{s(s+2)}$ (d) $\frac{0.5}{(s+1)(s+2)}$

**148.** How can the steady-state error in a system be reduced?

**[EC ESE - 2007]**

(a) By decreasing the type of system
(b) By increasing system gain
(c) By decreasing the static error constant
(d) By increasing the input

**149.** The characteristic polynomial of a system is
$q(s) = 2s^5 + s^4 + 4s^3 + 2s^3 + 2s + 1$
Which one of the following is correct?
The system is

**[EC ESE - 2007]**

(a) Stable (b) Marginally stable
(c) Unstable (d) Oscillatory

**150.** Match List-I (Time Function) with List-II (Laplace Transform) and select the correct answer using the code given below the lists:

**List-I**
A. 1
B. t
C. sin ωt
D. cos ωt

**List-II**
(i) $\frac{1}{s}$
(ii) $\frac{1}{s^2}$

(iii) $\frac{s}{s^2+\omega^2}$

(iv) $\frac{\omega}{s^2+\omega^2}$

**[EE ESE - 2007]**

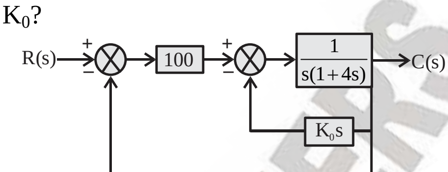
**Codes:**
(a) A-i, B-ii, C-iii, D-iv
(b) A-ii, B-i, C-iii, D-iv
(c) A-i, B-ii, C-iv, D-iii
(d) A-ii, B-i, C-iv, D-iii

**151.** For a unity feedback control system with forward path transfer function $G(s)=\frac{K}{s+5}$, what is error transfer function $w_e(s)$ used for determination of error coefficients ?

**[EE ESE - 2007]**

(a) $\frac{K}{s+5}$ (b) $\frac{K}{s+K+5}$
(c) $\frac{s+5}{s+K+5}$ (d) $\frac{K(s+5)}{s+K+5}$

**152.** Output rate control is used to improve the damping of the system given in the below figure. If the closed-loop system is required to have a damping factor of 0.5, what is the value of $K_0$?




**[EE ESE - 2007]**

(a) 4 (b) 19
(c) 1/4 (d) 6

**153.** For a second order system, natural frequency of oscillation of 10 rad/s and damping ratio is 0.1. What is the 2% settling time?

**[EE ESE - 2007]**

(a) 40 s (b) 10 s
(c) 0.4 s (d) 4 s

**154.** The impulse response of a second order under-damped system starting from rest is given by:
$C(t) = 12.5\, e^{-6t}\sin 8t;\ t \geq 0$
What are the value of natural frequency and damping factor of the system, respectively?

**[EE ESE - 2007]**

(a) 10 units and 0.6
(b) 10 units and 0.8
(c) 8 units and 0.6
(d) 8 units and 0.8

**155.** The input-output relationship of a linear time invariant continuous time system is given by

$$r(t)=\frac{d^2c(t)}{dt^2}+3\frac{dc(t)}{dt}+2c(t)$$

where r(t) and c(t) are input and output respectively. What is the transfer function of the system equal to?

**[EE ESE - 2007]**

(a) $\frac{1}{(s^2+s+2)}$ (b) $\frac{1}{(s^2+3s+2)}$
(c) $\frac{2}{(s^2+3s+2)}$ (d) $\frac{2}{(s^2+s+2)}$

**156.** The open loop transfer function for unity feedback system is given by

$$\frac{5(1+0.1s)}{s(1+5s)(1+20s)}$$

Consider the following statements:
(i)The steady state error for a step in put of magnitude 10 is equal to zero.
(ii)The steady -state error for a ramp input of magnitude 10 is 2.
(iii)The steady - state error for an acceleration input of magnitude 10 is infinite.
Which of the statements given above are correct?

**[EE ESE - 2007]**

(a) Only i and ii (b) Only i and iii
(c) Only ii and iii (d) i, ii and iii

**157.** A second-order control system has a transfer function

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\xi_n s + \omega_n^2}$$

For unit step input, match List-I (Time Domain Specification) with List-II (Expression) and select the correct answer using the code given below the lists:

**List-I**
A. Rise time
B. Peak time
C. Peak Overshoot
D. Settling time

**List-II**

(i) $\dfrac{\pi - \tan^{-1}\left(\dfrac{\sqrt{1-\delta^2}}{\delta}\right)}{\omega_n \sqrt{1-\delta^2}}$

(ii) $\dfrac{\pi}{\omega_n \sqrt{1-\delta^2}}$

(iii) $e^{(-\pi\delta\sqrt{1-\delta^2})}$

(iv) $\dfrac{4}{\delta\omega_n}$

**[EE ESE - 2007]**

**Codes:**
(a) A-i, B-ii, C-iii, D-iv
(b) A-iii, B-iv, C-i, D-ii
(c) A-i, B-iv, C-iii, D-ii
(d) A-iii, B-ii, C-i, D-iv

**158.** A particular control systems yielded a - state error of 0.20 for unit step input. A unit integrator is cascaded to this system and unit ramp input is applied to this modified system.
What is the value of steady – state error for this modified system?

**[EE ESE - 2006]**

(a) 0.10 (b) 0.15
(c) 0.20 (d) 0.25

**159.** A system function $N(s) = \dfrac{V(s)}{I(s)} = \dfrac{s+3}{4s+5}$

The system is initially at rest. If the excitation i(t) is a unit step, which of the following are the initial and steady-state values of v(t) ?

**[EE ESE - 2006]**

| | Initial value | Steady-state value |
|---|---|---|
| (a) | 0 | 3/5 |
| (b) | 1/4 | 0 |
| (c) | 3/5 | ¼ |
| (d) | 1/4 | 3/5 |

**160.** Consider the network function:

$$H(s) = \frac{2(s+3)}{(s+2)(s+4)}$$

What is the steady – state response due to a unit step input?

**[EE ESE - 2006]**

(a) 4/3 (b) 1/2
(c) 3/4 (d) 1

**161.** The system having characteristic equation:
$s^4 + 2s^3 + 3s^2 + 2s + K = 0$
is to be used as an oscillator. What are the values of K and the frequency of oscillation ω?

**[EC ESE - 2006]**

(a) K = 1 and ω = 1/r/s
(b) K = 1 and ω = 2 r/s
(c) K = 2 and ω = 1 r/s
(d) K = 2 and ω = 2 r/s

**162.** The unit step response of a system is $1 - e^{-t}(1 + t)$. Which is this system?

**[EC ESE - 2006]**

(a) Unstable (b) Stable
(c) Critically stable (d) Oscillatory

**163.** The open loop transfer function of a unity negative feedback control system is given by $G(s) = \dfrac{K}{(s+2)(s+4)(s^2+6s+25)}$. Which is the value of K which causes sustained oscillations in the closed loop system?

**[EC ESE - 2006]**

(a) 590 (b) 790
(c) 990 (d) 1190

**164.** The unit step response of a second order system is $= 1 - e^{-5t} - 5t\, e^{-5t}$

Consider the following statements:
1. The undamped natural frequency is 5 rad/s.
2. The damping ratio is 1.
3. The impulse response is $25\,t\,e^{-5t}$.
Which of the statements given above are correct?

**[EC ESE - 2006]**

(a) Only 1 and 2 (b) Only 2 and 3
(c) Only 1 and 3 (d) 1, 2 and 3

**165.** Which one of the following is a disadvantages of proportional controller?

**[EC ESE - 2006]**

(a) It destabilizes the system
(b) It produces offset
(c) It makes response faster
(d) It has very simple implementation

**166.** What is the value of K for a unity feedback system with $G(s)=\dfrac{K}{s(1+s)}$ to have a peak overshoot of 50%?

**[EC ESE - 2006]**

(a) 0.53 (b) 5.3
(c) 0.6 (d) 0.047

**167. Assertion (A)**: The impulse response is only a function of the terms in natural response.
**Reason (R)**: The differentiation and differencing operations eliminate the constant terms associated with the particular solution in the step response and change only the constants associated with exponential in the natural response.

**[EC ESE - 2006]**

(a) Both A and R are true and R is true and R is the correct explanation of A.
(b) Both A and R are true but R is NOT the correct explanation of A.
(c) A is true but R is false
(d) A is false but R is true.

**168.** A linear network has the system function
$$H\frac{(s+c)}{(s+a)(s+b)}$$
The outputs of the network with zero initial conditions for two different inputs are tabled as

| Input x(t) | Output y(t) |
|---|---|
| u(t) | $2 + De^{-t} + Ee^{-3t}$ |
| $e^{-2t}$ u(t) | $Fe^{-t} + Ge^{-3t}$ |

Then the values of c and H are, respectively

**[EC ESE - 2005]**

(a) 2 and 3 (b) 3 and 2
(c) 2 and 2 (d) 1 and 3

**169.** What is the steady state error for a unity feedback control system having $G(s)=\dfrac{1}{s(s+1)}$, due to unit ramp input?

**[EC ESE - 2005]**

(a) 1 (b) 0.5
(c) 0.25 (d) $\sqrt{0.5}$

**170.** Given a unity feedback system with $G(s)=\dfrac{K}{s(s+4)}$, What is the value of K for a damping ratio of 0.5?

**[EC ESE - 2005]**

(a) 1 (b) 16
(c) 4 (d) 2

**171.** Match List-I (System G(s)) with List-II (Nature of Response) and select the correct answer using the code given below the lists:

**List-I**

A. $\dfrac{400}{s^2+12s+400}$

B. $\dfrac{900}{s^2+90s+900}$

C. $\dfrac{225}{s^2+30s+225}$

D. $\dfrac{625}{s^2+625}$

**List-II**

(i) Undamped
(ii) Critically damped
(iii) Underdamped
(iv) Overdamped

**[EC ESE - 2005]**

**Codes**:
(a) A-iii, B-i, C-ii, D-iv

(b) A-iii, B-iv, C-iii, D-i
(c) A-iii, B-iv, C-ii, D-i
(d) A-ii, B-i, B-iii, D-iv

**172.** An underdamped second order system with negative damping will have the two roots

**[EC ESE - 2005]**

(a)On the negative real axis as real roots.
(b)On the left hand side of complex plane as complex roots
(c)On the right hand side of complex plane as complex conjugates.
(d)On the positive real axis as real roots.

**173.** With negative feedback in a closed loop control system, the system sensitivity to parameter variations:

**[EC ESE - 2005]**

(a) Increases (b) Decreases
(c) Becomes zero (d) Becomes infinite

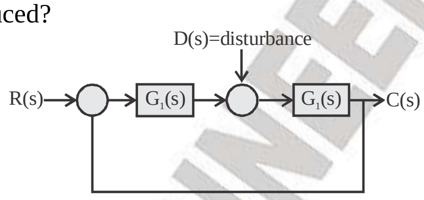
**174.** Which one of the following expresses the time at which second peak in step response occurs for a second order system?

**[EC ESE - 2005]**

(a) $\dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$ (b) $\dfrac{2\pi}{\omega_n\sqrt{1-\xi^2}}$

(c) $\dfrac{3\pi}{\omega_n\sqrt{1-\xi^2}}$ (d) $\dfrac{\pi}{\sqrt{1-\xi^2}}$

**175.** For the given system, how can be steady state error produced by step disturbance be reduced?




**[EC ESE - 2005]**

(a) By increasing dc gain of $G_1(s)\ G_2(s)$
(b) By increasing dc gain of $G_2(s)$
(c) By increasing dc gain of $G_1(s)$
(d) By removing the feedback

**176.** With regard to the filtering property, the lead compensator and the lag compensator are, respectively:

**[EC ESE - 2005]**

(a) Low pass and high pass filters
(b) High pass and low pass filters
(c) Both high pass filters
(d) Both low pass filters

**177.** In an RLC series circuit, if the resistance R and the inductance L are kept constant but capacitance C is decreased, then which one of the following statements is/are correct?
(i) Time constant of the circuit is changed.
(ii) Damping ratio decreases.
(iii) Natural frequency increases.
(iv) Maximum overshoot is unaffected.

**[EE ESE - 2007]**

(a) i and ii (b) ii only
(c) ii and iii (d) iii and iv

**178.** Match List-I with List-II and select the correct answer using the code given below the lists :

**List-I**
A. Imaginary axis of s-plane
B. Oscillatory time domain response
C. Overdamped time response
D. Poles at origin of s-plane

**List-II**
(i) imaginary axis poles in s - plane
(ii) Type of the system
(iii) Unit circle of z - plane
(iv) Poles of real axis of s - plane

**[EE ESE - 2005]**

**Codes:**
(a) A-i, B-iii, C-iv, D-ii
(b) A-i, B-iii, C-ii, D-iv
(c) A-iii, B-i, C-iv, D-ii
(d) A-iii, B-iv, C-i, D-ii

**179.** Match List-I (Response) with List-II (Parameter) and select the correct answer using the codes given below the lists:

**List-I**
A.Swiftness of transient response
B.Closeness of the response to the desired response
C.Reduction of steady state error

D.Number of integrators in loop transfer function

**List-II**

(i) Feedback control
(ii) Type number
(iii) Rise time and peak time
(iv) Overshoot and setting time

**[EE ESE - 2005]**

**Codes:**

(a) A-iii, B-iv, C-i, D-ii
(b) A-ii, B-i, C-iv, D-iii
(c) A-iii, B-i, C-iv, D-ii
(d) A-ii, B-iv, C-i, D-iii

**180.** $4\frac{d^2y}{dt^2} + 36y = 36x$

Consider the following statements in connection with the differential equation given above:
(i)The natural frequency of the response is 6 rad/s
(ii)The response is always oscillatory
(iii)The percentage overshoot is 10% and damping ratio of the system is 0.6
(iv)Both system time constant and setting time are infinite
Which of the statements given above are correct?

**[EE ESE - 2005]**

(a) i and iii (b) ii and iv
(c) i, ii and iii (d) ii, iii and iv

**181.** The open loop transfer function of a unity feedback control system is given by $G(s) = \frac{k}{s(s+1)}$. If gain k is increased to infinity, then damping ratio will tend to become

**[EE ESE - 2005]**

(a) Zero (b) 0.707
(c) Unity (d) Infinite

**182.** What are the order and type of close – loop system for the plant transfer function $G(s) = \frac{k}{s^2(1+Ts)}$ and with unity feedback?

**[EE ESE - 2005]**

(a) Two and two (b) Three and two
(c) Two and zero (d) Three and zero

**183.** Which one of the following is the steady state error of a control system with step error, ramp error and parabolic error constants $k_p$, $k_v$ and $k_a$ respectively for the input $(1 - t^2)\, 3u(t)$ ?

**[EE ESE - 2005]**

(a) $\frac{3}{1+k_p} - \frac{3}{2k_a}$ (b) $\frac{3}{1+k_p} + \frac{6}{k_a}$
(c) $\frac{3}{1+k_p} - \frac{3}{k_a}$ (d) $\frac{3}{1+k_p} - \frac{6}{k_a}$

**184.** Consider the following statements regarding advantages of using the generalized error coefficients:
(i) The generalized error coefficients provide a simple way of determining the nature of the response of a feedback control system to almost any arbitrary input.
(ii) The generalized error coefficients lead to the calculation of the steady-state response without actually solving the system differential equation.
(iii) The generalized error coefficients establish relationships among the various types of inputs.
Which of the above statements are correct?

**[EE ESE - 2005]**

(a) i, ii and iii (b) i and ii
(c) ii and iii (d) i and iii

**185.** Which one of the following equations gives the steady-state error for a unity feedback system excited by $u_S(f) + tu_S(t) + [t^2u_S(t)/2]$?

**[EE ESE - 2004]**

(a) $\frac{1}{(2+K_p)} + \frac{1}{K_v} + \frac{1}{K_a}$
(b) $\frac{1}{(1+K_p)} + \frac{1}{K_v} + \frac{2}{K_a}$
(c) $\frac{1}{K_p} + \frac{1}{K_v} + \frac{1}{K_a}$
(d) $\frac{1}{(1+K_p)} + \frac{1}{K_v} + \frac{1}{K_a}$

**186.** Consider the following transfer functions:

(i) $\frac{1}{(s^2+s+1)}$ (ii) $\frac{4}{(s^2+2s+4)}$

(iii) $\frac{2}{(s^2+2s+2)}$ (iv) $\frac{1}{(s^2+2s+1)}$

(v) $\frac{3}{(s^2+6s+3)}$

Which of the above transfer functions represent underdamped second order systems?

**[EE ESE - 2004]**

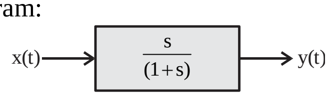
(a) iv and v (b) i, iv and v
(c) i, ii and iii (d) i, iii and v

**187.** The damping ratio and natural frequency of a second order system are 0.6 and 2 rad/s respectively. Which one of the following combinations gives the correct values of peak and setting time, respectively for the unit step response of the system?

**[EE ESE - 2004]**

(a) 3.33 s and 1.95 s (b) 1.95 s and 3.33 s
(c) 1.95 s and 1.5 s (d) 1.5 s and 1.95 s

**188.** Consider the following system shown in the diagram:




In the system shown in the above diagram x(t) = sin t. what will be the response y(t) in the steady state?

**[EE ESE - 2004]**

(a) $\frac{\sin(t-45°)}{\sqrt{2}}$ (b) $\frac{\sin(t+45°)}{\sqrt{2}}$

(c) $\sqrt{2}e^{-t}\sin(t)$ (d) sin t – cos t

**189.** A second order control system has

$$M(jw)=\frac{100}{100-\omega^2+10\sqrt{2}j\omega}$$

Its $M_P$ (Peak magnitude) is

**[EE ESE - 2004]**

(a) 0.5 (b) 1
(c) $\sqrt{2}$ (d) 2

**190.** Consider the following statements

Feedback in control system can be used
1. To reduce the sensitivity of the system to parameter variations and disturbances
2. To change time constant of the system
3. To increase loop gain of the system

Which of the statements given above are correct?

**[EC ESE - 2004]**

(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) 1 and 3

**191.** An open loop system has a transfer function $\frac{1}{s^3+1.5s^2+s-1}$. It is converted into a closed loop system by providing a negative feedback having transfer function 20 (s + 1). Which one of the following is correct?

The open loop and closed loop systems are, respectively.

**[EC ESE - 2004]**

(a) Stable and stable
(b) Stable and unstable
(c) Unstable and stable
(d) Unstable and unstable

**192.** Consider the following statements for a.c. series motors:
1. The rotor is designed so that its R/S ratio is small.
2. $dT/d\omega < 0$ where T and ω are torque and speed respectively.
3. The reference and control voltages should be in phase quadrature, but their magnitudes need not be equal.

Which of the statements given above are correct?

**[EC ESE - 2004]**

(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) 1 and 3

**193.** What is the unit step response of a unity feedback control system having forward path transfer function $G(s)=\frac{80}{s(s+18)}$?

**[EC ESE - 2004]**

(a) Overdamped
(b) Critically damped

(c) Under damped
(d) Underdamped oscillatory

**194.** When the time period of observation is large the type of the error is

**[EC ESE - 2003]**

(a) Transient error
(b) Steady state error
(c) Half-power error
(d) Position error constant

**195.** Assuming unit ramp input, match List-I (System Type) with List-II (Steady State Error) and select the correct answer using the codes given below the lists:

| **List-I** | **List-II** |
|---|---|
| A. 0 | (i) K |
| B. 1 | (ii) ∞ |
| C. 2 | (iii) 0 |
| D. 3 | (iv) 1/K |

**[EC ESE - 2003]**

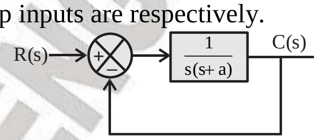
**Codes**:
(a) A-ii, B-iv, C-iii, D-i
(b) A-i, B-ii, C-iii, D-iv
(c) A-ii, B-i, C-iv, D-iii
(d) A-i, B-ii, C-iv, D-iii

**196.** Which one of the following is the transfer function of a linear system whose output is $t^2e^{-t}$ for a unit step input?

**[EC ESE - 2003]**

(a) $\frac{s}{(s+1)^3}$ (b) $\frac{2s}{(s+1)^3}$
(c) $\frac{1}{s^2(s+1)}$ (d) $\frac{2}{s(s+1)^2}$

**197.** Consider the unity feedback system as shown below. The sensitivity of the steady state error to change in parameter K and parameter a with ramp inputs are respectively.




**[EC ESE - 2003]**

(a) 1, –1 (b) –1, 1
(c) 1, 0 (d) 0, 1

**198.** The unit impulse response of a second order system is 1/6 $e^{-0.8t}$ sin(0.6t). Then the natural frequency and damping ratio of the system are respectively

**[EE ESE - 2003]**

(a) 1 and 0.6 (b) 1 and 0.8
(c) 2 and 0.4 (d) 2 and 0.3

**199.** Which one of the following statements is NOT correct?

**[EE ESE - 2003]**

(a) With the introduction of integral control, the steady state error increases.
(b) The generalized error coefficients provide a simple way of determining the nature of the response of a feedback control to any arbitrary input.
(c) The generalized error coefficients lead to calculation of complete steady state response without actually solving the system differential equation.
(d) For a type - 1, the steady state error for acceleration input is infinite

**200.** Consider the following statements with reference to a system with velocity error constant $K_v$ = 1000;
(i) The system is stable
(ii) The system is of type 1
(iii) The test signal used is a step input
Which of these statements are correct?

**[EE ESE - 2003]**

(a) i and ii (b) i and iii
(c) ii and iii (d) i, ii and iii

**201. Assertion (A):** A system may have no steady state error to a step input, but the same system may exhibit non zero steady state error to ramp input
**Reason (R):** The steady state error of a system depends on the 'type' of the open loop transfer function

**[EE ESE - 2002]**

(a) Both A and R are true and R is the correct explanation of A

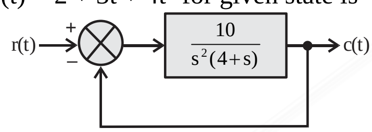
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**202.** If a ramp input is applied to Type - 2 system, the steady state error is

**[EE ESE - 2002]**

(a) Positive constant (b) Negative constant
(c) Zero (d) Positive infinity

**203.** The steady – state error resulting from an input $r(t) = 2 + 3t + 4t^2$ for given state is




**[EE ESE - 2002]**

(a) 2.4 (b) 4.0
(c) Zero (d) 3.2

**204.** Given the transfer function

$G(s) = \dfrac{121}{s^2 + 13.2s + 121}$ of a system. Which of the following characteristics does it have?

**[EE ESE - 2002]**

(a) Overdamped and settling time 1.1s
(b) Underdamped and settling time 0.6s
(c) Critically damped and settling time 0.8 s
(d) Underdamped and settling time 0.707 s

**205.** A system has a single pole at origin. Its impulse response will be

**[EC ESE - 2002]**

(a) Constant
(b) Ramp
(c) Decaying exponential
(d) Oscillatory

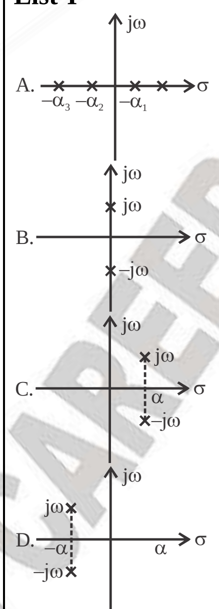
**206. Assertion (A)**: Addition of a pole to the open loop transfer function of a system increases the rise time of the closed loop step-response.
**Reason (R)**: Additional pole has the effect of reducing the bandwidth of the system.

**[EC ESE - 2002]**

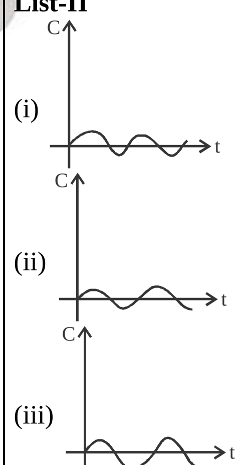
(a) Both A and R are true and r is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

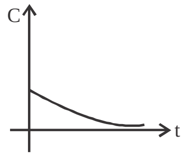
**207.** Match List-I (Pole-zero plot of linear control system) with List-II (Responses of the system) and select the correct answer:

**List-I**




**List-II**



(iv) 


**[EC ESE - 2002]**

**Codes**:
(a) A-iv, B-iii, C-i, D-ii
(b) A-iv, B-iii, C-ii, D-i
(c) A-iii, B-iv, C-ii, D-i
(d) A-iii, B-iv, C-i, D-ii

**208.** A third-order system approximated to an equivalent second order system. The rise time of this approximated lower order system will be.
**[EC ESE - 2001]**
(a)Same as original system for any input
(b)Smaller than the original system for any input
(c)Larger than the original system for any input
(d)Larger or smaller depending on the input.

**209.** The unit step response of a particular control system is given by $c(t) = 1 – 10e^{–t}$. Then its transfer function is
**[EC ESE - 2001]**
(a) $\frac{10}{s+1}$ (b) $\frac{s-9}{s+1}$
(c) $\frac{1-9s}{s+1}$ (d) $\frac{1-9s}{s(s+1)}$

**210.** Which one of the following is the steady-state error for s step input applied to a unity feedback system with the open loop transfer function $G(s) = \frac{10}{s^2+14s+50}$?
**[EC ESE - 2001]**
(a) $e_{SS} = 0$ (b) $e_{SS} = 0.83$
(c) $e_{SS} = 1$ (d) $e_{SS} = \infty$

**211.** Which one of the following is the response y(t) of a causal LTI system described by
$H(s) = \frac{(s+1)}{s^2+2s+2}$
For a given input $x(t) = e^{–t}\,u(t)$?
**[EC ESE - 2001]**
(a) $y(t) = e^{–t} \sin tu(t)$
(b) $y(t) = e^{–(t – 1)} \sin(t – 1)\, u(t – 1)$
(c) $y(t) = \sin (t – 1)\, u(t – 1)$
(d) $y(t)= e^{–t} \cos t\, u(t)$

**212.** Two identical first order systems have been cascaded non-interactively. The unit step response of the systems will be
**[EC ESE - 2001]**
(a) Overdamped (b) Underdamped
(c) Undamped (d) Critically damped

**213.** A linear time invariant system, initially at rest when subjected to a unit step input gave response $c(f) = te^{-t}$ $(t \geq 0)$. The transfer function of the system is
**[EE ESE - 2001]**
(a) $\frac{s}{(s+1)^2}$ (b) $\frac{1}{s(s+1)^2}$
(c) $\frac{1}{(s+1)^2}$ (d) $\frac{1}{s(s+1)}$

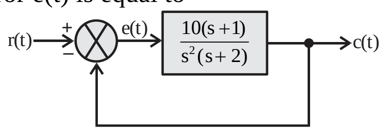
**214.** The open loop transfer function of a unity feedback system is given by $\frac{K}{s(s+1)}$. If the valve of gain K is such that the system is critically damped, the closed loop poles of the system will lie at
**[EE ESE - 2001]**
(a) – 0.5 an d – 0 .5 (b) $\pm j0.5$
(c) 0 and -1 (d) $0.5 \pm j0.5$

**215.** The steady state error due to a ramp input for a type two system is equal to
**[EE ESE - 2001]**
(a) Zero (b) Infinite
(c) Non - zero number (d) Constant

**216.** A second order control is defined by the following differential equation:
$$4\frac{d^2c(t)}{dt^2}+8\frac{dc(t)}{dt}+16c(t)=16\,u(t)$$
The damping ratio and natural frequency for this system are respectively
**[EE ESE - 2001]**

(a) 0.25 and 2 rad/s (b) 0.50 and 2 rad/s
(c) 0.25 and 4 rad/s (d) 0.50 and 4 rad/s

**217.** In the system shown in the given figure, r(t) = 1 + 2t(t ≥ 0). The steady - state value of the error e(t) is equal to

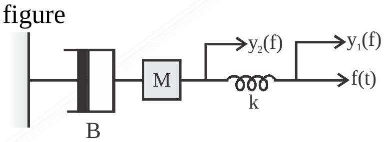



**[EE ESE - 2001]**

(a) Zero (b) $\frac{2}{10}$

(c) $\frac{10}{2}$ (d) Infinity

**218.** The mechanical system is shown in the given figure




**[EE ESE - 2001]**

(a) $M\frac{d^2y_1(t)}{dt^2}+B\frac{dy_1(t)}{dt}=k[y_2(t)-y_1(t)]+f(t)$

(b) $M\frac{d^2y_2(t)}{dt^2}+B\frac{dy_2(t)}{dt}=k[y_2(t)-y_1(t)]+f(t)$

(c) $M\frac{d^2y_1(t)}{dt^2}+B\frac{dy_1(t)}{dt}=k[y_1(t)-y_2(t)]+f(t)$

(d) $M\frac{d^2y_2(t)}{dt^2}+B\frac{dy_2(t)}{dt}=k[y_1(t)-y_2(t)]+f(t)$

**219.** A second order system has the damping ratio ξ and undamped natural frequency of oscillation $\omega_n$. The settling time at 2% tolerance band of the system is

**[EC ESE - 2000]**

(a) $2/\xi\omega_n$ (b) $3/\xi\omega_n$
(c) $4/\xi\omega_n$ (d) $\xi\omega_n$

**220.** Consider the following statements
1.The effect of feedback is to reduce the system error.
2.Feedback increases the gain of the system in one frequency range but decreases in another.
3.Feedback can cause a system that is originally stable to become unstable.
Which of these statements are correct?

**[EC ESE - 2000]**

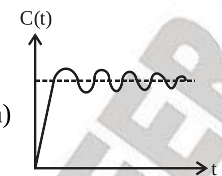
(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) 1 and 3

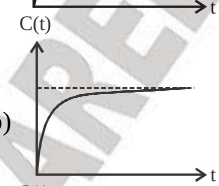
**221.** A step input is applied to a system with the transfer function

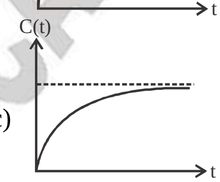
$$G(s)=\frac{e^{-s}}{1+0.5s}$$

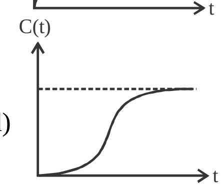
The output response will be

**[EC ESE - 1999]**

(a)



(b)



(c)



(d)



**222.** The system with the open-loop transfer function

$$G(s)H(s)=\frac{1}{s(1+s)} \text{ is}$$

**[EC ESE - 1999]**

(a) Type 2 and order 1
(b) Type 1 and order 1
(c) Type 0 and order 0
(d) Type 1 and order 2

**223.** The response c(t) of a system is described by the differential equation.

$$\frac{d^2c(t)}{dt^2}+4\frac{dc(t)}{dt}+5c(t)=0$$

The system response is

**[EC ESE - 1999]**

(a) Undamped (b) Underdamped
(c) Critical sampled (d) Oscillatory

**224.** First column elements of the Routh's tabulation are 3, 5, –3/4, ½, 2. It means than there.

**[EC ESE - 1999]**

(a) Is one root in the left half of s-plane
(b) Are two roots in the left half of s-plane
(c) Are two roots in the right half of s-plane
(d) Is one root in the right half of s-plane

**225.** Consider the following statements relating to synchros:
1. The rotor of the control transformer is either disc shaped.
2. The rotor of the transmitter s so constructed as to have a low magnetic reluctance.
3. Transmitter and control transformer pair is used as an error detector.
Which of these statements are correct?

**[EC ESE - 1999]**

(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) 1 and 3

**226.** For two-phase AC servomotor, if the rotor resistance and reactance are respectively R and X, its length and diameter are respectively L and D, then

**[EC ESE - 1999]**

(a) X/R and L/D are both small
(b) X/R is large but L/D is small
(c) X/R is small but L/D is large
(d) X/R and L/D are both large

**227.** When a human being tries to approach an object, his brain acts as

**[EC ESE - 1999]**

(a) An error measuring device
(b) A controller
(c) An actuator
(d) An amplifier

# SOLUTIONS

**Sol. 1. (a)**

$$e_{ss} = \frac{1}{1+kp}$$

$$1+K_p = \frac{1}{0.2}$$

$k_p = 4$

$$k_p = \lim_{s\to 0} GCSH(s) = 4$$

The error due to step i/p is made to zero so type of system would have increased

$$G(s) = \frac{G(S)H(S)}{S},\ K_v = \lim_{s\to 0}\ \&\ \frac{GCSH(S)}{s} = 4$$

$$k_v = \frac{1}{4} = 0.25$$

**Sol. 2. (b)**

CE. $1+\frac{25}{s(s+6)} = 0$

$$s^2+65+25=0$$

$\omega n = 5$

$$\xi = \frac{6}{2\times 5} = 0.6$$

Setting time

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{5\times 0.6} = 1.33\text{sec}$$

**Sol. 3. (d)**

$$CE.1+\frac{k}{s(s+a)} = 0$$

$S^2 + as + k = 0$

$2\xi\omega n = a$

$$\omega_n = \sqrt{k}$$

$$\xi = \frac{a}{2\sqrt{k}}$$

For undreamed system

$\xi < 1$

$$\frac{a}{2\sqrt{k}} < 0 \quad k > \frac{a^2}{4}$$

$$\sqrt{k} > \frac{a}{2}$$

**Sol. 4. (b)**

Settling time is defined as the time for the response to react and stay within 2% of its final value.

**Sol. 5. (a)**

$$k_P = \lim_{s\to 0} G(s)$$

$$= k_P = \lim_{s\to 0}\frac{k(s+2)}{s^2(s^2+75+12)} = \infty$$

$$k_v = \lim_{s\to 0} s.G(s)$$

$$= \lim_{s\to 0}\frac{k(s+2)}{s(s^2+75+12)} = \infty$$

$$K.G = \lim_{s\to 0} s^2 G(s)$$

$$= \lim_{s\to 0}\frac{k(s+2)}{s^2+75+12} = \frac{2k}{12} = \frac{k}{6}$$

**Sol. 6. (c)**

For open loop T.F.

Poles are lies at s = 0, 0, –2

Hence repeated poles at origin unstable

For close loop system

$$1+\frac{k(s+1)}{s^2(s+2)} = 0$$

$S^3 + 2s^2 + ks + k = 0$

| | | |
|---|---|---|
| $S^3$ | 1 | k |
| $S^2$ | 2 | k |
| $S^1$ | $\frac{2k-k}{2}$ | |
| $S^0$ | k | k>0 |

So for k > 0 close loop system is stable.

**Sol. 7. (b)**

Derivative compensation is phase lead compensation so damping factor (δ) increases $\omega_n$ (natural frequency) remains unchanged.

**Sol. 8. (None)**

$$G(s) = \frac{1}{s(1+sT)} \rightarrow \text{Type}-1$$

(i) Position Error constant.

$$K_p = \lim_{s\to0} G(s) = \lim_{s\to0} \frac{1}{s(1+sT)} = \infty$$

(ii) Acceleration Error constant.

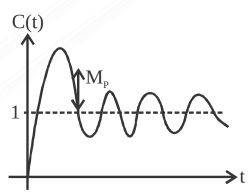
$$K_1 = \lim_{s\to0} s^2 G(s) = \lim_{s\to0} s \frac{1}{2s(1+sT)} = 0$$

(iii) r(t) = u(t)

Steady state Error

$$e_{ss} = \lim_{s\to0} s. \frac{1/s}{1+\frac{1}{s(1+ST)}} = 0$$

**Sol. 9. (c)**




The largest Error between reference input and output during transient periodis called peak over shoot.

$M_P = C(t_p) - C(\infty)$

$C(t_p) \Rightarrow$ Response at peak time

$C(\infty) \Rightarrow$ steady state Response

Peak overshoot is maximum overshoot over its steady state value.

**Sol. 10.(c)**

Given, characteristic equation is

$2s^2 + 6s + 6 = 0$ Or $s^2 + 3s + 3 = 0$

Comparing with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

Gives, $\omega_n = \sqrt{3}$, and $2\xi\omega_n = 3$

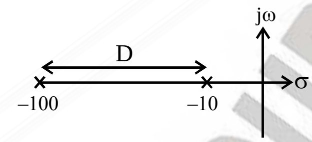
Or $2\xi \times \sqrt{3} = 3$ Or $\xi \times \frac{\sqrt{3}}{2} = 0.866 < 1$

Hence, system is underdamped.

**Sol. 11. (None)**

$$T(S) = \frac{2}{(s+10)(s+100)}$$

$$= \frac{2}{1000\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)}$$




Using Dominant Pole Concept

$$T(s) = \frac{0.002}{\left(1+\frac{s}{10}\right)} = \frac{K}{(1+sT)}$$

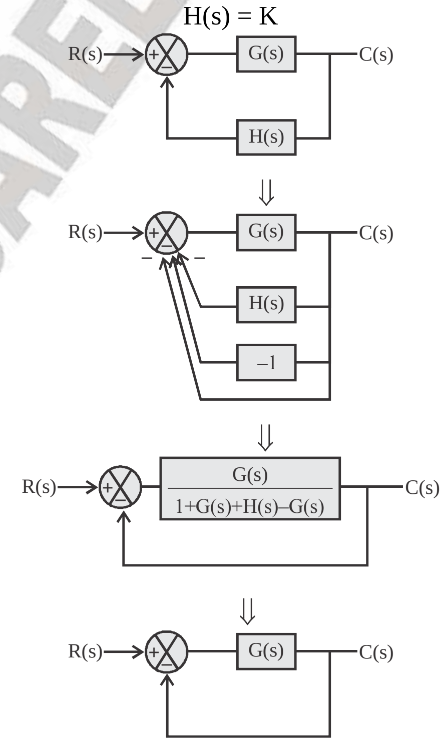
T = 0.1s

Setting Time = 4T for 2% criterion= 0.4s

**Sol. 12.(d)**

$$G(s) = \frac{10}{s(s+5)}$$

H(s) = K




$$G'(s) = \frac{G(s)}{1+G(s)H(s)-G(s)}$$

$$G'(s) = \frac{\frac{10}{s(s+5)}}{1+\frac{10}{s(s+5)}K - \frac{10}{s(s+5)}}$$

$$G'(s) = \frac{10}{s^2+5s+10(K-1)} \equiv \text{Type 'O' system}$$

$$K_P = \underset{s\to 0}{Lt}\, G'(s) = \underset{s\to 0}{Lt} \frac{10}{s^2+5s+10(K-1)} = \frac{1}{K-1}$$

Steady State Error

$$e_{ss} = \frac{1}{1+K_P} = \frac{1}{1+\frac{1}{K-1}} = \frac{K-1}{K}$$

$e_{SS} < 0.05$

$\frac{K-1}{K} < 0.05$

$K - 1 < 0.05\,k$

$0.95K < 1$

$\Rightarrow K < 1.052$

**Sol. 13. (a)**

The transfer function of system is $\left(\frac{4}{s^2+1.6s+4}\right)$ poles are at $\left(\frac{-4}{5} \pm \frac{2\sqrt{21}}{5}i\right)$.

The 2% tolerance ban setting time is $4\tau$.

So $4\tau \Rightarrow \left(\frac{4}{\xi\omega_n}\right) \Rightarrow \frac{4}{\frac{4}{5}} = 5$

**Sol. 14. (d)**

Open loop transfer function is $\frac{5}{s(s+4)}$

Since the $\beta$ = 1 and damping factor = 0.5

So closed loop function is $\left(\frac{k}{s^2+4s+k}\right)$

so $\omega_0 = \sqrt{k}$

and $2\times 0.5\times\sqrt{k} = 4$ so k = 16

**Sol. 15. (b)**

$$G(s) = \frac{40}{s(s+2)(s^2+2s+30)}$$

Since type of system is 1 so steady state error for $\frac{5t^2}{2}$ will be $\infty$

**Sol. 16. (c)**

(a) Adding a zero lead to decrease in the angle of asymptote so push root locus to left.

(b) Adding a pole lead to increase in the angle of asymptote so push root locus to right.

(c) Complementary root locus refer to root loci with negative k.

(d) Adding of pole in forward path transfer function increase maximum overshoot and adding a zero reduces maximum overshoot.

**Sol. 17. (b)**

For critically damped system the system should have poles which are purely real, equal and negative.

**Sol. 18. (c)**

$$G(s) = \frac{57.3k}{s(s+10)}$$

Input is 10 rpm and steady state error is 1°

steady state error is given by $\lim_{s\to 0} \frac{X(s)}{1+G(s)H(s)}$

so $\lim_{s\to 0} \frac{\frac{10\times 60}{s}}{\frac{(57.3)k}{s(s+10)}} = 1°$

so $k = \frac{10\times 60}{57.3} = 10.47$

**Sol. 19. (a)**

**Sol. 20. (c)**

$$T.F = \frac{1}{2S+1}$$

$$M = \frac{1}{\sqrt{4\omega^2+1}}$$

M at $\omega = 0$ is 1 and at $\omega = \infty$ is 0.

**Sol. 21. (a)**

$$TF = \frac{100}{s^2 + 10s + 100}$$

$2\xi\omega_n = \xi\omega_n = 5$

$$t_s(2\%) = \frac{4}{\xi\omega_n} = \frac{4}{5} = 0.8s$$

**Sol. 22. (c)**

$SR = 1 + 0.2\, e^{-60t} - 1.2\, e^{-10t}$

From SR the poles are at – 10, –60

Hence, q(s) = (S + 10) (S + 60) = 0

$$\therefore \; W_n = \sqrt{600} \approx 24.5$$

$2\xi\omega_n = 70$

$\therefore \xi = 1.43$

**Sol. 23. (c)**

$$GH(s) = \frac{25}{S(S+6)}$$

$q(s) = 1 + GH(s) = S^2 + 6S + 25 = 0$

$\omega_n = 5;\ \xi = 0.6\ \therefore\ \omega_d = 4$

$$\therefore \; t_p = \frac{\pi}{\omega d} = 0.79s$$

**Sol. 24. (a)**

$$TF = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

$$OLTH\ GH(s) = \frac{\omega_n^2}{S(S+2\xi\omega_n)}$$

Velocity error constant = Kv = $\lim_{s\to 0} SGH(s)$

$$\therefore \; Kv = \lim_{s\to 0} S\frac{\omega_n^2}{S(S+2\xi\omega_n)} = \frac{\omega_n}{2\xi}$$

**Sol. 25. (b)**

$$GH(s) = \frac{K}{1+ST};\ G_C(s) = k_p$$

$$CLTF = \frac{KK_p}{1+ST+KK_p} = \frac{\frac{KK_p}{1+KK_p}}{\frac{T}{KK_p}S+1}$$

$$\Rightarrow \text{Time constant} = \frac{T}{1+KK_p}$$

$$\Rightarrow e_{ss} = \frac{1}{KK_p}$$

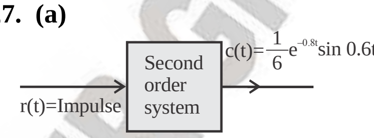
$\therefore$ If $K_p$ is increased then both time constant and $e_{ss}$ decreases.

**Sol. 26. (c)**

$\xi = 1$ means critically damped

Hence, roots are real equal negative.

**Sol. 27. (a)**




r(t) = Im pulse

here $\omega_d = 0.6,\ \xi\omega_n = 0.8$

$\omega_n\sqrt{1-\xi^2} = 0.6;$ ...(i)

$\xi\omega_n = 0.8$ …(ii)

$$\frac{I}{II} = \frac{\omega\sqrt{1-\xi^2}}{\xi\omega_n} = \frac{0.6}{0.8}$$

$$0.8\sqrt{1-\xi^2} = 0.6\xi$$

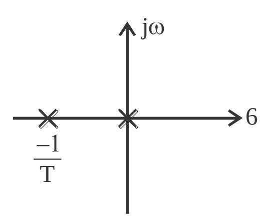
$\therefore \xi = 0.8$

$$\&\ \omega_n = \frac{0.8}{\xi} = \frac{0.8}{0.8} = 1$$

$\omega_n = 1$r/s

**Sol. 28. (c)**

$$G(s) = \frac{k}{s^2(1+sT)}$$




Two poles at origin ⇒ type –2 system

Total three characteristic equation roots $\Rightarrow$ order –3 = system

**Sol. 29. (d)**

$$\text{CLTF} = \frac{100}{s^2+8s+100}$$

$$\text{OLTF} = \frac{100}{(s^2+8s)}$$

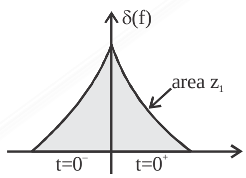
**Sol. 30. (a)**

**Sol. 31. (b)**

Parabolic function = c(t)

Ramp function $r(t) = \frac{dc(t)}{dt}$

**Sol. 32. (d)**




Unit impulse function

$\delta(t) = 1,\ t = 0$

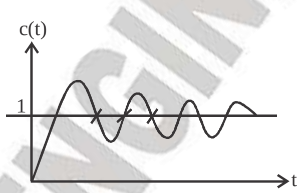
$0,\ t \neq 0$

$$G(t) = \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

**Sol. 33. (d)**

**Sol. 34. (b)**

**Sol. 35. (d)**




For an underdamped system $\xi < 1$; oscillation are damped by frequency.

**Sol. 36. (b)**

Step response of first order system

$c(t) = 1 - e^{-t/\tau}$

$0.9 = 1 - e^{-30/\tau}$

$\Rightarrow e^{-30/\tau} = 0.1;$

$$\Rightarrow \tau = \frac{-30}{\log_e(0,1)} = \frac{30}{2.3}$$

$\tau$ = 13 sec

**Sol. 37. (a)**

| Type of system | Unit step input | Unit ramp input | Unit parabolic input |
|---|---|---|---|
| 0 | $\frac{1}{1+K_p}$ | ∞ | ∞ |
| 1 | 0 | $\frac{1}{1+K_v}$ | ∞ |
| 2 | 0 | 0 | $\frac{1}{1+K_a}$ |

Statement 1 and 2 are correct

**Sol. 38. (c)**

Characteristic equation:

$s^2 + 4s + 16 = 0$

On comparing with general equation

$s^2 + 2\xi\omega_n s + \omega^2_n = 0$

we get, $\omega_n$ = 4 rad/sec,

$$\xi = \frac{1}{2}$$

**Sol. 39. (d)**

$c(t) = t.e^{-t},\ t \geq 0$

$r(t) = u(t) = 1,\ t \geq 0$

$R(s) = 1/s$

$$\frac{C(s)}{R(s)} = \frac{s}{(s+1)^2}$$

**Sol. 40. (c)**

Impulse response is

$c(t) = -4e^{-t} + 6e^{-2t}$

step response is

$$= \int c(t)\,dt$$

$$=\frac{4e^{-t}}{(-1)}+6\frac{e^{-2t}}{(-2)}+k$$

at t = 0, response = 0

$$0=\frac{-4(t)}{(-1)}+\frac{6(1)}{-2}+k$$

0 = 4 – 3 + k

k = – 1

response = $4e^{-t}-3e^{-2t}-1$

$$\int_0^t(-4e^{-t}+6e^{-2t})\,dt=\left(\frac{-4}{-1}\right)e^{-t}\Big|_0^t+\left(\frac{6}{-2}\right)e^{-2t}\Big|_0^t$$

$=4(e^{-t}-1)-3(e^{-2t}-1)$

$=4e^{-t}-3e^{-2t}-1$

**Sol. 41. (c)**

C.E. is $JS^2+BS+K=0$

$$\omega_n=\sqrt{K}$$

$$2\xi\omega_n=B\Rightarrow\xi\omega_n=\frac{B}{2}$$

$$\xi=\frac{B}{2\sqrt{K}}$$

Setting time, $\alpha\frac{1}{\xi\omega_n}\alpha\frac{1}{B}$

e.g. independent of gain

**Sol. 42. (a)**

**Sol. 43. (d)**

$$C(s)=\frac{20/s^2}{1+\frac{20}{s^2}\times(s+5)}\times\frac{1}{5}$$

$\lim_{s\to0}s\,C(s)=$ Final value theorem

$$=\lim_{s\to0}s.\frac{20}{s^2}\times\frac{s^2}{s^2+20(s+5)}\times\frac{1}{5}$$

$$=\lim_{s\to0}\frac{20}{100}=0.2$$

**Sol. 44. (a)**

$$e_{ss}=\lim_{s\to0}\frac{R(s)}{1+G(s)H(s)}$$

$$=\lim_{s\to0}\frac{1/s}{1+\frac{5(s+1)}{s^2+(s+5)(s+12)}}\quad=\frac{1}{s+\infty}=0$$

**Sol. 45. (c)**

$$k_p=\lim_{s\to0}\frac{50}{(1+0.1s)(1+2s)}=50$$

$$k_v=\lim_{s\to0}s.\frac{50}{(1+0.1s)(1+2s)}=0$$

**Sol. 46. (c)**

$$\frac{C(s)}{R(s)}=\frac{2}{s^2+3s+2}$$

For unit step input,

$$C(s)=\frac{1}{s}.\frac{2}{s^2+3s+2}$$

$$=\frac{1}{s}.\frac{2}{(s+2)(s+1)}$$

$$=\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+1}$$

$\Rightarrow A(s+2)(s+1)+B(s+1)+C(s+2)s=2$

$\Rightarrow A(s^2+3s+2)+B(s^2+5)=C(s^2+2s)=2$

A + B + C = 0,

3A + B + 2C = 0,

2A = 2

B + C = -1 …(i)

B + 2C = -3 …(ii)

A = 1

From (i) and (ii),

C = +2

C = -2

B = 1

$$C(s)=\frac{1}{s}+\frac{1}{s+2}+\frac{-2}{s+1}$$

$=1+e^{-2t}-2e^{-t}$

**Sol. 47. (c)**

Characteristics equations

$\Rightarrow 1+G(s)=0$

$$\Rightarrow1+\frac{9}{s(s+3)}=0$$

$$\Rightarrow s(s+3)+9=0$$

$\omega_n = 3$

$2\xi\omega_n = 3$

$$\Rightarrow \xi = \frac{3}{2\times 3} = \frac{1}{2} = 0.5$$

**Sol. 48. (c)**

$s = -\ 2 \ \pm j\ 2$

$2 \downarrow \xi\omega_n \downarrow \alpha$; $2 \downarrow \omega_d$

$$\omega_n^2 = \alpha^2 + \omega_d^2 = (2)^2 + (2)^2 = 8$$

$$\omega_n = 2\sqrt{2}\ \text{rad/sec.}$$

$\xi\omega_n = 2$

$$\xi = \frac{2}{2\sqrt{2}} = 0.707$$

**Sol. 49. (a)**

$\omega_d = \omega_n\sqrt{1-\varepsilon^2}$ ,When $\varepsilon = 0 \Rightarrow \omega_d = \omega_n$

**Sol. 50. (b)**

A u.f.b system $G(s) = \dfrac{k(s+12)}{(s+14)(s+18)}$

Since the given system is of type-0, it will have finite steady error only when step input have infinite steady error only when step input is applied. So for the above w.r.t. unit step

$$e_{ss} = \frac{1}{1+K_p} \text{ where } K_p = \underset{s\to 0}{\text{It}}\ G(s) = \frac{k}{21},$$

$$e \to = \frac{1}{1+\frac{k}{21}} = \frac{21}{21+k}$$

If we choose k = 1, $e_{ss} = \dfrac{21}{22}$

It is asked that for what value of k, we will have 10% error in steady state,

**Sol. 51. (b)**

For a u.f.b system $G(s) = \dfrac{K}{s(s+10)}$

The characteristics equation of above $s^2 + 10s + k = 0$

Comparing with characteristics equation

$(s^2 + 2\xi\omega_{ns} + \omega n^2 = 0)$

$2\xi\omega_{ns} = 10$, $\omega n^2 = k$

$$2\xi\omega_n = 10 \Rightarrow \omega_n = \frac{10}{2\xi} = 10$$

$$\Rightarrow \sqrt{k} = 10 \Rightarrow k = 100$$

**Sol. 52. (c)**

Assuming the transfer function is unity feedback system, the characteristics equation is

$s^3 + 6s^2 + 5s + k = 0$

Shortcut method comparing with std third order equation

$As^3 + bs^3 + cs + D = 0$

If bc = ad then sysem is marginally stable.

So, k = 6 × 5 = 30

**Sol. 53. (b)**

**Sol. 54. (c)**

**Sol. 55. (d)**

**Sol. 56. (d)**

**Sol. 57. (c)**

**Sol. 58. (c)**

∵ For unit step input, response = $(1 - e^{-3t})$

∴ Impulse response,

$$H(s) = \frac{\frac{1}{s} - \frac{1}{s+3}}{\frac{1}{s}} = \frac{3}{s+3}$$

∴ For $3u(t) + \delta(t)$.

$$\text{Response} = \left(\frac{3}{s}+1\right)\times\frac{3}{s+3} = \frac{3}{s}$$

i.e. In time domain, 3u(t)

**Sol. 59. (a)**

Unit impulse response = $-4e^{-t} + 6e^{-2t}$

Unit step response

$= \int$ (unit impulse response)

$= \int (-4e^{-t} + 6e^{-2t})\,dt$

$= 4e^{-t} - 3e^{-2t} + C$

⇒ Option (a) is correct.

**Sol. 60. (a)**

$$J\frac{d^2Q}{dt^2}+D\frac{dQ}{dt}+S\theta=T$$

$$\Rightarrow \theta(s)=\frac{T}{Js^2+Ds+S}=\frac{T/J}{s^2+\frac{D}{J}s+\frac{S}{J}}$$

On comparing the denominator with

$$s^2+2\xi\omega_n s+\omega_n^2=0$$

We have, $\omega_n=\sqrt{\frac{S}{J}}$

**Sol. 61. (b)**

$$H(t)=\frac{1}{6}e^{-0.8t}\sin(0.6t)$$

On comparing with standard notation

i.e. $K\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\phi)$

We have,

$\xi\omega_n = 0.8$ …(i)

$\omega_d=\omega_n\sqrt{1-\xi^2}=0.6\,\text{rad/sec}$ …(ii)

On solving equation (i) and equation (ii)

$\omega_n$ = 1 rad/sec and $\xi$ = 0.8

**Sol. 62. (b)**

As gain K increases, damping ratio $\xi$ decreases.

**Sol. 63. (c)**

$$CLTF=\frac{1}{1+sT}=\frac{OLTF}{1+OLTF}$$

(For unity feedback system)

$$\Rightarrow OLTF=\frac{1}{sT}$$

Input, r(t) = t u(t)

$$\Rightarrow R(s)=\frac{1}{s^2}$$

$$\therefore e_{ss}=\lim_{s\to 0}\frac{sR(s)}{1+OLTF}=\lim_{s\to 0}\frac{s.\frac{1}{s^2}}{1+\frac{1}{sT}}=T$$

**Sol. 64. (c)**

In transient response overshoot and damping ratio are measures of speed of response i.e. how fast the response is achieved.

**Sol. 65. (d)**

$f(t) = (t^2 - 2t)\,u(t-1)$

$=[(t-1)^2 - 1]\,u(t-1)$

$=(t-1)^2\,u(t-1) - u(t-1)$

$$\therefore F(s)=\frac{2e^{-s}}{s^3}-\frac{e^{-s}}{s}=\left(\frac{2-s^2}{s^3}\right)e^{-5}$$

**Sol. 66. (d)**

$$2\frac{d^2y}{dt^2}+4\frac{dy}{dt}+8y=8x$$

Taking Laplace transform

$2s^2\,Y(s) + 4s\,Y(s) + 8\,Y(s) = 8\,X(s)$

$\Rightarrow$ Transfer function

$$=\frac{Y(s)}{X(s)}=\frac{8}{2s^2+4s+8}$$

$$=\frac{4}{s^2+2s+4}$$

On comparing with standard second order transfer function i.e.

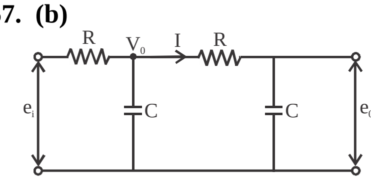
$$T.F.=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

We get, $\omega_n$ = 2 r/s

and damping ratio,

$\xi$ = 0.5

**Sol. 67. (b)**




$$E_0(s)=\frac{1}{sC}I(s)\quad .....(i)$$

$$I(s)=\frac{E_i(s)}{\frac{\left(R+\frac{1}{sC}\right)\times\frac{1}{sC}}{r+\frac{1}{sC}\times\frac{1}{sC}}+R}\times\frac{\frac{1}{sC}}{R+\frac{1}{sC}+\frac{1}{sC}}$$

(using current division rule)

$$= \frac{E_i(s)}{R + \frac{\frac{1}{sC}}{sCR+2} + R} \times \frac{1}{sCR+2}$$

$$I(s) = \frac{E_i(s)}{R + \frac{1}{sC} + sCR^2 + 2R} \times \frac{sCR+2}{sCR+2}$$

Using equation (i),

$$\frac{E_0(s)}{\frac{1}{sC}} = \frac{E_i(s)}{sCR^2 + \frac{1}{sC} + 3R}$$

$$\therefore \text{T.F.} = \frac{E_o(s)}{E_i(s)}$$

$$\frac{\frac{1}{sC}}{sCR^2 + 3R + \frac{1}{sC}}$$

$$= \frac{1}{s^2C^2R^2 + 3sCR + 1}$$

$$= \frac{1}{s^2T^2 + 3sT + 1} (\because T = RC)$$

**Sol. 68. (d)**

Non – minimum phase functions have their zeros in the right half of the s – plane.

**Sol. 69. (c)**

$$\text{OLTF} = G(s) = \frac{1}{(s+2)^2}$$

For unity feedback system, H(s) = 1

$$\therefore \text{CLTF} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{1}{(s+2)^2}}{1+\frac{1}{(s+2)^2}}$$

$$= \frac{1}{s^2+4s+5}$$

$\therefore$ Close loop poles will be the roots of $s^2 + 4s + 5 = 0$

i.e, s = – 2 + j and – 2 – j

**Sol. 70. (c)**

The highest power of the characteristic equation 1 + G(s) H(s) = 0, determines the order of the system.

$\therefore s^2(1 + sT) + K = 0$

$\Rightarrow$ order of the system is 3.

The type of the system is obtained from open loop transfer function G(s) H(s).

$$G(s)H(s) = \frac{K}{s^2(1+sT)}$$

$\Rightarrow$ Type 2 system.

**Sol. 71. (c)**

$$G(s) = \frac{10}{s+1}, H(s) = 1$$

$$G(s)H(s) = \frac{10}{s+1}$$

$$E(s) = R(s).\frac{1}{1+G(s)H(s)}$$

For unit step input signal

$$E(s) = \frac{1}{s}.\frac{1}{1+\frac{10}{(s+1)}} = \frac{s+1}{s(s+11)}$$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{s+1}{s+11} = \frac{1}{11}$$

**Sol. 72. (b)**

Transfer function = $\mathcal{L}$ [impulse response]

$$=\mathcal{L}(e^{-t}) = \frac{1}{s+1}$$

**Sol. 73. (c)**

Overall transfer function

$$M(s) = \frac{G(s)}{1+G(s)} = \frac{K}{K+s(s+8)}$$

Therefore characteristic equation

$S^2 + 8s + K = 0$

$$\Rightarrow \omega_n = \sqrt{K}, 2\xi\omega_n = 8$$

For critically damped system, $\xi = 1$

$$\therefore \omega_n = 4 = \sqrt{K}$$

$\Rightarrow$ K = 16

**Sol. 74. (c)**

$\xi = 0 \Rightarrow$ undamped
$\xi = 1 \Rightarrow$ critically damped
$\xi < 1 \Rightarrow$ underdamped
$\xi > 0 \Rightarrow$ overdamped

**Sol. 75. (d)**
$s^2 + 25 = 0$
$\Rightarrow \xi = 0$ and $\omega_n = 5$
$\Rightarrow$ undamped
$s^2 + 18s + 64 = 0$
$\Rightarrow \omega_n = 8$ and $\xi = \frac{9}{8} > 1$
$\Rightarrow$ Overdamped
$s^2 + 12s + 36 = 0$
$\Rightarrow \omega_n = 6$ and $\xi = 1$
$\Rightarrow$ Critically damped
$s^2 + 8s + 25 = 0$
$\Rightarrow \omega_n = 5$ and $\xi = \frac{4}{5} < 1$
Undamped

**Sol. 76. (a)**
$$G(s) = \frac{1}{s^2 + 0.1s + 1}$$
Characteristic equation
$$s^2 + 0.1s + 1 = 0$$
$$\Rightarrow \; 2\xi\omega_n = 0.1, \omega_n^2 = 1$$
$\Rightarrow \xi = 0.05$
$$\text{Setting time, } t_s = 3.\frac{1}{\omega_n \xi} = \frac{3}{0.05} = 60\,\text{sec}$$

**Sol. 77. (c)**
$$\% \text{ Overshoot} = e^{\frac{n\pi\xi}{\sqrt{1-\xi^2}}} \times 100$$
$$100 = e^{\frac{n\pi\xi}{\sqrt{1-\xi^2}}} \times 100$$
$\Rightarrow \xi = 0$

**Sol. 78. (b)**

**Sol. 79. (a)**
The characteristic equation is given as
$s^3 + 2s^2 + 6\,s + 12 = 0$

$$\begin{array}{c|cc} s^3 & 1 & 6 \\ s^2 & 2 & 12 \\ s^1 & 0 & 0 \\ s^0 & & \end{array}$$

When one full row becomes zero, then the system will be marginally stable (oscillatory) or unstable. To calculate this stability, we need to check the roots of auxiliary equation i.e. equation of $s^2$ terms.
so $2s^2 + 12 = 0$
$\Rightarrow s = \pm\, j\sqrt{6}$ rad/sec
$\omega = \pm\sqrt{6}$ rad/sec
As we can see that system is undamped oscillatory, so $\delta = 0$.

**Sol. 80. (b)**

**Sol. 81. (d)**
Integral controller reduced both the steady state error and the relative stability (because it adds one pole to the system).

**Sol. 82. (c)**
The roots of the second order control system is given as
$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$
If $\xi = 1$, then
$s_1, s_2 = -\omega_n$
Thus the poles are equal, negative and real.

**Sol. 83. (b)**
$$G(s)H(s) = \frac{4}{s^2(s+3)}$$
Type of system is found from open loop poles at origin. Hence type-2 system.

**Sol. 84. (a)**
Maximum overshoot, rise time and overall gain of the system determines the transient characteristics.

**Sol. 85. (b)**
$$G(s)H(s) = \frac{25}{s^2 + 5s + 25}$$

Comparing the above transfer function with the standard second order transfer function:

$$\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

So $\omega_n = 5$

$2\xi\omega_n = 5$

$\Rightarrow \xi = 0.5$

Settling time for 2% tolerance band $= \frac{4}{\xi\omega_n}$

$$= \frac{4}{2.5} = 1.6\,\text{sec.}$$

**Sol. 86. (b)**

The type of system is determined from the number of poles at origin for open loop transfer function.

**Sol. 87. (c)**

$$G(s) = \frac{K}{s(s+6)}$$

Characteristic's equation

$1 + G(s)\,H(s) = 0$

$$\Rightarrow 1 + \frac{K}{s(s+6)} = 0$$

$\Rightarrow s(s + 6) + K = 0$

$\Rightarrow s^2 + 6s + K = 0$

Comparing above equation with standard equation i.e.

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

we have, $\omega_n = \sqrt{K}$ and $2\xi\omega_n = 6$

It is given that $\xi = 0.75$; so

$2 \times 0.75 \times \sqrt{K} = 6$

$$\therefore \sqrt{K} = \frac{6}{1.5} = 4$$

K = 16

**Sol. 88. (b)**

Given, $\xi = 0.707 = \frac{1}{\sqrt{2}}$

Setting time $= \frac{3}{\xi\omega_n} = 0.60\,\text{sec}$

$$\xi\omega_n = \frac{3}{0.6} = \frac{30}{6} = 5$$

Poles are given as

$$s = -\xi\omega_n + \omega_n\sqrt{\xi^2 - 1}$$

$$= -5 \pm 5\sqrt{2}\sqrt{\frac{1}{2} - 1}$$

$$= -5 \pm j5\sqrt{2}.\frac{1}{2} = -5 \pm j5$$

**Sol. 89. (b)**

$$40\frac{dx}{dt} + 2x = f(t)$$

$\Rightarrow X(s)\,(40s + 2) = F(s)$

$$\therefore \frac{X(s)}{F(s)} = \frac{1}{40s + 2}$$

Pole will be at $s = -\frac{1}{20}$

Time constant is reciprocal of location of pole for a first order system.

**Sol. 90. (b)**

$$G(s)H(s) = \frac{2}{s(s^2 + 2s + 2)}$$

Since G(s) H(s) has one pole at origin, so given system is type-1 system.

**Sol. 91. (b)**

$$\frac{C(s)}{R(s)} = H(s) = \frac{1}{s(s+1)}$$

$$\Rightarrow H(s) = \frac{1}{s} - \frac{1}{s+1}$$

Taking inverse Laplace transform.

$h(t) = (1 - e^{-t})\,u(t)$

**Sol. 92. (b)**

$$\frac{C(s)}{R(s)} = \frac{10\left(\frac{1}{s+1}\right)}{1 + \frac{1}{s}.\frac{10}{(s+1)}}$$

$$\Rightarrow C(s) = \frac{10s}{s(s+1) + 10}R(s)$$

Given r(t) = u(t)

So $R(s) = \frac{1}{s}$

$$\therefore C(s) = \frac{10s}{s(s+1)+10}.\frac{1}{s}$$

$$\Rightarrow C(s) = \frac{10}{s(s+1)+10}$$

Steady state value of response

$$= \lim_{s\to 0} sC(s)$$

$$\lim_{s\to 0} s.\frac{10}{s(s+1)+10} = 0$$

**Sol. 93. (b)**

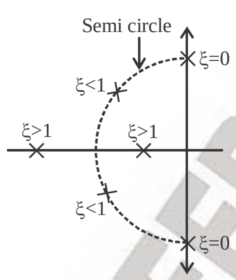
Roots present in RHS of the s – plane results in unstable system while roots in LHS of the s – plane results in stable system.

Hence, option (b) is correct.

**Sol. 94. (d)**

For $\xi < 1$ the system has underdamped response.

**Sol. 95. (d)**




**Sol. 96. (d)**

Roots on imaginary axis represents marginally stable system, roots in RHS of s-plane represents unstable system. Hence, option (d) is correct.

**Sol. 97. (a)**

**Sol. 98. (b)**

$$G(s) = \frac{k}{s(s+4)}$$

Characteristic equation 1+ g(s) H(s) = 0\

$$1+\frac{K}{s(s+4)} = 0$$

$s^2 + 4s + K = 0$

$\omega_n = \sqrt{K}$

$2\xi\omega_n = 4$

$2 \times 0.5\ \omega_n = 4$ given $\xi = 0.5$

$\omega_n = 4$ rad/sec

$L = 4^2 = 16$

**Sol. 99. (a)**

$$G(s) = \frac{k(s+1)}{s(s+2)(s+3)}$$

$$e_{ss} = \lim_{s\to 0}\frac{sR(s)}{G(s)H(s)}$$

$$= \lim_{s\to 0}\frac{s\times\frac{1}{s}}{1+\frac{K(s+1)}{s(s+2)(s+3)}}$$

$e_{ss} = 0$

**Sol. 100.(a)**

$$T.F. = \frac{(s+1)}{(s+1-j)\ (s+1+j)}$$

$$\frac{C(s)}{R(s)} = \frac{(s+1)}{(s+1)^2+1}$$

$$R(s) = \frac{1}{s}$$

$$\underset{s\to 0}{C(s)} = s\times R(s)(T.F.)$$

$$= \frac{s+1}{(s+1)^2+1} = 0.5\angle 0^\circ$$

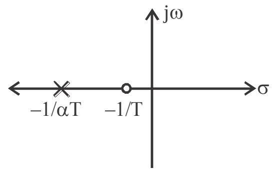
**Sol. 101.(b)**

Negative feedback increases stability but not positive feedback.

**Sol. 102.(b)**

Integral controller improves steady state performance while derivative controller improves transient state response.

**Sol. 103.(c)**




Transfer function of lead compensator

$$= \frac{\alpha(1+Ts)}{(1+\alpha Ts)}$$

Where, $\alpha < 1$

**Sol. 104.(b)**

$$G(s) = \frac{10}{(3s+1)} = \frac{10}{(1+3s)}$$

T = 3

$\Rightarrow$ corner frequency = $\frac{1}{3}$ = 0.33 rad/sec

**Sol. 105.(c)**

$$G(s) = \frac{1}{s+1}$$

$$g(j\omega) = \frac{1}{1+j\omega}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

and $\angle G(j\omega) = -\tan^{-1}\omega$

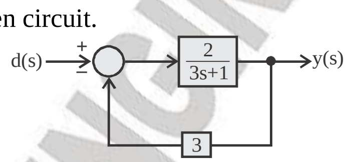
$$|G(j\omega)_{\omega=1} = \frac{1}{\sqrt{2}}$$

and $\angle G(j\omega)|_{\omega=1} = -\tan^{-1} 1$

$= -45°$

**Sol. 106.(a)**

To calculate $\frac{y(s)}{d(s)}$, set R(s) = 0 and now redraw the given circuit.




$$\therefore \frac{y(s)}{d(s) = \cfrac{\frac{2}{3s+1}}{1+3\times\frac{2}{3s+1}}} = \frac{2}{3s+7}$$

**Sol. 107.(d)**

$$G(s) = \frac{4}{s^2+0.4s}; H(s) = 1$$

Characteristic equation

1 + G(s) H(s) = 0

$$\Rightarrow 1+\frac{4}{s^2+0.4s}.1 = 0$$

$\Rightarrow s^2 + 0.4s + 4 = 0$ …(i)

Comparing equation (i) with standard equation second order system i.e.

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

we have

$2\xi\omega_n = 0.4 \Rightarrow \xi\omega_n = 0.2$

setting time within 2% tolerance band

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.2}$$

$\Rightarrow t_s = 20$ sec

**Sol. 108.(d)**

$$F(s) = \frac{5}{s(s^2+3s+2)}$$

This initial value of f(t) is

$$\underset{t\to 0}{f(t)} = \underset{s\to\infty}{sF(s)}$$

$$\underset{s\to\infty}{s}.\frac{5}{s(s^2+3s+2)}$$

$$= \frac{5/s^2}{s\to\infty\left(1+\frac{3}{s}+\frac{2}{s^2}\right)}$$

$$\underset{t\to 0}{f(t)} = 0$$

**Sol. 109.(c)**

Given

Input x (t) = u(t)

and output y (t) = $t.e^{-t}$, t > 0

Taking Laplace transform

$$X(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{(s+1)^2}$$

Therefore transfer function

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{1}{(s+1)^2}}{\frac{1}{s}}$$

$$h(s) = \frac{s}{(s+1)^2}$$

**Sol. 110.(a)**

$$M(s) = \frac{G(s)}{1+G(s)H(s)}$$

Sensitivity of M to the variation in G is

$$\frac{dM}{dG} \times \frac{G}{M}$$

$$\frac{dM}{dG} = \frac{1+G(s)H(s) - G(s)H(s)}{\{1+G(s)H(s)\}^2}$$

$$\frac{dM}{dG} \times \frac{G}{M} = \frac{1}{\{1+G(s)H(s)\}^2} \times \frac{G(s)}{G(s)1+G(s)}$$

$$= \frac{1}{1+G(s)H(s)}$$

**Sol. 111.(d)**

Output voltage = $V_{in} \frac{A}{1+AB}$

$$= 5 \times \frac{1}{1+(1\times 1)} = 2.5\,V$$

**Sol. 112.(c)**
High gain feedback can lead to instability problem.

**Sol. 113.(d)**
In a good control system, output is sensitive to input variation but insensitive to parameter variations.

**Sol. 114.(a)**

Steady state value $= \lim_{s\to 0} s \frac{1}{(s+1)} = 0$

**Sol. 115.(d)**
This is the Laplace transform of sin t.
So, f(t) = sin t
Steady-state value of f(t) is undetermined because poles of f(s) are not in LHS of s-plane. Therefore, steady-state value will vary between –1 and +1.

**Sol. 116.(b)**

**Sol. 117.(a)**

$$C(s) = \frac{12.5 \times 8}{(s+6)^2 + 8^2}$$

$$C(s) = \frac{100}{s^2 + 12s + 100}$$

$$\therefore \omega_n^2 = 100$$

$\omega_n$ = 10 rad/sec
and $2\xi\omega_n$ = 12
∴ $\xi$ = 0.6

**Sol. 118. (b)**
1 + G(s) = 0

$$1 + \frac{20}{s(s+5)} = 0$$

$S^2 + 5s + 20 = 0$

$$\therefore \quad t_s = \frac{4}{\xi\omega_n}$$

$2\xi\omega_n = 5$
$\xi\omega_n = 2.5$

$$\therefore t_s = \frac{4}{2.5} = 1.60\,\text{sec}$$

**Sol. 119.(d)**

**Sol. 120.(c)**

**Sol. 121.(c)**
Addition of pole increases the rise time but decrease the band – width.

$$\therefore \quad t_r \alpha \frac{1}{B.W.}$$

**Sol. 122.(a)**

Response to unit impulse = $\frac{4}{s+2}$

Response to step input = $\frac{1}{s}\times\frac{4}{s+2}$

$=\frac{4}{2}\left(\frac{1}{s}-\frac{1}{s+2}\right)$

Taking inverse laplace transformation,

Response = $2(1 - e^{-2t})\, u(t)$

**Sol. 123.(d)**

Resonant frequency ($\omega_r$) $=\omega_n\sqrt{1-2\xi^2}$

$=3\sqrt{1-2\times\frac{1}{4}}=2.4\,\text{rad/sec}$

Resonant peak ($m_r$) $=\frac{1}{2\xi\sqrt{1-\xi^2}}$

$=\frac{1}{2\times\frac{1}{2}\sqrt{1-\frac{1}{4}}}=1.16$

**Sol. 124.(c)**
2 poles at origin

**Sol. 125.(c)**
Shifting theorem

**Sol. 126.(d)**

**Sol. 127.(a)**
Analysis relives on find value theorem.

**Sol. 128. (c)**
Positive correctly torque is related to $\xi$

**Sol. 129.(b)**

$r(t)=\frac{d^2c(t)}{dt^2}+\frac{3dc(t)}{dt}+2c(t)$

Taking laplace transform,
$R(s) = s^2c(s) + 3sC(s) + 2C(s)$

$\frac{C(s)}{R(s)}=\frac{1}{s^2+3s+2}$

**Sol. 130.(d)**
$f(t) = \sin t$

**Sol. 131.(a)**

**Sol. 132.(a)**
Pole at origin is one

**Sol. 133.(a)**
Velocity error coefficient $K_v = \lim_{s\to 0}$ and G(s) H(s) = ∞ for type 2. Hence error = $1/K_v$.

**Sol. 134.(c)**

$\frac{K}{s(s+4)}$ on comparing with $\frac{\omega_n^2}{s(s+2\xi\omega_n)}$

$2\xi\omega_n = 4$; $\omega_n=\sqrt{k}$, $\xi = 0.5$ as given

$2\times 0.5\times\sqrt{k}=4;\ k=16$

**Sol. 135.(a)**

$c(t)=12.5\frac{8}{(s+6)^2+8^2}$

$=\frac{100}{s^2+36+12s+64}$

$\text{T.F.}=\frac{100}{s^2+12s+100}$

$\therefore\ \omega_n = 10$ and $2\xi\omega_n = 12$

$\xi=\frac{2}{10\times 2}=0.6$

**Sol. 136.(b)**

**Sol. 137.(c)**

**Sol. 138.(a)**
The response of an amplifier with three (or more) poles is determined approximately by the two lowest poles, $p_1$ and $p_2$ provided that $|p_3/p_2| \geq 4$.

**Sol. 139.(a)**
Characteristic equation is
$s(s + 1)(s + 5) + K = 0$
i.e. $s^3 + 6s^2 + 5s + K = 0$

Routh array

| | | |
|---|---|---|
| $s^3$ | 1 | 5 |
| $s^2$ | 6 | K |
| $s^1$ | $\frac{30\text{-}K}{6}$ | D |
| $s^0$ | K | |

For marginal stability

$$\frac{30-K}{6} = 0 K = 30$$

For frequency of sustained oscillation

$6s^2 + K = 0$

$\Rightarrow 6s^2 + 30 = 0$

$\Rightarrow s^2 + 5 = 0$

$\Rightarrow (j\omega)^2 + 5 = 0$

$\Rightarrow -\omega^2 + 5 = 0$

$\Rightarrow \omega^2 = 5$

$\Rightarrow \omega = \sqrt{5}\ \text{rad/s}$

**Sol. 140.(c)**

Comparing the transfer function

$$\frac{16}{s^2+4s+16} \text{ with } \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\omega_n^2 = 16 \Rightarrow \omega_n = 4$ rad/s

$2\xi\omega_n = 4 \Rightarrow = \frac{4}{2\omega_n}$

$\Rightarrow \xi = \frac{4}{2\times 4}$

Time for first overshoot

$$tp = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{4\sqrt{1-\frac{1}{4}}} = \frac{\pi}{2\sqrt{3}} s$$

**Sol. 141.(b)**

Impulse response function

$$G(s) = \frac{1}{\left(s+\frac{1}{\sqrt{2}}+\frac{j}{\sqrt{2}}\right)\left(s+\frac{1}{\sqrt{2}}-\frac{j}{\sqrt{2}}\right)}$$

Input, r(t) = u(t)

$\Rightarrow R(s) = \frac{1}{s}$

Steady, state deformation

$$e_{ss} = \lim_{s\to 0}\frac{sR(s)}{1+G(s)}$$

$$= \lim_{s\to 0}\frac{s.1/s}{1+\left(s+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\frac{-1}{\sqrt{2}}\right)}$$

$$= \frac{1}{1+\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{1}{1+1} = 1/2 = 0.5$$

**Sol. 142.(c)**

Impulse response,

$g(t) = e^{-t}, t > 0$

Transfer function,

$$G(s) = L\{g(t)\} = \frac{1}{s+1}$$

**Sol. 143.(d)**

**Sol. 144.(c)**

Given that,

Input r(t) = 10t

$\Rightarrow R(s) = 10/s^2$

$$G(s) = \frac{100}{s^2(s+4)}, H(s) = 1$$

$$\therefore e_{ss} = \lim_{s\to 0}\frac{s.10/s^2}{1+\frac{10}{s^2(s+4)}}$$

$$= \lim_{s\to 0}\frac{10s(s+4)}{s^2(s+4)+10} = 0$$

**Sol. 145.(b)**

Characteristic equation

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

If $\xi = 0$, then $s_2 + \omega_n^2 = 0$

$\Rightarrow s = \pm j\omega_n$

It is clear that the closed-loop poles are purely imaginary.

**Sol. 146.(b)**

**Sol. 147.(a)**

Step response, $g_1(t) = -0.5\,(1 + e^{-2t})$

Its impulse response,

$$\frac{dg_1(t)}{dt} = -0.5 \times (-2)\,e^{-2t} = e^{-2t}$$

Another impulse response, $g_2(t) = e^{-t}$

Transfer function of cascaded combination,

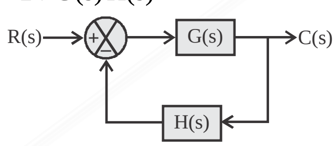
$$= L\left\{\frac{dg_1(t)}{dt}\right\} L\{g_2(t)\}$$

$$= \frac{1}{s+2}\cdot\frac{1}{s+1} = \frac{1}{(s+1)(s+2)}$$

**Sol. 148.(b)**

Steady state error,

$$e_{ss} = \lim_{s\to 0}\frac{sR(s)}{1+G(s)H(s)}$$




(i) By increasing the input r(s), $e_{ss}$ increases.

(ii) By increasing the type of system, $e_{ss}$ increases.

(iii) $e_{ss} \propto \frac{1}{\text{static error constant}}$

Therefore, by decreasing the static error constant ($K_p$, $K_v$ or $K_a$), $e_{ss}$ increases.

**Sol. 149.(c)**

$q(s) = 2s^5 + s^4 + 4s^3 + 2s^2 + 2s + 1 = 0$

$= (2s + 1)s^4 + (2s + 1)\,2s^2 + (2s + 1) = 0$

$= (2s + 1)\,(s^4 + 2s^2 + 1) = 0$

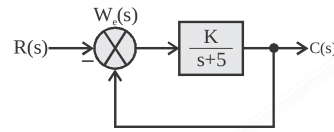
$= (2s + 1)\,(s^2 + 1)^2 = 0$

Therefore, the roots of the characteristic equation is $s = -1/2$, $s = \pm j$, $s = \pm j$. Since the poles of the system are repeated on $j\omega$-axis, therefore, the system is unstable.

**Sol. 150.(c)**

**Sol. 151.(c)**

$$\frac{W_e(s)}{R(s)} = \frac{R(s)-C(s)}{R(s)}$$




$$\text{Error T.F.} = \frac{W_e(s)}{R(s)} = 1 - \frac{C(s)}{R(s)}$$

$$= 1 - \frac{\frac{k}{s+5}}{1+\frac{k}{s+5}} = 1 - \frac{k}{s+5+k} = \frac{s+5}{s+5+k}$$

**Sol. 152.(b)**

$$\frac{C(s)}{R(s)} = \frac{100.\frac{1}{s(1+4s)}}{1+k_0 s.\frac{1}{s(1+4s)}} \Bigg/ 1+\frac{\frac{100}{s(1+4s)}}{1+\frac{k_0 s}{s(1+4s)}}$$

$$= \frac{100}{4s^2 + s(1+k_0) + 100}$$

$$= \frac{25}{s^2 + \frac{s(1+k_0)}{4} + 25}$$

$\Rightarrow \omega_n = 5$ rad/sec

$$2\xi \times \omega_n = \frac{1+k_0}{4} = 2\times 0.5\times 5$$

$1 + k_0 = 20$

$k_0 = 20 - 1 = 19$

**Sol. 153. (d)**

$$t_s = \frac{4}{\xi\omega_n} \text{ for } 2\% = \frac{4}{10\times 0.1} = 4 \text{ second}$$

**Sol. 154.(a)**

$$C(s) = \frac{12.5\times 8}{(s+6)^2+8^2} = \frac{100}{s^2+12s+100}$$

$\omega_n^2 = 100$ ; $\omega_n = 10$ rad/sec

$$2\xi\omega_n = 12;\ \xi = \frac{12}{2\times 10} = 0.6$$

**Sol. 155.(b)**

By taking Laplace

**Sol. 156.(d)**

$$e_{ss} = \underset{s\to 0}{Lt}\frac{sR(s)}{1+G(s)H(s)}$$

**Sol. 157.(a)**

**Sol. 158.(d)**
For unit step input steady state error $\neq 0$
for Type '0' system

$$\frac{1}{1+K_p} = 0.20 \Rightarrow K_p = 4$$

$\therefore$ With unit integrator system becomes Type '1'. For Type '1' system with Ramp input steady state error = $\frac{1}{K_v} = \frac{1}{4} = 0.25$

**Sol. 159.(d)**

Steady state value $= \lim_{s\to 0} s.\frac{1}{s}.\frac{(s+3)}{4s+5} = \frac{3}{5}$

Initial value $= \lim_{s\to \infty} s.\frac{1}{s}.\frac{(s+3)}{(4s+5)} = \frac{1}{4}$

**Sol. 160.(c)**
Apply final value theorem

$$\lim_{s\to 0} sC(s) = \lim_{s\to 0}\frac{2s(s+3)}{(s+2)(s+4)}.\frac{1}{s} = \frac{3}{4}$$

**Sol. 161.(c)**
Routh array is

| | | |
|---|---|---|
| $s^4$ | 1 | 3 |
| $s^3$ | 2 | 2 |
| $s^2$ | 2 | K |
| $s^1$ | 2-K | 0 |
| $s^0$ | K | |

For oscillations,
$2 - K = 0$
$\Rightarrow K = 2$
For oscillations,
$2s^2 + K = 0$
Putting $s = j\omega$ and $K = 2$,
$-2\omega^2 + 2 = 0$
$\Rightarrow \omega^2 = 1$
$\Rightarrow \omega = 1$ rad/s

**Sol. 162.(c)**

$$C(s) = \frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

$$= \frac{(s+1)^2 - s(s+1) - s}{s(s+1)^2} = \frac{1}{s(s+1)^2}$$

$$R(s) = \frac{1}{s}$$

$$G(s) = \frac{C(s)}{R(s)} = \frac{1}{(s+1)^2} = \frac{1}{s^2+2s+1}$$

$\therefore \xi = 1$
So given system is critically stable.

**Sol. 163.(a)**

$$\frac{G(s)}{1+G(s)} = \frac{K}{(s+2)(s+4)(s^2+6s+25)+K}$$

Characteristic equation is
$s^4 + 12s^3 + 69s^2 + 198s + 200 + K = 0$
Routh array is

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 69 | K+200 |
| $s^3$ | 2 | 33 | |
| $s^2$ | 52.5 | K+200 | |
| $s^1$ | $\frac{1332.5-2K}{52.5}$ | 0 | |

For oscillations,
$1332.5 - 2K = 0$
$\Rightarrow K = 666.25$

**Sol. 164.(d)**

$$C(s) = \frac{1}{s} - \frac{1}{s+5} - \frac{5}{(s+5)^2}$$

$$= \frac{(s+5)^2 - (s+5)s - 5s}{s(s+5)^2} = \frac{25}{s(s+5)^2}$$

$$C(s) = \frac{25}{s(s^2+10s+25)}$$

$$R(s) = \frac{1}{s}$$

$$G(s) = \frac{C(s)}{R(s)} = \frac{25}{s^2+10s+25}$$

$$\omega_n = \sqrt{25}$$

$\Rightarrow \omega_n = 5$ rad/s

$$\xi = \frac{10}{2\times5} = 1$$

Impulse response = $\frac{d}{dt}(1 - e^{-5t} - 5te^{-5t})$

$= 5e^{-5t} - 5e^{-5t} + 25\ te^{-5t}$

$= 25\ te^{-5t}$

**Sol. 165.(b)**

Main disadvantage of proportional controller is, it produces a permanent error called offset error.

**Sol. 166.(b)**

$$\frac{G(s)}{1+G(s)} = \frac{K}{s^2+s+K}$$

$$\xi = \frac{1}{2\sqrt{K}}$$

$$\Rightarrow \frac{\pi^2}{4K} = 0.48\left(1-\frac{1}{4K}\right)$$

$$\Rightarrow 4K - 1 = \frac{\pi^2}{0.48}$$

$\Rightarrow 4K = 21.56$

$\Rightarrow K = 5.39$

**Sol. 167.(a)**

**Sol. 168.(a)**

**Sol. 169.(a)**

For type 1, ramp input

$$e_{ss} = \frac{1}{K_v}$$

Where $K_v = \lim_{s\to0} sG(s)$

$$\lim_{s\to0} s.\frac{1}{s(s+1)} = 1$$

So, $e_{ss} = \frac{1}{K_v} = 1$

**Sol. 170.(b)**

$$\frac{G(s)}{1+G(s)} = \frac{K}{s^2+4s+K}$$

$$\xi = \frac{4}{2\sqrt{K}} = 0.5$$

$$\Rightarrow \sqrt{K} = \frac{4}{2\times0.5} = 4$$

$\Rightarrow K = 16$

**Sol. 171.(c)**

$s^2 + 12s + 400 = 0$

$$\Rightarrow \xi = \frac{12}{2\sqrt{400}} = \frac{12}{40} < 1 \Rightarrow \text{underdamped}$$

$s^2 + 90s + 900 = 0$

$$\Rightarrow \xi = \frac{90}{2\sqrt{900}} = \frac{90}{2\times30} > 1 \Rightarrow \text{overdamped}$$

$s^2 + 30s + 225 = 0$

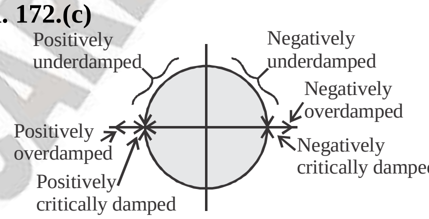
$$\Rightarrow \xi = \frac{30}{2\sqrt{225}} = \frac{30}{2\times15} = 1$$

$\Rightarrow$ Critically damped

$s^2 + 625 = 0$

$\Rightarrow \xi = 0 \Rightarrow$ undamped.

**Sol. 172.(c)**




**Sol. 173.(b)**

**Sol. 174.(c)**

Time for peak overshoots are

$$t_p = \frac{n\pi}{\omega_n\sqrt{1-\xi^2}} \quad n = 1, 3, 5, .......$$

For first peak overshoot, n = 1

$$t_{p1} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

For second peak overshoot, n = 3

$$t_{p2} = \frac{3\pi}{\omega_n\sqrt{1-\xi^2}}$$

**Sol. 175.(c)**

Output due to disturbance D(s) is

$$C_D(s) = \frac{G_2}{1+G_1G_2}.D(s)$$

$$C_D(s) \equiv \frac{G_2}{G_1G_2}.D(s)[\because G_1 G_2 >> 1]$$

$$C_D(s) \approx \frac{1}{G_1(s)}.D(s)$$

Thus effect of disturbance can be reduced by increasing $G_1(s)$.

**Sol. 176.(b)**
Lead compensator is a high pass filter.
Lag compensator is a low pass filter.

**Sol. 177.(c)**

$$\xi = \frac{R}{s}\sqrt{\frac{C}{L}}$$

as C decreases, ξ decreases
i.e. damping ratio decreases

$$\omega_n = \frac{1}{\sqrt{LC}}$$

as C decreases, ξ decreases

Time constant $= \frac{1}{\xi\omega_n} = \frac{2L}{R}$

As C decreases, time constant remains unaffected.
∴ Natural Frequency increases.

**Sol. 178.(c)**

**Sol. 179.(a)**

**Sol. 180.(b)**

**Sol. 181.(a)**
Characteristic equation of the system is
$s^2 + s + k = 0$

∴ $2\xi\omega_n = 1$ and $\omega_n = \sqrt{k}$

$$\therefore \ \xi = \frac{1}{2\sqrt{k}} = 0 \text{ as } k \to \infty$$

**Sol. 182.(b)**

**Sol. 183.(d)**

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)\ H(s)}$$

Where, $R(s) = \left(\frac{3}{s} - \frac{6}{s^3}\right)$

$$\therefore e_{ss} = \lim_{s\to 0}\frac{s.\frac{3}{s}}{1+G(s)H(s)} - \lim_{s\to 0}\frac{s.\frac{6}{s^3}}{1+G(s)H(s)}$$

$$= \frac{3}{1+\lim_{s\to 0} G(s)H(s)} - \frac{6}{\lim_{s\to 0} s^2 \lim_{s\to 0} s^2 G(s)H(s)}$$

$$= \frac{3}{1+K_p} - \frac{6}{K_a}$$

**Sol. 184.(b)**
The disadvantages of static error constants ($K_p$, $K_v$, $K_a$) is that they do not give information on the steady – state error when inputs are other than the three basic types step, ramp and parabolic. Another difficulty is that the error constants fail to indicate the exact manner in which error function changes with time. The dynamic error may be evaluated using the dynamic error coefficients – the concept generalized to include inputs of almost any arbitrary function of time.

**Sol. 185.(d)**

$$E(s) = \frac{R(s)}{1+G(s)\ H(s)}$$

For a unity feedback system

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)}$$

$$\therefore R(s) = \frac{1}{s} + \frac{1}{s^2} + \frac{1}{s^3}$$

$$\therefore e_{ss} = \lim_{s\to 0} s.\frac{1}{1+G(s)}.\frac{1}{s} + \lim_{s\to 0} s.\frac{1}{1+G(s)}.\frac{1}{s^2}$$

$$+\lim_{s\to 0} s.\frac{1}{1+G(s)}.\frac{1}{s^3}$$

$$= \frac{1}{1+K_p} + \frac{1}{K_v} + \frac{1}{K_a}$$

**Sol. 186.(c)**

For underdamped; $0 < \xi < 1$

**Sol. 187.(b)**

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 1.96s$$

and $t_s = \frac{4}{\xi\omega_n} = 3.33s$

**Sol. 188.(b)**

$$Y(s) = \frac{s}{(1+s)}.\frac{1}{(s^2+1)}$$

$$= \frac{\left(-\frac{1}{2}\right)}{(s+1)} + \frac{\frac{1}{2}}{(s^2+1)} + \frac{\frac{s}{2}}{(s^2+1)}$$

$$\Rightarrow\ y(t) = -\frac{1}{2}e^{-t} + \frac{1}{2}(\cos t + \sin t)$$

y(t) in the steady state

$$= \frac{1}{\sqrt{2}}\cos(t-45°)\ \text{as}\ e^{-t} = 0$$

$$= \frac{1}{\sqrt{2}}\sin(t+45°)$$

**Sol. 189.(b)**

From given control system we can find,

$$2\xi\omega_n = 10\sqrt{2},\ \omega_n = 10r/s$$

$$\Rightarrow \xi = \frac{1}{\sqrt{2}}$$

$$M_p = \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)} = 1$$

**Sol. 190.(b)**

(i) In open-loop system, transfer function T = G

Sensitivity of open-loop system is

$$S_G T = \frac{\partial T}{\partial G}\times\frac{G}{T} = 1 \quad [\because T = G]$$

In closed-loop system, transfer function

$$T = \frac{G}{1+GH}$$

$$S_G{}^T = \frac{\partial T}{\partial G}\times\frac{G}{T}$$

$$= \frac{1+GH-GH}{(1+GH)^2}\times\frac{G}{G/(1+GH)}$$

$$S_G{}^T = \frac{1}{1+GH}$$

Thus feedback is used to reduce the sensitivity of the system.

**Sol. 191.(c)**

$$G(s) = \frac{1}{s^3+1.5s^2+s-1}$$

The coefficient of $s^0$ is negative. So the open-loop systems is unstable.

$$\frac{G(s)}{1+G(s)H(s)} = \frac{\frac{1}{s^3+1.5s^2+s-1}}{1+\frac{20s+20}{s^3+1.5s^2+s-1}}$$

$$= \frac{1}{s^3+1.5s^2+21s+19}$$

Since all the coefficient of the denominator are positive, hence the closed loop system is stable.

**Sol. 192.(c)**

**Sol. 193.(a)**

$$\frac{G(s)}{1+G(s)} = \frac{80}{s^2+18s+80}\quad \omega_n = \sqrt{80}$$

$$\xi = \frac{18}{2\sqrt{80}} = 1.00623$$

So, the system is overdamped.1

**Sol. 194.(b)**

Steady state error is the error at $t \to \infty$.

**Sol. 195.(a)**

Table for steady state error

| Type \ Input | Unit step | Unit Ramp | Unit Parabola |
|---|---|---|---|
| Type 0 | $\frac{1}{1+K_p}$ | $\infty$ | $\infty$ |
| Type 1 | 0 | $\frac{1}{K_v}$ | $\infty$ |
| Type 2 | 0 | 0 | $\frac{1}{K_a}$ |

Where $K_p = \lim_{s\to0} sG(s)\,H(s)$

$$K_V = \lim_{s\to0} sG(s)H(s)$$

$$K_a = \lim_{s\to0} s^2G(s)\,H(s)$$

**Sol. 196.(b)**

$C(t) = t^2 e^{-t}$

$$C(s) = \frac{2}{(s+1)^3}$$

$$R(s) = \frac{1}{s}$$

Transfer function

$$G(s) = \frac{C(s)}{R(s)} = \frac{2/(s+1)^3}{1/s}$$

$$\Rightarrow G(s) = \frac{2s}{(s+1)^3}$$

**Sol. 197.(b)**

$$R(s) = \frac{1}{s^2}$$

Steady state error

$$e_{ss} = \lim_{s\to0} \frac{sR(s)}{1+G(s)\,H(s)}$$

$$\Rightarrow e_{ss} = \lim_{s\to0} \frac{s.1/s^2}{1+\frac{K}{s(s+1)}.1}$$

$$\Rightarrow e_{ss} = \lim_{s\to0} \frac{s+a}{s(s+a)+K}$$

$$\Rightarrow e_{ss} = \frac{a}{K}$$

Sensitivity of $e_{ss}$ to change in K is

$$S_K^{e_{ss}} = \frac{de_{ss}}{dK} \times \frac{K}{e_{ss}} = \frac{-a}{K^2 \times \frac{K}{a/K}}$$

$$\Rightarrow S_K^{e_{ss}} = -1$$

Now, $s_a^{e_{ss}} = \frac{de_{ss}}{da} \times \frac{a}{e_{ss}} = \frac{1}{K} 5 \frac{a}{a/K}$

$$\Rightarrow S_a^{e_{ss}} = 1$$

**Sol. 198.(b)**

$$\therefore\ T(s) = \frac{1}{6} \frac{0.6}{(s+0.8)^2 + (0.6)^2}$$

$$= \frac{1}{10}\left(\frac{1}{s^2+1.6s+1}\right)$$

$$\therefore\ \omega_n^2 = 1 \Rightarrow \omega_n = 1 \text{rad/s}$$

And $2\xi\omega_n = 1.6 \Rightarrow \xi = 0.8$

**Sol. 199.(a)**

(i) With the introduction of integral control, the steady state error decrease. As the type of system becomes higher (i.e. increasing number of integrations), progressively more steady - state errors are eliminated. However, additional integrations introduces a distinct possibility of system instability.

(ii) The disadvantages of static error constants ($K_p$, $K_v$, $K_a$) is that they do not give information on the steady -state error when inputs are other than the three basic types - step, ramp and parabolic. Another difficulty is that the error constants fail to indicate the exact manner in which error function changes with time. The dynamic error may be evaluated using the dynamic error may be evaluated using the dynamic error coefficients - the concept generalized to include inputs of almost any arbitrary function of time.

**Sol. 200.(a)**

Static velocity error constant $K_v$ is associated with a ramp input not with a step input. Further, $K_v = 0$ and $\infty$ for Type '0' and Type ' 2' system respectively.

**Sol. 201.(a)**

**Sol. 202.(c)**

**Sol. 203.(d)**

$$E(s) = \left[1 - \frac{C(s)}{R(s)}\right]$$

$$R(s) = \frac{2}{s} + \frac{3}{s^2} + \frac{8}{s^3}$$

$$\frac{C(s)}{R(s)} = \frac{10}{4s^2 + s^3 + 10}$$

$$e_{ss} = \lim_{s\to 0}[s.\,E(s)]$$

$$\therefore e_{ss} = \lim_{s\to 0}\left[s.\left(\frac{2}{s}+\frac{3}{s^2}+\frac{8}{s^3}\right).\left(\frac{4s^2+s^3}{4s^2+s^3+10}\right)\right]$$

= 3.2

**Sol. 204.(b)**

$$\omega_n = \sqrt{121} = 11\text{rad/s},\ 2\xi\omega_n = 13.2$$

$\Rightarrow \xi = 0.6$ (< 1, underdamped)

$$\therefore\ T_s = \frac{4}{\xi\omega_n} = 0.606\text{s}$$

**Sol. 205.(a)**

$$G(s) = \frac{1}{s} \Rightarrow g(t) = 1$$

The impulse response of the system is constant.

**Sol. 206.(a)**

**Sol. 207.(b)**

**Sol. 208.(b)**

**Sol. 209.(a)**

Step response

$c(t) = 1 - 10\ e^{-t}$

Impulse response,

$$h(t) = \frac{d}{dt}\text{(step response)}$$

$$h(t) = \frac{d}{dt}(1 - 10\ e^{-t})$$

$h(t) = 10e^{-t}$

$$H(s) = \frac{10}{s+1}$$

**Sol. 210.(b)**

$$G(s) = \frac{10}{s^2+14s+50}$$

It is type 0 system. Input is step input.

$$e_{ss} = \frac{1}{1+K_p}$$

Where $K_p = \lim_{s\to 0} G(s)H(s)$

$$K_p = \lim_{s\to 0}\frac{100}{s^2+14s+50} = \frac{10}{50} = 0.2$$

$$e_{ss} = \frac{1}{1+0.2} = \frac{1}{1.3} = 0.83$$

**Sol. 211.(a)**

$$X(s) = \frac{1}{s+1}$$

Y(s) = X(s) H(s)

$$= \frac{1}{(s+1)}.\frac{(s+1)}{\{(s+1)^2+1\}}$$

$$= \frac{1}{(s+1)^2+1}$$

$\Rightarrow y(t) = e^{-t}\sin t\ u(t)$

**Sol. 212.(d)**

$$\left(\frac{1}{s+\tau}\right).\left(\frac{1}{s+\tau}\right) = \frac{1}{(s+\tau)^2}$$

Since both are cascaded non-interactively, the overall unit step response will be shown above. It is clear that the above response is critically damped.

**Sol. 213.(a)**

$$C(s) = \frac{1}{(s+1)^2},\ R(s) = \frac{1}{s}$$

$$\therefore\qquad T(s) = \frac{s}{(s+1)^2}$$

**Sol. 214.(a)**

Characteristic equation $s^2 + s + K = 0$

$2\xi\omega_n = 1, \qquad \omega_n = \sqrt{K}$

for $\xi = 1 \Rightarrow K = \frac{1}{4}$

$$\therefore\ s^2 + s + \frac{1}{4} = 0(s+0.5)^2 = 0$$

**Sol. 215.(a)**

**Sol. 216.(b)**

$\omega_n^2 = 4$ and $2\xi\omega_n = 2$

**Sol. 217.(a)**

$$e_{ss} = \underset{s\to0}{Lt}\left[\frac{s\times R(s)}{1+G(s)H(s)}\right]$$

**Sol. 218.(d)**

**Sol. 219.(c)**

Settling time at 2% of tolerance band of the system,

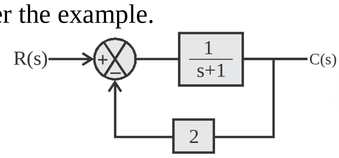
$$t_s = \frac{4}{\xi\omega_n}$$

Settling time at 5% of tolerance band of the system,

$$t_s = \frac{3}{\xi\omega_n}$$

**Sol. 220.(d)**

Feedback is applied to reduce the system error. Consider the example.




$$\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)} = \frac{\frac{1}{s+1}}{1-\frac{2}{s-1}} = \frac{1}{s-1}$$

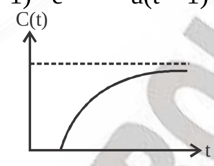
Thus, we see that the closed loop system is unstable while the open loop system is stable.

**Sol. 221.(d)**

$$C(s) = G(s)\,.\,R(s) = \frac{e^{-s}}{1+0.5s}.\frac{1}{s}$$

$$\Rightarrow C(s) = \frac{2e^{-s}}{s(s+2)} \quad \Rightarrow (s) = \frac{e^{-s}}{s} - \frac{e^{-s}}{s+2}$$

$$\Rightarrow c(t) = u(t-1) - e^{-2(t-1)}\,u(t-1)$$




**Sol. 222.(d)**

In the pole zero form,

$$G(s)H(s) = \frac{k(s+z_1)(s+z_2).......}{s^n(s+p_1)(s+p_2).....}$$

The type of the system is 'n' and order of the system is the highest power of s in the denominator.

**Sol. 223.(b)**

$$\omega_n = \sqrt{5}\,\text{rad/s}$$

$$2\xi\omega_n = 4 \Rightarrow \xi \;\; \frac{4}{2\sqrt{5}} < 1$$

$\Rightarrow$ System response is underdamped.

**Sol. 224.(c)**

No. of roots in the right half of s-plane = no. of sign changes.

**Sol. 225.(c)**

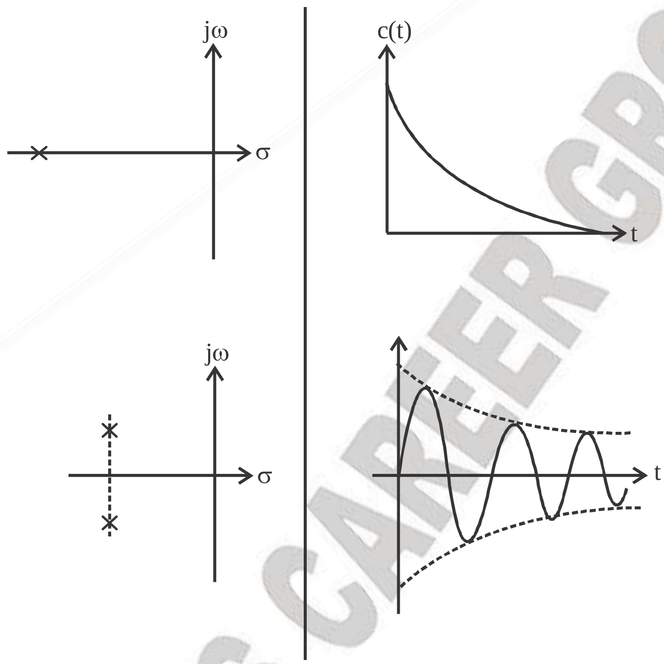
**Sol. 226.(c)**

**Sol. 227.(b)**

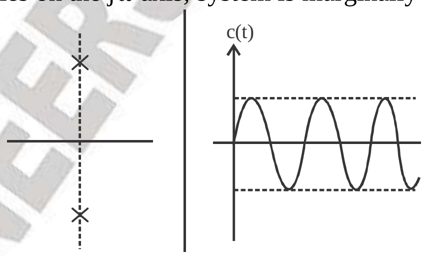
# CHAPTER - 5
# *STABILITY ANALYSIS OF CONTROL SYSTEM*

## 5.1 INTRODUCTION
If all the poles of the system lie in the left half of s plane, then the system is stable.

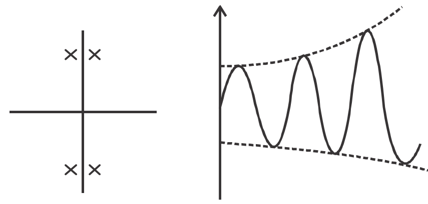
**Location of poles (roots)** **Response**




**Case-I.**
If there are non – repeated poles on the jω axis, system is marginally stable.

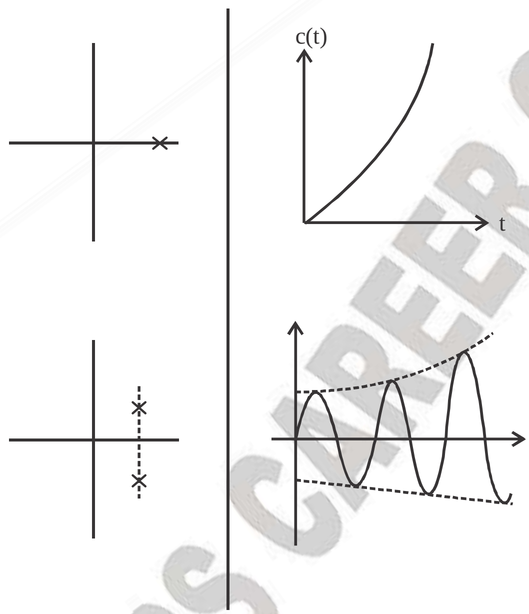



**Case-II.**
If there are repeated poles of the system on jω axis, system is unstable.



**Case-III.**
If there is one or more than one pole in R.H of s plane, system is unstable.

**Case-IV.**




**Definition**
The system is said to be stable if
(a) Bounded input gives bounded o/op
(b) o/p should reduce to zero when input is removed

**5.2 ROUTH'S STABILITY CRITERION**
If we write polynomial in s in the following form
$a_0s^n + a_1s^{n-1} + \ldots.. + a_{n-1}\,s + a_n = 0$ …(i)
Where the coefficients are real quantities. We assume that $a_n \neq 0$; that is any zero root has been removed.

**5.2.1 Routh's Stability Criterion States**
The necessary and sufficient condition that all roots of equation (i) lie in the left – half s plane is that all the coefficients of equation (i) be positive and all terms in the first column of the array have positive signs. The number of roots lying in the right half is given by the number of sign changes in the first column of Routh array.
Let us apply Routh's stability criterion to the following polynomial.

$a_0s^5 + a_1s^4 + a_2s^3 + a_2s + a_5 = 0$

Where all the coefficients are positive numbers. The array of coefficients (Routh array) becomes.

| $S^5$ | $A_0$ | $A_2$ | $A_4$ |
|---|---|---|---|
| $S^4$ | $A_4$ | $A_3$ | $A_5$ |
| $S^3$ | $B_1$ | $B_2$ | 0 |
| $S^2$ | $C_1$ | $C_2$ | 0 |
| $S^1$ | $D_1$ | | |
| $S^0$ | $A_5$ | | |

Where ,

$$b_1 = \frac{a_4a_2 - a_0a_3}{a_1}$$

$$b_2 = \frac{a_1a_4 - a_0a_5}{a_1}, C_1 = \frac{b_1a_3 - a_1b_2}{b_1}$$

$$C_2 = \frac{b_1a_5 - a_1 \times 0}{b_1}$$

$$d_1 = \frac{C_1b_2 - b_1c_2}{c_1}$$

The conditions that all roots have negative real parts or system stability is given by

$a_4\,a_2 > a_0a_3$

$b_1a_3 > a_1b_2$

**Example.** Consider the following polynomial:

$s^4 + 2s^2 + 3s^2 + 4s + 5 = 0$

**Solution.**

Let us follow the procedure just presented and construct the array of coefficients. (The first two rows can be obtained directly from the given polynomial. The remaining terms are obtained from these. If any coefficients are missing, they may be replaced by zeros in the array.)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $s^4$ | 1 | 3 | 5 | $s^4$ | 1 | 3 | 5 |
| $s^3$ | 2 | 4 | 0 | $s^3$ | 2 | 4 | 0 |
| | | | | | 1 | 2 | 0 |
| $s^2$ | 1 | 5 | | $s^2$ | 1 | 5 | |
| $s^1$ | –6 | | | $s^1$ | –3 | | |
| $s_0$ | 5 | | | $s^0$ | 5 | | |

In this example, the number of changes in sign of the coefficients in the first column is two. This means that there are two roots with positive real parts. Note that the result is unchanged when the coefficients of any row are multiplied or divided a positive number in order to simplify the computation. **Since there are two sign changes indicating two roots lying in the right half of the s plane. So the system is unstable.**

## 5.3 DIFFICULTIES WITH ROUTH ARRAY

**Difficulty-1.** If a first – column term in any row is zero, but the remaining terms are not zero or there is no remaining term, then Routh stability test does not work. Then the zero term is replaced by a very small positive number $\in$ and the rest of the array is evaluated.

**Example.** Consider the following equation:

$s^3 + 2s^2 + s + 2 = 0$ …(ii)

The array of coefficient is

| | | |
|---|---|---|
| $S^3$ | 1 | 1 |
| $s^2$ | 2 | 2 |
| $s^1$ | $0 \approx \in$ | |
| $s^0$ | 2 | |

If the sign of the coefficient above the zero ($\in$) is the same as that below it, it indicates that there are a pair of imaginary roots. Actually,

Equation (II) has two roots at $s = \pm j$.

If, however the sign of the coefficient above the zero ($\in$) is opposite that below it, it indicates that there is a sign change and the system is unstable.

**Example.** For the following equation;

$s^3 - 3s + 2 = (s - 1)^2 (s + 2) = 0$

the array of coefficient is

| | | |
|---|---|---|
| $S^3$ | 1 | -3 |
| $s^2$ | $0 \approx \in$ | 2 |
| $s^1$ | $-3-(2/\in)$ | |
| $s^0$ | 2 | |

There are two sign changes of the coefficients in the first column indicating that the system is unstable. This agrees with the correct result indicated by the factored form of the polynomial equation.

**Difficulty-II.** If all the coefficients in any derived row are zero, it indicates that there are roots of equal magnitude lying radically opposite in the s –plane, that is, two real roots with equal magnitudes and opposite signs and /or two conjugate imaginary roots. In such a case, the evaluation of the rest of the array can be continued by forming an auxiliary polynomial with the coefficients of the last row and by using the coefficients of the derivative of this polynomial in the next row.

Such roots with equal magnitudes and lying radially opposite in the plane can be found by solving the auxiliary polynomial, which is always even .For a 2n – degree auxiliary polynomial, there are n pairs of equal and opposite roots.

**Example.** Consider the following equation

$S^5 + 2s^4 + 24s^3 + 48 s^2 - 25 s - 50 = 0$

The array of coefficients is

| | | | |
|---|---|---|---|
| $s^5$ | 1 | 24 | $-25$ |
| $s^4$ | 2 | 48 | $-50 \leftarrow$ Auxiliary polynomial P(s) |
| $s^3$ | 0 | 0 | |

The first term of the fifth row has a value of -2 as $\varepsilon \Rightarrow 0$. Thus there 2 sign changes indicating that the system is unstable.

The terms in this $s^3$ row are all zero. The auxiliary polynomial is then formed from the coefficients of the $s^4$ row. The auxiliary polynomial P(s) is

$P(s) = 2s^4 + 48s^2 - 50$

Which indicates that there are two pairs of roots of equal magnitude and opposite sign. These pairs are obtained by solving the auxiliary polynomial equation P(s) = 0. The derivative of P(s) with respect to s is

$$\frac{dP(s)}{ds} = 8s^4 + 96s$$

The terms in the $s^3$ row are replaced by the coefficients of the last equation that is 8 and 96. The array of coefficient then becomes

| | | | |
|---|---|---|---|
| $s^5$ | 1 | 24 | –25 |
| $s^4$ | 2 | 48 | -50 |
| $s^3$ | 8 | 96 | ← coefficient of dP(s) /ds |
| $s^2$ | 24 | –50 | |
| $s^1$ | 112.7 | 0 | |
| $s^0$ | –50 | | |

We see that there is one change in sign in the first column of the new array.

Thus, the original equation has one root with a positive real part. By solving for roots of the auxiliary polynomial equation,

$2s^4 + 48s^2 – 50 = 0$

We obtain $s^2 = 1$, $s^2 = –25$

or $s = \pm 1$, $s = \pm j5$

These two pairs of roots are a part of the roots of the original equation. As a matter of fact, the original equation can be written in factored form as follows:

$(s + 1) (s – 1) (s + j5) (s–j5) (s + 2) = 0$

This is obtained by long division method. By dividing the original equation with $2s^4 + 48s^2 – 50 = 0$

Clearly, the original equation has one root with a positive real part.

**Example.** The open – loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K}{s(sT_1+1)(sT_2+1)}$$

Applying Ruth =- Hurwitz criterion determine the value of K in term of $T_1$ and $T_2$ for the system to be stable.

**Solution.**

The characteristic equation is given by

$S(sT_1 + 1) (sT_2 + 1) + K = 0$

or $T_1T_2S^3 + (T_1 + T_2)s^2 + s + K = 0$

The Ruth's array is formed below

| | | |
|---|---|---|
| $s^3$ | $T_1T_2$ | 1 |
| $s^2$ | $(T_1 + T_2)$ | K |
| $s^1$ | $\frac{[(T_1+T_2)-KT_1T_2}{(T_1+T_2)}$ | |
| $s^0$ | K | |

For the system to be stable

(i) K > 0 …(i)

(ii) $\frac{[(T_1+T_2)-KT_1T_2]}{(T_1+T_2)} > 0$

or $[(T_1 + T_2) - KT_1T_2] > 0$

or $KT_1T_2 < (T_1 + T_2)$

$\therefore\ K < \left(\frac{1}{T_1}+\frac{1}{T_2}\right)$ …(ii)

In view of relations (1) and (2) following condition for stability is obtained below;

$0 < K < \left(\frac{1}{T_1}+\frac{1}{T_2}\right)$

**Example.** For a unity feedback system with $G(s) = \frac{K(s^2+1)}{(s+1)(s+2)}$ find the range of K for which the system is stable.

**Solution.**

The characteristic $eq^n$ is

$1 + G(s) = 0 \Rightarrow (s+1)(s+2) + K(s^2 + 1) = 0$

$S^2 + 3s + 2 + Ks^2 + K = 0$

$S^2 + 3s + 2 + Ks^2 + K = 0$

$S^2 (1 + K) + 3s + (K + 2) = 0$

The Routh's array is :

| | | |
|---|---|---|
| $S^2$ | 1+K | K+2 |
| $S^1$ | 3 | |
| $S^0$ | K+2 | |

For the system to be stable, (1 + K) > 0 i.e. K > -1 and (K + 2) > 0 i.e. K > -2. Combining both conditions, K > -1

**Example.** Determine the value of K that will cause sustained oscillations in the closed loop system which has the following characteristic equation:-

$S^4+4s^3 + 4s^2 + 3s + K = 0$

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 4 | K |
| $S^3$ | 4 | 3 | |
| $S^2$ | 13/4 | K | |
| $S^1$ | $\frac{\frac{39}{4}-4K}{\frac{13}{4}}$ | | |
| $s^0$ | K | | |

When $K = \frac{39}{16}$, there will be a zero at the first entry in the fourth row. This indicates presence of imaginary roots, So K = 39/16 will cause sustained oscillations. Put this value of K in the third row.

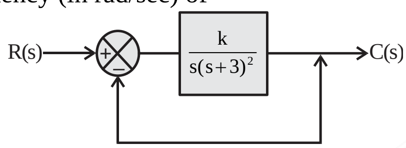
$\frac{13}{4}s^2 + \frac{39}{16} = 0$

$s \pm jo.75$

So frequency of oscillation is 0.75 rad/sec.

# ASSIGNMENT

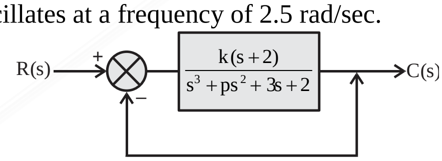
**1.** By properly choosing the value of the 'k' the output c(t) of the system as shown in the figure can be made to oscillate sinusodially at a frequency (in rad/sec) of




(a) 3 (b) 2.5
(c) 4 (d) 1.25

**Linked Statement for Q.2 & Q.3**

**2.** Determine the values of $K_{mar}$. if the system oscillates at a frequency of 2.5 rad/sec.




(a) 1.25 (b) 3.25
(c) 1.36 (d) 3.5

**3.** Also find the value of P.

(a) 1.36 (b) 2.36
(c) 3.5 (d) 1.45

**4.** The open-loop transfer function of a feedback control system is given by

$$G(s)H(s) = \frac{K}{s(s+4)(s^2+2s+2)}$$

Determine the stability of the system when k = 12 and find the range of the values of k for stability.

(a) Stable and $0 < k \le 11.56$
(b) Stable and $0 < k \le 12.56$
(c) Unstable and $0 < k \le 11.56$
(d) Unstable and $0 < k \le 12.56$

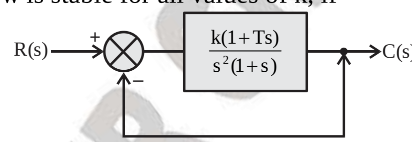
**5.** The open-loop transfer function of a feedback control system is given by

$$G(s) = \frac{e^{-sT}}{s(s+2)}$$

Find the range of the values of T for stability.

(a) T > 2 (b) T < 2
(c) T < 2.5 (d) None

**6.** A feedback control system shown in figure below is stable for all values of k, if




(a) T = 0 (b) 0 < T < 1
(c) T < 0 (d) T = 2

**7.** The number of sign changes in the entries in the first column of Routh's array donates

(a) The number of roots of the characteristic polynomial in RHP.
(b) The number of open-loop poles in RHP.
(c) The number of zeros of the system in RHP.
(d) The number of open-loop zeros in RHP.

**8.** The closed loop transfer function of a system is $T(s) = \frac{(s+8)(s+6)}{s^5 - s^4 + 4s^3 - 4s^2 + 3s - 2}$

The number of poles in right half plane and in left half plane are

(a) 2, 3 (b) 4, 1
(c) 3 (d) 4

**9.** The first two rows of Routh's tabulation of a fourth-order system are

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 10 | 5 |
| $s^3$ | 2 | 20 | |

The number of roots of the system lying on the right half of s-plane is

(a) Zero (b) 2
(c) 3 (d) 4

**10.** Consider the following statements regarding stability analysis by Routh-Hurwitz criterion:

**I.** For a system to be stable, all the coefficients of the characteristic equation must be present and be of the same sign.

**II.** If a system is to be stale, there should not be any sign change in the first column of the Routh's array.

**III.** The order of the auxiliary equation obtained from the elements of the Routh's table is always odd. Of these statements:

(a) I and II are correct
(b) II and III are correct
(c) I and III are correct
(d) I, II and III are correct

**11.** Which of the following represent a stable system?

**I.** Impulse response of the system decreases exponentially.

**II.** Area within the impulse response is finite.

**III.** Eigen-values of the system are positive and real.

**IV.** Roots of the characteristic equation of the system are real and negative.

Select the correct answer using the code given below:

(a) I and IV  (b) I and III
(c) II, III and IV  (d) I, II and IV

**12.** The value of k for which the unity feedback system $G(s) = \frac{k}{s(s+2)(s+4)}$ crosses the imaginary axis at

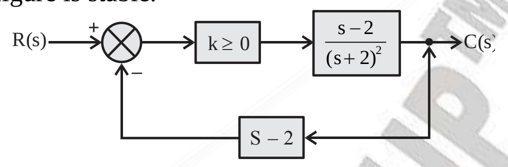
(a) 2  (b) 4
(c) 8  (d) 48

**13.** Consider the following statement, Routh Hurwitz criterion gives:

I. Absolute stability.
II. The number of roots lying inf the RHP?
III. The gain margin and phase margin.
Which of these statements are correct?

(a) I, II and III  (b) I and II
(c) II and III  (d) I and III

**14.** The feedback control system shown in figure is stable.




(a) For all $k \geq 0$  (b) Only if $k \geq 0$
(c) Only if $0 \leq k < 1$  (d) Only if $0 \leq k \leq 1$

**15.** Find the values of k such that the following system has roots with real parts more negative than –1.

(a) $k > 0.63$  (b) $k > 0.53$
(c) $k < 0.53$  (d) $k < 0.43$

**16.** The open-loop transfer function with ufb are given below for different systems. The unstable system is

(a) $\frac{1}{s+3}$  (b) $\frac{1}{s^2(s+3)}$
(c) $\frac{1}{s(s+3)}$  (d) $\frac{(s+1)}{s(s+3)}$

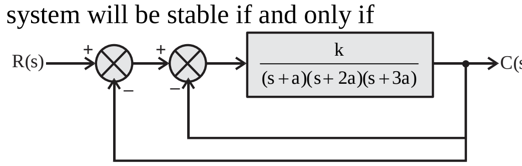
**17.** The open-loop transfer function of a ufb control system is

$$G(s) = \frac{k(s+2)}{(s+1)(s-7)}$$

For k > 6, the stability characteristic of the open loop and closed-loop configuration of the system are respectively

(a) Stable and unstable
(b) Stable and stable
(c) Unstable and stable
(d) Unstable and unstable

**18.** For the block diagram shown in figure below, the limiting value of k for stability of the inner loop is found to be X < k < Y. the overall system will be stable if and only if



(a) $4X < k < 4Y$  (b) $2X < k < 2Y$

(c) $X < k < Y$  (d) $\frac{X}{2} < k < \frac{Y}{2}$

**19.** A unity feedback control system has an open-loop transfer function

$$G(s) = \frac{k}{s(s^2+7s+12)}$$

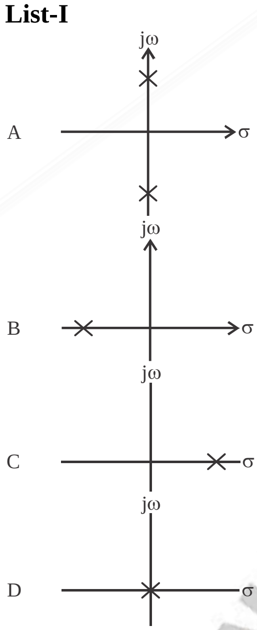
The gain k for which s = – 1 + j1 will lie on the root locus of the system is

(a) 10  (b) 100

(c) 12  (d) None

**20.** Match List-I (Roots in s-plane) with List-II (Impulse response) and select the correct code given below:

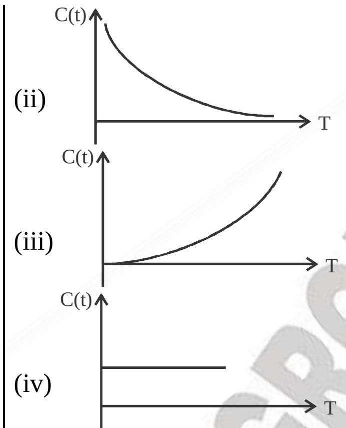
**List-I**




**List-II**

(i)






**Codes:**

(a) A-i, B-ii, C-iii, D-iv

(b) A-iv, B-ii, C-iii, D-iv

(c) A-i, B-iii, C-ii, D-iv

(d) A-iv, B-iii, C-ii, D-i

**21.** The first element of each of the rows of a Routh-Hurwitz stability test showed the sign as follows:

| Rows | I | II | III | IV | V | VI | VII |
|---|---|---|---|---|---|---|---|
| Signs | + | – | + | + | + | – | + |

The number of roots of the system lying in the right half of s-plane is

(a) 2  (b) 3

(c) 4  (d) 5

**22.** A closed-loop system is shown in the following figure:

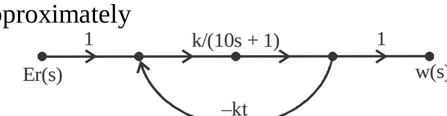
The largest possible value of β for which this system would be stable is:

(a) 1  (b) 1.1

(c) 1.2  (d) 2.3

**23.** Given: $Kk_t = 99$; S = j 1 rad/s

The sensitivity of the closed loop system(shown in the figure) to variation in parameter k is approximately




(a) 0.01  (b) 0.1

(c) 1.0  (d) 10

**24.** The open-loop transfer function of unity feedback control system is

$$G(s) = \frac{K}{s(s+1)(s+b)}$$

The system is stable is

(a) $0 < K < \frac{(a+b)}{ab}$

(b) $0 < K < \frac{ab}{(a+b)}$

(c) $0 < K < ab(a + b)$

(d) $0 < K < a/b\ (a + b)$

**25.** Which one of the following characteristic equations of result in the stable operation of the feedback system?

(a) $S^3 + 4s^2 + s - 6 = 0$

(b) $S^3 + s^2 + 5s + 6 = 0$

(c) $S^3 + 4s^2 + 10s + 11 = 0$

(d) $S^4 + s^3 + 2s^2 + 4s + 6 = 0$

**26.** The given characteristic polynomial $s^4 + s^3 + 2s^2 + 2s + 3 = 0$ has

(a) Zero root in RHS of s-plan

(b) One root in RHS of s-plane

(c) Two roots in RHS of s-plane

(d) Three roots in RHS of s-plane

**27.** The characteristic equation of a control system is given as

$S^4 + 4s^3 + 4s^2 + 3s + K = 0$

What is the value of K for which this system is marginally stable?

(a) $\frac{9}{16}$ (b) $\frac{19}{16}$

(c) $\frac{29}{16}$ (d) $\frac{39}{16}$

**28.** In closed loop control system, what is the sensitivity of the gain of the overall system, M to the variation in G?

(a) $\frac{1}{1+G(s)H(s)}$ (b) $\frac{1}{1+G(s)}$

(c) $\frac{G(s)}{1+G(s)\ H(s)}$ (d) $\frac{G(s)}{1+G(s)}$

**29.** The characteristics equation of a system is given by $3s^4 + 10s^3 + 5s^2 + 2 = 0$. This system is

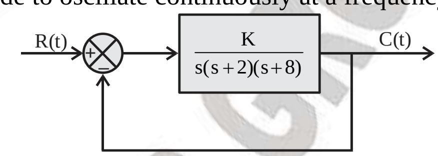
(a) Stable

(b) Marginal stable

(c) Unstable

(d) Data is insufficient

**30.** By a suitable choice of the scalar parameter 'K' the system shown in fig given below can be made to oscillate continuously at a frequency is




(a) 1 rad/sec (b) 2 rad/sec

(c) 4 rad/sec (d) 8 rad/sec

**31.** The characteristics equation of closed loop control system is given as $s^2 + 4s + 16 = 0$. Then resonant frequency in radian/sec of the system is

(a) 2 (b) $2\sqrt{3}$

(c) 4 (d) $2\sqrt{2}$

**32.** An amplifier with resistive negative feedback has two left half plane poles in its open-loop transfer function. The amplifier

(a) Will always be unstable at high frequency

(b) Will be stable for all frequency

(c) May be unstable, depending on the feedback factor.

(d) Will oscillate at low frequency

**33.** A system described by the transfer function

$$H(s) = \frac{1}{s^3 + \alpha s^2 + Ks + 3}$$ is stable.

The constraints on $\alpha$ and k are,

(a) $\alpha > 0, \alpha K < 3$ (b) $\alpha > 0, \alpha K > 3$

(c) $\alpha < 0, \alpha K > 3$ (d) $\alpha < 0, \alpha K < 3$

**34.** The number of roots of $s^3 + 5s^2 + 7s + 3 = 0$ in the left half of the s-plane are.

(a) 0 (b) 1

(c) 2 (d) 3

**35.** The open-loop transfer function of a unity feedback system is

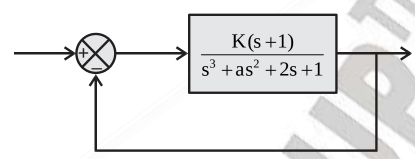
$$G(s) = \frac{k}{[s(s)]^2 + s + 2)(s+3)}$$

the range of 'k' for which the system is stable.

(a) $\frac{21}{4} > k > 0$ (b) $13 > k > 0$

(c) $\frac{21}{4} > k > \infty$ (d) $-6 < k < \infty$

**36.** The feedback system shown below oscillates at 2 rad/s when.




(a) K = 2 and a = 0.75
(b) K = 3 and a = 0.75
(c) K = 4 and a = 0.5
(d) K = 2 and a = 0.5

## ANSWER KEY

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | a | **2.** | b | **3.** | a | **4.** | c | **5.** | b | **6.** | d | **7.** | a | **8.** | c | **9.** | b | **10.** | a |
| **11.** | d | **12.** | d | **13.** | b | **14.** | c | **15.** | b | **16.** | b | **17.** | c | **18.** | d | **19.** | a | **20.** | a |
| **21.** | c | **22.** | b | **23.** | b | **24.** | c | **25.** | c | **26.** | c | **27.** | d | **28.** | a | **29.** | c | **30.** | c |
| **31.** | d | **32.** | b | **33.** | b | **34.** | d | **35.** | a | **36.** | a | | | | | | | | |

# SOLUTIONS

**Sol. 1.**
The characteristic equation, 1 + G(s) H(s) = 0

$$1+\frac{k}{s(s+3)^2}=0$$

Now, the Routh's table is

| | | |
|---|---|---|
| $s^3$ | 1 | 9 |
| $s^2$ | 6 | k |
| $s^1$ | $\frac{54-k}{6}$ | |
| $s^0$ | k | |

For the system to be sinusoidally oscillate so

$$\frac{54-k}{6}\geq 0$$

$k \geq 54$
$\therefore$ Auxiliary equation becomes
$6s^2 + k = 0$
$6s^2 + 54 = 0$
$s^2 = -9$
$s = \pm 3j$
$\therefore$ s = ± 3j
$\therefore$ Frequency of oscillations is 3 rad/sec.

**Sol.2.**
Since the system oscillates, it is marginally stable. The characteristic equation of the system becomes.

$$1+\frac{k(s+2)}{s^3+ps^2+3s+2}=0$$

$S^3 + ps^2 + (k + 3) s + 2 (k + 1) = 0$
Now the Routh's Array is

| | | |
|---|---|---|
| $s^3$ | 1 | $(k+3)$ |
| $s^2$ | p | $(2k+1)$ |
| $s^1$ | $\frac{p(k+3)-2(k+1)}{p}$ | 0 |
| $s^0$ | $2(k+1)$ | |

At marginal value of k,

$$\frac{p(k+3)-2(k+1)}{p}=0$$

$$p=\frac{2(k+1)}{k+3}$$

Again, at this value of p,
$A(s) = ps^2 + 2 (k + 1) = 0$

$$s^2=-\frac{2(k+1)}{p}\frac{-2(k+1)(k+3)}{2(k+1)}$$

$s^2 = -(k+3)$
s
Given, ω = 2.5 rad/sec, therefore

$$\sqrt{(k+3)}=2.5$$

k + 3 = 6.25
k = 3.25

**Sol.3.**
From the above solution:

$$p=\frac{(k+1)}{(k+3)}$$

at k = 3.5 then,

$$p=\frac{2(3.25+1)}{(3.25+3)}=1.36$$

**Sol. 5.**
The characteristic equation of the system is
1 + G(s) H(s) = 0

$$1+\frac{e^{-\tau}}{s(s+2)}=0$$

$s^2 + 2s + e^{-esT} = 0$
$s^2 + 2s + (1 - Ts) = 0$
$s^2 + s(2 - T) + 1 = 0$
Routh's array becomes:

| | | |
|---|---|---|
| $s^2$ | 1 | 1 |
| $s^1$ | $(2+T)$ | 0 |
| $s^0$ | 1 | |

The system will be stable if
$2 - T > 0$
$2 > T$
$T < 2$

**Sol.6.**

The characteristic equation, 1 + G(s) H(s) = 0

$$1+\frac{k\left(1+T_s\right)}{s^2\left(1+s\right)}=0$$

$$s^3+s^2kTs+k=0$$

Now Routh's Array is

$$\begin{array}{c|cc} s^3 & 1 & kT \\ s^2 & 1 & k \\ s^1 & k\left(T+1\right) & \\ s^0 & k & \end{array}$$

The system will stable when,

k(T – 1) > 0

k > and T > 1 or k < 0 and T < 1

In options only (d) option is satisfy the condition T > 1.

**Sol. 9.**

$$\begin{array}{c|ccc} s^4 & 1 & 10 & 5 \\ s^3 & 2 & 20 & \\ s^2 & 0(\xi) & 5 & \\ s^1 & \frac{20\xi-10}{5} & & \\ s^0 & 5 & & \end{array}$$

Now, $\xi \to 0, \ \frac{20\xi-10}{\xi} \to -\infty$

There are two sign change in the first column, so two roots of the system lying on the right half of s-plane.

**Sol. 12.**

The characteristic equation 1 + G(s) H(s) = 0

$$1+\frac{k}{s\left(s+2\right)\left(s+4\right)}=0$$

Routh's array is

$$\begin{array}{c|cc} s^3 & 1 & 8 \\ s^2 & 6 & k \\ s^1 & \frac{48-k}{6} & \\ s^0 & k & \end{array}$$

$$\frac{48-k}{6}=0$$

k = 48

**Sol. 14**.

The characteristic equation, 1 + G(s) H(s) = 0

$$1+\frac{k\left(s-2\right)^2}{\left(s+2\right)^2}$$

$s^2$ (1 + k) + s(4 – 4k) + (4k + 4) = 0

The Routh's Array is

$$\begin{array}{c|cc} s^2 & \left(1+k\right) & \left(4k+k\right) \\ s^1 & \left(4+4k\right) & \\ s^0 & 4\left(k+4\right) & \end{array}$$

For stable,

4 – 4 k > 0

∴ Range of k for stability 0 ≤ k < 1.

**Sol. 15.**

Put s = (p – 1) then the system becomes:

$(p – 1)^3 + 3(k + 1)(p – 1)^2 + (7k + 5)(p – 1) + 4k + 7 = 0$

$p^3 + 3kp^2 + p(k + 2) + 4 = 0$

The Rooth's array is

$$\begin{array}{c|cc} p^3 & 1 & \left(k+2\right) \\ p^2 & 3k & 4 \\ p^1 & \frac{3k^2+6k-4}{3k} & \\ p^0 & 4 & \end{array}$$

For stability:

3k > 0 and $\frac{3k^2+6k-4}{3k}$

k > 0 $3k^2 + 6k – 4 > 0$

k > 0.53

**Sol. 16.**

In characteristic equation $s^3 + 3s^2 + 1 = 0$, the terms 's' missing. Hence the system is unstable.

**Sol. 17.**

In open loop system there is a pole in RHP. System is unstable.

In closed loop system.

The characteristic equation 1 + G(s) H(s) = 0

$$1+\frac{k\left(s+2\right)}{\left(s+1\right)\left(s-7\right)}=0$$

s2 + (k – 6)s + (2k – 7) = 0

Now the Routh's array is

$$\begin{array}{c|cc} s^2 & 1 & (2k-7) \\ s^1 & (k-6) & 0 \\ s^0 & (2k-7) & \end{array}$$

For stability,
$k - 6 > 0 \Rightarrow k > 6$
and $2k - 7 > 0$
$k > 3.5$
so for $k > 6$, the closed-loop system is stable.

**Sol. 18**.
For inner loop:

$$\text{Transfer function } \frac{\frac{k}{(s+a)(s+2a)(s+3a)}}{\frac{k}{(s+a)(s+2a)(s+3a)}}$$

$$\frac{k}{(s+a)(s+2a)(s+3a)+k} = \frac{k}{p(s)+k}$$

For outer loop:

$$\text{Transfer fucntion } \frac{\frac{k}{(s+a)(s+2a)(s+3a)+k}}{1+\frac{k}{(s+a)(s+2a)(s+3a)+k}}$$

$$= \frac{k}{(s+a)(s+2a)(s+3a)k} = \frac{k}{p(s)+2k}$$

$$= \frac{k}{p(s)+k}$$

Therefore, if inner loop is stable for $X < k < Y$
Then outer loop will be stale fo $X < 2k < y$

$$\text{i.e.} \frac{X}{2} < k < \frac{Y}{2}$$

**Sol. 19.**
The characteristic equation is 1 + G(s) H(s)s) = 0

$$1+\frac{k}{s(s^2+7s+12)} = 0$$

$s(s^2 + 7s + 12) + k = 0$
point $s = -1 + j1$ lie on root locus if it satisfy above equation.
i.e$(-1 + j)\{(-1 + j)^2 + 7(-1 + j) + 12\} + k = 0$

$\therefore$ k = 10

**Sol. 21.**
Number of roots of the system lying in the right half of s-plane = total number of sign change in first column = 4.

**Sol. 22. (b)**

$$\frac{C(s)}{R(s)} = \frac{10}{1+\frac{10\beta}{s^2+4s^2+3s+1}}$$

$$= \frac{10}{s^2+4s^2+3s+1+10\beta}$$

Characteristic equation is
$s^2 + 4s^2 + 3s + (10\beta + 1) = 0$
From R – H Criteria

$$\begin{array}{lll} s^3 & 1 & 3 \\ s^2 & 4 & 10\beta + 1 \\ s^1 & \frac{12-10\beta-1}{4} & \\ s^0 & 10\beta + 1 & \end{array}$$

i.e. $11 - 10\beta \geq 0$
$10\beta \leq 11$
For β mas

$$\beta = \frac{11}{10} = 1.1$$

**Sol. 23. (b)**

$$S_K^M = \frac{dM/M}{dK/K} = \frac{K}{M}\frac{dM}{dK}$$

$$M = \frac{G}{1+GH} = \frac{K/(10s+1)}{1+KK_t/(10s+1)}$$

$$= \frac{K}{10s+1+KK_t}$$

$$\frac{dM}{dK} = \frac{1\times(10s+1+KK_t)-KK_t}{(10s+1+KK_t{}^2)}$$

$$\frac{K}{M}\frac{dM}{dK} = \frac{10s+1}{(10s+1+KK_t)}$$

$$= \frac{(10s+2)}{(10s+1+KK_t)} = \frac{10s+1}{10(s+10)}$$

$\therefore \text{magnitude} = \frac{\sqrt{100+1}}{10\sqrt{100+1}} = 0.1$

**Sol. 24. (c)**

$$\frac{G(s)}{1+G(s)} = \frac{K}{s(s+a)(s+b)+K}$$

Characteristic equation is

$S^3 + (a + b)s^2$ abs + K

Routh array is

| | | |
|---|---|---|
| $s^3$ | 1 | $a+b$ |
| $s^2$ | ab | k |
| $s^1$ | $\frac{ab(a+b)-K}{ab}$ | 0 |
| $s^0$ | K | |

For the system to be stable

K > 0

Ab(a + b) – K > 0 $\Rightarrow$ K < ab(a + b)

So 0 < K < ab(a + b)

**Sol. 25. (c)**

For stable operation, all coefficients of the characteristic equation should be real and have the same sign. Furthermore, none of the coefficients should be zero.

**Sol. 26. (c)**

Routh array is

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 2 | 3 |
| $s^3$ | 1 | 2 | |
| $s^2$ | $\in$ | 3 | |
| $s^1$ | $\frac{2\in-3}{\in}$ | 0 | |
| $s^0$ | 3 | | |

Since the sign changes twice, so there are two roots in RHS of s-plane

**Sol. 27. (d)**

Routh array:

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 4 | K |
| $s^3$ | 4 | 3 | |
| $s^2$ | $\frac{16-3}{4}$ | K | |
| $s^1$ | $\frac{\frac{39}{4}-4K}{\frac{13}{4}}$ | 0 | |
| $s^0$ | K | | |

For the system to be marginally stable,

$$\frac{39}{4} - 4K = 0 \Rightarrow K = \frac{39}{16}$$

**Sol. 28. (a)**

$$M(s) = \frac{G(s)}{1+G(s)H(s)}$$

Sensitivity of M to the variation in G is

$$\frac{dM}{dG} \times \frac{G}{M}$$

$$\frac{dM}{dG} = \frac{1+G(s)H(s)-G(s)H(s)}{\{1+G(s)H(s)\}^2}$$

$$\frac{dM}{dG} \times \frac{G}{M} = \frac{1}{\{1+G(s)H(s)\}^2} \times \frac{G(s)}{\frac{G(s)}{1+G(s)}}$$

$$= \frac{1}{1+G(s)H(s)}$$

**Sol. 29. (c)**

**Sol. 30. (c)**

$$\frac{C(S)}{R(s)} = \frac{\frac{k}{(s^2+2s)(s+8)}}{1+\frac{k}{(s^2+2s)(s+8)}}$$

$$= \frac{k}{s^3+10s^2+16s+k}$$

For the system to be marginally stable

$$10 - \frac{k}{16} = 0 \text{ i.e. } 10\frac{k}{16}$$

$\Rightarrow$ k = 16

Characteristic eq$^n$ is $s^3 + 10s^2 + 16s + k = 0$

| | | |
|---|---|---|
| $s^3$ | 1 | 16 |
| $s^2$ | 10 | k |
| $s^1$ | $10-\frac{k}{16}$ | |
| $s^0$ | k | |

$\therefore$ The system will oscillate at a frequency of :-
$10s^2 + 160 = 0$
$s^2 + 16 = 0$
$s = \pm j4$
or $w = \pm 4$ rad/sec

**Sol. 31. (d)**
Characteristic equation is $s^2 + 4s + 16 = 0$ comparing it with $s^2 + 2\xi w_n^2 = 0$
$\omega_n = 4$ rad/sec

$$\Rightarrow \xi = \frac{4}{2\times 4} = 0.5$$

$\therefore$ resonant frequency

$$\omega_f = \omega_n\sqrt{1-2\xi^2}$$
$$\omega_f = 4\sqrt{1-2\times 0.25}$$
$$= 2\sqrt{2}\ \text{rad/sec}$$

**Sol. 32. (b)**
For resistive network feed pack factor is always less than unity. So overall gain decreases

**Sol. 33. (b)**

1 K $\qquad \frac{\alpha K-3}{\alpha}$

For system to be stable

$$\alpha > 0, \frac{\alpha K-3}{\alpha} > 0$$

$\alpha K > 3$.

**Sol . 34. (d)**
**RH- Criteria**
Characteristic equation $s^3 + 5s^2 + 7s + 3 = 0$

| | | |
|---|---|---|
| $s^3$ | 1 | 7 |
| $s^2$ | 5 | 3 |
| $s^1$ | 32 / 5 | |
| $s^0$ | 3 | |

There are no sign change in the 1st column, therefore all the three roots lie in left half of the s-plane.

**Sol. 35. (a)**
The characteristic equation is.

$$\frac{k}{s(s^2+s+2)(s+3)+k} = 0$$

$(s^3 + s^2 + 2s)(s + 3) + k = 0$
$s^4 + s^3 + 2s^2 + 3s^3 + 3s^3 + 6s + k = 0.$
$s^4 + 4s^3 + 5s^2 + 6s + k = 0$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 5k | |
| $s^3$ | 4 | 6 | |
| $s^2$ | 7 / 2 | k | |
| $s^1$ | $\frac{21-4K}{7}\times 2 > 0$ | $s^1$ | |
| $s^0$ | k > | $s^0$ | |

For the system to be stable, k > 0 and
(21-4k) . 2/7 > 0
21 – 4k > 0 = k < 21/7
21/4 > k > 0.

**Sol. 36. (a)**

# GATE QUESTIONS

**1.** Consider $p(s) = s^3 + a_2s^2 + a_1s + a_0$ with all real coefficients. It is known that its derivative $p'(s)$ has no real roots. The number of real roots of $p(s)$ is

**[GATE - 2018]**

(a) 0 (b) 1
(c) 2 (d) 3

**2.** A closed loop system has the characteristic equation given by $s^3 + Ks^2 + (K+2)s + 3 = 0$. For this system to be stable, which one of the following conditions should be satisfied?

**[GATE - 2017]**

(a) $0 < K < 0.5$ (b) $0.5 < K < 1$
(c) $0 < K < 1$ (d) $K > 1$

**3.** Which one of the following options correctly describes the locations of the roots of the equation $s^4 + s^2 + 1 = 0$

**[GATE - 2017]**

(a)Four left half plane (LHP) roots
(b)One right half plane (RHP) root, one LHP root and two roots on the imaginary axis
(c)Two RHP roots and two LHP roots
(d)All four roots are on the imaginary axis

**4.** Given the following polynomial equation
$s^3 + 5.5s^2 + 8.5s + 3 = 0$
the number of roots of the polynomial which have real parts strictly less than -1 is __________.

**[GATE - 2016]**

**5.** The phase cross-over frequency of the transfer function $G(s) = \dfrac{100}{(s+1)^3}$ in rad / s

**[GATE - 2016]**

(a) $\sqrt{3}$ (b) $\dfrac{1}{\sqrt{3}}$
(c) 3 (d) $3\sqrt{3}$

**6.** The open loop transfer function of a unity feedback control system is given by $G(s) = \dfrac{K(s+1)}{s(1+Ts)(1+2s)}$, $K > 0, T > 0$. The closed loop system will be stable if,

**[GATE - 2016]**

(a) $0 < T < \dfrac{4(K+1)}{K-1}$ (b) $0 < K < \dfrac{4(T+2)}{T-2}$
(c) $0 < K < \dfrac{T+2}{T-2}$ (d) $0 < T < \dfrac{8(K+1)}{K-1}$

**7.** The first two rows in the Routh table for the characteristic equation of a certain closed-loop control system are given as

| | | |
|---|---|---|
| $S^3$ | 1 | (2K+3) |
| $S^2$ | 2K | 4 |

The range of K for which the system is stable is

**[GATE - 2016]**

(a) $-2.0<K<0.5$ (b) $0<K<0.5$
(c) $0<K<\infty$ (d) $0.5<K<\infty$

**8.** The transfer function of a linear time invariant systems is given by
$H(s) = 2s^4 - 5s^3 + 5s - 2$
The number of zeros in the right half of the s-plane is _____.

**[GATE - 2016]**

**9.** A closed-loop control system is stable if the Nyquist plot of the corresponding open-loop transfer function

**[GATE - 2016]**

(a)Encircles the s-plane point $(-1 + j0)$ in the counterclockwise direction as many times as the number of right-half s-plane poles.
(b)Encircles the s-plane point $(0 - j1)$ in the clockwise direction as many times as the number of right-half s-plane poles.

(c)Encircles the s-plane point (–1 +j0) in the counterclockwise direction as many times as the number of left-half s-plane poles.
(d)Encircles the s-plane point (–1+j0) in the counterclockwise direction as many times as the number of right-half s-plane zeros.

**10.** The characteristic equation of an LTI system is given by $F(s) = s^5 + 2s^4 + 3s^3 + 6s^2 – 4s – 8 = 0$.
The number of roots that lie strictly in the left half s-plane is __________

**[GATE - 2015]**

**11.** Negative feedback in a closed-loop control system DOES NOT

**[GATE - 2015]**

(a) Reduce the overall gain
(b) Reduce bandwidth
(c) Improve disturbance rejection
(d) Reduce sensitivity to parameter variation

**12.** Consider a transfer function $G_P(s) = \dfrac{ps^2 + 3ps - 2}{s^2 + (3+p)s + (2-p)}$ with p a positive real parameter. The maximum value of p until which $G_p$ remains stable is ____.

**[GATE - 2014]**

**13.** The open loop transfer function G(s) of a unity feedback control system is given as

$$G(s) = \frac{K\left(s + \frac{2}{3}\right)}{s^2(s+2)}$$

From the root locus, at can be inferred that when K tends to positive infinity.

**[GATE - 2011]**

(a) Three roots with nearly equal real parts exist on the left half of the s-plane
(b) One real root is found on the right half of the s-plane
(c) The root loci cross the jω axis for a finite value of K : K ≠ 0
(d) Three real roots are found on the right half of the s-plane

**14.** The characteristic equation of a closed – loop system is s(s+1) (s+3) k(s+2) = 0, k > 0. Which of the following statements is true?

**[GATE - 2010]**

(a) Its root are always real
(b) It cannot have a breakaway point in the range –1 < Re[s] < 0
(c) Two of its roots tend to infinity along the asymptotes Re[s] = –1
(d) It may have complex roots in the right half – plane.

**15.** The first two rows of Routh's tabulation of a third order equation are as follows.

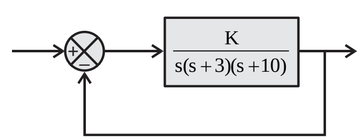
$s^3$ 2 2
$s^2$ 4 4

This means there are

**[GATE - 2009]**

(a) Two roots at s = ±j and one root in right half s-plane
(b) Two roots at s = ±j2 and one root in left half s-plane
(c) Two roots at s = ± j2 and one root in right half s-plane
(d) Two roots at s = ±j and one root in left half s-plane

**16.** Figure shows a feedback system where K > 0

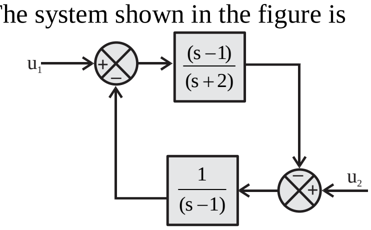



The range of K for which the system is stable will be given by

**[GATE - 2008]**

(a) 0 < K < 30 (b) 0 < K < 39
(c) 0 < K < 390 (d) K > 390

**17.** The system shown in the figure is



**[GATE - 2007]**

(a) Stable
(b) Unstable
(c) Conditionally stable
(d) Stable for input $u_1$, but unstable for input $u_2$.

**18.** A unity feedback system, having an open loop gain $G(s)H(s) = \frac{K(1-s)}{(1+s)}$ becomes stable when

**[GATE - 2005]**

(a) $|K|>1$ (b) $K > 1$
(c) $|K| < 1$ (d) $K < -1$

**19.** The open loop transfer function of a unity feedback system is $G(s) = \frac{K}{s(s^2+s+2)(s+3)}$

The range of K for which the system is stable is

**[GATE - 2004]**

(a) $\frac{21}{4} > K > 0$ (b) $13 > K > 0$
(c) $\frac{21}{4} < K < \infty$ (d) $-6 < K < \infty$

**20.** For the polynomial $P(s) = s^2 + s^4 + 2s^3 + 2s^2 + 3s + 15$ the number of roots which lie in the right half of the s-plane is

**[GATE - 2004]**

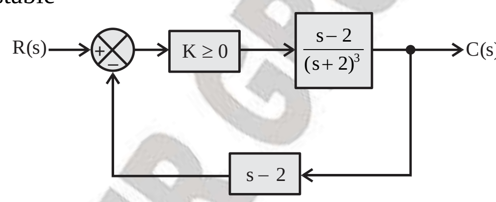
(a) 4 (b) 2
(c) 3 (d) 1

**21.** The loop gain GH of a closed loop system is given by the following expression $\frac{K}{s(s+2)(s+4)}$. The value of K for which the system just becomes unstable its

**[GATE - 2003]**

(a) K = 6 (b) K = 8
(c) K = 48 (d) K = 96

**22.** The feedback control system in the figure is stable




**[GATE - 2001]**

(a) For all $K \geq 0$ (b) Only if $K \geq 0$
(c) Only if $0 \leq K < 1$ (d) Only if $0 \leq K \leq 1$

**23.** A system described by the transfer function $H(s) = \frac{1}{s^3 + \alpha s^2 + ks + 3}$ is stable.

**[GATE - 2000]**

(a) $> 0, \alpha k < 3$ (b) $\alpha > 0, \alpha k > 3$
(c) $\alpha < 0, \alpha k > 3$ (d) $\alpha > 0, \alpha k < 3$

# SOLUTIONS

**Sol. 1. (b)**
If p(s) has "r" real roots, then p′(s) will have at least "r – 1" real roots.

**Sol. 2. (d)**

| | | |
|---|---|---|
| $S^3$ | 1 | K+2 |
| $S^2$ | K | 3 |
| $S^1$ | $\frac{K^2+2K-3}{k}$ | |
| $S^0$ | 3 | |

For stability $K > 0$ and $K^2 + 2K - 3 > 0$
So, $K^2 + 2K - 3 > 0 \Rightarrow$ i.e, $K > 1$

**Sol. 3. (c)**
CE $S^4 + S^2 + 1 = 0$

| | | | | |
|---|---|---|---|---|
| $S^4$ | $1S^4$ | $1S^2$ | $1S^0$ | |
| $S^3$ | $0^{\to 4}$ | $0^{\to 2}$ | 0 | 1 ROZ |
| $S^2$ | 0.5 | 1 | | |
| $S^1$ | $\frac{-3}{0.5} = -6$ | | | |
| $S^0$ | 1 | | | |

(sign changes: 1 between $S^2$ and $S^1$, 2 between $S^1$ and $S^0$)

$\Rightarrow$ 2 sign changes and 1 ROZ
$\Rightarrow$ 2 poles in right half of S = plane
And symmetrical poles in the LHS – plane.

**Sol. 4. (2)**
$(Z-1)^3 + (5.5)(Z-1)^2 + 8.5(Z-1) + 3 = 0$
$Z^3 - 3Z^2 + 3Z - 1 + 5.5Z^2 - 11Z + 5.5 + 8.5Z - 8.5 + 3 = 0$
$Z^3 + 2.5Z^2 + 0.5Z - 1 = 0$

| | | |
|---|---|---|
| $+Z^3$ | 1 | 0.5 |
| $+Z^2$ | 2.5 | –1 |
| $+Z^1$ | $\frac{2.25}{2.5}$ | |
| $-Z^0$ | –1 | |

Two roots real parts are less than -1.

**Sol. 5. (a)**
$$\angle \frac{100}{(j\omega+1)^3} = -180°$$
$-3\tan^{-1}\omega_{pc} = -180$
$\omega_{pc} = \sqrt{3}$

**Sol. 6. (c)**
$$CE = 1 + \frac{Ks+K}{(s+Ts^2)(1+2s)}$$
$s + Ts^2 + 2s^2 + 2Ts^3 + Ks + K = 0$
$2Ts^3 + (T+2)s^2 + (K+1)s + K = 0$

| | | |
|---|---|---|
| $S^3$ | 2T | K+1 |
| $S^2$ | T+2 | K |
| $S^1$ | $\frac{(T+2)(K+1)-2TK}{T+2}$ | |
| $S^0$ | K | |

$K > 0$
$(T + 2)(K+1) - 2TK > 0$
$KT + 2K + T + 2 - 2TK > 0$
$K(T - 2T + 2) > -(T + 2)$
$K(T-2) < T + 2$
$$K < \frac{T+2}{T-2}$$

**Sol. 7. (d)**

| | | |
|---|---|---|
| $S^3$ | 1 | 2k+3 |
| $S^2$ | 2k | 4 |
| $S^1$ | $\frac{2k(2k+3)-4}{2k} > 0$ for stability | |
| $S^0$ | 4 | |

$4k^2 + 6k - 4 > 0$
$k > -2,\ k > 0.5$
$0.5 < k < \infty$

**Sol. 8. (3)**

TF $H(s) \Rightarrow 2s^4 - 5s^3 + 5s - 2$

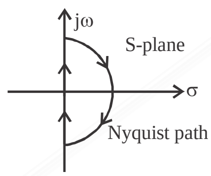
RH – Criteria

| | | | |
|---|---|---|---|
| ① $+S^4$ | 2 | 0 | –2 |
| ② $-S^3$ | –5 | +5 | |
| $+S^2$ | 2 | –2 | |
| ③ $+S^1$ | 0(2) | | |
| $-S^0$ | –2 | | |

3 Sign Changes

3 Roots (Zeros) in the RH -S-Plane.

**Sol. 9. (a)**




N = P – Z

For closed loop stability Z = 0, N = P

∴ (-1, j0) should be encircled in Counter clock wise direction equaling P poles in RHP.

**Sol. 10. (2)**

$$F(s) = s^5 + 2s^4 + 3s^3 + 6s^2 - 4s - 8 = 0$$

| | | | |
|---|---|---|---|
| $s^5$ | 1   3 | | –4 |
| $s^4$ | 2   6 | | –8 |
| $s^3$ | 0(E) | 0(t) | |
| $s^2$ | | | Auxiliary equation |
| $s^1$ | | | $A(s) = 2s^4 + 6s^2 - 8$ <br> $\frac{dA(s)}{ds} = 8s^3 + 125 - 0$ |
| $s^0$ | | | |
| $s^4$ | 2 | 6 | –8 |
| $s^3$ | 8 | 12 | 0 |
| $s^2$ | 3 | –8 | 0 |
| $s^1$ | $\frac{100}{3}$ | 0 | |
| $s^0$ | –8 | 0 | |

$A(s) = 2s^4 + 6s^2 - 8$

$$\frac{-6 \pm \sqrt{36 - 64}}{2 \times 2}$$

2 complex are 1 right and 2 left way

**Sol. 11. (b)**

Since for any system gain x bandwidth is always a constant quantity.

Negative feedback reduces overall gain of system & hence bandwidth increases.

**Sol. 12. (2)**

Given $G_p(s) = \dfrac{ps^2 + 3ps - 2}{s^2 + (3+p)s + (2-p)}$

By R–H criteria

The characteristic eqution is $s^2 + (3 + p)s + (2 - p) = 0$

i.e. $s^2 + (3 + p)s + (2 - p) = 0$

By forming R-H array,

| | | |
|---|---|---|
| $s^2$ | 1 | $(2-p)$ |
| $s^1$ | $(3+\phi)$ | 0 |
| $s^0$ | $(2-p)$ | |

For stability, first column elements must be positive and non – zero.

i.e. (1) (3 + p) ? 0 ⇒ p > –3

and (2)(2–p) > 0 is ⇒ p < 2

i.e. –3 < p < 2

The maximum value of p unit which $G_p$ remains stable is 2.

**Sol. 13. (a)**

$$G(s) = \frac{K\left(s + \frac{2}{3}\right)}{s^2(s+2)}$$

Steps for plotting the root – locus

1. Root loci starts at s = 0, s = 0 and s = -2
2. n > m, therefore, number of branches of root locus b = 3
3. Angle of asymptotes is given by

$$\frac{(2q+1)180°}{n-m}, q = 0.1$$

(I) $\dfrac{(2\times 0+1)180°}{(3-1)} = 90°$

(II) $\dfrac{(2\times 1+1)180°}{(3-1)} = 270°$

4. The two asymptotes intersect on real axis at centroid

$$x = \frac{\sum \text{Poles} - \sum \text{Zeroes}}{n-m} = \frac{-2-\left(-\frac{2}{3}\right)}{3-1} = -\frac{2}{3}$$

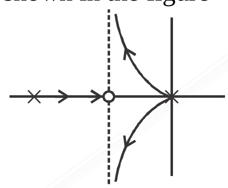
5. Between two open – loop poles s = 0 and s =–2 there exist a breakaway point.

$$K = -\frac{s^2(s+2)}{\left(s+\frac{2}{3}\right)}$$

$$\frac{dk}{ds} = 0$$

S = 0

Root locus is shown in the figure



Three roots with nearly equal parts exist on the left half of s-plane.

**Sol. 14. (c)**

Given characteristic equation

s (s + 1)(s +3) + K(s +2) = 0

$s(s^2 - + 4s + 3) + K(s +2) = 0$

$s^3 + 4s^2 + (3 + K)s + 2K = 0$

From Routh's tabulation method

| | | |
|---|---|---|
| $s^3$ | 1 | 3+K |
| $s^2$ | 4 | 2K |
| $s^1$ | $\frac{4(3+K)-2K(1)}{4}$ $=\frac{12+2K}{4}>0$ | |
| $s^0$ | 2K | |

There is no sign change in the first column of routh table. So not root is lying in right half of s-plane.

For plotting root locus, the equation can be written as

$$1+\frac{K(s+2)}{s(s+1)(s+3)} = 0$$

Open loop transfer function

$$G(s) = \frac{K(s+2)}{s(s+1)(s+3)}$$

Root locus is obtain in following steps:

1. No. of poles n = 3, at s = 0, s = –1 and s= –3
2. No of zeroes m = 1, at s = –2
3. The root locus on real axis lies between s = 0 and s = –1, between s = –3 and s = –2.
4. Breakaway point lies between open loop poles of the system. Here breakaway point lies in the range –1 < Re[s] < 0
5. Asymptotes meet on real axis at a point c, given by

$$C = \frac{\sum \text{real part of poles} - \sum \text{real parts of zeroes}}{n-m}$$

$$= \frac{(0-1-3)-(-2)}{3-1} = -1$$

As no. of poles is 3, so two root loci branches terminates at infinity along asymptotes Re(s)= –1.

**Sol. 15. (d)**

Given Routh's tabulation.

| | | |
|---|---|---|
| $s^3$ | 2 | 2 |
| $s^2$ | 4 | 4 |
| $s^1$ | 0 | 0 |

So the auxiliary equation is given by

$4s^2 + 4 = 0$

$s^2 = -1$

$s = \pm J$

From table we have characteristic equation as

$2s^3 + 2s + 4s^2 + 2 = 0$

$s^3 + s + 2s^2 + 2 = 0$

$s(s^2 + 1) + 2(s^2 + 1) = 0$

s = –2, s = ± j

**Sol. 16. (c)**

Characteristic equation for the system

$$1+\frac{K}{s(3+3)(s+10)} = 0$$

s (s +3)(s+10) + K =0

$s^3 + 13s^2 + 30s + K = 0$

Applying Routh's stability criteria

| | | |
|---|---|---|
| $s^3$ | 1 | 30 |
| $s^2$ | 13 | K |

| | | |
|---|---|---|
| $s^1$ | $\frac{(13\times 30)-K}{13}$ | |
| $s^0$ | K | |

For stability there should be no sign change in first column

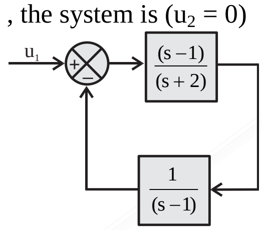
So, $390 - K > 0 \Rightarrow K < 390$

$K > 0$

$0 < K < 90$

**Sol. 17. (d)**

For input $u_1$ , the system is ($u_2 = 0$)

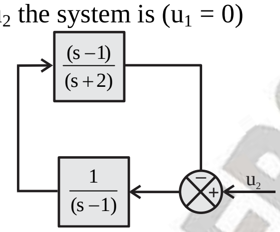



system response is

$$H_1(s) = \frac{\frac{(s-1)}{(s+2)}}{1+\frac{(s-1)}{(s+2)(s-1)}} = \frac{(s-1)}{(s+3)}$$

Poles of the system is lying at s = –3 (negative s-plane) so this is stable.

For input $u_2$ the system is ($u_1 = 0$)




System response is

$$H_2(s) = \frac{\frac{1}{(s-1)}}{1+\frac{1}{(s-1)}\frac{(s-1)}{(s+2)}} = \frac{(s+2)}{(s-1)(s+3)}$$

One pole of the system is lying in right half of s-plane, so the system is unstable.

**Sol. 18. (c)**

Characteristic equation for the given system

$1 + G(s) H(s) = 0$

$$1+K\frac{(1-s)}{(1+s)}=0$$

$(1 + s) + K(1 – s) = 0$

$S(1 – K) + (1 + K) =0$

For the system to be stable, coefficient of characteristic equation should be of same sign.

$1 – K > 0, K + 1 > 0$

$K < 1, K > –1$

$–1 < K < 1$

$|K| < 1$

**Sol. 19. (a)**

For ufb system the characteristics equation is

$1 + G(s) = 0$

$$1+\frac{K^{1+G(s)}}{s(s^2+2s+2)(s+3)}=0$$

$s^4 + 4s^3 + 5s^2 + 6s + K = 0$

The routh table is shown below. For system to be stable.

$$0 < K \text{ and } 0 < \frac{(21-4K)}{2/7}$$

This gives $0<K<\frac{21}{4}$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 5 | K |
| $s^3$ | 4 | 6 | 0 |
| $s^2$ | $\frac{7}{2}$ | K | |
| $s^1$ | $\frac{21-4K}{7/2}$ | 0 | |
| $s^0$ | K | | |

**Sol. 20. (b)**

We have

$P(s) = s^5 + s^4 + 2s^3 + 3s + 15$

The routh table is shown below.

If $\varepsilon \to 0^+$ then $\frac{2\varepsilon+12}{\varepsilon}$ is positive and $\frac{-15\varepsilon^2-24\varepsilon-144}{2\varepsilon+12}$ is negative. Thus there are two sign change in first column. Hence system has 2 root on RHS of plane.

| | | | |
|---|---|---|---|
| $s^5$ | 1 | 2 | 3 |
| $s^4$ | 1 | 2 | 15 |
| $s^3$ | ε | –12 | 0 |

| | | | |
|---|---|---|---|
| $s^2$ | $\frac{2\varepsilon+12}{\varepsilon}$ | 15 | 0 |
| $s^1$ | $\frac{-15\varepsilon^2-24s-144}{2\varepsilon+12}$ | | |
| $s^0$ | 0 | | |

**Sol. 21. (c)**
Characteristic equation of the system is given by

$1 + GH = 0$

$$1+\frac{K}{s(s+2)(s+4)}=0$$

$s(s+2)(s+4)+K=0$

$s^3 + 6s^2 + 8s + K = 0$

Applying routh's criteria for stability

| | | |
|---|---|---|
| $s^3$ | 1 | 8 |
| $s^2$ | 6 | K |
| $s^1$ | $\frac{K-48}{6}$ | |
| $s^0$ | K | |

System becomes unstable if

$$\frac{K-48}{6}=0 \Rightarrow K=48$$

**Sol. 22. (c)**
From the Block diagram transfer function is

$$T(s)=\frac{G(s)}{1+G(s)H(s)}$$

Where $G(s)=\frac{K(s-2)}{(s+2)}$ And $H(s) = (s-2)$

The characteristic equation is

$1+ G(s) H(s) = 0$

$$1+\frac{K(s-2)}{(s+2)^2}(s-2)=0$$

Or $(s+2)^2 + K(s-2)^2 = 0$

or $(1 + K)s^2 + 4(1 - K)s + 4K + 4 = 0$

Routh Table is shown below. For system to be stable $1 + k > 0$, and $4 + 4k > 0$ and $4 - 4k > 0$. This gives $-1 < K < 1$

As per question for $0 \le K < 1$

| | | |
|---|---|---|
| $s^2$ | $1+k$ | $4+4k$ |
| $s^1$ | $4-4k$ | 0 |
| $s^0$ | $4+4k$ | |

**Sol. 23. (b)**
The characteristics equation is $s^2 + \alpha s^2 + ks + 3 = 0$

The Routh Table is shown below

For system to be stable $\alpha > 0$ and $\frac{\alpha K-3}{\alpha} > 0$

Thus $\alpha > 0$ and $\alpha K > 3$

| | | |
|---|---|---|
| $s^3$ | 1 | K |
| $s^2$ | $\alpha$ | 3 |
| $s^1$ | $\frac{\alpha K-3}{\alpha}$ | 0 |
| $s^0$ | 3 | |

# ESE OBJ QUESTIONS

**1.** What is the open –loop transfer function for the system, whose characteristic equation is
$F(s) = s^3 + 3s^2 + (K+2)s + 5K = 0$?

**[EE ESE - 2017]**

(a) $G(s)H(s) = \frac{5K}{s(s+1)(s+3)}$

(b) $G(s)H(s) = \frac{Ks}{s(s+1)(s+2)}$

(c) $G(s)H(s) = \frac{K(s+5)}{s(s+1)(s+2)}$

(d) $G(s)H(s) = \frac{5K}{s(s+1)(s+2)}$

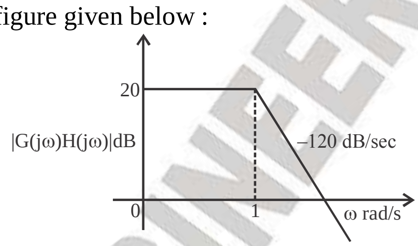
**2.** The closed-loop transfer function of a system is $\frac{C(s)}{R(s)} = \frac{s-2}{s^3 - 8s^2 + 19s + 12}$

The system is

**[EE ESE - 2017]**

(a) Stable
(b) Unstable
(c) Conditionally stable
(d) Critically stable

**3.** The magnitude plot for the open – loop transfer function of a control system is shown in the figure given below :




Its open – loop transfer function, G(s)H(s), is

**[EC ESE - 2017]**

(a) 10(s+1)
(b) $\frac{1}{s+1}$
(c) $\frac{10}{s+1}$
(d) 20(s+1)

**4.** When gain K of the open-loop transfer function of order greater than unity is varied from zero to infinity, the closed-loop system

**[EE ESE - 2016]**

(a) May become unstable
(b) Stability may improve
(c) Stability may not be affected
(d) Will become highly stable

**5.** In a closed-loop control system

**[EE ESE - 2016]**

(a) Control action is independent of output
(b) Output is independent of input
(c) There is no feedback
(d) Control action is dependent on output

**6.** The characteristic polynomial of a system can be defined as

**[EE ESE - 2016]**

(a)Denominator polynomial of given transfer function
(b)Numerator polynomial of given transfer function
(c)Numerator polynomial of a closed-loop transfer function
(d)Denominator polynomial of a closed-loop transfer function.

**7.** The characteristic equation of a certain feedback control system is given by $s^4 + 4s^3 + 13s^2 + 36s + k = 0$. The range of values of k for which the feedback system is stable is given by

**[EC ESE - 2016]**

(a) $0 < k < 4$
(b) $4 < k < 36$
(c) $0 < k < 36$
(d) $13 < k < 36$

**8.** The feedback system with characteristic equation $s^4 + 20Ks^3 + 5s^2 + 10s + 15 = 0$ is

**[EC ESE - 2015]**

(a) Stable for all values of K
(b) Stable for positive values of K
(c) Stable for $7.0 < K < \infty$
(d) Unstable for any value of K

**9.** The oscillation frequency of the system with the characteristic equation
$s^6 + 2s^5 + 3s^4 + 3s^2 + 2s + 1 = 0$ is
**[EC ESE - 2015]**
(a) + 1 radian/sec (b) –1 radian/sec
(c) j1 radian/sec (d) –j1 radian/sec

**10.** None of the poles of a linear control system lies in the right – half of s – plane. For a bounded input, the output of this system
**[EE ESE - 2015]**
(a) Is always bounded
(b) Could be unbounded
(c) Always tends to zero
(d) None of the above

**11.** How may roots of the following equation lie in the right – half of s – plane ?
$2s^4 + s^3 + 2s^2 + 5s + 10 = 10$
**[EE ESE - 2015]**
(a) 1 (b) 2
(c) 3 (d) 4

**12.** The first element of each of the rows of a Routh Hurwitz stability test showed the signs as follows

| **Row** | I | II | III | IV | V |
|---|---|---|---|---|---|
| **Sign** | + | + | – | + | – |

Consider the following statements:
(i) The system has three roots in the right – half of s-plane.
(ii) The system has three roots in the left – half of s-plane.
(iii) The system is stable
(iv) The system is unstable
Which of the above statements about the system are correct?
**[EE ESE - 2015]**
(a) i and iii (b) i and iv
(c) ii and iii (d) ii and iv

**13.** Consider the following statements with respect to Routh- Hurwitz criterion :
(i)It can be used to determine relative stability.
(ii)It is valid only for real coefficients of the characteristic equation
(iii)It is applicable only for non – linear systems.
(iv) It does not provide the exact location of closed - loop poles in left or right - half of s - plane.
Which of the above statements are correct?
**[EE ESE - 2015]**
(a) i, ii and iii only (b) iii and iv only
(c) i, ii and iv only (d) i, ii, iii and iv

**14.** The characteristic equation of a feedback system is $s^3 + Ks^2 + 5s + 10 = 0$. For a stable system, the value of K should be less than
**[EE ESE - 2015]**
(a) 1 (b) 2
(c) 3 (d) 4.5

**15.** The characteristic equation of a feedback control system is $s^4 + s^3 + 2s^2 + 4s + 15 = 0$. The number of roots in the right half of the s - plane is
**[EE ESE - 2014]**
(a) 4 (b) 3
(c) 2 (d) 1

**16.** A feedback system with characteristic equation $s^4 + 20ks^3 + 5s^2 + 10s + 15 = 0$ is:
**[EC ESE - 2013]**
(a) Stable for all value of K
(b) Stable only for $K \geq 0$
(c) Stable for $\infty > K \geq 70$
(d) Unstable for all values of K

**17.** The system having the characteristic equation $s^3 + 4s^2 + s – 6 + K = 0$ will be stable for
**[EC ESE - 2013]**
(a) $K > 6$ (b) $0 < K < 6$
(c) $6 < K < 10$ (d) $0 < K < 10$

**18.** Consider the following statements about Routh- Hurwitz criterion:
If all the elements in one row of Routh array are zero, then there are
(i) Pairs of conjugate roots on imaginary axis.
(ii) Pairs of equal real roots with opposite sign.
(iii) Conjugate roots forming a quadrate in the s -plane.
Which of these statements are correct?
**[EE ESE - 2013]**

(a) i and ii only (b) i and iii only
(c) ii and iii only (d) i, ii and iii

**19.** A unity feedback system has forward transfer function

$$G(s) = \frac{K}{s(s+3)(s+10)}$$

The range of K for the system to be stable is

**[EE ESE - 2012]**

(a) $0 < K < 390$ (b) $0 < K < 39$
(c) $0 < K < 3900$ (d) None of above

**20.** The characteristic equation of a control system is given below
$F(s) = s^4 + s^3 + 3s^2 + 2s + 5 = 0$
The system is

**[EE ESE - 2012]**

(a) Stable (b) Critically stable
(c) Conditionally stable (d) Unstable

**21. Statement (I)**:All the systems which exhibit overshoot in transient response will also exhibit resonance peak in frequency response.
**Statement (II)**: A large resonance peak in frequency response corresponds to a large overshoot in transient response.

**[EC ESE - 2012]**

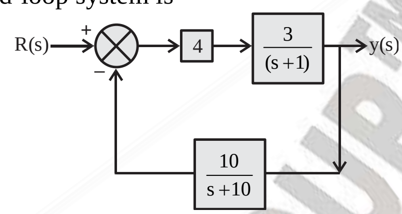
(a) Both Statement (I) and Statement (II) are individually true and statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
(c) Statement (I) is true but Statement (II) is false
(d) Statement (I) is false but Statement (II) is true.

**22.** The characteristic equation of control system is given as
$S^4 + 8s^3 + 24s^2 + 32s + K = 0$
What is the value of K for which the system is unstable?

**[EC ESE - 2011]**

(a) 10 (b) 20
(c) 60 (d) 100

**23.** The characteristic equation of the below closed-loop system is




**[EC ESE - 2010]**

(a) $s^2 + 11s + 10 = 0$
(b) $s^2 + 11s + 130 = 0$
(c) $s^2 + 10s + 120 = 0$
(d) $s^2 + 10s + 12 = 0$

**24. Assertion (A):** All the coefficients of the characteristics equation should be positive and no term should be missing in the characteristic equation for a system to be stable.
**Reason (R):** If some of the coefficients are zero or negative then the system is not stable.

**[EE ESE - 2010]**

(a)Both A and r are true and R is the correct explanation of A.
(b)Both a and R are true but R is NOT the correct explanation of A.
(c)A is true but R is false
(d)A is false but R is true

**25.** Consider the following statements in connection with pole location
(i) A distinct pole always lies on the real axis.
(ii) A dominant constant pole has a large time
Which of the above statements is/are correct?

**[EE ESE - 2010]**

(a) Both i and ii (b) Neither i nor ii
(c) i only (d) ii only

**26.** Consider the following statements: in connection with 'the closed - loop poles of feedback control system
(i)Poles on jω - axis will make the output amplitude neither decaying nor growing in time.
(ii)Dominant closed – loop poles occur in the form of a complex conjugate pair.
(iii)The gain of a higher order system is adjusted so that there will exist a pair of complex conjugate closed – loop on jω - axis.

(iv)The presence of complex conjugate closed-loop poles reduces the effects of such non-linearities as dead zones, backlash and coulomb friction.

**[EE ESE - 2010]**

(a) ii only (b) ii, iii and iv only
(c) i, ii and iv only (d) i, ii, iii and iv

**27.** The feedback control system represented by the open loop transfer function

$$G(s)H(s)=\frac{10(s+2)}{[(s+1)(s+3)(s-5)]} \text{ is}$$

**[EE ESE - 2010]**

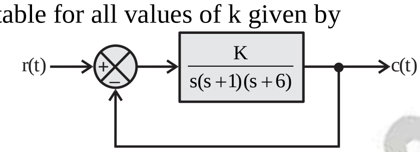
(a) Unstable (b) Stable
(c) Marginally stable (d) Insufficient data

**28.** Using Routh's criterion, the number of roots characteristic equation in the right half s – plane for the characteristic equation:
$s^4 + 2s^3 + 2s^2 + 3s + 6 = 0$

**[EE ESE - 2010]**

(a) One (b) Two
(c) Three (d) Four

**29.** The feedback system shown in figure below is stable for all values of k given by




**[EE ESE - 2010]**

(a) $k > 0$ (b) $k < 0$
(c) $0 < k < 42$ (d) $0 < k < 60$

**30.** The unit step response of a system is $[1 - e^{-t}(1 + t)]\, u(t)$. What is the nature of the system in turn of stability ?

**[EE ESE - 2009]**

(a) Unstable (b) Stable
(c) Critically stable (d) Oscillatory

**31.** The characteristic equation of a feedback control system is given by:
$s^2 + 6s^2 + 9s + 4 = 0$
What are the number of roots in the left – half of the s – plane ?

**[EE ESE - 2009]**

(a) Three (b) Two
(c) One (d) Zero

**32.** Consider the following statements:
(i)A system is said to be stable if its output is bounded for any input.
(ii)A system is stable if all the roots of the characteristic equation lie in the left half of the s – plane.
(iii)A system is stable if all the roots of the characteristic equation have negative real parts.
(iv)A second order system is always stable for finite positive values of open loop gain.
Which of the above statements is/are correct?

**[EE ESE - 2009]**

(a) ii, iii and iv (b) i only
(c) ii and iii only (d) iii and iv only

**33.** Which one of the following statements is correct for the open – loop transfer function ?

$$G(s)=\frac{K(s+3)}{s(s-1)} \text{ for } K>1$$

**[EE ESE - 2009]**

(a) Open - loop system is stable but the closed - loop system is unstable.
(b)Open - loop system is unstable but the closed – loop system is stable.
(c) Both open - loop and closed - loop systems are unstable.
(d) Both open - loop and closed - loop systems are stable.

**34.** The characteristic equation of a control system is given as
$s^4 + 4s^3 + 4s^2 + 3s + K = 0$
What is the value of K for which this system is marginally stable?

**[EC ESE - 2009]**

(a) $\frac{9}{16}$ (b) $\frac{19}{16}$
(c) $\frac{29}{16}$ (d) $\frac{39}{16}$

**35.** How many number of branches the root loci of the equation.
$s (s + 2) (s + 3) + K (s + 1) = 0$ have?

**[EC ESE - 2009]**

(a) Zero (b) One

(c) Two (d) Three

**36.** The characteristic equation of a control system is given as
$s^4 + 8s^3 + 24s^2 + 32s + K = 0$
What is the range of value of K for this system to be stable?
**[EC ESE - 2009]**
(a) $0 \le K < 80$ (b) $0 \le K < 100$
(c) $0 \le K < 300$ (d) $0 \le K < 600$

**37.** How many roots with positive real parts do the equation $s^3 + s^2 - s + 1 = 0$ have?
**[EC ESE - 2009]**
(a) Zero (b) One
(c) Two (d) Three

**38.** For what positive value of K does the polynomial,
$s^4 + 8s^3 + 24s^2 + 32s + K$ have roots with zero real parts?
**[EC ESE - 2009]**
(a) 10 (b) 20
(c) 40 (d) 80

**39.** The characteristic equation of a control system is given by
$s^5 + s^4 + 2s^3 + 2s^2 + 4s + 6 = 0$
What is the number of roots of the equation which lie in the right half of s-plane?
**[EC ESE - 2008]**
(a) Zero (b) 1
(c) 2 (d) 3

**40.** Consider the unity feedback system with
$$G(s) = \frac{K}{(s^2+2s+2)(s+2)}.$$
The system is marginally stable. What is the radian frequency of oscillation?
**[EC ESE - 2008]**
(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) $\sqrt{5}$ (d) $\sqrt{6}$

**41.** The open loop transfer function of a unity feedback control system is
$$G(s) = \frac{K}{s(s+1)(s+5)}$$
What is the value of K for its stable operation?
**[EC ESE - 2008]**
(a) $0 < K < 5$ only (b) $0 < K < 6$ only
(c) $1 < K < 5$ only (d) $0 < K < 30$

**42.** Consider the following statements:
When all the elements in one row of the Routh's tabulation are zero then this conditions indicates:
(i) One pair of real roots with opposite sign in s – plane.
(ii) One pair of conjugate roots on the imaginary axis in s – plane
(iii) Conjugate roots forming a quadrate in s-plane
Which of the statements given below is/are correct?
**[EE ESE - 2008]**
(a) i only (b) ii only
(c) iii only (d) i, ii and iii

**43.** What is the range of K for which the open loop transfer function
$$G(s) = \frac{K}{s^2(s+a)}$$
represents an unstable closed loop system ?
**[EE ESE - 2008]**
(a) $K > 0$ (b) $K = 0$
(c) K , 0 (d) $-\infty < K < \infty$

**44.** The characteristic polynominal of a discrete time system is given by $z^2 + z + a$. For what value of 'a' is the system stable ?
**[EE ESE - 2008]**
(a) 2 (b) 0.5
(c) 1.5 (d) – 0.5

**45.** In the time domain analysis of feedback control systems which one pair of the following is not correctly matched ?
**[EE ESE - 2008]**
(a)Under damped : Minimizes the effect of non-linearities
(b)Dominant poles: Transients die out more Rapidly

(c)For away poles : Transients die out more
To the left half of rapidly s – plane
(d)A pole near to : Magnitude of transient is the left of domin-small-ant complex poles and near a zero.

**46.** Which of the following transfer functions is/are minimum phase transfer function(s) ?

(i) $\frac{1}{(s-1)}$

(ii) $\frac{(s-1)}{(s+3)(s+4)}$

(iii) $\frac{(s+2)}{(s+3)(s-4)}$

Select the correct answer using the code given below:

**[EE ESE - 2008]**

(a) i and iii (b) i only
(c) ii and iii (d) None of these

**47.** If the poles of a system lie on the imaginary axis, the system will be

**[EE ESE - 2008]**

(a) Stable
(b) Conditionally stable
(c) Marginally stable
(d) Unstable

**48.** Which one of the following is the correct statement?

**[EE ESE - 2008]**

A non-minimum phase network is one whose transfer function has
(a) Zeros in the left hand plane and poles in the right hand plane
(b) Zeros and poles in the left hand plane
(c) Zeros in the right hand plane and poles in the left hand plane
(d) Arbitrary distribution of zeroes and poles in the s - plane

**49.** The open – loop transfer function of a unity feedback control system is given by $G(s) = Ke^{-Ts}$, where K and T are constant and these are greater than zero. The stability of close-loop system depends on which of the following ?

**[EE ESE - 2007]**

(a) K only
(b) Both K and T
(c) T only
(d) Neither on K nor on T

**50.** Consider the following statements regarding Routh-Hurwitz criterion for stability:
(i) Routh-Hurwitz criterion is a necessary and sufficient condition for stability.
(ii) The relative stability is dictated by the location of the roots of the characteristic equation.
(iii) A stable system is a dynamic system with a bounded response to a bounded input.
Which of the statements given above are correct?

**[EE ESE - 2007]**

(a) i and ii (b) ii and iii
(c) i and iii (d) i,ii and iii

**51.** The characteristic equation of a system is given as $s^3 + 25s^2 + 10s + 50 = 0$.
What is the number of roots in the right half s-plane and on the jω axis, respectively ?

**[EE ESE - 2007]**

(a) 1, 1 (b) 0,0
(c) 2, 1 (d) 1, 2

**52.** The transfer function of a system is (1 – s)/(1 + s). The system is then which one of the following ?

**[EE ESE - 2007]**

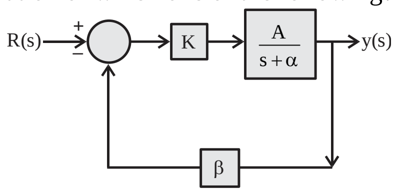
(a) Non-minimum phase system
(b) Minimum phase system
(c) Low-pass system
(d) Second- order system

**53.** Which one of the following is the correct statement ?
A minimum phase transfer function has

**[EE ESE - 2007]**

(a) Poles in the right half of s-plane
(b) Zeroes in the right half of s-plane
(c) Poles in the left half of s-plane and zeroes in the right half of s-plane
(d) No poles of zeros in the right half of the s-plane or on the jω-axis excluding the origin.

**54.** For the system given below, the feedback does not reduce the closed-loop sensitivity due to variation of which one of the following?




**[EC ESE - 2007]**

(a) K  (b) A
(c) $K\alpha$  (d) $\beta$

**55. Assertion (A)**: The closed loop stability can be determined from the poles of an open loop system and the polar plot of frequency response.
**Reason (R)**: Unstable system has right half-poles.

**[EC ESE - 2006]**

(a)Both A and R are true and R is the correct explanation of A
(b)Both A and R are true but R is NOT the correct explanation of A
(c)A is true but R is false.
(d)A is false but R is true.

**56.** The characteristic equation of a control system is
$s^5 + 15s^4 + 85s^3 + 225s^2 + 274s + 120 = 0$.
What are the number of roots of the equation which lie to the left of the line s + 1 = 0?

**[EC ESE - 2006]**

(a) 2  (b) 3
(c) 4  (d) 5

**57.** The characteristic equation of second-order sampled data system is given by
$F(z) = a_2Z^2 + a_1z + a_0 = 0,\ a_2 > 0$
What are the stability constraints for this system?
(i) $a_2 + a_1 + a_0 > 0$  (ii) $a_2 - a_1 + a_0 > 0$
(iii) $|a_0| < a_2$  (iv) $|a_0| > a_2$
(v) $|a_1| < a_2$
Select the correct answer using the code given below:

**[EE ESE - 2006]**

(a) Only i, ii and iii  (b) Only i, ii and iv
(c) Only i, iii and v  (d) Only ii, iii and v

**58.** An electromechanical closed-loop control system has the transfer function

$$\frac{C(s)}{R(s)} = \frac{k}{s(s^2+s+1)(s+4)+k}$$

Which one of the following is correct ?

**[EE ESE - 2006]**

(a)The system is stable for all positive values of k.
(b)The system is unstable for all values of k.
(c)The system is stable for values of k between zero and 3.36.
(d)The system is stable for values of k between 1.6 and 2.45

**59.** For a discrete-time system to be stable, all the poles of the Z-transfer function should lie

**[EE ESE - 2006]**

(a) Within a circle of unit radius
(b) Outside the circle of unit radius
(c) On left-half of z-plane
(d) On right-half of z-plane

**60. Assertion (A):** For a stable feedback control system, the zeros of the characteristic equation must all be located in the left-half of the s-plane.
**Reason (R):** The poles of the closed-loop transfer function are the zeros of the characteristic equation.

**[EE ESE - 2006]**

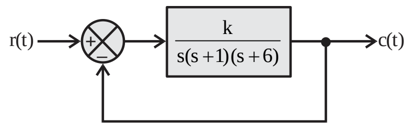
(a)Both A and R are true and R is the correct explanation of A.
(b)Both A and R are true but R is NOT the correct explanation of A.
(c)A is true but R is false
(d)A is false but R is false.

**61.** Consider the following equation:
$2s^4 + s^3 + 3s^2 + 5s + 10 = 0$
How many roots does this equation have in the right half of s – plane?

**[EE ESE - 2006]**

(a) One  (b) Two
(c) Three  (d) Four

**62.** For which of the following values of k, the feedback system shown in the below figure is stable ?




**[EE ESE - 2005]**

(a) k > 0 (b) k < 0
(c) 0< k< 42 (d) 0 < k< 60

**63.** The characteristic equation for a third-order system is $q(s) = a_0s^3 = a_1s^2 + a_2s + a_3 = 0$. For the third-order system to be stable, besides that all the coefficients have to be positive, which one of the following has to be satisfied as a necessary and sufficient condition ?

**[EE ESE - 2004]**

(a) $a_0a_1 > a_2a_3$ (b) $a_1a_2 \geq a_0a_3$
(c) $a_2a_3 \geq a_1a_0$ (d) $a_0a_3 \geq a_1a_2$

**64.** A control system is defined in s – domain. Following points regarding the poles of the transfer function obtained from the characteristic equation were noted :
(i) Poles with positive real part denote stable system
(ii) Complex poles always occur in pairs
(iii) A pole s = - σ(σ > 0) means that the transient response contains exponential decay.
Which of the above are correct?

**[EE ESE - 2004]**

(a) i and ii (b) i and iii
(c) ii and iii (d) i, ii and iii

**65. Assertion (A):** Stability of a system deteriorates when integral control is incorporated in it.
**Reason (R):** With integral control order of the system increases, and higher the order of the system the more the system tends to become unstable.

**[EE ESE - 2003]**

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false.
(d) A is false but R is true.

**66.** Consider the following statements:
(i) A discrete - time system is said to be stable if and only if its response of unit impulse δ(t) decays with k.
(ii) Routh – Hurwitz testing may be applied to determine the stability of discrete – data system using bilinear transformation $Z = \frac{1+\omega}{1-\omega}$
(iii) A discrete data system is unstable if any of roots of the characteristic equation lies within the unit circle on the complex plane.
Which of these statements is/are correct?

**[EE ESE - 2003]**

(a) i and ii (b) i and iii
(c) iii only (d) ii and iii

**67. Assertion (A):** Relative stability of a system reduces due to the presence of transportation lag.
**Reason (R):** Transportation lag can be conveniently handled by Bode plot.

**[EE ESE - 2002]**

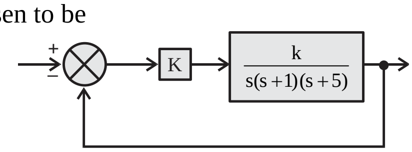
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**68.** The characteristic equation of a system is given by $3s^4 + 10s^3 + 5s^2 + 2 = 0$. This system is

**[EE ESE - 2002]**

(a) Stable (b) Marginally stable
(c) Unstable (d) Neither a, b nor c

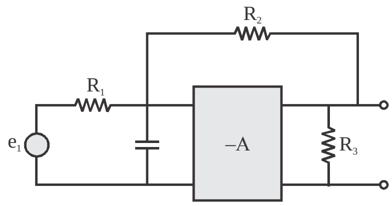
**69.** The closed loop system shown below becomes marginally stable if the constant K is chosen to be



**[EE ESE - 2002]**

(a) 10 (b) 20
(c) 30 (d) 40

**70.** The feedback amplifier shown in the figure below:




**[EC ESE - 2002]**

(a) Is stable for all value of R and C
(b) Is stable only for $R_1R_2 = R_3$
(c) Is stable only for $R_1C = R_2R_3$
(d) Is stable is $R_1/R_2 = C/R_3$

**71.** The given characteristic polynomial $s^4 + s^3 + 2s^2 + 2s + 3 = 0$ has

**[EC ESE - 2001]**

(a) Zero root in RHS of s-plane
(b) One root in RHS of s-plane
(c) Two roots in RHS of s-plane
(d) Three roots in RHS of s-plane

**72.** In order to use Routh - Hurwitz Criterion for determining the stability of sampled data system, the characteristic equation 1 + G(z) H(z) = 0 should be modified by using bilinear transform of

**[EE ESE - 2001]**

(a) Z = r + 1 (b) z = r – 1
(c) $z = \frac{r-1}{r+1}$ (d) $z = \frac{r+1}{r-1}$

**73. Assertion (A):** For a system to be stable, all coefficients of the characteristic polynominal must be positive.
**Reason (R):** All positive coefficients of the characteristic polynomial of a system is a sufficient conditions for stability.

**[EE ESE - 2001]**

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**74.** Consider the following statements:
Routh–Hurwitz criterion gives.
1.Absolute stability
2.The number of roots lying on the right half of the s-plane
3.The gain margin and phase margin
Which of these statements are correct?

**[EC ESE - 2000]**

(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) 1 and 3

**75.** The Routh-Hurwitz criterion cannot be applied when the characteristic equation of the system contains any coefficients which is

**[EC ESE - 2000]**

(a) Negative real and exponential functions of s
(b)Negative real, both exponential and sinusoidal function of s
(c)Both exponential and sinusoidal function of s
(d)Complex, both exponential and sinusoidal functions of s

**76.** The open-loop transfer function of unity feedback control system is

$$G(s) = \frac{K}{s(s+a)(s+b)}, 0 < a < b$$

The system is stable is

**[EC ESE - 2000]**

(a) $0 < K < \frac{(a+b)}{ab}$
(b) $0 < K < \frac{ab}{(a+b)}$
(c) 0 < K < ab (a + b)
(d) 0 < K < a/ba (a + b)

# SOLUTIONS

**Sol.1. (c)**

The given characteristic equation is

$s^3 + 3s^2 + (K+2)s + 5K = 0$

or $s^3 + 3s^2 + 2s + K(s+5) = 0$

or $1 + \frac{K(s+5)}{s^3 + 3s^2 + 2s} = 0$

or $1 + \frac{K(s+5)}{s(s+1)(s+2)} = 0$

$\therefore\ G(s)H(s) = \frac{K(s+5)}{s(s+1)(s+2)}$

**Sol.2. (a)**

The characteristic equation of given system is

$s^3 + 8s^2 + 19s + 12 = 0$

Routh table is

| | | |
|---|---|---|
| $s^3$ | 1 | 19 |
| $s^2$ | 8 | 12 |
| $s^1$ | 17.5 | 0 |
| $s^0$ | 12 | |

No sign change in the first column. Hence, system is stable.

**Sol.3. (c)**

The initial slope of the plot is 0dB/decade hence the system is type 0.

20 log k = 20

$\therefore$ k = 10

At ω = 1 rad/sec., the slope of the plot changes by –20 dB/decode. Hence the corresponding term of the transfer function is

$1/(sT+1)$, where, $T = \frac{1}{\omega} = \frac{1}{1} = 1\text{sec}$

$\therefore$ Open loop transfer function

$G(s)H(s) = \frac{10}{(1+s)}$

**Sol.4. (a)**

When gain k of the system is varied from 0 to ∞ then the closed loop system may became unstable, because the poles may go to the right half of s plane.

**Sol.5. (d)**

Since closed loop system is having a feedback so the control system action depends on output.

**Sol.6. (d)**

Since poles are most important to determine properties of a system so determinator of closed loop system is called characteristic polynomial of a system.

**Sol.7. (c)**

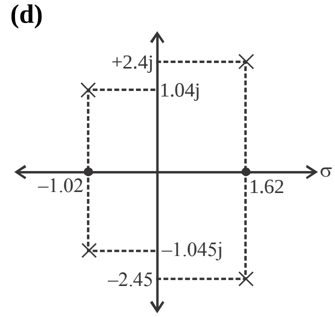
$q(s) = s^4 + 4s^3 + 13s^2 + 36S + K = 0$

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 13 | K |
| $S^3$ | 4 | 36 | |
| $S^2$ | 4 | K | |
| $S^1$ | $\frac{36\times4-4K}{4}$ | | |
| $S^0$ | K | | |

For stability K > 0 and K < 36

$\therefore\ 0 < K < 36$

**Sol.8. (d)**




$20ks^3 + (s^4 + 5s^2 + 15) = 0$

$GH = \frac{20ks^3}{(s^4 + 5s^2 + 10s + 15)}$

Z=3; poles $+1.02 \pm 2.44j$;
$-1.02 \pm 1.044j$

The system is unstable.

**Sol.9. (a)**

**Sol.10. (d)**
If multiple pole lies on jω - axis then system become unstable. Hence it could be stable or unstable for bounded input.

**Sol.11. (b)**
Using Routh's criteria

| | | | |
|---|---|---|---|
| $S^4$ | 2 | 2 | 10 |
| $S^3$ | 1 | 5 | |
| $S^2$ | –8 | 10 | |
| $S^1$ | $\frac{50}{8}$ | 0 | |
| $S^0$ | 10 | | |

No. of sign change occurs = 2 times.
Hence no. of roots of equation lying in the right half of s plane = 2

**Sol.12. (b)**
Total number of changes of sign = 3
i.e. number of root at R.H.S. = 3
⇒ system is unstable

**Sol.13. (c)**

**Sol.14. (b)**
Using Routh's stability criteria,

| | | |
|---|---|---|
| $s^3$ | 1 | 5 |
| $s^2$ | K | 10 |
| $s^1$ | $\frac{5K-10}{K}$ | 0 |
| $s^0$ | 10 | – |

For stability,

$$\frac{5K-10}{K} \geq 0$$

$\Rightarrow K \geq 2$

**Sol.15. (c)**

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 2 | 15 |
| $S^3$ | 1 | 4 | 15 |
| $S^2$ | $\left(\frac{2-4}{1}=2\right)$ | 15 | 0 |
| $S^1$ | $\left(\frac{-8-15}{-2}=\frac{23}{2}\right)$ | 0 | |
| $S^0$ | 15 | | |

$s^4 + s^3 + 2s^2 + 4s + 15 = 0$
two sign change
so, 2 roots in the right half of the s – plane.

**Sol.16. (*)**

**Sol.17. (*)**

**Sol.18. (d)**
All the elements in one row of Routh array are zero means system is either marginally stable or unstable i.e. roots will lie either on imaginary axis or on right hand side, nothing can be said perfectly. So, in this case all the 3 statements can be correct.

**Sol.19. (a)**
Characteristic equation
$s(s + 3)(s + 10) + K = 0$
$s^3 + 13s^2 + 30s + K = 0$
Routh array

| | | |
|---|---|---|
| $S^3$ | 1 | 30 |
| $S^2$ | 13 | K |
| $S^1$ | $\frac{390-K}{13}$ | 0 |
| $S^0$ | K | |

For system to be stable
K >0 and 390 = K > 0
⇒ $0 < K < 390$

**Sol.20. (d)**
Characteristic equation
$s^4 + s^3 + 2s^2 + 2s + 5 = 0$

**Routh array table :**

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 3 | 5 |
| $S^3$ | 1 | 2 | |
| $S^2$ | 1 | 5 | |
| $S^1$ | –3 | | |
| $S^0$ | 5 | | |

**Sol.21. (d)**

**Sol.22. (d)**
$s^4 + 8s^2 + 24s^2 + 32s + k = 0$
Routh array

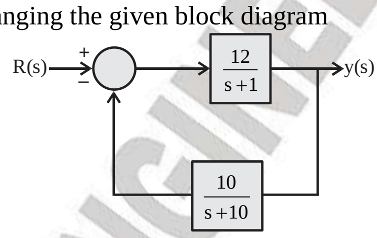
| | | | |
|---|---|---|---|
| $S^4$ | 1 | 24 | K |
| $S^3$ | 8 | 32 | |
| $S^2$ | 20 | K | |
| $S^1$ | $\frac{20\times 32-8K}{20}$ | | |
| $S^0$ | K | | |

For marginally stability

$$\frac{20\times 32-8K_{mar}}{20}=0$$

$\Rightarrow 8K_{mar} = 20 \times 32$
$\Rightarrow K_{mar} = 80$
Hence system will be unstable for $K > K_{mar}$ so option (d) is the correct answer.

**Sol.23. (b)**
Rearranging the given block diagram




Hence, $G(s)=\frac{12}{s+1}$

and, $H(s)=\frac{10}{s+10}$

The characteristic equation is given by
1 + G(s) H(s) = 0

$$1+\left(\frac{12}{s+1}\right)\left(\frac{10}{s+10}\right)=0$$

$\Rightarrow s^2 + 11s + 130 = 0$

**Sol.24. (a)**

**Sol.25. (a)**

**Sol.26. (c)**

**Sol.27. (a)**
Characteristic equations
1 + G(s) H(s) = 0

$$1+\frac{10(s+2)}{(s+1)(s+3)(s-5)}=0$$

$s^3 – s^2 – 7s + 5 = 0$
By Routh array

| | | |
|---|---|---|
| $S^3$ | 1 | –7 |
| $S^2$ | –1 | K |
| $S^1$ | | |
| $S^0$ | | |

There is – ve sign in 1st column of Routh array means roots over laying RHS. So system is unstable.

**Sol.28. (b)**
$s^4 {}_{+\,2s}{}^3 + 2s^2 + 3s + 3s + 6 = 0$
By Routh criterion

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 2 | 6 |
| $S^3$ | 2 | 3 | |
| $S^2$ | $\frac{4-3}{2}$ | 6 | |
| $S^1$ | $\frac{3/2-12}{1/2}$ | | |
| $S^0$ | 6 | | |

There are 2 sign changes 1st coloumn of Routh array. So number of roots in RHS = 2

**Sol.29. (c)**

Characteristic equation

$1 + G(s)\,H(s) = 0$

$$1+\frac{K}{s(s+1)(s+6)}=0$$

$s^3 + 7s^2 + 6s + K = 0$

By RH criterion

| | | |
|---|---|---|
| $S^3$ | 1 | 6 |
| $S^2$ | 7 | K |
| $S^1$ | $\frac{42-K}{7}$ | |
| $S^0$ | K | |

For stable system

$42 - K > 0;\ K > 0$

$42 > K;\ K > 0$

$0 < K < 42$

**Sol.30. (b)**

Impulse response = $\frac{d}{dt}$ (step response)

$$= \frac{d}{dt}[1-e^{-t}(1+t)]$$

Impulse response $= te^{-t}$

L.T. of impulse response = T.F.

$$\therefore\ \text{T.F.}=\frac{1}{(s+1)^2}$$

$1+G(s)\,H(s) = 0$

$$1+\frac{1}{(s+1)^2}=0$$

$\therefore\ s^2 + 1 + 2s + 1 = 0$

$s^2 + 2s + 2 = 0$

$\therefore\ s = (-1 + i)$ and $(-1 - i)$

$\therefore$ It has two roots on left half of s – plane.

Hence the system is absolutely stable.

**Sol.31. (a)**

By Routh's Array

| | | |
|---|---|---|
| $S^3$ | 1 | 9 |
| $S^2$ | 6 | 4 |
| $S^1$ | $\frac{54-4}{6}$ | |
| $S^0$ | 4 | |

All are positive in 1st column. Hence all the three roots lie in the left half of s – plane.

**Sol.32. (c)**

A system is stable if its output is bounded for bounded input.

**Sol.33. (b)**

Open loop is unstable because of pole in R.H.S.

For closed loop system

$1 + G(s) = 0$

$$1+\frac{k(s+3)}{s(s-1)}=0$$

$s^2 + s(k - 1) + 3k = 0$

Routh array

| | | |
|---|---|---|
| $s^2$ | 1 | 3k |
| $s^1$ | $k-1$ | 0 |
| s | 3k | |

$\because$ k > 1 elements of 1st row are greater than zero. Hence it is stable.

**Sol.34. (d)**

Routh array:

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 4 | K |
| $S^3$ | 4 | 3 | |
| $S^2$ | $\frac{16-3}{4}=\frac{13}{4}$ | K | |
| $S^1$ | $\frac{39/4-4K}{13/4}$ | 0 | |
| $S^0$ | K | | |

For the system to be marginally stable,

$$\frac{39}{4}-4K=0 \Rightarrow K=\frac{39}{16}$$

**Sol.35. (d)**

$s(s + 2)(s + 3) + K(s + 1) = 0$

$\Rightarrow G(s)H(s) = \dfrac{K(s+1)}{s(s+2)(s+3)}$

Since there are 3 poles and 1 zero, therefore, in the root loci one branch will be from a pole to zero and two more branches will be from rest of the poles towards infinity.

**Sol.36. (a)**

Routh array:

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 24 | K |
| $S^3$ | 8 | 32 | |
| $S^2$ | 1<br>20 | 4<br>K | |
| $S^1$ | $\frac{80-K}{20}$ | 0 | |
| $S^0$ | K | | |

For the system to be stable,

$\dfrac{80-K}{20} > 0$ and $K \geq 0$

$\Rightarrow 0 \leq K < 80$

**Sol.37. (c)**

Routh array:

| | | |
|---|---|---|
| $S^3$ | 1 | –1 |
| $S^2$ | 1 | 1 |
| $S^1$ | –2 | 0 |
| $S^0$ | 1 | |

Since the sign changes two times in the first column, therefore, two roots have positive parts.

**Sol.38. (d)**

Routh array:

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 24 | K |
| $S^3$ | 8 | 32 | |
| $S^2$ | 1<br>20 | 4<br>K | |
| $S^1$ | $\frac{80-K}{20}$ | 0 | |
| $S^0$ | K | | |

For roots with zero real parts,

$\dfrac{80-K}{20} = 0$

$\Rightarrow K = 80$

**Sol.39. (c)**

Characteristic equation is

$s^5 + s^4 + 2s^3 + 2s^2 + 4s + 6 = 0$

Putting s = 1/z

$6z^5 + 4z^4 + 2z^3 + 2z^3 + z + 1 = 0$

Routh array:

| | | | |
|---|---|---|---|
| $Z^4$ | 6 | 2 | 1 |
| $Z^3$ | 4 | 2 | 1 |
| $Z^2$ | $\frac{8-12}{4} = -1$ | $\frac{4-6}{4} = \frac{-1}{2}$ | |
| $Z^1$ | 0 (Let us take $\in$) | | |
| $Z^0$ | 1 | | |

Since only two sign changes in the first column of Routh array, therefore, two roots of the equation lie in the right half of s-plane.

**Sol.40. (d)**

$$\frac{G(s)}{1+G(s)} = \frac{K}{\left(s^2+2s+2\right)(s+2)+K}$$

Characteristic equation is

$s^3 + 4s^2 + 6s + 4 + K = 0$

Routh array:

| | | |
|---|---|---|
| $S^3$ | 1 | 6 |
| $S^2$ | 4 | 4+K |
| $S^1$ | $\frac{24-4-K}{4}$ | 0 |
| $S^0$ | 4+K | |

For the system to be marginally stable, all elements of the row of s should be zero.

$\therefore\ 24 - 4 - k = 0$

$\Rightarrow K = 20$

$4s^2 + 4 + K = 0$

$4s^2 + 4 + 20 = 0$

$s^2 + 6 = 0$

$\Rightarrow \omega^2 = 6$

$\Rightarrow \omega = \sqrt{6}$ rad / s

**Sol.41. (d)**

$$\frac{G(s)}{1+G(s)} = \frac{\frac{K}{s(s+1)(s+5)}}{1+\frac{K}{s(s+1)(s+5)}}$$

$$= \frac{K}{s(s+1)(s+5)}$$

Characteristic equation is

$s^3 + 6s^2 + 5s + K = 0$

Routh array:

| | | |
|---|---|---|
| $s^3$ | 1 | 5 |
| $s^2$ | 6 | K |
| $s^1$ | $\frac{30-K}{6}$ | |
| $s^0$ | K | |

For stable operation, each element of the first column of the Routh array should have the same sign.

Therefore, 0 < K < 30.

**Sol.42. (d)**

One row zero means system is either unstable or marginally stable.

**Sol.43. (d)**

Characteristic equation 1 + G(s) = 0

$\Rightarrow s^3 + s^2a + K = 0$

Routh Array

| | | |
|---|---|---|
| $s^3$ | 1 | 0 |
| $s^2$ | a | K |
| $s^1$ | $-\frac{K}{a}$ | |
| $s^0$ | K | |

For K > 0 number of sign change = 2

For K < 0 number of sign change = 1

Hence option (d) is correct.

**Sol.44. (b)**

Put, z = s – a

$z^2 + z + a = 0$

$(s - a)^2 + (s - a) + a = 0$

$s^2 + a^2 - 2as + s - a + a = 0$

$s^2 + s(1 - 2a) + a^2 = 0$

$$\frac{da}{ds} = 0$$

$\Rightarrow 2s + 1(1 - 2a) + 0 = 0$

$2s + 1 - 2a = 0$

$2s + 1 = 2a$

$$a = \frac{2s}{2} + \frac{1}{2}$$

Leave s, and take

$$a = \frac{1}{2} = 0.5$$

**Sol.45. (b)**

Time constant, will be less for system with pole for away to left of s – plane. So match at ‘C’ is correct hence at B not correct.

**Sol.46. (d)**

For minimum phase transfer function any of the zeros or poles should not lie on right side of s – plane.

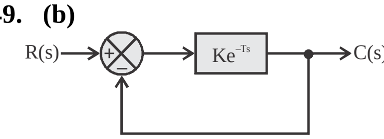
For non minimum T.F. zeros lie on RHS and poles lie on LHS.

**Sol.47. (c)**

**Sol.48. (c)**

Definition of non – minimum phase transfer function.

**Sol.49. (b)**




Characteristic equation 1 + G(s) H(s) = 0

$1 + Ke^{-Ts} = 0$

$e^{-Ts} = 1 - Ts$ (Approx Value)

So, change equation $1 + K(1 - Ts) = 0$

$\Rightarrow = 1 + K - KTs$

Hence, stability depend on K and T

**Sol.50. (d)**

**Sol.51. (b)**
Characteristic equation $s^3 + 25s^2 + 10s + 50 = 0$ has not sign change so number of roots on right is zero. Where as to know roots on $j\omega$ routh array is required but here only one option has 0 roots on right side.

**Sol.52. (a)**
Pole on left and zero on right half of s – plane.

**Sol.53. (d)**
Definition of minimum phase transfer function.

**Sol.54. (d)**

**Sol.55. (b)**

**Sol.56. (c)**
$s^5 + 15s^4 + 85s^3 + 225s^2 + 274s + 120 = 0$
Put $s = z - 1$
$(z-1)^5 + 15(z-1)^4 + 85(z-1)^3 + 225(z-1)^2 + 274(z-1) + 120 = 0$
$\Rightarrow z^5 + 10z^4 + 35z^3 + 50z^2 + 24z = 0$
Routh array is

| | | | |
|---|---|---|---|
| $Z^5$ | 1 | 35 | 24 |
| $Z^4$ | 10 | 50 | |
| $Z^4$ | 1 | 5 | |
| $Z^3$ | 30 | 24 | |
| $Z^2$ | 4.2 | 0 | |
| $Z^1$ | 24 | 0 | |
| $Z^0$ | 0 | | |

There are 4 roots which lie to the left of the line s + 1 = 0 and one root lies on s + 1 = 0.

**Sol.57. (a)**
Apply Jury's stability test.

**Sol.58. (c)**
Apply Routh – Hurwitz criteria.

**Sol.59. (a)**

**Sol.60. (a)**

**Sol.61. (b)**

| | | | |
|---|---|---|---|
| $S^4$ | 2 | 3 | 10 |
| $S^3$ | 1 | 5 | |
| $S^2$ | $3 - 10 = -7$ | 10 | |
| $S^1$ | $\frac{-35-10}{-7} = \frac{45}{7}$ | 0 | |
| $S^0$ | 10 | | |

There are two sign changes, so two poles on R.H.S.

**Sol.62. (c)**
$1 + G(s)H(s) = 1$

$$1 + \frac{k}{s(s+1)(s+6)} = 0$$

| | | |
|---|---|---|
| $S^3$ | 1 | 6 |
| $S^2$ | 7 | K |
| $S^1$ | $\frac{42-K}{7}$ | |
| $S^0$ | 0 | |

$$\therefore \frac{42-k}{7} = 0$$

$\therefore k = 42$
$k > 0$
$\therefore$ Range $0 < k < 42$

**Sol.63. (b)**
Apply Routh – Hurwitz stability criteria.

**Sol.64. (c)**
Poles with positive real part denote unstable system.

**Sol.65. (a)**

**Sol.66. (a)**
A discrete data system is stable if all the roots of the characteristic equation lie within the unit circle on the complex plane.

**Sol.67. (b)**
Transportation lag can be conveniently handled on Bode plot as well without the need to make Any approximation. The log magnitude of transportation lag is 20 log $|e^{-j\omega T}| = 0$.
Thus the open – loop log – magnitude plot of a system is unaffected by the presence of transportation lag. The lag, of course, contributes a phase angle of $-(\omega T \times 80^0)/\pi$, thereby causing the modification of the phase plot.

**Sol.68. (c)**
There is a missing co – efficient so system is unstable.

**Sol.69. (c)**
$1 + G(s)H(s) = 0$
$s^3 + 5s^2 + 6s + K = 0$

| | | |
|---|---|---|
| $S^3$ | 1 | 6 |
| $S^2$ | 7 | K |
| $S^1$ | $\frac{30-K}{5}$ | 0 |
| $S^0$ | K | |

For marginal stability $\frac{30-K}{5} = 0$

$\therefore K = 30$

**Sol.70. (a)**

**Sol.71. (c)**
Routh array is

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 2 | 3 |
| $S^3$ | 1 | 2 | |
| $S^2$ | $\in$ | 3 | |
| $S^1$ | $\frac{2\in-3}{\in}$ | 0 | |
| $S^0$ | 3 | | |

Since the sign changes twice, so there are two roots in RHS of s-plane.

**Sol.72. (d)**

**Sol.73. (c)**
All positive coefficients of the characteristic polynominal of a system is a necessary condition not a sufficient condition for stability.

**Sol.74. (b)**
Routh-Hurwitz criterion gives absolute stability and the number of roots lying on the right half of the s-plane but it does not tell about the gain margin and phase margin.

**Sol.75. (b)**
A necessary (but not sufficient) condition for stability of a linear system is that all the coefficients of its characteristic equation be real and have the same sign. Furthermore, none of the coefficients should be zero.

**Sol.76. (c)**

$$\frac{G(s)}{1+G(s)} = \frac{K}{s(s+a)(s+b)+K}$$

Characteristic equation is
$s^3 + (a+b)s^2 + abs + K$
Routh array is

| | | |
|---|---|---|
| $S^3$ | 1 | a+b |
| $S^2$ | ab | K |
| $S^1$ | $\frac{ab(a+b)-K}{ab}$ | 0 |
| $S^0$ | K | |

For the system to be stable.
$K > 0$
$ab(a+b) - K > 0 \Rightarrow K < ab(a+b)$
So $0 < K < ab(a+b)$

# CHAPTER - 6
# *ROOT LOCUS*

## 6.1 INTRODUCTION

The Routh's criterion gives a satisfactory answer to the question of stability but its adoption to determine the relative stability is not satisfactory and requires trial and error procedure even in the analysis problem.

A simple technique, known as the root locus technique, for finding the roots of the characteristic equation, introduced by W.R. Evans, is extensively used in control engineering practice. This technique provides a graphical method of plotting the locus of the roots in the s-plane as a given system parameter is varied over the complete range of values (may be from zero to infinity). The roots corresponding to a particular value of the system parameter can then be located on the locus or the value of the parameter for a desired root location can be determined from the locus.

## 6.2 RULES OF DRAWING THE ROOT LOCUS

1. Root locus start from open loop poles and ends on open loop zeros or $\infty$ with $K = \infty$

Let no. of poles = n (open loop poles)

No. of open loop zeros = m

(i) No. of root loci ending on $\infty = n - m,\ n > m$

2. Root locus is always symmetrical about real axis.

3. A point on real axis lies on the root locus if no. of poles + zeros to the right of the point are odd.

4. Asymptotes are the paths along which root locus moves towards $\infty$.

(i) No. of asymptotes = (n – m)

(ii) Angle of asymptotes

$$\theta_A = \frac{(2x+1)180°}{n-m}$$

x = 0, 1, 2,….. n – m – 1

(iii) Centroid : It is the point of intersection of asymptotes with the real axis.

$$\sigma_A = \frac{\sum(\text{real part of poles}) - \sum(\text{real part of zeros})}{n-m}$$

5. Determination of Breakaway or break in point : On the root locus between two adjacent poles the two poles move towards each other with K=0 and move at a point where K is maximum and the root locus will break away into two parts. This point is called the breakaway point and it is determined by:

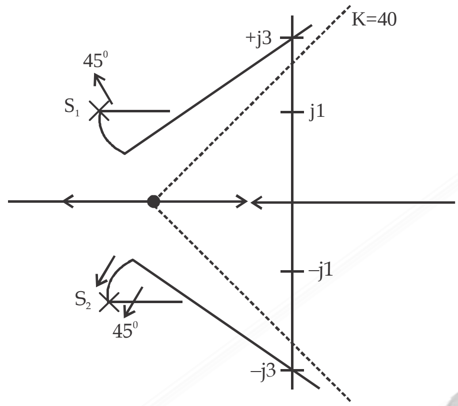
$\text{Put}\left(\frac{dK}{ds} = 0\right)$ and find out the value of 's'

6. Angle of departure or Angle of arrival

angle made by root locus with real axis when it departs from a complex open loop poles is called angle of departure.

$$\begin{Bmatrix} \phi_D\ (\text{angle of departure}) = 180° + \angle GH' \\ \phi_A\ (\text{angle of arrival} = 180° - \angle GH') \end{Bmatrix}$$

GH' is value of function excluding the concerned poles at the poles itself




**Example.**

$$G(s)H(s) = \frac{K}{(s+1)(s^2+2s+2)} = \frac{K}{(s+1)\underbrace{(s+1+j1)}_{s_2}\underbrace{s+1-j1}_{s_1}} GH' = \frac{K}{(s_1+1)(s_1+1)+J1}$$

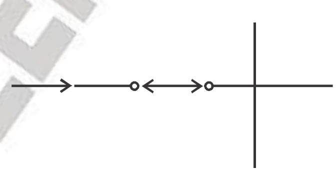
at $(s_1 = -1 + j1)$, $GH' = \dfrac{K}{(j1)\,(j2)}$

Just calculate for one ($s_1$ or $s_2$) and you can write for the other by putting negative sign.

7. The intersection of the root loci with the imaginary axis is calculated using the Routh's stability criteria. By using Routh's criteria gives frequency at that point of 'K' which has been found out.

**Example.**

$$G(s)H(s) = \frac{(s+1)(s+4)}{(s+3)(s+5)}$$



**Solution.**

No. of open loop poles = 2

No. of open loop zeros = 2

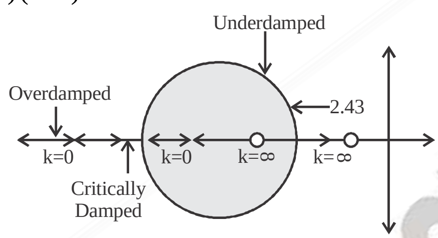
No. of root loci ending on $\infty$ = 0

No. of asymptotes = 0

Angle of asymptotes $= \dfrac{(2x+1)180°}{0}$

Break away / breaking point
No breakaway / breaking point

**Example.** $G(s)H(s) = \frac{K(s+1)(s+3)}{(s+4)(s+5)}$




**Over damped system**
**Breakaway point / Breaking point**
1 + G(s) H(s) = 0

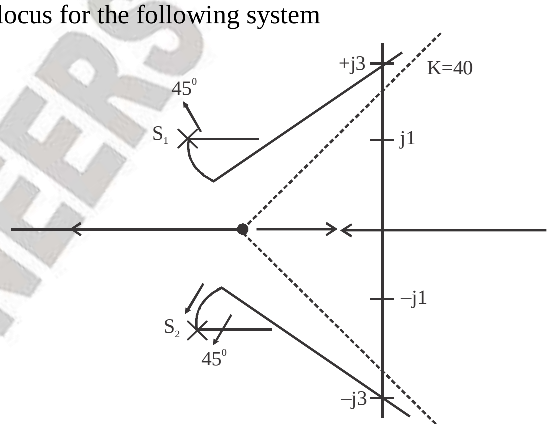
$$\therefore\ 1+\frac{K(s+1)(s+3)}{(s+4)(s+5)} = 0$$

$$K = -\frac{s^2+9s+20}{s^2+4s+3}$$

$$\frac{dK}{ds} = \frac{\begin{Bmatrix}(s^2+4s+3)(2s+9) \\ -(s^2+9s+20)(2s+4)\end{Bmatrix}}{(s^2+4s+3)^2} = 0$$

$(s_1 = -2.43, -4.3)$

**Example.** Draw the root locus for the following system




$$G(s)H(s) = \frac{K}{(s+1)(s^2+4s+5)}$$

$$s = \frac{-4 \pm \sqrt{4^2 - 20}}{2}$$

$$= \frac{-4 \pm 2j}{2} = -2 + j$$

$$= \frac{K}{(s+1)(s+2+j)(s+2j)}$$

No of open loop poles = 3
No of zeros = 0
No. of root loci ending on $\infty$ = 3 – 0 = 3
Root locus on real axis
No. of asymptotes = 3 – 0 = 3
Angle of asymptotes

$$\theta_A = \frac{(2x+1)180°}{3} = 60^0, 180^0, 300^0 \qquad x = 0, 1, 2$$

$$\text{Centroid} = \frac{(-1)+(-2)(-2)-0}{3} = \left(\frac{-5}{3}\right) = -1.66$$

$$\text{Angle of departure at } s_1 = \frac{k}{(-2+j+1)(-2+j+2+j)}$$

$$\angle GH' = -90° - \left\{\pi - \tan^{-1}\frac{1}{1}\right\} = -90° - 180° + 45° = -225°$$

$$\left(-a + jb = \pi - \tan^{-1}\frac{b}{a}\right)$$

$\phi_D = 180^0 - 225^0 = -45^0$
jω crossover
Characteristic equation

$$1 + \frac{K}{(s+1)(s^2+4s+5)} = 0$$

$s^3 + 5s^2 + 9s + (5 + K) = 0$

**Routh's array**

| | | |
|---|---|---|
| $s^3$ | 1 | 9 |
| $s^2$ | 5 | (5+k) |
| $s^1$ | $\left(\frac{45-5-K}{5}\right)$ | |
| $s^0$ | (5+K) | |

$$\therefore \frac{40-K}{5} > 0$$

40 – K > 0 $\quad\therefore$ K < 40
at jω crossover,
K = 40
Auxiliary equation
$5s^2 + 45 = 0$

$s = \pm j3$

**Example.** $G(s)\,H(s) = \dfrac{K}{(s+1)\,(s+3)\,(s^2+4s+8)}$

**Solution.**

$s^2 + 4s + 8 = 0$

$$s = \frac{-4 \pm \sqrt{16-32}}{2} = \frac{-4 \pm 4j}{2}$$

$= -2 \pm 2j$

No. of poles = 4

No. of zeros = 0

$\therefore$ No of asymptotes = 4

$\sigma_A$ = centroid $= \dfrac{(-1)+(-3)+(-2)+(-2)}{4} = (-2)$

**Angle of asymptotes**

$$\theta_A = \frac{(2q+1)180°}{(n-m)}\; q = 0, 1, 2, 3$$

$$\theta_A = \frac{180°}{4} = 45°, 135°, 225°, 315°$$

**Determination of breakaway or Breaking point**

$$1 + \frac{K}{(s+1)\,(s+3)\,(s^2+4s+8)} = 0$$

$(s + 1)\,(s + 3)\,(s^2 + 4s + 8) + K = 0$

$K = -(s\;\;1)\,(s + 3)\,(s^2 + 4s + 8)$

$= -((s^2 + 4s + 3)\,(s^2 + 4s + 8))$

$$\frac{dK}{ds} = \left\{(s^2+4s+3)\,(2s+4) + (s^2+4s+8)\right\} = -\left((s^2+4s+8)\,(2s+4)\right) = 0$$

$\therefore$ $s = -2$ or $2s^2 + 8S + 11 = 0$

$$S = \frac{-8 \pm \sqrt{64-88}}{4} = \frac{-8 \pm 2\sqrt{6}j}{4} = \frac{4 \pm 2\sqrt{6}j}{2}$$

To find the angle of departure

$\phi_D = 180^0 + \angle GH'$

$$GH'\big|_{ats \to (-2+2j)} = \frac{K}{(s+1)(s+3)(s-(-2+2j))}$$

$$= \frac{K}{(-2+2j+1)(-2+2j+3)(-2+2j+2+2j)}$$

$$= \frac{K}{(-1+2j)\,(1+2j)\,(4j)}$$

$\angle GH'|_{ats \to (-2+2j)} = -(180° - \tan^{-1}) - 90° - \tan^{-1} 2$

$= -180^0 - 90^0 = 270^0$

$\phi_d = 180^0 - 270^0 = -90^0$

jω cross over

$1 + G(s) H(s) = 0$

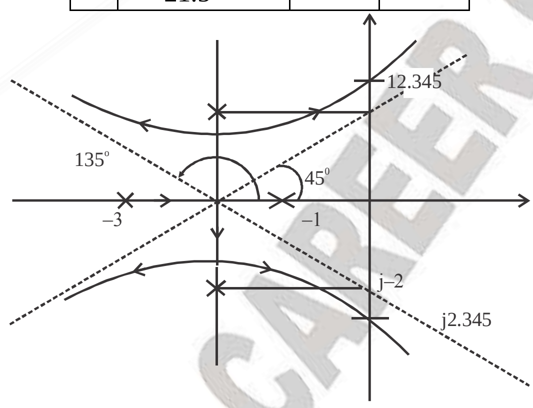
$S^4 + 4s^3 + 8s^2 + 4s^3 + 16s^2$

$+ 32s + 3s^2 + 12s + 24 + K = 0$

$\Rightarrow s^4 + 8s^3 + 27s^2 + 44s + 24 +$

$K = 0$

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 27 | 24+K |
| $s^3$ | 8 | 44 | |
| $s^2$ | 21.5 | 24+K | |
| $s^1$ | $35.06-\frac{8K}{21.5}$ | 0 | |
| $s^0$ | $\frac{754-8K}{21.5}$ | | |




For the system to be marginally stable

$$\frac{754-8K}{21.5}=0$$

$$K=\frac{754}{8}=94.25$$

∴ Auxiliary equation

$21.5 s^2 + (24 + K) = 0$

$21.5 s^2 + (24 + 94.25) = 0$

$$s^2=\frac{-188.25}{21.5}$$

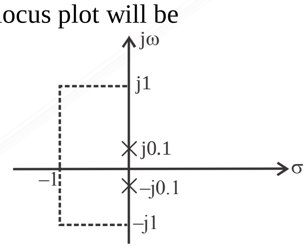
∴ Cross over frequency s = 2.345

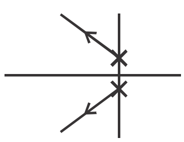
# ASSIGNMENT

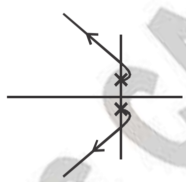
**1.** If the open-loop transfer function of a feedback system is given by, $G(s)H(s) = \dfrac{k}{s(s+2)(s^2+2s+5)}$, then the centroid of the asymptotes will be

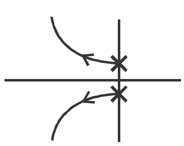
(a) –1, 0 (b) 1, 0
(c) 0, –1 (d) 0, 1

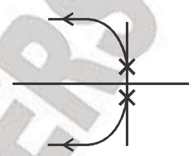
**2.** A plot of open-loop poles/zeros of a unity feedback system is shown in the given figure. Its roots-locus plot will be



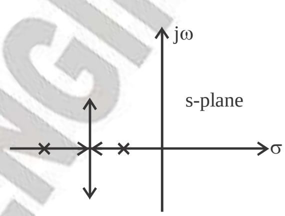

(a)


(b)


(c)


(d)


**3.** The figure shown below root-loci of open-loop transfer function G(s) H(s) of a system. Consider the following statement drawn from this figure.



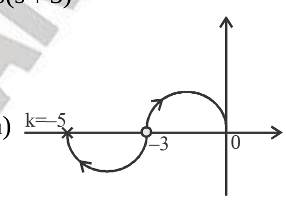

I. It has no zero.
II. It is a stable system
III. It is a second order system.
Which of these statements are correct?

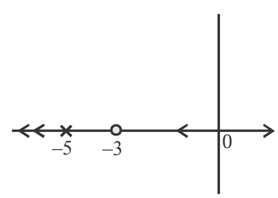
(a) I, II and III (b) I and II
(c) II and III (d) I and III

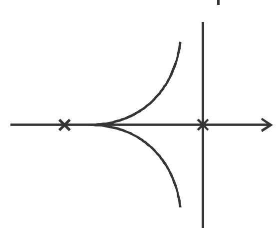
**4.** The transfer function for a system is $G(s) = \dfrac{k(s+2)}{(s+1)(s+2)(s+4)}, k > 0.$ Two branches (loci) of the plot directed along asymptotes are centroid at a point.

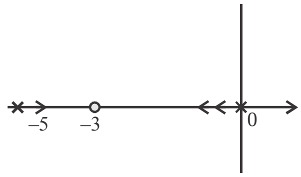
(a) –1 (b) 3
(c) –2 (d) –3

**5.** An open-loop transfer function is given by $\dfrac{k(s+3)}{s(s+5)}$. Its root-loci will be as in

(a)



(b)


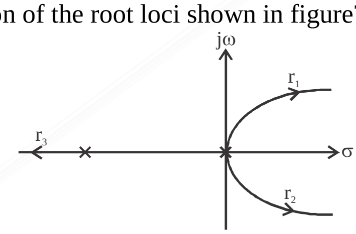

(c)


(d) 


**6.** The loop transfer function of a closed loop system is given by $G(s)H(s)=\dfrac{k}{s^2(s^2+2s+2)}$, the angle of departure of the root locus at s = –1 + j is

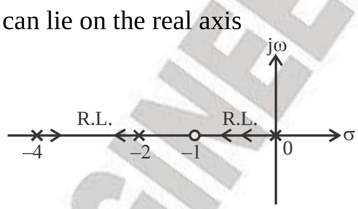
(a) Zero (b) + 180°
(c) –90° (d) –180°

**7.** Which of the following is the open-loop function of the root loci shown in figure?




(a) $\dfrac{k}{s(s+T_1)^2}$ (b) $\dfrac{k}{s(s+T_1)(s+T_2)^2}$
(c) $\dfrac{k}{s(s+T_1)^3}$ (d) $\dfrac{k}{s^2(sT_1+1)}$

**8.** A control system has $G(s)H(s)=\dfrac{k(s+1)}{s(s+2)(s+4)}$. Root locus of the system can lie on the real axis




(a) Between s = –1 and s = –2
(b) Between s = 0 and s = –4
(c) Between s = –2 and s = –4
(d) Towards left of s = –4

**9.** An open-loop transfer function is given by $G(s)H(s)=\dfrac{k(s+1)}{s(s+3)(s^2+2s+2)}$. It has

(a) One zero at infinity
(b) Two zero at infinity
(c) Three zeros at infinity
(d) Four zeros at infinity

**10.** Consider the following statements regarding root loci

I. All root loci start from the respective poles of G(s) H(s)
II. All root loci end at the respective zeros of G(s) H(s) or go to infinity.
III. The root loci are symmetrical about the imaginary axis of the s-plane.

(a) I, II and III are correct
(b) I and II are correct
(c) II and III are correct
(d) I and III are correct

**11.** If the open-loop transfer function is a ratio of a numerator polynomial of degree m and a denominator polynomial of degree n, then the integer (n-m) represents the number of

(a) Breakaway (b) Unstable poles
(c) Separate root loci (d) Asymptotes

**Common data for Q. 12 to Q.14**

A system having $G(s)=\dfrac{K}{(s+1)}$

and $H(s)=\dfrac{(s+1)}{(s^2+4s+5)}$ for k > 0.

**12.** Find the centroid of the system.

(a) –2 (b) –1.33
(c) –1.66 (d) –1

**13.** The angle of asymptotes are

(a) 90°, 270° (b) 60°, 180°, 315°
(c) 180°, 300° (d) 60°, 180°, 300°

**14.** Angle of departure from pole –2 + j is

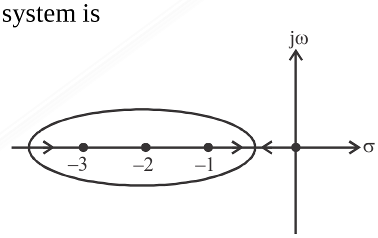
(a) 45° (b) –45°
(c) –30° (d) 30°

**15.** If the gain (k) of a system becomes zero, the root will

(a) Move away from zeros
(b) Move away from the poles
(c) Coincide with the zeros
(d) None of these

**16.** The root locus plot is symmetrical about the real axis because
(a) Roots occur simultaneously in LH and RH planes
(b) Complex roots occur in conjugate pairs
(c) All roots occur in pairs
(d) None of these

**17.** The root locus of a unity feedback system is shown in figure below. The open-loop transfer function
of the system is

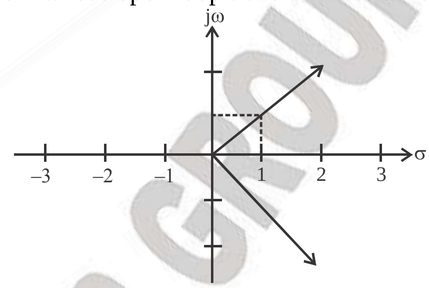



(a) $\frac{k}{s(s+1)(s+3)}$ (b) $\frac{k(s+1)}{s(s+3)}$
(c) $\frac{k(s+3)}{s(s+1)}$ (d) $\frac{ks}{(s+1)(s+3)}$

**18.** The root locus of the system having the loop transfer function
$G(s)H(s) = \frac{k}{s(s+4)(s^2+4s+5)}$ has
(a) 3 breakaway point
(b) 3 breaking point
(c) 2 breaking and 1 breakaway point
(d) 2 breakaway and 1 breaking point

**19.** A unity feedback control system has an open-loop transfer function
$G(s) = \frac{k}{s(s^2+7s+12)}$
The gain k for which s = –1 + j will lie on the root locus of the system is
(a) 10 (b) 5.5
(c) 6.5 (d) 20

**20.** Figure shown below the root-locus plot (location of poles not given) of a third order system whose open loop transfer function is




(a) $\frac{k}{s^3}$ (b) $\frac{k}{s^2(s+1)}$
(c) $\frac{k}{s(s^2+1)}$ (d) $\frac{k}{s(s^2-1)}$

**21.** The characteristic equation of a closed-loop system is s(s + 1) (s + 3) + k(s + 2) = 0, k > 0. Which of the following statements is true?
(a) Its roots are always real
(b) It cannot have a breakaway point in the range –1 < Re(s) < 0
(c) Two of its roots tend of infinity along the asymptotes Re(s) = –1
(d) If may have complex roots in the right half plane

**22.** How many number of branches the root loci of the equation s(s + 2) (s + 3) + k(s + 1) = 0 have
(a) Zero (b) One
(c) Two (d) Three

**23.** The closed loop transfer function of a control system has the following poles and zeros

| **Poles** | **Zeros** |
|---|---|
| $P_1 = -0.5$ | $Z_1 = -7$ |
| $P_2 = -1.0$ | $Z_2 = -9$ |
| $P_3 = -5$ | |
| $P_4 = -10$ | |

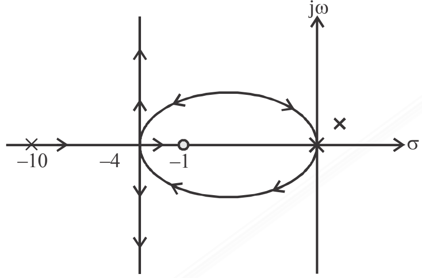
The closed loop response can be closely approximated by considering which of the following?

(a) $P_3$ and$P_4$ (b) P and $Z_1$

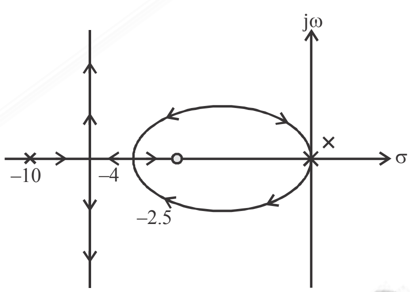
(c) $P_1$ and $P_2$ (d) $P_4$ and $Z_2$

**24.** The open-loop transfer function of a system is $\frac{k(s+1)}{s^2(s+10)}$. The root locus of the system is

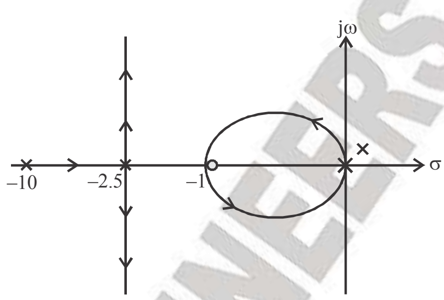
(a)




(b)



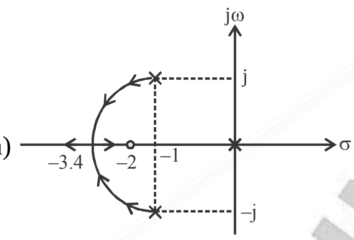

(c)



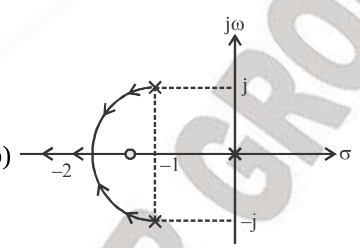

(d) None

**25.** The open-loop transfer function of a system is $G(s)H(s)=\frac{k(s+2)}{s^2+2s+2}$. Indicate the correct root locus diagram is

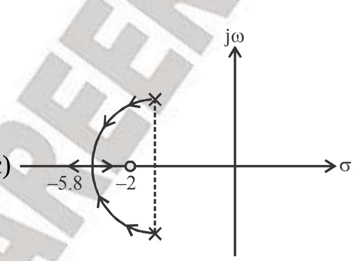
(a)




(b)




(c)




(d) None

**26.** The open loop transfer function of a unity feedback control system is given by

$$G(s)=\frac{k}{s(s+1)}$$

If the gain K is increased to infinity, then the damping ratio will tend to become

(a) $\frac{1}{\sqrt{2}}$ (b) 1

(c) 0 (d) $\infty$

**27.** Given as open-loop transfer function

$$G(s)H(s)=\frac{k(s+1)}{s(s-b)(s^2+2\xi\omega_n s+\omega_n^2)}$$

$a > 0, b > 0$

for $a = b = 1$, $\xi = 0.5$ and $\omega_n = 4$

Find the value of k at which the root loci cross the imaginary axis.

(a) k = 30 and k = 20

(b) k = 30
(c) k = 35.7
(d) k = 35.7 and k = 23.3

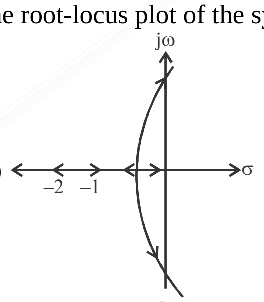
**28.** Consider the following statements:
1. In root-locus plot, the breakaway points
2. Need not always be on the real axis alone
3. Must lie on the root loci
4. Must lie between 0 and -1

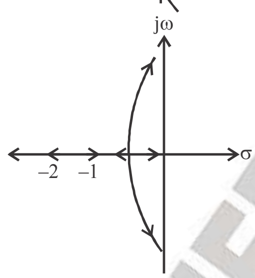
Which of these statements are correct
(a) 1, 2 and 3 (b) 1 and 2
(c) 1 and 3 (d) 2 and 3

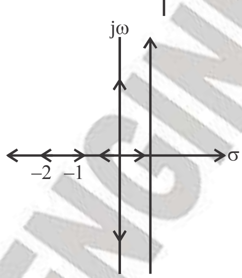
**29.** For a unity negative feedback control system, the open loop transfer function is

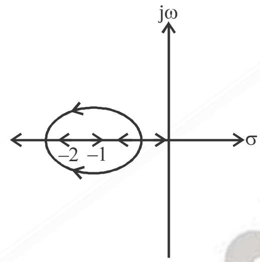
$$G(s) = \frac{K}{s(s+1)(s+2)}$$

The root-locus plot of the system is

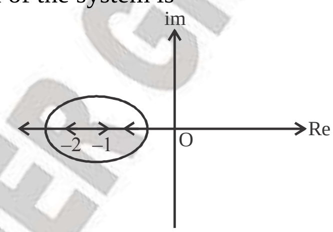
(a) 


(b) 


(c) 


(d) 

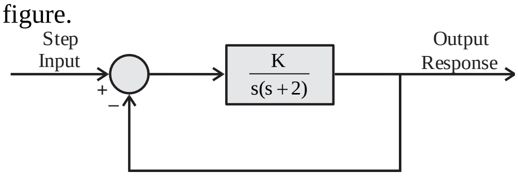

**30.** The below figure shows the roots locus of a unity feedback system. The open loop transfer function of the system is




(a) $\frac{K}{s(s+1)(s+2)}$ (b) $\frac{Ks}{(s+1)(s+2)}$
(c) $\frac{K(s+1)}{s(s+2)}$ (d) $\frac{K(s+2)}{s(s+1)}$

**31.** Which one of the following open-loop transfer functions has root locus parallel to imaginary axis?
(a) $K/(s+1)$ (b) $K(s+1)/(s+2)^2$
(c) $K/(s+2)^2$ (d) $K(s+2)/(s+1)^2$

**32.** A closed loop system is shown in the figure.




What is the ratio of output frequencies $\frac{\omega(\text{for } K = 32)}{\omega(\text{for } K = 16)}$?
(a) 1.40 (b) 1.42
(c) 1.44 (d) 1.46

**33.** How many number of branches the root loci of the equation
S(s + 2) (s + 3)+ K (s + 1) =0 have?
(a) Zero (b) One
(c) Two (d) Three

**34.** The intersection of asymptotes of root loci of a system with open loop transfer function $G(s)H(s) = \frac{K}{s(s+1)(s+3)}$ is
(a) 1.44 (b) 1.33
(c) −1.44 (d) −1.33

**35.** Consider the loop transfer function
$G(s).H(s) = \frac{K(s+6)}{(s+3)(s+5)}$
In the root – locus diagram, the centroid will be located at
(a) –4 (b) –1
(c) –2 (d) –3

**36.** The loop transfer function of a closed-loop is given by $G(s)H(s) = \frac{K}{s^2(s^2+2s+2)}$
The angle of departure of the root locus at s = 1 + j is
(a) Zero (b) 90°
(c) -90° (d) -180°

**37.** Match List-I (Loop transfer function) with List-II (Points (s) of root-locus plot) and select the correct answer using the codes given below in the lists:
**List-I**
A. $\frac{K(s+1)}{s^2(s+10)}$
B. $\frac{K}{s(s+2)(s^2+2s+2)}$
C. $\frac{K}{s(s+2)(s^2+2s+5)}$
D. $\frac{K}{s(s+4)(s^2+4s+5)}$
**List-II**
(i) One real breakaway point
(ii)Two real breakaway points
(iii)Three real breakaway points
(iv)One real and one pair of complex conjugate breakaway points
**Codes:**
(a) A-i, B-ii, C-iv, D-iii
(b) A-i, B-ii, C-iii, D-iv
(c) A-ii, B-i, C-iv, D-iii
(d) A-ii, B-i, C-iii, D-iv

# SOLUTIONS

**Sol. 1.**
Poles = 0, –2, –1, + 2j, –1 –2j
Total number of poles, P = 4
Total number of zero, Z = 0
$\therefore$ P – Z = 4

$$\therefore \text{ Centroid } = \frac{\sum P - \sum Z}{P - Z} = \frac{0-2-1-1}{4} = -1$$

= (–1, 0)

**Sol. 2.**

$$\text{Angle of Asymptotes } = \frac{(2q+1)180°}{P-Z}$$

P – Z = 2
$\therefore$The root-locus plot moves according to the angle of asymptotes i.e. 90° and 270°.

**Sol. 3.**

$$G(s)H(s)\frac{k}{(s+p_1)(s+p_2)}$$

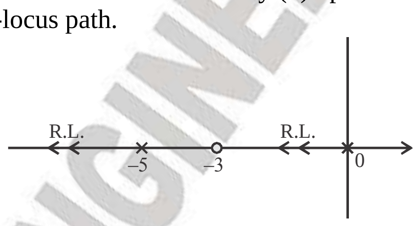
This is the type '0' and second order system

**Sol. 4.**

$$\text{Centroid} = \frac{\sum P - \sum Z}{P - Z}$$

$$= \frac{-1-3-4+2}{2} = \frac{-6}{2} = -3$$

**Sol. 5.**
The Root-locus path lie between 0and -3 and in between −5 and - ∞. So only (b) option is valid root-locus path.




**Sol. 6.**

$$G(s)H(s) = \frac{k}{s^2(s^2+2s+2)}$$

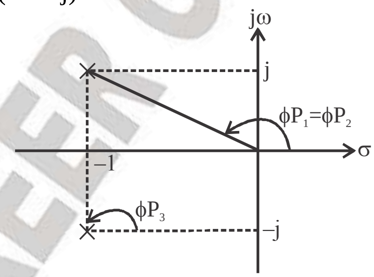
Poles = 0, 0, −1 + j, −-1+j
Angle of departure, $\phi_D$ = 180° – $\phi$
Where, $\phi = \sum\phi_p - \sum\phi_z$

$$\phi_{p_1} = 180° - \tan^{-1}(1) = 135°$$

$$\phi_{p_2} - \phi_{p_1} = -135°$$

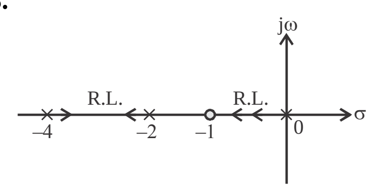
$$\phi_{p_1} = 90°$$

$\therefore$ $\phi$ = 135° + 135° + 90° = 360°
$\therefore$ $\phi_D$ (–1 + j) = 180° – 360° = 180°

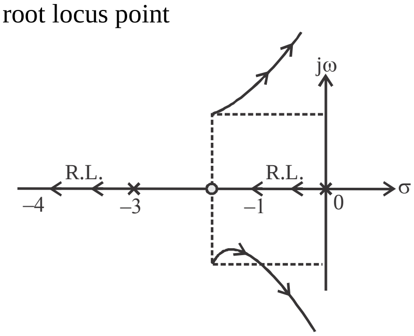



**Sol. 7.**
From the root-locus plot, two poles originates from origin. So it is type-2 system. In option (d), the transfer-function is type-2 system.

**Sol. 8.**




**Sol. 9.**
The root locus point



From the Root-locus point, it is shown that three zeroes at infinity.

**Sol. 10. (b)**

$G(s)H(s) = \frac{h(s+a)}{(s+b)}$

$1 + G(s)H(s) = 1 + \frac{k(s+a)}{(s+b)} = 0$

(s+b) + k(s+a) = 0

$k = \frac{-(s+b)}{(s+a)}$

At = s = –b; K = 0 (Start from open loop poles)
at s = a; k = ∞ (End at open loop zero)

**Sol. 11. (d)**

$Q_A = \frac{(2q+1)180}{p-z}$

q = 0 to (P–Z)–1

**Sol. 12.**

In this problem cancellation of pole of G(s) at $s = -1$ and zero of H(s) at $s = -1$ taken place. So as a closed-loop at $s = -1$ pole must be added.

$G(s)H(s) = \frac{k(s+1)}{(s+1)(s^2+4s+5)}$

Poles = $-1, -2+j, -2-j$

Zero = $-1$

Centroid $= \frac{\sum P - \sum Z}{P - Z}$

$= \frac{-1-2-2-1}{2} = -2$

**Sol. 13.**

Total number of poles, P = 3
Total number of zero, Z = 1
The number of asymptotes $(\theta) = P - Z = 2$

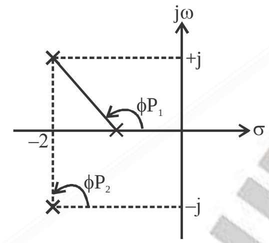
Angle of asymptotes

$= \frac{(2q+1)180°}{P-Z}, q = 0, 1, 2......$

90°, 270°

**Sol. 14.**

$\phi_{P_1} = 180° - \tan^{-1}(1) = 135°$




$\phi_{P_2} = 90°$

Angle of departure, $\phi_D = 180° - \phi$
Where $\phi = \sum\phi_p - \sum\phi_Z$
$\phi = 135° + 90° = 225°$
$\phi_D = 180° - 225° = -45°$

**Sol. 15. (d)**

**Sol. 16. (b)**

**Sol. 17.**

From the root locus plot:
There is a pole which originate from origin so the open-loop transfer function is type 1 system. There is another pole which originate from –1 and one zero terminate at –3

So, the transfer function = $\frac{k(s+3)}{s(s+1)}$

**Sol. 18.**

Poles = 0, –4, –2 ± j
Characteristic equation,
1 + G(s) H(s) = 0

$1 + \frac{k}{s(s+4)(s^2+4s+5)} = 0$

$s(s + 4)(s^2 + 4s + 5) = 0$
$k = -s^4 - 8s^3 - 21s^2 - 20s$
To find the breakaway/breaking point

$\frac{dk}{ds} = 0$

$4s^3 + 24s^2 + 42s + 20 = 0$
$2s^3 + 12s + 21s + 10 = 0$
s = –2, –0.775, –3.225
It can be checked that k is a maxima at s = –0.775, –3.2225and minima at s = –2. Hence maxima points are breakaway point and minima points is a break-in point.

Hence 2 breakaway and 1 breaking point.

**Sol. 19.**

The characteristics equation, 1 + G(s) H(s) = 0

$$1+\frac{k}{s(s^2+7s+12)}=0$$

$s(s^2 + 7s + 12) + K = 0$

Point s = –1 + j lie on root locus if it satisfy above equation i.e.

$(-1 + j) [(-1 + j)^2 + 7 + k0 (-1 + j) + 12] + k = 0$

$(-1 + j) [(5 + 5j)] + k = 0$

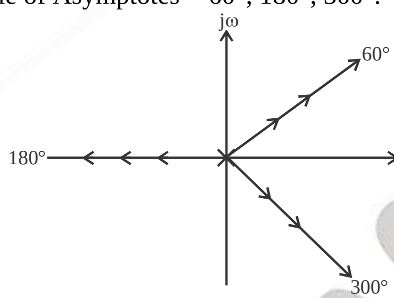
$-5 - 5j - 5 + k = 0$

$k = 10.$

**Sol. 20**.

The root-locus of $\frac{k}{s^3}$ is:

Angle of Asymptotes = 60°, 180°, 300°.




**Sol. 21.**

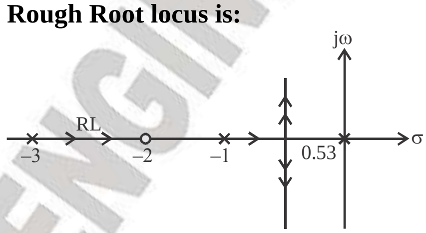
Characteristic equation,

s(s + 1) (s + 3) + k(s + 2) = 0

$$1+\frac{k(s+2)}{s(s+1)(s+3)}=0$$

$$G(s)H(s)=\frac{k(s+2)}{s(s+1)(s+3)}$$

**The Rough Root locus is:**




From the Root-locus, it is seen that the two of its poles tends to infinitely and one of pole terminate at zero.

**Sol. 22.**

Given, 1 + H(s) H(s) = 0

s(s + 2) (s + 3) + k(s + 1) = 0

$$1+\frac{k(s+1)}{s(s+2)(s+3)}=0$$

$$\therefore G(s)H(s)=\frac{k(s+1)}{s(s+2)(s+3)}$$

Total number of poles = 3

Total number of zeros = 1

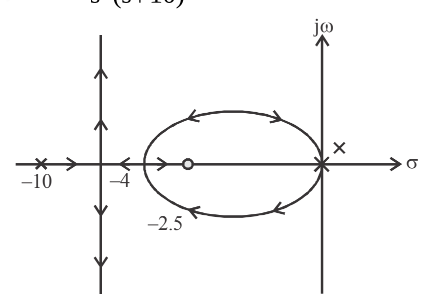
The number of branches of the root loci is equal to the total number of poles 3.

**Sol. 23.**

Because of concept of dominant pole. Here $P_1$ and $P_2$ are dominant pole and $P_3$ and $P_4$ are insignificant poles.

**Sol. 24.**

$$G(s)H(s)=\frac{k(s+1)}{s^2(s+10)}$$




Poles = 0, 0, -10

Zero = – 1

$$\text{Centroid } =\frac{\sum P \sum Z}{P-Z}=\frac{-10+1}{2}=4.5$$

Angle of Asymptotes

$$=\frac{(2q+1)180°}{P-Z}=90°,\ 270°$$

Breakaway point:

1 + G(s) + H(s) = 0

$s^2 (s + 10) + k(s + 1) = 0$

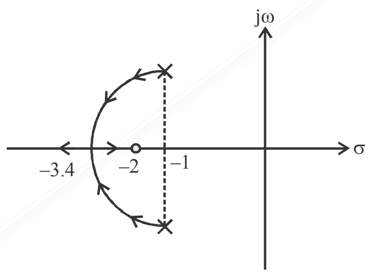
$$k = \frac{-s^3 - 10s^2}{s+1}$$

$$\frac{dk}{ds} = 0 \Rightarrow \frac{(s+1)(-3s^2 - 20s) - (-s^3 - 10s^2)}{(s+1)^2} = 0$$

$-3s^3 - 3s^2 - 20s^2 - 20s + s^3 + 10s^2 = 0$
$-2s^3 - 3s^2 - 20s^2 - 20s + s^3 + 10s^2 = 0$
$2s^2 - 13s^2 - 20s = 0$
$s = -2.5, -4$
Now the root-locus plot is

**Sol. 25.**

$$G(s)H(s) = \frac{k(s+2)}{s^2 2s + 2}$$




Poles = $-1 + j, -1 - j$
Zero = $-2$
Breakaway point:

$$1 + \frac{k(s+2)}{s^2 + 2s + 2} = 0$$

$s^2 + s + 2 + k(s + 2) = 0$

$$k = \frac{-s^2 - 2s - 2}{s+2}$$

$$\frac{dk}{ds} = 0$$

$$\frac{(s+2)(-2s-2) - (-s^2 - 2s - 2)}{(s+2)^2} = 0$$

$-2s^2 - 4s - 2s - 4 + s^2 + 2s + 2 = 0$
$-s^2 - 4s - 2 = 0$
$s = -0.58, -3.4$
$s = -3.4$ is a valid breakaway point.

**Sol. 26. (c)**

$$G(s) = \frac{K}{s(s+1)}$$

The equation corresponding to unity feed back control loop is $s(s + 1) + K = 0$
$\therefore s^2 + s + K = 0$
The may be written as

$$s^2 + 2\xi\omega_n s + \omega_n^s = 0$$

Where $\xi$ is the damping ratio

$$\omega_n = \sqrt{K}, \xi = \frac{1}{2\sqrt{K}}$$

thus if $K \to \infty$, $\xi \to 0$

**Sol. 27. (d)**
The open-loop transfer function for the system is

$$G(s)H(s) = \frac{K(s+1)}{s(s-1)(s^2 + 4s + 16)}$$

Using routh's stabilitycriterion, ch. Equation is
$s^4 + 3s^3 + 12s^2 + (K - 16)s + K = 0$
The routh array becomes

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 12 | K |
| $s^3$ | 3 | $K-16$ | 0 |
| $s^2$ | $\frac{52-K}{3}$ | K | 0 |
| $s^1$ | $\frac{-K^2 + 59K - 832}{3(52-K)}$ | 0 | |
| $s^0$ | K | | |

Values of K that make $s^1$ term in the first column equal to zero are K = 35.7 and K = 23.3.

**Sol. 28. (b)**
The breakaway points not always be on the real axis alone but it must lie on the root loci. It is not necessary that break away points must lie between 0 and -1.

**Sol. 29. (a)**

$$\frac{G(s)}{1+G(s)} = \frac{\frac{k}{s(s+1)(s+2)}}{\frac{k}{s(s+1)(s+2)}} = \frac{k}{s(s+1)(s+2)k}$$

Characteristic equation us
$S(s+1)(s+2) + k = 0$
Or $s^3 + 3s^2 + 2s + k = 0$
Routh array is

| | | |
|---|---|---|
| $s^3$ | 1 | 2 |
| $s^2$ | 3 | k |
| $s^1$ | $\frac{6-k}{3}$ | 0 |
| $s^0$ | k | |

For margin stability,

$$\frac{6-k}{3}=0 \Rightarrow 6=6$$

$3s^2 + k = 0 \Rightarrow 3\,(j\omega)^2 + 6 = 0$

$\Rightarrow -\omega^2 + 2 = 0 \Rightarrow \omega^2 = 2$

$\Rightarrow \ \omega=\sqrt{2}\ \text{rad/s}$

So, the root locus intersects with the imaginary axis at $\pm j\sqrt{2}$

**Sol. 30. (d)**

Root locus shows the transfer function has poles at s = 0, -1 and zero at s = -2.

So, $G(s)=\frac{K(s+2)}{s(s+1)}$

**Sol. 31. (c)**

For the root locus to be parallel to the imaginary axis, the angle of asymptotes should be ±90°

**Sol. 32. (c)**

**Sol. 33. (d)**

$S(s + 2)(s + 3) + K(s + 1) = 0$

$$\Rightarrow G(s)\ H(s)=\frac{K(s+1)}{s(s+2)(s+3)}$$

Since there are 3 poles and 1 zero, therefore, in the root loci one branch will be form a pole to zero and two more branches will be from rest of the poles towards infinity.

**Sol. 34. (d)**

$$G(s)H(s)=\frac{K}{s(s+1)(s+3)}$$

No. of poles = 3

at s = 0, –1, –3

No. of zeros = 0

$\therefore$ Inter section of asymptotes of root loci with open loop transfer function is also called centroid.

$$\therefore \text{Centroid} = \frac{\Sigma P-\Sigma Z}{P-Z}$$

$$=\frac{0-1-3}{3}=-\frac{4}{3}=-1.33$$

**Sol. 35. (c)**

**Sol. 36. (d)**

**Sol.37. (c)**

# GATE QUESTIONS

**1.** The range of K for which all the roots of the equation $s^3 + 3s^2 + 2s + K = 0$ are in the left half of the complex s-plane is

**[GATE - 2017]**

(a) $0 < K < 6$  (b) $0 < K < 16$
(c) $6 < K < 36$  (d) $6 < K < 16$

**2.** The root locus of the feedback control system having the characteristic equation $s^2 + 6Ks + 2s + 5 = 0$ where $K > 0$, enters into the real axis at

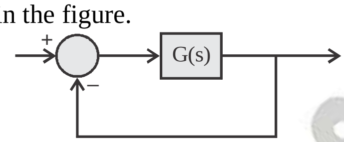
**[GATE - 2017]**

(a) $s = -1$  (b) $s = -\sqrt{5}$
(c) $s = -5$  (d) $s = \sqrt{5}$

**3.** A linear time invariant (LTI) system with the transfer function

$$G(s) = \frac{K(s^2 + 2s + 2)}{(s^2 - 3s + 2)}$$

Is connected in unity feedback configuration as shown in the figure.



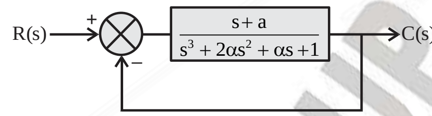

For the closed loop system shown, the root locus for $0 < K < \infty$ intersects the imaginary axis for K = 1.5. the closed loop system is stable for

**[GATE - 2017]**

(a) $K > 1.5$
(b) $1 < K < 1.5$
(c) $0 < K < 1$
(d) No positive value of K

**4.** A closed-loop system is shown in the figure. The system parameter $\alpha$ is not known. The condition for asymptotic stability of the closed loop system is

**[GATE - 2017]**




(a) $\alpha < -0.5$  (b) $-0.5 < \alpha < 0.5$
(c) $0 < \alpha < 0.5$  (d) $\alpha > 0.5$

**5.** The gain at the breakaway point of the root locus of a unity feedback system with open loop transfer function $G(s) = \frac{Ks}{(s+1)(s-4)}$ is

**[GATE - 2016]**

(a) 1  (b) 2
(c) 5  (d) 9

**6.** The forward-path transfer function and the feedback-path transfer function of a single loop negative feedback control system are given as

$$G(s) = \frac{K(s+2)}{s^2 + 2s + 2}$$

and H(s) = 1 respectively. If the variable parameter K is real positive, then the location of the breakaway point on the root locus diagram of the system is ______.

**[GATE - 2016]**

**7.** The open-loop transfer function of a unity-feedback control system is

$$G(s) = \frac{K}{s^2 + 5s + 5}$$

The value of K at the breakaway point of the feedback control system's root-locus plot is ________.

**[GATE - 2016]**

**8.** The open loop poles of a third order unity feedback system are at 0, –1. –2. Let the frequency corresponding to the point where the root locus of the system transits to unstable region be K. Now suppose we introduce a zero in the open loop transfer function at –3, while keeping all the earlier open loop poles intact.

Which one of the following is TRUE about the point where the root locus of the modified system transits to unstable region?

**[GATE - 2015]**

(a) It corresponds to a frequency greater than K
(b) It corresponds to a frequency less than K
(c) It corresponds to a frequency K
(d) Root locus of modified system never transits to unstable region

**9.** An open loop transfer function G(s) of a system is

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

For a unity feedback system, the breakaways point of the root loci on the real axis occurs at,

**[GATE - 2015]**

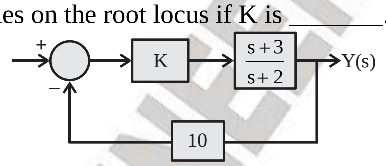
(a) - 0.42
(b) –1.58
(c) - 0.42 and – 1.58
(d) None of the above

**10.** The open-loop transfer function of a plant in a unity feedback configuration is given as

$G(s) = \frac{K(s+4)}{(s+8)(s^2-9)}$. The value of the gain K(>0) for which –1+j2 lies on the root locus is _____.

**[GATE - 2015]**

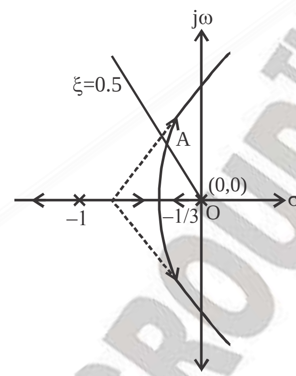
**11.** For the system shown in the figure s = –2.75 lies on the root locus if K is _______.




**[GATE - 2015]**

**12.** The characteristic equation of a unity negative feedback system is 1 + KG(s) = 0. The open loop transfer function G(s) has one pole at 0 and two poles at –1. The root locus of the system for varying K is shown in the figure.

**[GATE - 2014]**



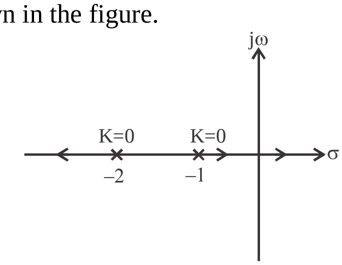

The constant damping ratio line, for ξ = 0.5. intersects the root locus at point A. The distance from the origin to point a is given as 0.5. The value of K at point A is _________.

**13.** In the formation of Routh-Hurwitz array for a polynomial. All the elements of a row have zero values. This premature termination of the array indicates the presence of

**[GATE - 2014]**

(a) Only one root at the origin
(b) Imaginary roots
(c) Only positive real roots
(d) Only negative real roots

**14.** The root locus of a unity feedback system is shown in the figure.




The closed loop transfer function of the system is

**[GATE - 2014]**

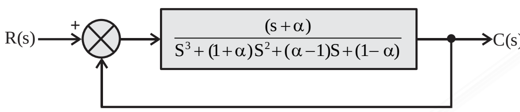
(a) $\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)}$

(b) $\frac{C(s)}{R(s)} = \frac{-K}{(s+1)(s+2)+K}$

(c) $\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)-K}$

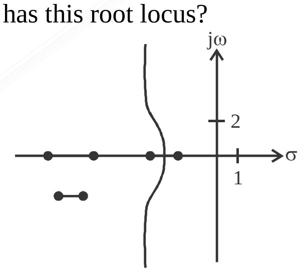
(d) $\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)+K}$

**15.** For the given system, it is desired that the system be stable. The minimum value of $\alpha$ for this condition is ______




**[GATE - 2014]**

**16.** In the root locus plot shown in the figure, the pole /zero marks and the arrows have been removed. Which one f the following transfer functions has this root locus?




**[GATE - 2014]**

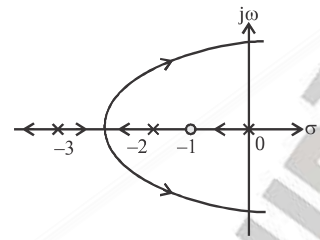
(a) $\frac{s+1}{(s+2)(s+4)(s+7)}$

(b) $\frac{s+4}{(s+1)(s+2)(s+7)}$

(c) $\frac{s+7}{(s+1)(s+2)(s+4)}$

(d) $\frac{(s+1)(s+2)}{(s+7)(s+4)}$

**17.** The root locus plot for a system is given below. The open loop transfer function corresponding to this plot is given by




**[GATE - 2011]**

(a) $G(s)H(s) = k\frac{s(s+1)}{(s+2)(s+3)}$

(b) $G(s)H(s) = k\frac{(s+1)}{s(s+2)(s+3)}$

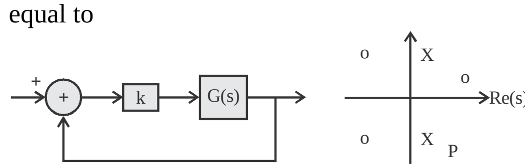
(c) $G(s)H(s) = k\frac{1}{s(s-1)(s+2)(s+3)}$

(d) $G(s)H(s) = k\frac{(s+1)}{s(s+2)(s+3)}$

**18.** The feedback configuration and the pole-zero locations of

$G(s) = \frac{s^2-2s+2}{s^2+2s+2}$

are shown below. The root locus for negative values of k, i.e. for $-\infty < k < 0$, has break always/break in points and angle of departure at pole P(with respect to the positive real axis) equal to




**[GATE - 2009]**

(a) $\pm\sqrt{2}$ and $0^o$

(b) $\pm\sqrt{2}$ and $45°$

(c) $\pm\sqrt{3}$ and $0°$

(d) $\pm\sqrt{3}$ and $45°$

**19.** A unity feedback control system has an open – loop transfer function

$G(s) = \frac{k}{s(s^2+7s+12)}$

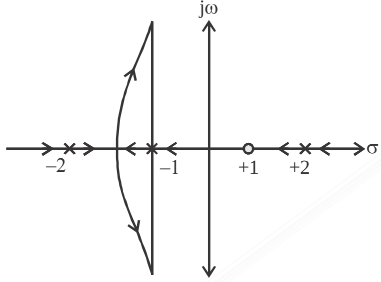
The gain K for which s = 1 + j1 will lie on the root locus of this system is

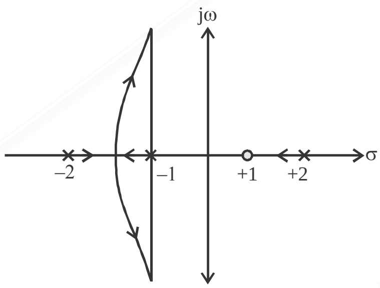
**[GATE - 2007]**

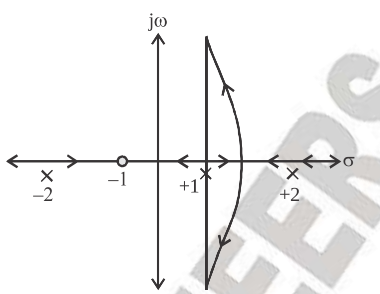
(a) 4 (b) 5.5
(c) 6.5 (d) 10

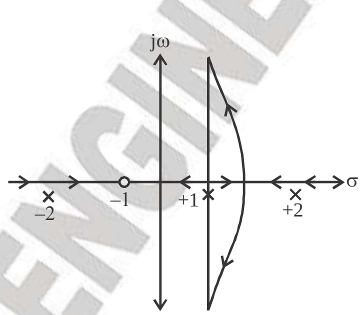
**20.** A closed-loop system has the characteristic function $(s^2 - 4)(s+1) + K(s-1) = 0$. Its root locus plot against K is

**[GATE - 2006]**

(a)


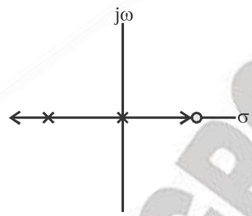

(b)


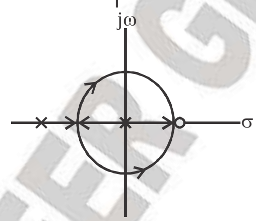

(c)


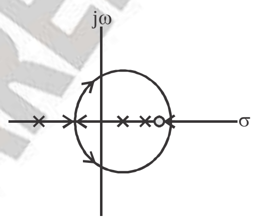

(d)


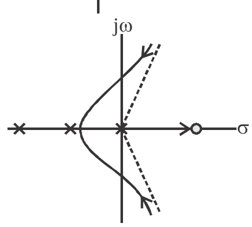

**21.** An unity feedback system is given as $G(s) = \frac{K(1-s)}{s(s+3)}$. Indicate the correct root locus diagram.

**[GATE - 2005]**

(a)



(b)



(c)


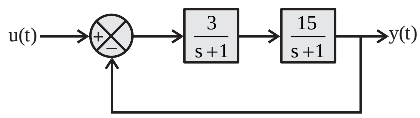

(d)



**22.** Given $G(s)H(s) = \frac{K}{s(s+1)(s+3)}$. The point of intersection of the asymptotes of the root loci with the real axis is

**[GATE - 2004]**

(a) –4 (b) 1.33
(c) – 1.33 (d) 4

**23.** The roots of the closed loop characteristic equation of the system shown above




**[GATE - 2003]**

(a) –1 and –15 (b) 6 and 10
(c) –4 and –15 (d) –6 and –10

**24.** The root locus of system $G(s)H(s) = \dfrac{K}{s(s+2)(s+3)}$ has the breakaway point located at

**[GATE - 2003]**

(a) (–0.5, 0) (b) (–2.548, 0)
(c) (–4, 0) (d) (–0.784, 0)

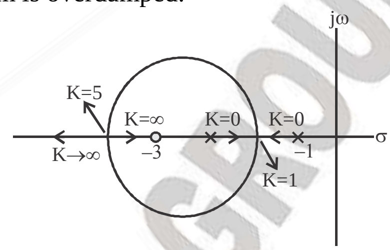
**25.** Which of the following points is NOT on the root locus of a system with the open loop transfer function

$$G(s)H(s) = \frac{K}{s(s+1)(s+3)}$$

**[GATE - 2002]**

(a) $0^o$ (b) $63.4^o$
(c) $90^o$ (d) ∞

**26.** The root-locus diagram for a closed-loop feedback system is shown in the figure. The system is overdamped.




**[GATE - 2001]**

(a) Only if 0 ≤ k ≤ 1
(b) Only if 1 < k < 5
(c) Only if k > 5
(d) If 0 ≤ k < 1 or k > 5

# SOLUTIONS

**Sol. 1. (a)**

| | | |
|---|---|---|
| $S^3$ | 1 | 2 |
| $S^2$ | 3 | k |
| $S^1$ | 6–k | |
| $S^0$ | $k^3$ | |

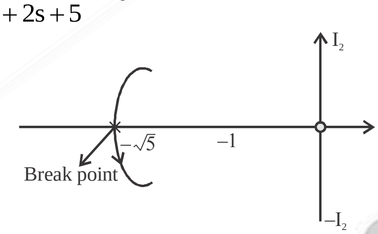
k > 0 and k < 6

0 < k < 6

**Sol. 2. (b)**

CF = $s^2 + 6ks + 2s + 5 = 0$

$$1+\frac{6ks}{s^2+2s+5} = 0$$




1 + G(s) H(s) = 0

$$G(s)H(s) = \frac{6ks}{s^2+2s+s} = 0$$

$$\frac{d}{ds}\left[\frac{6s}{s^2+2s+s}\right] = 0$$

$(s^2 + 2s + s)(6) - 6s(2s+2) = 0$

$6s + 125 + 30 - 12s^2 - 12s = 0$

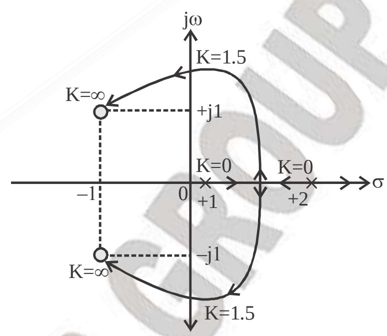
$-6s^2 + 30 = 0$

$s = \pm\sqrt{5}$

$s = -\sqrt{5}$ is a breakpoint

**Sol. 3. (a)**

For CL system stability system gain is greater than 1.5.




**Sol. 4. (d)**

Characteristics equation

C.E = $s^3 + 2\alpha s^2 + (\alpha + 1)\, s + 1 + \alpha = 0$

Condition for stability

$2\alpha(\alpha + 1) \,.\, 1 + \alpha$

$2\alpha^2 + 2\alpha - \alpha - 1 > 0$

$2\alpha^2 + \alpha - 1 \,.\, 0$

$2\alpha^2 + 2\alpha - \alpha - 1 > 0$

$(2\alpha - 1)(\alpha + 1) > 0$

$2\alpha - 1 > 0$

$\alpha + 1 > 0$

$\alpha > ½$

$\alpha > -1$

$\alpha > 0.5$

**Sol. 5. (a)**

$$\frac{d}{ds}\left(\frac{s}{s^2-5s+4}\right) = 0$$

$(s^2 - 5s + 4) - s[2s - s] = 0$

$s^2 - 5s + 4 - 2s^2 + 5s = 0$

$s^2 - 4 = 0$

$s = \pm 2$

$$K\Big|_{s=2} = \frac{(s-1)(s-4)}{s}\Bigg|_{s=2} = \left|\frac{(2-1)(2-4)}{2}\right| = 1$$

K=1

**Sol. 6. (-3.41)**

Given $G(s) = \dfrac{K(s+2)}{s^2+2s+2}, H(s) = 1$

Break away point

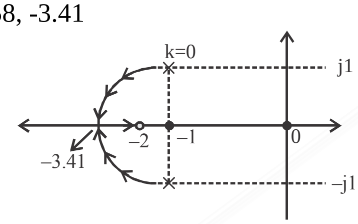
$\Rightarrow \frac{dk}{ds} = 0$

$\frac{d}{ds}\left(\frac{(s+2)}{s^2+2s+2}\right) = 0$

$\Rightarrow \left[\frac{1(s^2+2s+2)-(s+2)(2s+2)}{(s^2+2s+2)^2}\right] = 0$

$\Rightarrow$ $-s^2-4s-2=0$

$\Rightarrow$ -0.58, -3.41




Valid BAP is –3.41

**Sol. 7. (1.25)**

Break away point $\frac{dk}{ds} = 0$

$\frac{d}{ds}\left(\frac{1}{s^2+5s+5}\right) = 0$

0 – (2s+5) = 0

s = – 2.5 is a break away point

K Value is Obtain from Magnitude Condition

$\left|\frac{K}{s^2+5s+5}\right|_{s=-2.5} = 1$

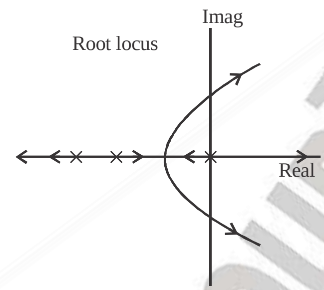
$\left|\frac{K}{6.25-12.5+5}\right| = 1$

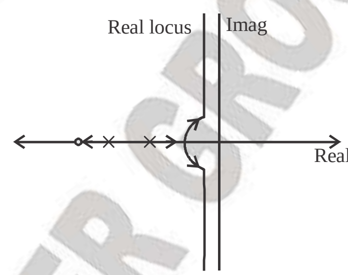
K=1.25

**Sol. 8. (d)**

$G(B) = \frac{1}{s(s+1)(s+2)}$

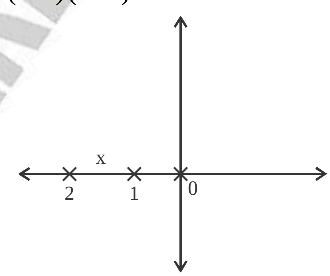
$G_2(3) \Rightarrow \frac{s+3}{s(s+1)(s+2)}$




**Sol. 9. (a)**

$G(s) = \frac{k}{s(s+1)(s+2)}$




H(s) = 1

Characteristic equation is given by q(s) = 1+ GH(s)

$q(s) = s^3 + 3s^2 + 2s + k = 0$

$k = -s^3 - 3s^2 - 2s$

$\frac{dk}{ds} = -3s^2 - 65 - 2 = 0$

s = –0.42 and –1.5.8c

s = – 0.42 will be on root locus

s = – 1.58 will not lie on Root locus.

**Sol. 10. (25.54)**

$G(s) = \frac{K(s+4)}{(s+8)(s^2-9)}$

S = –1 + j2 on root locus so it must satisfy characteristic equation

Q(s) = 1 + 4(s) = (s + 8) $(s^2 - 9)$ + K (s + 4)

$Q(s)|_{s=-1+2j} = 0$

$(-1 + 2j + 8)\,((-1 + 2j)^2 - 9) + (-1 + 2j + 4) = 0$

K = 25.5385 + 0. 3077 j

|K| = 25.54

**Sol. 11. (0.3)**

$$\frac{C(s)}{R(s)} = \frac{K\left(\frac{s+3}{s+2}\right)}{1+10k\left(\frac{s+3}{s+2}\right)}$$

1 + G(s) H(s) = 0

$$G(s)\,H(s) = K(10)\left(\frac{s+3}{s+2}\right)$$

(s + 2) + 1 + 10 K (s + 3) = 0

s + 2 + 10 ks + 30 = 0

10 k + 1 s

$$\frac{10k(s+3)}{(s+2)} = 10k\frac{(-2.75+3)}{(2.75+2)} = 1$$

$$\frac{10k(0.25)}{(-0.7.5)} = 10k\times\frac{25}{25} = 1$$

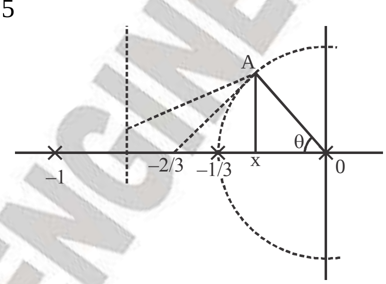
$$k = \frac{3}{10} = 3 \qquad k = 0.3$$

**Sol. 12. (0.375)**

We know that the co-ordinate of point A of the given root locus i.e., magnitude condition |G(s)H(s)|=1

Here, the damping factor ξ = 0.5 and the length of 0A = 5

ξ = 0.5




Then in the right angle triangle

$$\cos\theta = \frac{OX}{OA} \Rightarrow \cos 60 = \frac{OX}{0.5} \Rightarrow OX = \frac{1}{4}$$

$$\Rightarrow \sin\theta = \frac{AX}{OA} \Rightarrow \sin 60 = \frac{AX}{0.5} \Rightarrow AX = \frac{\sqrt{3}}{4}$$

So, the co-ordinate of point A is $\frac{-1}{4} + \frac{j\sqrt{3}}{4}$

Substituting the above value of A in the transfer function and equating to 1

i.e. by magnitude condition .

$$\left|\frac{k}{s(s+1)^2}\right|_{s=-\frac{1}{4}+\frac{j\sqrt{3}}{4}} = 1$$

$$k = \sqrt{\frac{1}{16}+\frac{3}{16}}.\left(\sqrt{\frac{9}{16}+\frac{3}{16}}\right)^2$$

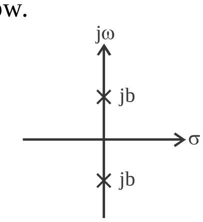
K = 0.375

**Sol. 13. (b)**

In the given Routh-Hurwitz array of polynomial, all the elements of a row have zero value. This is due to symmetrical location of the roots in the s-plane with respect to origin. The system is either marginally stable or unstable. Now, we check this characteristic for all the given

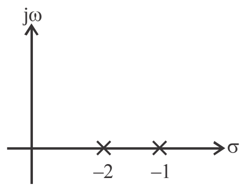
**Option (a):** Only one root is at origin. So, it does not satisfy the symmetrical condition.

**Option (b):** Since, the system has imaginary roots, so we get the pole – zero location diagram as shown below.

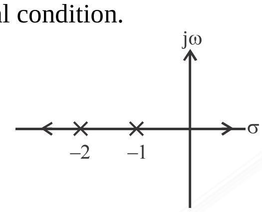



The imaginary roots on jω (imaginary) axis are symmetrical with respect to origin. Hence this option is correct

**Option (c)**: The system has only positive real roots as shown below. So, the root location diagram does not satisfy the symmetrical condition.

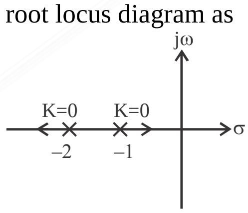



**Option (d)**: Again, the system has only negative real roots, as shown below. So, the root location diagram does not satisfy the symmetrical condition.




**Sol. 14. (c)**

We have the root locus diagram as




As the root locus have poles s = –1, –2 and root lies in even multiple of poles, so it is converse of the main transfer function. Hence, gain should be negative, i.e.

$$G(s)H(s) = \frac{-K}{(s+1)(s+2)}$$

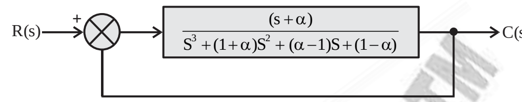
This is open loop transfer function and closed loop transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)H(s)}{1+G(s)H(s)} = \frac{\frac{-K}{(s+1)(s+2)}}{1+\frac{-K}{(s+1)(s+2)}}$$

$$= \frac{-K}{(s+1)(s+2)-K}$$

**Sol. 15. (0.618)**

The Block diagram of given system is




The open loop transfer function is

$$G(s)H(s) = \frac{(s+\alpha)}{s^3+(1+\alpha)s^2+(\alpha-1)+(1-\alpha)}$$

So, we obtain the character equation as

$1 + G(s)H(s) = 0$

or $1+\dfrac{(s+\alpha)}{s^3+(1+\alpha)s^2+(\alpha-1)s+(1-\alpha)} = 0$

or $s^3 + (1 + \alpha)s^2 + (\alpha-1)s + (1-\alpha) + (s+\alpha) = 0$

or $s^3 + (1 + \alpha)s^2 + (\alpha - 1 + 1)s + 1 - \alpha + \alpha = 0$

or $s^3 + ( 1 + \alpha) s^2 + \alpha s + 1 = 0$

For the characteristic equation, we form the Routh's array as

| | | |
|---|---|---|
| $s^3$ | $1$ | $\alpha$ |
| $s^2$ | $1 + \alpha$ | $1$ |
| $s^1$ | $\dfrac{\alpha(1+\alpha)-1}{1-\alpha}$ | $0$ |
| $s^0$ | $1$ | |

For stable system, the required condition is

$1 + \alpha > 0$

or $\alpha > -1$ or $\dfrac{\alpha(1+\alpha)}{1+\alpha} > 0$

or $\alpha (1 + \alpha) - 1 > 0$

Solving the inequality, we obtain the roots

$$\alpha = \frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}$$

So, we get the result for inequality as

$\alpha > 0.618$ and $\alpha < -1.62$

i.e. the minimum value of $\alpha$ is

$\alpha = 0.618$

**Sol. 16. (c)**

Shifting in time domain does not change PSD. Since PSD is Fourier transform of autocorrelation function of WSS process, autocorrelation function depends on time difference.

$X(t) \leftrightarrow R_x(z0 \leftrightarrow S_x(f)$

$Y(t) = X(2t -1) \leftrightarrow R_y(2\zeta ) \leftrightarrow \frac{1}{2}S_x\left(\frac{f}{2}\right)$

[Time scaling property of Fourier transform]

**Sol. 17. (b)**

For given plot root locus exists from –3 to ∞, so there must be odd number of poles and zeros. There is a double pole at s = –3

Now  Poles = 0, –2, –3, –3

Zeros = –1 Thus transfer function

$$G(s)H(s) = \frac{k(s+1)}{s(s+2)(s+3)^2}$$

**Sol. 18. (b)**

The characteristic equation is

1 + G(s) H(s) = 0

$$\text{or } 1+\frac{K(s^2-2s+2)}{s^2+2s+2}=0$$

or $s^2 + 2s + 2 + K(s^2 – 2s +2) = 0$

$$\text{or } K = -\frac{s^2+2s+2}{s^2-2s+2}$$

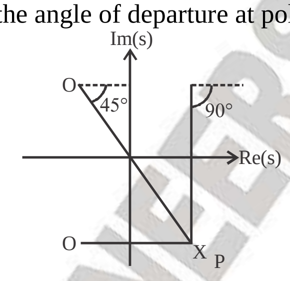
For break away &^ break in point differentiating above w.r.t s we have

$$\frac{dK}{ds} = -\frac{(s^2-2s+2)(2s+2)-(s^2+25+2)(2s-2)}{(s^2-2s+2)^2}$$

Thus $(s^2 –2s + 2)(2s +2) – (s^2 + 2s +2)(2s –2)=0$

Or $s = \pm\sqrt{2}$

Let $\theta_d$ be the angle of departure at pole P, then




$-\theta_d - \theta_{p1} + \theta_{z1} + \theta_{z2} = 180^o$

$-\theta_d = 180^o - (-\theta_{p1} + \theta_{z1} + \theta_2)$

$= 180^o - (90^o + 180 - 45^o) = -45°$

**Sol. 19. (d)**

For ufb system the characteristics equation is

1 + G(s) = 0

$$\text{Or } 1+\frac{K}{s\left(s^2+7s+12\right)}=0$$

Or $s(s^2 + 7s + 12) + K = 0$

Point s = –1 + j lie on root locus if it satisfy above equation i.e,

$(–1 + j) [(–1 + j)^2 + 7(–1 + j) + 12) + K] = 0$

Or K = +10

**Sol. 20. (b)**

Given characteristic equation

$(s^2 – 4) (s +1) + K(s -1) = 0$

$$\text{Or } 1+\frac{K(s-1)}{(s^2-4)(s+1)}=0$$

So, the open loop transfer function of for the system

$$G(s) = \frac{K(s-1)}{(s-2)(s+2)(s+1)}$$

No. of poles n = 3

No. of zeroes m = 1

Steps for plotting the root – locus

(1) Root loci starts at s = 2, s = –1 , s = –2

(2) n > m, therefore, number of branches of root locus b = 3

(3) angle of asymptotes is given by

$$\frac{(2q+1)}{n-m}, q = 0, 1$$

(I) $\frac{(2\times 0+1)180°}{(3-1)} = 90°$

(II) $\frac{(2kt1+1)}{(3-1)} = 270°$

(4) The two asymptotes intersect on real axis at

$$x = \frac{\sum \text{Poles} - \sum \text{Zeroes}}{n-m}$$

$$= \frac{(-1-2+2)-(1)}{3-1} = -1$$

(5) between two open – loop poles s = –1 and s = –2 thee exist a breakaway point.

$$K = -\frac{(s^2-4)(s+1)}{(s-1)} \quad \frac{dK}{ds} = 0$$

s = –1.5

**Sol. 21. (c)**

Any point on real axis of s – is part of root locus if number of OL poles and zeros to right of that point is even. Thus (b) and (c) are possible option.
The characteristics equation is
1 + G(s) H(s) = 0

Or $1+\frac{K(1-s)}{s(s+3)}=0$ or $K=\frac{s^2+3s}{1-s}$

For break away & break in point

$$\frac{dK}{ds}=(1-s)(2s+3)+s^2+3s=0$$

Or $-s^2 + 2s + 3 = 0$ which gives s = 3, –1
Here –1 must be the break away point and 3 must be the break in point.

**Sol. 22. (c)**
Centroid is the point where all asymptotes intersects.

$$\sum \text{real of open loop pole}$$

$$\sigma=\frac{-\sum \text{Real part of open loop pole}}{\sum \text{No. of open loop pole}}$$

$$-\sum \text{No. of open loop zero}$$

$$=\frac{-1-3}{3}=-1.33$$

**Sol. 23. (c)**
Characteristic equation is given by
1 + G(s) H(s) = 0
Here H(s) = 1 (unity feedback)

$$G(s)=\left(\frac{3}{s+15}\right)\left(\frac{15}{s+1}\right)$$

$$\text{So, } 1+\left(\frac{3}{s+15}\right)\left(\frac{15}{s+1}\right)=0$$

$(s+15)(s+1) + 45 = 0$  $s^2 + 16s + 60 = 0$
$(s+6)(s+10) = 0$  $s = -6, -10$

**Sol. 24. (d)**
We have 1 + G (s) H(s) = 0

$$\text{Or } 1+\frac{K}{s(s+2)(s+3)}=0$$

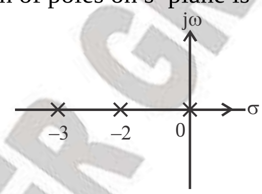
Or $K = -s(s^2 + 5s^2 + 6s)$

$$\frac{dK}{ds}=-(3s^2+10s+6)=0$$

Which gives

$$s=\frac{-10\pm\sqrt{100-72}}{6}=-0.784, 2.548$$

The location of poles on s–plane is




**Sol. 25. (b)**
Any point on real axis lies on the root locus if total number of poles and zeros to the right of that point is odd. Here s = –1.5 does not lie on real axis because there are total two poles and zeros (0 and –1) to the right of s = –1.5.

**Sol. 26. (d)**
It roots of characteristics equation lie on negative axis at different positions (i.e. unequal), then system response is over damped.
From the root locus diagram we see that for 0 < K < 1, the roots are on imaginary axis and for 1 < K < 5 roots are on complex plain. For K > 5 roots are again on imaginary axis.
Thus system is over damped for $0 \le K < 1$ and $K > 5$.

# ESE OBJ QUESTIONS

**1.** While forming a Routh array, the situation of a row of zeros indicates that the system

**[EE ESE - 2017]**

(a) Has symmetrically located roots
(b) Is stable
(c) Is insensitive to variations in gain
(d) Has asymmetrically located roots

**2.** A unity feedback system has open loop transfer function with two of its poles located at –0.1, 1; and two zeroes located at –2 and –1 with a variable gain K. For what value (s) of K would the closed – loop system have one pole in the right half of the s-plane?

**[EE ESE - 2017]**

(a) $K > 0.3$ (b) $K < 0.05$
(c) $0.05 < K < 0.3$ (d) $K > 0$

**3.** The open-loop transfer function of a unity feedback control system is

$$G(s)H(s) = \frac{10}{s(s+2)(s+K)}$$

Here, K is a variable parameter. The system will be stable for all values of

**[EC ESE - 2017]**

(a) $K > -2$ (b) $K > 0$
(c) $K > 1$ (d) $K > 1.45$

**4.** Consider that in a system loop transfer function, addition of a pole results in the following:
1. Root locus gets pulled to the right-hand side.
2. Steady – state error is increased.
3. system responses gets slower.
Which of the above statements are correct?

**[EC ESE - 2017]**

(a) 1, 2 and 3 (b) 1 and 2 only
(c) 1 and 3 only (d) 2 and 3 only

**5.** Consider the system with $G(s) = \frac{K(s+2)}{s^2+2s+3}$ and H(s) = 1. The breakaway point(s) of the root loci is/are at

**[EC ESE - 2016]**

(a) –0.265 only
(b) –3.735 only
(c) –0.3735 and –0.265
(d) There is no breakaway point

**6.** The main objectives of drawing the root-locus plot are
1.To obtain a clear picture of the open-loop poles and zeros of the system.
2.To obtain a clear picture of the transient response of the system for varying gain K.
3.To find the range of K to make the system stable.
Which of the above statements are correct?

**[EC ESE - 2016]**

(a) 1, 2 and 3 (b) 1 and 2 only
(c) 1 and 3 only (d) 2 and 3 only

**7.** A unity feedback system has open-loop poles at s = –2 ± j2, s = –1 and s = 0 and a zero at s = –3. What are the angles made by the root-loci asymptotes with the real axis?

**[EC ESE - 2016]**

(a) 60°, 180° and –60°
(b) 30°, 90° and 60°
(c) 60°, 120° and –30°
(d) 30°, 60° and 180°

**8. Statement (I):** A root locus is obtained using the closed- loop poles.
**Statement (II):** A root locus is plotted using the open – loop poles.

**[EE ESE - 2015]**

**Codes:**
(a)Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b)Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).

(c)Statement (I) is true but Statement (II) is false.
(d)Statement (I) is false but Statement (II) is true.

**9. Statement (I):**At breakaway point, the system is critically damped.
**Statement (II):**At the point where root loci intersect with the imaginary axis, the system is marginally stable.
**[EE ESE - 2015]**
**Codes:**
(a)Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b)Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c)Statement (I) is true but Statement (II) is false.
(d)Statement (I) is false but Statement (II) is true.

**10. Statement (I):** Centroid is the point where the root loci break from the real axis.
**Statement (II):** Centroid is the point of the real axis where all the asymptotes intersect.
**[EE ESE - 2015]**
**Codes:**
(a)Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b)Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c)Statement (I) is true but Statement (II) is false.
(d)Statement (I) is false but Statement (II) is true.

**11. Statement (I):** Inverse root locus is the image of the direct root locus.
**Statement (II):** Root locus is symmetrical about the imaginary axis.
**Codes:**
(a)Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b)Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c)Statement (I) is true but Statement (II) is false.
(d)Statement (I) is false but Statement (II) is true.

**12.** The open – loop transfer function of a feedback control system is given by

$$G(s)H(s) = \frac{K(s+8)}{s(s+4)(s^2+4s+8)}$$

In the root locus diagram of the system, the asymptotes on the root loci for large values of K meet at a point in the s – plane. Which one of the following is the set of coordinates of that point ?
**[EE ESE – 2015]**
(a) (– 1, 0) (b) (– 2, 0)
(c) (1, 0) (d) (2, 0)

**13.** Consider the following statements about root locus:
(i) The root locus is symmetrical about real axis.
(ii) If a root locus branch moves along the real axis from an open – loop pole to zero or to infinity, this root locus branch is called real root branch
(iii) The breakaway points of the root locus are the solutions of $\frac{dK}{ds} = 0$
Which of the above statements are correct?
**[EE ESE - 2015]**
(a) i and ii only (b) i and iii only
(c) ii and iii only (d) i, ii and iii

**14.** A unity feedback system has open-loop transfer function

$$G(s) = \frac{K(s+4)}{(s+1)(s+2)}$$

The portions of the real axis that lie on the root loci are between
**[EE ESE - 2015]**
(a) s = – 2 and s = – 4; s = –1 and + ∞
(b) s = –1 and s = –2; s = –4 and –∞
(c) s = 0 and s = –2; beyond s = –4

(d) s = 0 and s = –1

**15.** Which of the following points is not on the root locus of a system with the given open – loop transfer function ?

$$G(s)H(s)=\frac{K}{s(s+1)(s+3)}$$

**[EE ESE - 2014]**

(a) $s=-j\sqrt{3}$ (b) $s=-1.5$
(c) $s=-3$ (d) $s=-\infty$

**16.** The characteristic equation of a control system is given by
s(s + 4) (s + 5) (s + 6) + K(s + 3) = 0
The number of asymptotes and the centroid of the asymptotes of this control system are

**[EE ESE - 2014]**

(a) 3 and (4, 0) (b) –3 and (–, 0)
(c) –3 and (–12, 0) (d) 3 and (–4, 0)

**17.** Consider the transfer function G(s) H(s) $=\frac{K}{s^3+4s^2+s-6}$. The root-locus plot of the system passes through s = 0. The value of K at this point will be:

**[EC ESE - 2013]**

(a) 10 (b) 0
(c) 6 (d) 8

**18.** A system has its open-loop transfer function of $\frac{K}{s(s^2+6s+10)}$. The break points are s = –1.18 and s = –2.82, the centroid is at s = –2, while the asymptotic angles are ±60° and ± 180°. The value of K for the closed loop system to be oscillatory and the frequency of oscillations are respectively:

**[EC ESE - 2013]**

(a) 600 and 10 rad/sec
(b) 120 and 5 rad/sec
(c) 60 and 3.16 rad/sec
(d) 30 and 3.16 rad/sec

**19.** If root loci plots of a particular control system do not intersect imaginary axis at any point, then the gain margin of the system will be

**[EC ESE - 2012]**

(a) Zero (b) 0.707
(c) 1.0 (d) Infinite

**20.** If a feedback control system has its open-loop transfer function.

$G(s)H(s)=\frac{K}{\left[s(s+2)(s^2+2s+5)\right]}$ the coordinates of the centroid of the asymptotes of its root-locus are

**[EC ESE - 2012]**

(a) –1 and 0 (b) 1 and 0
(c) 0 and –1 (d) 0 and 1

**21. Statement (I)**: Root loci are symmetrical with respect to real axis of the s-plane.
**Statement (II)**: Root loci are normally symmetrical with respect to the loop transfer function.

**[EC ESE - 2012]**

(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

**22. Statement (I):** The network function N(s) is denoted with scale factor multiplied with the ratio of zero factors with pole factors.
**Statement (II):** When there are n zeroes and m poles, then the poles at infinity are of multiplicity or degree of (n – m). similarly when n <m, then the zeroes at infinity are of multiplicity of degree of (m – n).

**[EE ESE - 2012]**

(a)Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b)Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c)Statement (I) is true but Statement (II) is false.
(d)Statement (I) is false but Statement (II) is true.

**23.** The open-loop transfer function of the feedback control system is given by

$$G(s) = \frac{K}{(s+1)(s+2)(s+3)}$$

The breakaway point in its root locus will be

**[EE ESE - 2012]**

(a) Between – 2 and –3
(b) Between –1 and –2
(c) Between 0 and –1
(d) Beyond –3

**24.** Addition of open-loop poles results into which of the following ?

**[EE ESE - 2012]**

(a) Root locus shifts towards imaginary axis.
(b) Root locus shifts away from imaginary axis.
(c) System stability increases.
(d) System becomes less oscillatory.

**25.** The angle between two adjacent asymptotes in a root locus diagram

**[EE ESE - 2012]**

(a) $\frac{\pi}{n+m}$ (b) $\frac{2\pi}{n+m}$
(c) $\frac{\pi}{n-m}$ (d) $\frac{2\pi}{n-m}$

**26.** Consider the following statements regarding root loci plot:
(i)When gain K is zero, the roots coincide with the poles.
(ii)When K is increased, the roots move away from the poles.
(iii)A root locus diagram is always symmetric about the imaginary axis.
(iv)The number of branches terminates of infinity is open loop poles plus zeros.
Which of these statements are correct?

**[EE ESE - 2011]**

(a) i and ii (b) ii and iii
(c) iii and iv (d) i, ii, iii and iv

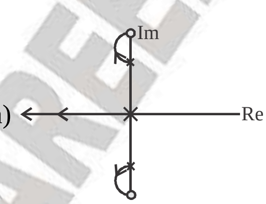
**27.** The breakaway point in the root loci plot for the loop transfer function G(s) = $\frac{K}{s(s+3)^2}$ is

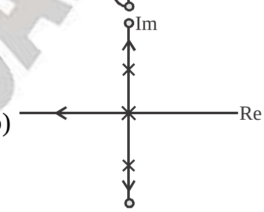
**[EE ESE - 2011]**

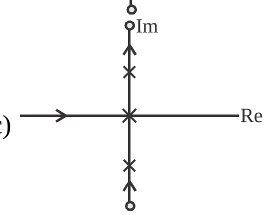
(a) –2.5 (b) –2.0
(c) –1.0 (d) –0.5

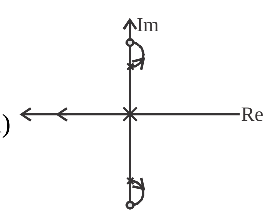
**28.** Loop transfer function is unity feedback system is $G(s) - \frac{K(s^2+64)}{s(s^2+16)}$. The correct root locus diagram for the system is

**[EE ESE - 2011]**

(a)


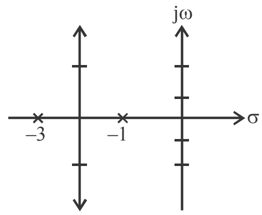

(b)



(c)



(d)



**29.** Given the root locus of a system

$$G(s)=\frac{4K}{(s+1)(s+3)}$$




What will be the gain for obtaining the damping ratio 0.707?

**[EC ESE - 2011]**

(a) 1/4 (b) 5/4

(c) –3/4 (d) 11/4

**30.** Where are the K = 0 points on the root loci of the characteristic equation of the closed loop control system located at?

**[EC ESE - 2011]**

(a) Zero of G(s) H(s)

(b) Poles of G(s) H(s)

(c) Both Zero and Poles of G(s) H(s)

(d) Neither at Zeros nor at Poles of G(s) H(s)

**31.** The characteristic equation of a control system is given as

$$1+\frac{K(s+1)}{s(s+4)(s^2+2+s)}=0$$

For large value of s, the root loci for $K \geq 0$ are asymptotic to asymptotes, where do the asymptotes intersect on the real axis?

**[EC ESE - 2011]**

(a) $\frac{5}{3}$ (b) $-\frac{2}{3}$

(c) $-\frac{5}{3}$ (d) $\frac{4}{3}$

**32.** Where are the $K = \pm\infty$ points on the root loci of the characteristic equation of the closed loop control system located at?

**[EC ESE - 2011]**

(a) Poles of G(s) H(s)

(b) Zeros of G(s) H(s)

(c) Both Zeros and Poles of G(s) H(s)

(d) Neither at Zeros nor at Poles of G(s) H(s)

**33.** Consider the equation

$s^2 + 2s + 2 + K(s + 2) = 0$

Where do the roots of this equation break on the root loci plot?

**[EC ESE - 2009]**

(a) – 3.414 (b) – 2.414

(c) – 1.414 (d) – 0.414

**34.** The open loop transfer function of a closed loop control system is given as:

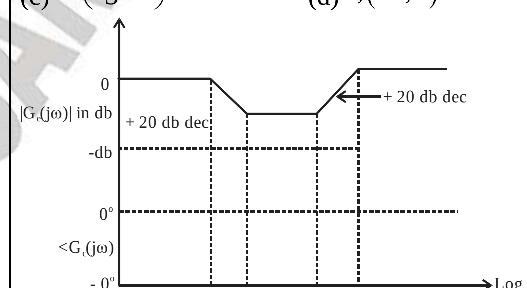
$$G(s)H(s)=\frac{K(s+2)}{s(s+1)(s+4)^2}$$

What are the number of asymptotes and the centroid of the asymptotes of the root – loci of closed loop system?

**[EE ESE - 2009]**

(a) $-3;\left(\frac{7}{3},0\right)$ (b) $-2;(2,0)$

(c) $3;\left(\frac{-7}{3},0\right)$ (d) $2;(-2,0)$




**35.** Root locus of $s(s + 2) + K(s + 4) = 0$ is a circle. What are the co – ordinates of the centre of this circle?

**[EE ESE - 2009]**

(a) –2, 0 (b) –3, 0

(c) –4, 0 (d) –5, 0

**36.** Which one of the following describes correctly the effect of adding a zero to the system ?

**[EE ESE - 2009]**

(a) System becomes oscillatory

(b) Root locus shifts toward imaginary axis

(c) Relative stability of the system increases
(d)Operating range of K for stable operation decreases

**37.** Which one of the following is correct ?
The value of the system gain at any point on a root locus can be obtained as a

**[EE ESE - 2008]**

(a) Product of lengths of vectors from the poles to that point
(b) Product of lengths of vectors from the zeroes to that point
(c) Ratio of product of lengths of vectors from poles to that point to the product of length of vectors from zeroes to that point
(d) Product of lengths of vectors from all poles to zeroes.

**38.** Which one of the following is correct?
The root locus is the path of the roots of the characteristic equation traced out in the s-plane.

**[EC ESE - 2008]**

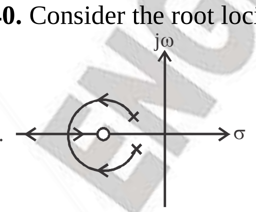
(a) As the input of the system is changed
(b) As the output of the system is changed
(c) As a system parameter is changed
(d) As the sensitivity is changed

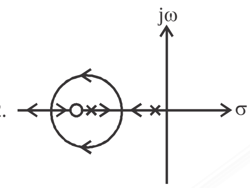
**39.** Which one of the following statements is correct for root loci?

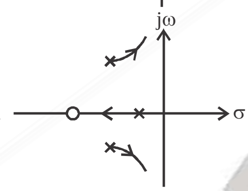
**[EC ESE - 2008]**

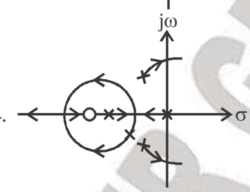
(a)The root loci are asymmetrical with respect to the real axis.
(b)The root loci are symmetrical with respect to the real axis
(c)The root locus lies in a section of the real axis to the left of which an even number of poles and zeros of open system are present.
(d)The root locus lies in a section of the real axis to the right of which an even number of poles and zeros of open loop system are present.

**40.** Consider the root loci plots:

1.



2.



3.



4.



Which one of the above plots is not correct?

**[EC ESE - 2008]**

(a) 1 (b) 2
(c) 3 (d) 4

**41.** What is the number of root-locus segments which do not terminate on zeros?

**[EC ESE - 2007]**

(a)The number of poles
(b)The number of zeros
(c)The difference between the number of poles and the number of zeros
(d)The sum of the number of poles and the number of zeros

**42.** Which one of the following open-loop transfer functions has root locus parallel to imaginary axis?

**[EC ESE - 2007]**

(a) $K/(s + 1)$ (b) $K (s + 1)/(s + 2)^2$
(c) $K/(s + 2)^2$ (d) $K(s + 2)/(s + 1)^2$

**43.** Consider the following statements made on the basis of root locus analysis:
(i) The intersection of asymptotes must always be on the real axis.
(ii) The asymptotes of root loci refer to the angle of root loci when gain K is zero.

Which of the statements given above is/are correct?

**[EE ESE - 2007]**

(a) i only (b) ii only
(c) Both i and ii (d) Neither i nor ii

**44.** Which one of the following statements is not correct?

**[EE ESE - 2007]**

(a) Root loci can be used for analyzing stability and transient performance.
(b) Root loci provide insight into system stability and performance.
(c) Shape of root locus gives idea of type of controller needed to meet design specification.
(d) Root locus can be used to handle more than one variable at a time.

**45. Assertion (A):** Adding a pole to the open–loop transfer function G(s) H(s) has the effect of pushing the root loci towards the R.H.S. in s–plane.
**Reason (R):** If the number of poles increases the angle of asymptotes for the complex roots is reduced.

**[EE ESE - 2007]**

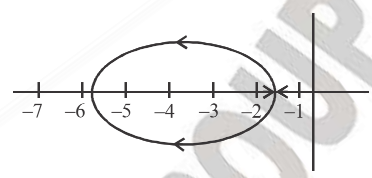
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and r are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**46.** Consider the following statements in connection with the addition of a pole to the forward path transfer function:
(i) Closed-loop system becomes less stable.
(ii) Rise time of the system increases.
(iii) Bandwidth of the system increases.
Which of the statements given above are correct?

**[EE ESE - 2006]**

(a) Only i and ii (b) Only ii and iii
(c) Only i and iii (d) i, ii and iii

**47.** What is the open-loop transfer function for a unity feedback having root locus shown in the following figure?




**[EE ESE - 2006]**

(a) $\frac{k(s+5)}{(s+1)(s+2)}$ (b) $\frac{k(s+1)}{(s+5)(s+6)}$

(c) $\frac{k}{s(s+1)(s+5)}$ (d) $\frac{k(s+2)}{(s+1)(s+5)}$

**48.** For a given unity feedback system with $G(s) = \frac{K(s+3)}{s(s+1)(s+2)(s+5)}$, what is the real axis intercept for root locus asymptotes?

**[EC ESE - 2006]**

(a) 2/3 (b) 1/4
(c) –5/3 (d) –3/2

**49.** In root locus, what is the number of separate loci?

**[EC ESE - 2006]**

(a)The number of zeros of the open loop transfer function.
(b)The number of poles of G(s) H(s)
(c)The number of roots of the characteristic equation with positive real part.
(d)The number of zeros of the characteristic equation with the negative real parts

**50.** Which one of the following is not a property of root loci?

**[EC ESE - 2005]**

(a)The root locus is symmetrical about $j\omega$ axis.
(b)They start from the open loop poles and terminate at the open loop zeros.
(c)The breakaway points are determined from dK/ds = 0.

(d)Segment of the real axis are part of the root locus, if and only, the total number of real poles and zeros to their right is odd.

**51.** The open loop transfer function of a feedback system has m poles and n zeroes (m > n).
Consider the following statements:
(i) The number of separate root loci is m.
(ii) The number of separate root loci is n.
(iii) The number of root loci approaching infinity is (m – n).
(iv) The number of root loci approaching infinity is (m + n).
Which of the statements given above are correct?

**[EE ESE - 2005]**

(a) i and iv (b) i and iii
(c) ii and iii (d) ii and iv

**52.** The characteristic equation of a control system is given by
$s(s + 4)(s^2 + 2s + s) + k(s + 1) = 0$
What are the angles of the asymptotes for the root loci for $k \geq 0$ ?

**[EE ESE - 2005]**

(a) 60°, 180°, 300° (b) 0°, 180°, 300°
(c) 120°, 180°, 240° (d) 0°, 120°, 240°

**53. Assertion (A):** An addition of real zero at $s = z_0$ in the transfer function G(s)H(s) of a control system results in the increase stability margin.
**Reason (R):** An addition of real zero at $s = -z_0$ in the transfer function g(s)H(s) will make the resultant root loci bend towards the left.

**[EE ESE - 2004]**

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is NOT the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

**54.** A control system has
$G(s)H(s) = K/[s(s+4)(s^2 + 4s + 20)]$ $(0 < K < \infty)$
What is the number of breakaway points in the root locus diagram ?

**[EE ESE - 2004]**

(a) One (b) Two
(c) Three (d) Zero

**55. Assertion (A)**: The number of branches of root locus terminating on infinity is equal to the number of open loop poles minus the number of zeros.
**Reason (R)**: Segment of the real axis having an odd number of real axis open loop poles plus zeros to their right are parts of the root locus.

**[EC ESE - 2004]**

(a) Both A and R are true and r is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

**56. Assertion (A)**: In the error detector configuration using a synchro transmitter and syncho control transformer, the latter is connected to the error amplifier.
**Reason (R)**: Synchro control transformer has almost a uniform reluctance path between the rotor and the stator.

**[EC ESE - 2004]**

(a) Both A and R are true and r is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

**57.** The root loci of a feedback control system for large values of s are asymptotic to the straight lines with angles θ to the real axis given by which one of the following?

**[EC ESE - 2004]**

(a) $\frac{(p-z)\pi}{2k+1}$ (b) $\frac{(2k+1)\pi}{p-z}$
(c) 2k(p – z) (d) $\frac{2k}{p}z$

where p = number of finite poles of G(s) H(s), z = Number of finite zeros of G(s) H(s) and k = 0, 1, 2, ….

**58.** The characteristic equation of a control system is given by $s^6 + 2s^5 + 12s^3 + 20s^2 + 16s + 16 = 0$. The number of the roots of the equation which lie on the imaginary axis of s-plane is

**[EC ESE - 2003]**

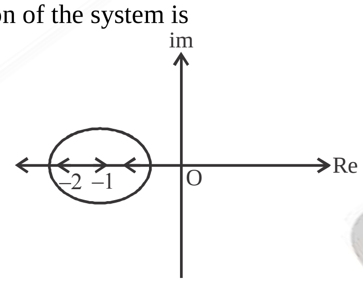
(a) Zero (b) 2
(c) 4 (d) 6

**59.** Which one of the following application softwares is used to obtain an accurate root locus plot?

**[EC ESE - 2003]**

(a) LISP (b) MATLAB
(c) dBase (d) Oracle

**60.** The below figure shows the root locus of ε unity feedback system. The open loop transfer function of the system is




**[EC ESE - 2003]**

(a) $\frac{K}{s(s+1)(s+2)}$ (b) $\frac{Ks}{(s+1)(s+2)}$
(c) $\frac{K(s+1)}{s(s+2)}$ (d) $\frac{K(s+2)}{s(s+1)}$

**61.** The loop transfer function of a system is given by:

$$G(s)H(s) = \frac{K(s+10)^2(s+100)}{s(s+25)}$$

The number of loci terminating at infinity is

**[EE ESE - 2003]**

(a) 0 (b) 1
(c) 2 (d) 3

**62.** Consider the following statements with reference to the root loci of the characteristic equation of unity feedback control system with an open loop transfer function of

$$G(s) = \frac{K(s+1)(s+3)(s+5)}{s(s+2)}$$

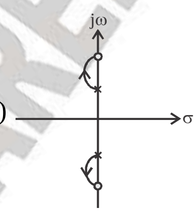
(i) Each locus starts at an open loop zero and ends either at an open loop pole or infinity.
(ii) Each locus starts at an open loop pole or infinity and ends at an open loop zero.
(iii) There are three separate root loci.
(iv) There are five separate root loci.
Which of these statements are correct?

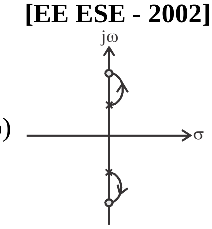
**[EE ESE - 2003]**

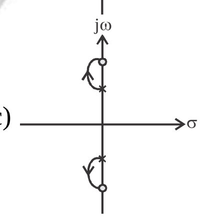
(a) ii and iii (b) ii and iv
(c) i and iii (d) i and iv

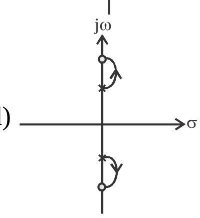
**63.** Identify the correct root locus from the figures given below referring to poles and zero at ± j10 respectively of G(s)H(s) of a single (closed) – loop control system.

**[EE ESE - 2002]**

(a) 


(b) 


(c) 


(d) 


**64.** Which of the following are the characteristics of the root locus of
(i) It has one asymptote
(ii) It has intersection with jω - axis
(iii) It has two real axis intersections
(iv) It has two zeros at infinity
Select the correct answer using the codes given below:

**[EE ESE - 2002]**

(a) i only (b) ii and iii
(c) iii and iv (d) i and iii

**65.** A control system has $G(s)H(s) = \frac{K(s+1)}{s(s+3)(s+4)}$

Root locus of the system can lie on the real axis.

**[EC ESE - 2002]**

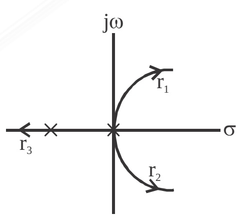
(a) Between s = –1 and s = –3
(b) Between s = 0 and s = –4
(c) Between s = –3 and s = –4
(d) Towards left of s = –4

**66.** The instrument used for plotting the root locus is called

**[EC ESE - 2002]**

(a) Slide rule (b) Spirule
(c) Synchro (d) Selsyn

**67.** Which of the following is the open loop transfer function of the root loci shown in figure?




**[EC ESE - 2002]**

(a) $\frac{K}{s(s+T_1)^2}$ (b) $\frac{K}{(s+T_1)(s+T_2)^2}$

(c) $\frac{K}{(s+T_1)^3}$ (d) $\frac{K}{s^2(sT_1+1)}$

**68.** An open loop transfer function is given by $G(s)H(s) = \frac{K(s+1)}{s(s+2)(s^2+2s+2)}$. It has

**[EC ESE - 2001]**

(a) One zero at infinity
(b) Two zeros at infinity
(c) Three zeros at infinity
(d) Four zeros at infinity

**69.** The root locus plot of the system having the loop transfer function

$G(s)H(s) = \frac{K}{s(s+4)(s^2+4s+5)}$ has

**[EC ESE - 2001]**

(a) No breakaway points
(b) Three real breakaway points
(c) Only one breakaway point
(d) One real and two complex breakaway points

**70.** The characteristic equation of a feedback control system is given by $s^3 + 5s^2 + (K + 6)s + K = 0$. In the root loci diagram, the asymptotes of the root loci for large 'K' meet at a point in the s – plane whose coordinates are

**[EE ESE - 2001]**

(a) (2, 0) (b) (– 1, 0)
(c) (– 2, 0) (d) (– 3, 0)

**71. Assertion (A):** The number of separate loci or poles of the closed loop system corresponding to $G(s)H(s) = \frac{K(s+4)}{s(s+1)(s+3)}$ is three.

**Reason (R):** Number of separate loci is equal to number of finite poles of G(s) G(s) if the latter is more than the number of finite zeroes of G(s) H(s).

**[EE ESE - 2001]**

(a) Both A and R are true and r is the correct explanation of A.
(b) Both A and R are true but R is NOT the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

**72.** Which one of the following characteristic equations of result in the stable operation of the feedback system?

**[EC ESE - 2000]**

(a) $s^3 + 4s^2 + s – 6 = 0$
(b) $s^3 + s^2 + 5s + 6 = 0$
(c) $s^3 + 4s^2 + 10s + 11 = 0$
(d) $s^4 + s^3 + 2s^2 + 4s + 6 = 0$

**73.** The intersection of asymptotes of root-loci of a system with open-loop transfer function

$G(s)H(s) = \frac{K}{s(s+1)(s+3)}$ is

**[EC ESE - 2000]**

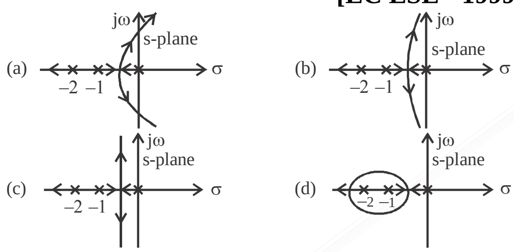
(a) 1.44 (b) 1.33
(c) –1.44 (d) –1.33

**74.** For a unity negative feedback control system, the open loop transfer function is

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

The root-locus plot of the system is

**[EC ESE - 1999]**




**75.** Consider the following statements:
In root-locus plot, the breakaway points

1. Need not always be on the real axis alone.
2. Must lie on the root loci
3. Must lie between 0 and –1

Which of these statements are correct?

**[EC ESE - 1999]**

(a) 1, 2 and 3
(b) 1 and 2
(c) 1 and 3
(d) 2 and 3

**76.** Consider the loop transfer function

$$G(s)H(s) = \frac{K(s+6)}{(s+3)(s+5)}$$

In the root-locus diagram, the centroid will be located at

**[EC ESE - 1999]**

(a) –4
(b) –1
(c) –2
(d) –3

# SOLUTIONS

**Sol.1. (a)**
All the elements of a row in Routh's tabulation being zero indicate a pair of conjugate root on imaginary axis. i.e., system has symmetrically located roots.

**Sol.2. (d)**
Given , H(s) = 1 and $G(s)=\frac{k(s+2)(s+1)}{(s+0.1)(s-1)}$ the characteristic equation is
(s +0.1)( s–1) + k(s+1) (s+2) = 0
Or $s^2(1 + k) + s(3k – 0.9) + (k –0.1) = 0$
RH table;

$s^2 \quad (1+k) \quad (k-0.1)$
$s \quad (3k-0.9)$
$s^0 \quad (k-0.9)$

For closed loop system to have one pole in the right half of s-plane, only option (d) satisfies.

**Sol.3. (d)**
The characteristic equation for given feedback control system is
1 + G(s)H(s) = 0

or $1+\frac{10}{s(s+2)(s+k)}=0$

or $s[s^2 + (k + 2)s + 2k] + 10 = 0$
or $s^3 + (k + 2)s^2 + 2ks + 10 = 0$
The routh table is formed below:

| | | |
|---|---|---|
| $s^3$ | 1 | 2k |
| $s^2$ | $(k+2)$ | 10 |
| $s^1$ | $2k-\frac{10}{k+2}$ | 0 |
| $s^0$ | 10 | |

For system to be stable

$k+2>0$ and $2k-\frac{10}{k+2}>0$
Or k > –2 and $2k^2 + 4k – 10 > 0$
$K^2 + 2k – 5 . 0$
$(k+1)^2 > 6$
$k>-1+\sqrt{6}$
K > 1.45

**Sol.4. (c)**
The effect of addition of a pole in a system loop transfer function are:
(i) Root locus gets pulled to the right – hand side.
(ii) System response gets slower.
(iii) System becomes more oscillatory in nature
(iv) System stability relatively decreases.

**Sol.5. (b)**
$GH(s)=\frac{K(S+2)}{S^2+2S+3}$

Breakaway point is solution of $\frac{dK}{dS}$ that lies on root locus.

Solution of $\frac{dK}{dS}$ are $-2\pm\sqrt{3}$

$\therefore$ $-2-\sqrt{3}$ = –3.73 lies on root locus

**Sol.6. (d)**
Root locus obtained mainly to obtain response and stability of system.

**Sol.7. (a)**
Total number of poles = 4
Total number of zeros = 1

Angles of asymptotes = $\frac{(2K+1)180°}{P-Z}$

K = 0, 1, 2
$\therefore$ Angles = 60°, 180°, 300° (or) 60°

**Sol.8. (d)**
Open loop poles and zeros are used for root locus plotting.
1 + GH(s) = 0

**Sol.9. (b)**

At breakaway point, $\frac{dK}{ds} = 0$; where K = 1; so the system is critically damped.
Also at imaginary axis, where the root locus intersect with it the value of K = 0, i.e. system is marginally stable as beyond RHS of imaginary axis system becomes unstable.

**Sol.10. (d)**
The root locus breaks from the real axis at breakaway points and at centroid all asymptotes intersect one another at real axis them.

**Sol.11. (c)**
Inverse root locus is obtained by K is varied from direct root locus. Whereas root locus is symmetrical about real axis but not symmetrical about imaginary axis.

**Sol.12. (*)**

$$GH_{(s)} = \frac{K(s+8)}{s(s+4)(s^2+4s+8)}$$

n = O.L. Poles; s = 0; – 4 ; –2 ± j2
m = O.L. zeros; s = –8

$$\text{Centroid} = \frac{\sum p - \sum z}{n-m};$$

Here, n – m = 4 – 1 = 3,

$$\text{Centroid} = \frac{-4-2+j2-2-j2-(-8)}{3}$$

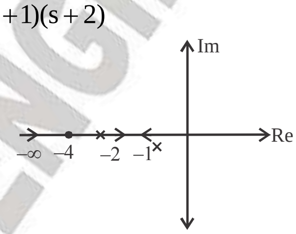
$$= \frac{-8+8}{3} = 0$$

Hence centroid = (0, 0)

**Sol.13. (d)**

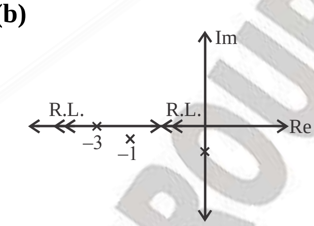
**Sol.14. (b)**

$$G(s) = \frac{K(s+4)}{(s+1)(s+2)}$$




The root locus is lying on real axis. The total number of poles and zeros lying right hand side to the root locus must be odd.

**Sol.15. (b)**




$$G(s)H(s) = \frac{k}{s(s+1)(s+3)}$$

s = – 1.5 is not on the root local.

**Sol.16. (d)**
Character equation
=(s+4) (s+5) (s+6) +K(s+3) = 0

$$\Rightarrow \text{OLTF} = \frac{K(s+3)}{s(s+4)(s+5)(s+6)}$$

∴ zero = – 3
i.e. Z = 1
Poles = 0, –4, –5, –6
i.e. P = 4
number of asymptotes = P – Z = 4 – 1 = 3

$$\text{Centroid} = \frac{\sum\begin{pmatrix}\text{real part of open}\\ \text{loop poles}\end{pmatrix} - \sum\begin{pmatrix}\text{real part of open}\\ \text{loop zero}\end{pmatrix}}{P-Z}$$

$$= \frac{(0-4-5-6)-(-3)}{4-1} = -4$$

**Sol.17. (*)**

**Sol.18. (*)**

**Sol.19. (d)**

**Sol.20. (a)**

$$\text{Centroid} = \frac{\begin{matrix}\Sigma \text{ real part of} \\ \text{open loop poles}\end{matrix} \quad \begin{matrix}\Sigma \text{real part of open} \\ \text{loop zeros}\end{matrix}}{P-Z}$$

P is number of open loop poles.
Z is number of open loop zeros.

**Sol.21. (a)**
Root loci are symmetrical about the real axis (σ-axis).
Also we know that, the roots of the characteristic equation are either real or complex conjugate or combination of both. Therefore their locus must be symmetrical about the σ-axis of the s-plane.

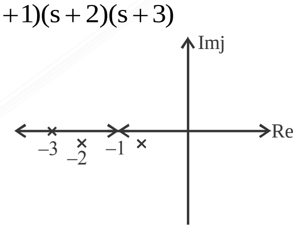
**Sol.22. (a)**

$$N(s) = \frac{K(s-z_1)(s-z_2)...(s-z_n)}{(s-P_1)(s-P_2)...(s-P_m)}$$

and the statement II is the correct explanation of statement I.

**Sol.23. (b)**

$$G(s) = \frac{K}{(s+1)(s+2)(s+3)}$$




Whenever there are two adjacently place poles on real axis with the section of real axis between them as a part of root locus then there exist a break – away point between the adjacently placed poles therefore break – away point will be between – 1 and – 2 at z = $-2 + \frac{1}{\sqrt{3}}$.

**Sol.24. (a)**
On the addition of open – loop poles results decrease in stability and shifts the root locus towards imaginary axis.

**Sol.25. (d)**
Angle of asymptotes

$$= \frac{(2q+1)\pi}{P-z}; q = 0,1,....(P-Z-1)$$

Therefore angle between adjacent asymptotes

$$= \frac{2\pi}{P-Z}$$

**Sol.26. (a)**
A root locus diagram is symmetric about the real axis, not about the imaginary axis.
The number of branches terminates on infinity is open loop poles minus zero.
Root locus starts from pole and ends at zeros as K is increased from 0 to ∞.
Hence, option (a) is correct.

**Sol.27. (c)**
Characteristic equation = 1 + G(s) H(s)

$$= 1 + \frac{K}{s(s+3)^2} = 0$$

$K = -s(s+3)^2$
$= -(s^3 + 6s^2 + 9s)$
for break away point

$$\frac{dK}{ds} = 0$$

$$\Rightarrow \frac{d}{ds}(s^3 + 6s^2 + 9s) = 3s^2 + 12s + 9$$

$S_{1,2} = -1, -3$
But –3 does not lie on root locus.
Hence, option (c) is correct.

**Sol.28. (d)**
Poles are at ± j4 and 0, zeroes at ± j8.
Roots locus starts from pole and ends at zeros.
Number of poles = 3, Number of zeros = 2
So one branch terminate at infinity.
Hence, option (d) is correct.
Gain at s = j6

$$|G(s)| = \frac{K(-36+64)}{6(-36+16)} < 0$$

⇒ Option (a) is wrong and option (d) is correct.

**Sol.29. (b)**
Characteristic equation of the given system is
1 + G(s) H(s) = 0

$$\Rightarrow 1 + \frac{4K}{(s+1)(s+3)}.1 = 0$$

$\Rightarrow s^2 + 4s + 4K + 3 = 0$
Comparing with standard equation
$2\xi\omega_n = 4$

$$\Rightarrow \omega_n = \frac{4}{2\xi} = \frac{4}{2\times\frac{1}{\sqrt{2}}} = 2\sqrt{2}$$

$$\omega_n^2 = 4K + 3$$

or 4K + 3 = 8

$\Rightarrow$ K = 5/4

**Sol.30. (b)**

Root loci starts from poles of G(s) H(s) for K = 0.

**Sol.31. (c)**

Characteristic equation

$$1 + \frac{K(s+1)}{s(s+4)(s^2+2s+2)} = 0$$

Comparing with standard equation

1 + G(s) H(s) = 0, we have;

$$G(s)H(s) = \frac{K(s+1)}{s(s+4)(s^2+2s+2)}$$

So open loop poles = 0; –4; –1 ± j

And open loop zero = –1

So

$$\text{Centroid} = \frac{\sum \begin{matrix}\text{real part of}\\ \text{open loop}\\ \text{poles}\end{matrix} - \sum \begin{matrix}\text{real part of}\\ \text{open loop}\\ \text{zeros}\end{matrix}}{P - Z}$$

$$= \text{Centroid} = \frac{(0-4-1-1)-(-1)}{4-1} = -\frac{5}{3}$$

**Sol.32. (b)**

Root loci starts from poles for K = 0 and ends at zero for K = ± ∞.

**Sol.33. (a)**

$s^2 + 2s + 2 + K(s + 2) = 0$

$$K = \frac{-(s^2+2s+2)}{s+2}$$

$$\frac{dK}{dS} = -\left[\frac{(s+2)(2s+2)-(s^2+2s+2)}{(s+2)^2}\right]$$

$$\frac{dK}{dS} = 0$$

$\Rightarrow 2s^2 + 2s + 4s + 4 - s^2 - 2s - 2 = 0$

$\Rightarrow s^2 + 4s + 2 = 0$

$$\Rightarrow s = \frac{-4 \pm \sqrt{16-8}}{2}$$

= – 0.586, –3.414

Therefore, break-away point is s = –3.414.

**Sol.34. (c)**

Number of Asymptotes = P – Z = 4 – 1 = 3

$$\text{Centroid} = \frac{\sum \begin{matrix}\text{Real parts of}\\ \text{open loop poles}\end{matrix} - \sum \begin{matrix}\text{Real parts of}\\ \text{open loop zeroes}\end{matrix}}{P - Z}$$

$$= \frac{(-1)+(-4)+(-4)-(-2)}{4-1} = \frac{-7}{3}$$

**Sol.35. (c)**

s(s + 2) + k(s + 4) = 0

$$1 + \frac{k(s+4)}{s(s+2)} = 0$$

$$\therefore\ G(s)H(s) = \frac{k(s+4)}{s(s+2)} = \frac{k(s+b)}{s(s+a)}$$

Centre = (– b, 0) = (– 4, 0)

**Sol.36. (c)**

**Sol.37. (c)**

**Sol.38. (c)**

The root locus is the locus of closed-loop poles of the system (i.e., the roots of characteristic equation) when the parameter is varied from 0 to ∞.

**Sol.39. (b)**

The root locus is symmetrical about the real axis. Segments of the real axis having an odd number of real axis open-loop poles plus zeros to their right are parts of the root locus.

**Sol.40. (d)**

Considering poles at s = 0, –1 ±j1, and zero at s = –4,

$$\text{Centroid} = \frac{(-1-1-3)-(-4)}{4-1} = \frac{-1}{3}$$

Which is not justified in the diagram. Angle of departure is also not justified.

**Sol.41. (c)**
The number of root-locus segments ending at infinity are equal to n-m, where
n = number of open-loop poles
and m = number of open-loop zeros

**Sol.42. (c)**
For the root locus to be parallel to the imaginary axis, the angle of asymptotes should be ±90°.
Angle of asymptotes,

$$\phi_A = \frac{(2q+1)180°}{n-m}; q = 0, 1, 2, ....$$

where n = number of open-loop poles
m = number of open-loop zeros
$\phi_A$ for $G(s) = K/(s + 2)^2$ is

$$\phi_A = \frac{(2q+1)180°}{2}$$

= 90° for q = 0
= 270° or – 90° for q = 1

Therefore, $\frac{K}{(s+2)^2}$ has root locus parallel to imaginary axis.

**Sol.43. (a)**
Intersection of asymtotes is centroid always lies on real axis.

**Sol.44. (d)**
For more than one variable state space is used.

**Sol.45. (a)**

**Sol.46. (a)**

**Sol.47. (a)**
Poles at – 1 and – 2
and zero at – 5

$$\therefore \quad T.F. = \frac{K(s+5)}{(s+1)(s+2)}$$

Poles move towards each other and break. After which one pole go to zero and other goes to infinity.

**Sol.48. (c)**

$$-\sigma = \frac{\sum(\text{Real part of poles}) - \sum(\text{Real parts of zeros})}{\text{no. of poles} - \text{no. of zeros}}$$

$$-\sigma = \frac{-1-2-5-(-3)}{4-1}$$

$$-\sigma \frac{-5}{3}$$

The real axis intercept is $\frac{-5}{3}$.

**Sol.49. (b)**

**Sol.50. (a)**
The root locus is symmetrical about real axis but it is not symmetrical about jω axis.

**Sol.51. (b)**

**Sol.52. (a)**

$$\theta = \frac{(2k+1)}{P-Z}180^0 = \frac{2k+1}{(4-1)}.180^0$$

$$= 60^0; 180^0; 300^0$$

For k = 0, 1, 2 respectively

**Sol.53. (a)**
Since an isolated zero is not physically realizable, we must add a pole along with the compensating zero so as to achieve physical realisability. The compensator having such transfer function is known as a lead compensator as the pole must of course be added for away from the jω - axis such that it has relatively negligible effect on the root locus in the region. As we know a lead compensator speeds up the transient response and increases the margin of stability of a system.

**Sol.54. (c)**
Breakaway points are s = –2, s = (–2 ±2.45j)

**Sol.55. (b)**

**Sol.56. (c)**

**Sol.57. (b)**

The (p – z) branches of the root loci which tend to infinity, so along straight line asymptotes whose angles are given by

$$\phi_A = \frac{(2K+1)\pi}{p-z}, K = 0, 1, 2, \ldots, p - z - 1$$

**Sol.58. (c)**
Characteristics equation is
$s^5 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$
The Routh array is

| | | | | |
|---|---|---|---|---|
| $s^6$ | 1 | 8 | 20 | 16 |
| $s^5$ | 2 | 12 | 16 | |
| $s^4$ | 2 | 12 | 16 | |
| $s^3$ | 0 | 0 | 0 | |

Auxiliary polynomial $A(s) = s^4 + 6s^2 + 8$
Solving for the roots of auxiliary polynomial–
$s^4 + 6s^2 + 8 = 0$
$\Rightarrow (s^2 + 2)(s^2 + 4) = 0$
$\Rightarrow s = \pm j\sqrt{2}$ and $\pm j2$
These two pairs of roots are also the roots of the original characteristic equation. Thus the characteristic equation has 4 roots on the imaginary axis of s-plane.

**Sol.59. (b)**

**Sol.60. (d)**
Root locus shows that transfer function has poles at s = 0, –1 and zero at s = –2.

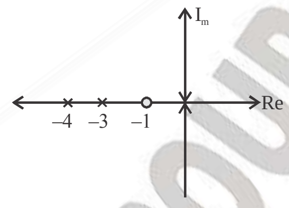
**Sol.61. (b)**
Z > P
Z – P = 3 – 2 = 1

**Sol.62. (c)**
Each locus starts at open loop zero and ends at open loop pole or infinity as number of zeros are more than no of poles. Number of separate root loci is equal to no of poles or zeros whichever is larger.

**Sol.63. (a)**
By solving Routh Array, it will always form circular path and hence for stability it always passes from LHS.

**Sol.64. (a)**

**Sol.65. (c)**




Root locus can lie on the real axis between s = –3 and s = –4 because the no. of poles + zero to the right are odd.

**Sol.66. (b)**

**Sol.67. (d)**
A point on the real axis lies on the locus if the number of open loop poles plus zeros on the real axis to the right of this point is odd.

**Sol.68. (c)**
No. of poles, n = 4
No. of zeros, m = 1
Since m < n, no. of zeros at infinity
= n – m = 4 – 1 = 3

**Sol.69. (b)**
$1 + G(s)H(s) = 0$
$\Rightarrow s(s + 4)(s^2 + 4s + 5) + K = 0$
$\Rightarrow K = -s(s + 4)(s^2 + 4s + 5)$
For breakaway points,
$\frac{dK}{ds} = 0$
$\Rightarrow (s + 4)(s^2 + 4s + 5) + s(s^2 + 4s + 5) + s(s + 4)(2s + 4) = 0$
$\Rightarrow (2s + 4)(s^2 + 4s + 5) + (s^2 + 4s)(2s + 4)$
$\Rightarrow (2s + 4)(2s^2 + 8s + 5) = 0$
$\Rightarrow (2s + 4)(s + 0.775)(s + 3.225) = 0$
$\Rightarrow s = -2, -0.775, -3.225$
Thus the system has three real breakaway points.

**Sol.70. (c)**
Characteristic equation can be rearranged as:
$s^3 + 5s^2 + 6s + k(s+1) = 0$

$$\Rightarrow 1 + \frac{k(s+1)}{s(s+3)(s+2)} = 0$$

$$\Rightarrow \sigma_A = \frac{0-3-2-(-1)}{2} = -2$$

**Sol.71. (a)**

**Sol.72. (c)**
For stable operation, all coefficients of the characteristic equation should be real ad have the same sign and none of the coefficients should be zero.

**Sol.73. (d)**
Intersection of asymptotes, i.e. centroid

$$= \frac{\sum \text{real parts of pole} - \sum \text{real parts of zeros}}{\text{no. of poles} - \text{no. of zeros}}$$

$$= \frac{-1-3}{3} = \frac{-4}{3} = -1.33$$

**Sol.74. (a)**

$$\frac{G(s)}{1+G(s)} = \frac{\frac{K}{s(s+1)(s+2)}}{1+\frac{K}{s(s+1)(s+2)}}$$

$$= \frac{K}{s(s+1)(s+2)+K}$$

Characteristic equation is
$s(s+1)(s+2) + K = 0$

Or $s^3 + 3s^2 + 2s + K = 0$
Routh array is

| | | |
|---|---|---|
| $S^3$ | 1 | 2 |
| $S^2$ | 3 | K |
| $S$ | $\frac{6-K}{3}$ | 0 |
| $S^0$ | K | |

For marginal stability,

$$\frac{6-K}{3} = 0 \Rightarrow K = 6$$

$3s^2 + K = 0 \Rightarrow 3(j\omega)^2 + 6 = 0$
$\Rightarrow -\omega^2 + 2 = 0 \Rightarrow \omega^2 = 2$
$\Rightarrow \omega = \sqrt{2}$ rad/s
So, the root locus intersects with the imaginary axis at $\pm j\sqrt{2}$.

**Sol.75. (b)**
Breakaway points need not always be on the real axis alone but it must lie on the root loci. It is not necessary that break away points must lie between 0 and –1.

**Sol.76. (c)**
Centroid,

$$-\sigma = \frac{\sum \text{real parts of poles} - \sum \text{real parts of zeros}}{\text{no. of poles} - \text{no. of zeros}}$$

$$-\sigma = \frac{-3-5-(-6)}{2-1}$$

$$-\sigma = \frac{-8+6}{1} = -2$$

So, the centroid will be located at –2.

# CHAPTER - 7
# *CONTROLLERS*

## 7.1 INTRODUCTION

While designing a system, the designer selects the reasonable values for the peak overshoot, rise time and the settling time. The designer is never sure of the final design of the system as to whether it is good or not. For example, if the system has been designed for minimum overshoot, the rise time increases and on the other hand if the rise time chosen is small, peak overshoot will be large. A system thus requires modification in order to meet even two independent specifications. This is called compensation and is achieved by the help of proportional, derivative or integral or derivative feedback control. In practice a combination of derivative and integral control is employed.

Let us consider a system whose block diagram is shown in Figure. It has a controller whose output signal will have an effect on the system performance. Its purpose is to measure the error between the output and the desired output.

The transfer function of the controller is

$$K = \frac{Y(s)}{E(s)}$$

Where

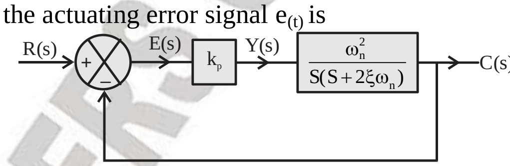
E(s) = R(s) – B(s)

or E(s) = R(s) – H(s)C(s)

this relationship is termed as control action relationship. We will now discuss various control actions as available to the control system engineer for improvement of system performance.

## 7.2 PROPORTIONAL CONTROL ACTION

In this the actuating signal is proportional to the error signal. The relationship between the output of the controller, $y_{(t)}$ and the actuating error signal $e_{(t)}$ is




$y_{(t)} = Ke_{(t)}$

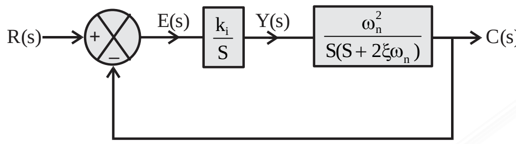
In Laplace-transform form, it can be written as

$Y_{(s)} = KE_{(s)}$

$$\text{Or } K_p = \frac{Y(s)}{E(s)}$$

## 7.3 INTEGRAL CONTROL ACTION

In this value of the controller output $y_{(t)}$ is altered at a rate proportional to the error signal $e_{(t)}$ . The output $y_{(t)}$. The output $y_{(t)}$ depends upon the integral of the error signed $e_{(t)}$.




Mathematically,

$$\frac{dy_{(t)}}{dt} = K_i e_{(t)}$$

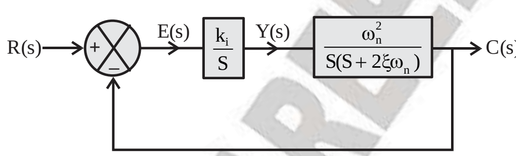
$$\text{or } y_{(t)} = K_i \int_0^t e_{(t)}^{dt} \text{ or } y(s) = \frac{K_i E(s)}{s} \quad \text{or } \frac{Y(s)}{E(s)} = \frac{K_i}{s}$$

Block diagram representation of integral control action is shown in figure.

**7.4 PROPORTIONAL PLUS INTEGRAL CONTROL ACTION**

Integral control action itself is not sufficient, as it introduces hunting in the system. Therefore, a combination of proportional plus integral control system is used. The actuating signal consists of proportional error signal added with an integral of the error signal.

Mathematically,




$$y_{(t)} = e_{(t)} + K\int_0^t e_{(t)}^{dt}$$

Where $e_{(t)}$ = error signal and

$\int_0^t = e_{(t)}^{dt}$ = integral of error signal

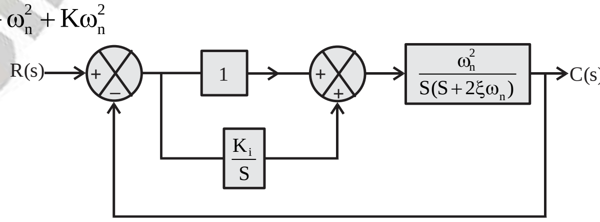
$$\text{or } Y(s) = E(s)\left[1+\frac{K}{S}\right]$$

$$\frac{Y(s)}{E(s)} = \left(1+\frac{K}{s}\right)$$

For a **second order unity feedback control system** employing proportional plus integral control action, the block diagram representation is shown in figure.

The transfer function of such a system is given by

$$\frac{C(s)}{R(s)} = \frac{(s+K)\omega_n^2}{s^3+2\xi\omega_n s^2+\omega_n^2+K\omega_n^2}$$




The characteristic equation is given by

$s^3 + 2\xi\omega_n s^2 + \omega_n^2 s + K\omega_n^2$

For the system to be stable

(a) $2\xi\omega_n > 0$ i.e. $\xi > 0$ and $\omega_n > 0$,

(b) $K\omega_n^2 > 0$ I.e. $K > 0$ and $\omega_n > o$, and

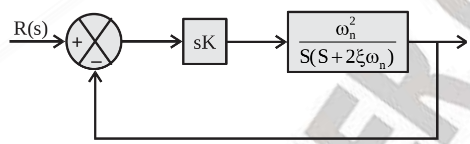
(c) $2\xi\omega_n^3 - K\omega_n^2 > 0$ i.e. $2\xi\omega_n > K$

Therefore, for a system to be stable $2\xi\omega_n > K$

**7.5 DERIVATIVE CONTROL ACTION**

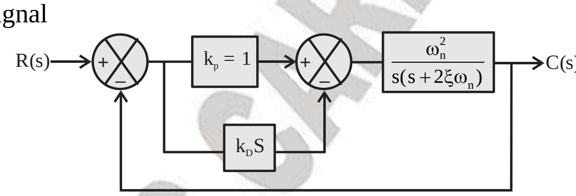
A control system is said to have a derivative control action if the output $y_{(t)}$ depends upon the rate of change of error signal. Mathematically

$$y_{(t)} = \frac{Kd.e_{(t)}}{dt}$$




**7.6 PROPORTIONAL PLUS DERIVATIVE CONTROL ACTION**

In this type of control action, the actuating signal $y_{(t)}$ depends upon the proportional error signal and derivative error signal




$$y_{(t)} = e_{(t)} = \frac{Kd.e_{(t)}}{dt}$$

or Y(s) = E(s) (1 + sK)

The block diagram of such a control action is shown in figure

The characteristic equation is

$$s^2 + (2\xi\omega_n + \omega_n^2 K)s + \omega_n^2 = 0$$

The characteristic equation of a second order control system without using derivative control action is

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

Therefore, $\xi' = \frac{2\xi\omega_n + \omega_n^2 K}{2\omega_n} \xi' = \xi + \frac{\omega_n K}{2}$

Therefore, it is seen that the damping ratio is increased by a factor $\frac{\omega_n K}{2}$

The overall transfer function given in equation can show now be rewritten as

$$\frac{C(s)}{R(s)} = \frac{\left(s+\frac{1}{k}\right)\omega_n^2 K}{s^2 + 2\xi' \omega s + \omega^2}$$

Comparing the transfer function derived in equation above and comparing with overall transfer function of a second order control system without any control action as given in equation below,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega^2}$$

Comparing transfer function given in equations , it is seen that

1.There is no change in natural frequency of oscillations $\omega_n$.

2.The damping ratio increases by a factor $\frac{\omega_n K}{2}$

3.The transfer function with derivative control action contains a zero at $s = -\frac{1}{K}$

4.Steady-state error remains unchanged even with derivative control action.

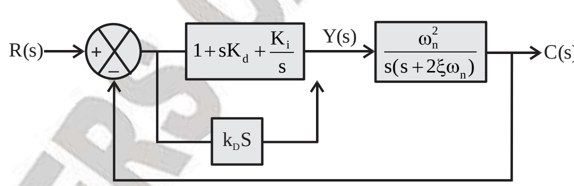
6.The derivative control action has an impact on transient response. The increase in damping ratio reduces the peak overshoot.

## 7.7 PID CONTROL ACTION

This type of control action employs proportional, integral and derivative control action together in a control system so as to derive the advantages of all the control actions into one. Mathematically,

$$y_{(t)} = e_{(t)} + K_d \frac{de_{(t)}}{dt} + K_i \int e_{(t)} dt \qquad Y(s) = E(s)\left(1 + sK_d + \frac{Ki}{s}\right)$$

The block diagram of a control system with unity feedback employing PID control action in figure.




## 7.8 CONCLUSION

**1. Proportional controller**

$G_c(s) = K_p$ = OLTF with controller

It is used to vary the transient response of a system. Proportional controller is usually an amplifier with gain $K_p$

**2. Integral Controller**

$G_c(s) = K_1/s$

It is used to decrease the steady state error by increasing the type of the system Disadvantage: Stability decreases

**3. Derivative controller**

$G_c(s) = K_D \cdot S$

It is used to increase the stability of the system. Stability of any system is increased by adding zeros.

Disadvantage: Steady state error increases, since type of the system decreases.

**4. Proportional + Integral (PI) controller**

$G_c(s) = K_D. S + K_P$

It is used to increase the stability without effecting steady error. Since type is not changed and a zero is added.

**5. Proportional + Derivative (PD) controller**

$G_c(s) = K_P + K_1/S + K_D .S$

$$\frac{G_C(s) = K_D.S^2 + K_p.S + K_1}{S}$$

. It is used to decrease the steady state error and to increase the stability. Since pole at origin and two zeros are added. One zero compensate the pole and zero will increase the stability.

# ASSIGNMENT

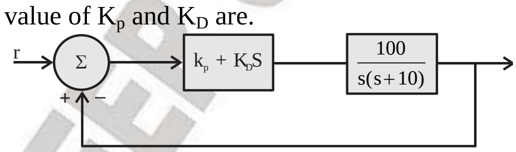
**1.** Consider the following statements:
1. A Proportional plus derivative controller.
2. Increase the stability of the system
3. Improves the steady-state accuracy
Which of these statements are correct?
(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) 1 and 3

**2.** The transfer function of simple RC network as a controller is $G_C(s) = \frac{s + z_1}{s + p_1}$. The condition for the RC network to act as a phase lead controller is
(a) $p_1 < z_1$ (b) $p_1 = 0$
(c) $p_1 = z_1$ (d) $p_1 > z_1$

**3.** The industrial controller having the best steady
state accuracy is
(a) A derivative controller
(b) An integral controller
(c) A rate feed back controller
(d) A proportional controller

**4.** The transfer function of a phase lead controller is $\frac{1+3Ts}{1+Ts}$. The maximum value of phase provided by this controller is
(a) 90° (b) 60°
(c) 45° (d) 30°

**5.** A control system with a PD controller is shown in fig. if the velocity error constant $K_v$ = 1000 and the damping ratio $\zeta$ = 0.5, then the value of $K_p$ and $K_D$ are.




(a) $K_P$ = 100, $K_D$ = 0.09
(b) $K_p$ = 100, $K_D$ = 0.9
(c) $K_p$ = 10, $K_D$ = 0.09
(d) $K_p$ = 10, $K_D$ = 0.9

**6.** A controller transfer function is given by C(s) = (2s + 1)/(09.2s + 1). What is its nature and parameter?
(a) Lag controller, $\alpha$ = 10
(b) Lag controller, $\alpha$ = 2
(c) Lead controller, $\beta$ = 0.1
(d) Lead controller, $\beta$ = 0.2

**ANSWER KEY**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | b | **2.** | d | **3.** | b | **4.** | d | **5.** | b | **6.** | c |

# SOLUTIONS

**Sol. 1. (b)**

**Sol. 2. (d)**

**Sol. 3. (b)**

**Sol. 4. (d)**

$$G(s)=\frac{1+3Ts}{1+Ts}$$

Comparing it with $G(s)=\frac{1+s\tau}{1+\alpha s\tau}$

$\tau = 3T$

$$\alpha\tau = T \Rightarrow \alpha=\frac{D}{3D}=\frac{1}{3}$$

The max value of phase is

$$\phi_m=\sin^{-1}\left[\frac{1-\alpha}{1+\alpha}\right]$$

$\phi_m = 30°$

**Sol. 5. (b)**

**Sol. 6. (c)**

C(s) = (2s + 1)/ (0.2 s + 1)

Comparing with sinusoidal transfer function of the lead controller

$$G_C(s)=\frac{1+s\tau}{1+\beta s\tau};\beta<1$$

$\tau = 2$

$\beta\tau = 0.2$

$$\Rightarrow \beta=\frac{0.2}{2}=0.1$$

# GATE QUESTIONS

**1.** The terms hysteresis is associated with

**[GATE - 2017]**

(a) On - OFF controller
(b) P - I control
(c) Feed - forward control
(d) Ratio control

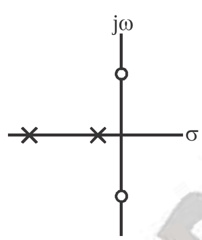
**2.** The transfer function of a first-order controller is given as $G_C(s) = \frac{K(s+a)}{s+b}$

Where K, a and b are positive real numbers. The condition for this controller to act as a phase lead

**[GATE - 2015]**

(a) a < b
(b) a > b
(c) K< ab
(d) K > ab

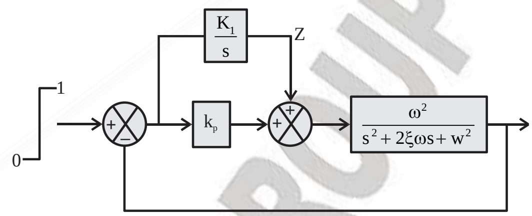
**3.** The pole – zero given below correspond to a




**[GATE - 2008]**

(a) Law pass filter
(b) High pass filter
(c) Band filter
(d) Notch filter

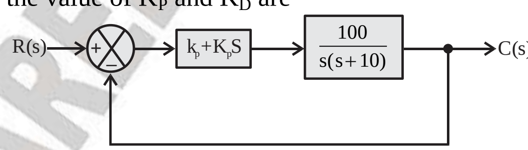
**4.** Consider the feedback system shown below which is subjected to a unit step input. The system is stable and has following parameters $K_p = 4$, $K_i = 10$, $\omega = 500$ and $\xi = 0.7$. The steady state value of Z is




**[GATE - 2007]**

(a) 1
(b) 0.25
(c) 0.1
(d) 0

**5.** A control system with PD controller is shown in the figure. If the velocity error constant $K_V = 1000$ and the damping ratio $\zeta = 0.5$, then the value of $K_P$ and $K_D$ are




**[GATE - 2007]**

(a) $K_P = 100$, $K_D = 0.09$
(b) $K_P = 100$, $K_D = 0.9$
(c) $K_P = 10$, $K_D = 0.09$
(d) $K_P = 10$, $K_D = 0.9$

**6.** The transfer function of phase lead compensator is given by $G_C(s) = \frac{1+3Ts}{1+Ts}$ where $T > 0$. The maximum phase shift provide by such a compensator is

**[GATE - 2006]**

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{6}$

**7.** A double integrator plant $G(s) = K/s^2$, $H(s)=1$ is to be compensated to achieve the damping ratio $\zeta = 0.5$ and an undamped natural frequency, $\omega_n = 5$ rad/sec which one of the following compensator $G_e(s)$ will be suitable?

**[GATE - 2005]**

(a) $\frac{s+3}{s+99}$ (b) $\frac{s+99}{s+3}$

(c) $\frac{s-6}{s+8.33}$ (d) $\frac{s-6}{s}$

**8.** A PD controller is used to compensate a system. Compared to the uncompensated system, the compensated system has

**[GATE - 2003]**

(a) A higher type number
(b) Reduced damping
(c) Higher noise amplification
(d) Larger transient overshoot.

# SOLUTIONS

**Sol.1. (a)**
Hysteresis term is associated with ON–OFF controller only.

**Sol.2. (a)**

$$G_c(s) = \frac{k(s+a)}{(s+b)}$$

$G_c(s)$ for lead compensator is given by

$$G_c(s) = \frac{Ts+1}{\alpha Ts+1} \Rightarrow \frac{\frac{s}{\alpha}+\frac{1}{T\alpha}}{s+\frac{1}{\alpha T}} = \frac{\frac{1}{\alpha}\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{T\alpha}\right)}$$

$$\frac{\alpha<1}{a = \frac{1}{T}}$$

$$b = \frac{1}{T\alpha} \; b > a$$

**Sol.3. (c)**
Percent overshoot depends only on damping ratio ξ.

$$M_p = e^{-\xi\pi\sqrt{1-\xi^2}}$$

If $M_P$ is same then ξ is also same and we get
ξ = cos θ
Thus θ = constant
The option (C) only have same angle.

**Sol.4. (a)**
For the given system Z is given by

$$Z = E(s)\frac{K_i}{s}$$

Where E(s) → steady state error of the system

Here $E(s) = \lim_{s\to 0}\frac{sR(s)}{1+G(s)H(s)}$

Input $R(s) = \frac{1}{s}$ (Unit step)

$$G(s) = \left(\frac{K_i}{s}+K_p\right)\left(\frac{\omega^2}{s^2+2\xi\omega s+\omega^2}\right)$$

H(s)= 1 (Unity feed back)
So,

$$Z = \lim_{s\to 0}\left[\frac{s\left(\frac{1}{s}\right)}{1+\left(\frac{K_i}{s}+K_p\right)\frac{\omega^2}{(s^2+2\xi\omega s+\omega^2)}}\right]\left(\frac{K_i}{s}\right)$$

$$= \lim_{s\to 0}\left[\frac{K_i}{s+(K_i+K_p s)\frac{\omega^2}{(s^2+2\xi\omega s+\omega^2)}}\right]$$

$$= \frac{K_i}{K_i} = 1$$

**Sol.5. (b)**
We have $K_v = \lim_{s\to s} sG(s)\,H(s)$

Or $1000 = \lim_{x\to\infty} s\frac{(K_p+K_D s)100}{s(s+100)} = K_p$

Now characteristics equations is
1 + G(s)H(s) = 0

$$1+\frac{(100+K_D S)100}{s(s+10)} = 0 \; K_p = 100$$

Or
$s^2 + (10 + 100K_D)s + 10^4 = 0$
Comparing with $s^2 + 2\xi\omega_n + \omega_n^2 = 0$ we get
$2\xi\omega_n = 10 + 100\,K_D$
Or $K_D = 0.9$

**Sol.6. (d)**
The transfer function of given compensator is

$$G_c(s) = \frac{1+3TS}{1+Ts}$$

Comparing with $G_c(s) = \frac{1+aTs}{1+Ts}$

We get a = 3
The maximum phase shift is

$$\phi = \tan^{-1}\frac{a-1}{2\sqrt{s}} \quad = \tan^{-1}\frac{3-1}{2\sqrt{3}} = \tan^{-1}\frac{1}{\sqrt{3}}$$

Or $\phi_{max} = \frac{\pi}{6}$

**Sol.7. (a)**

**Sol.8. (c)**
PD Controller may accentuate noise at higher frequency. It does not effect the type of system and it increases the damping. It also reduce the maximum overshoot.

# ESE OBJ QUESTIONS

**1.** The transfer function G(s) of a PID controller is

**[EE ESE - 2018]**

(a) $K_1 + K_2 s + K_3 s^2$ (b) $K_1 + \frac{K_2}{s} + K_3 s$

(c) $K_1 + \frac{K_2}{s}$ (d) $K_1 s + K_2 s^2 + K_3 s^3$

**2.** The characteristics of a mode of controller are summarized:
1. If error is zero, the output from the controller is zero.
2. If error is constant in time, the output from the controller is zero.
3. For changing error in time, the output from the controller is |K|% for every 1% $sec^{-1}$ rate of change of error.
4. for positive rate of change of error, the output is also positive.

The mode of controller is

**[EC ESE - 2017]**

(a) Integral controller
(b) Derivative controller
(c) Proportional derivative
(d) Proportional integral

**3. Statement (I):**
PID control system performs better than most predictive control methods in the context of measured disturbances.

**[EC ESE - 2017]**

**4.** For derivative control action, the actuating signal consists of proportional

**[EC ESE - 2017]**

(a) Derivative of the error signals
(b) Integral of the error signals
(c) Steady - state error
(d) A constant which is a function of the system type

**5.** Consider the following statements regarding a PID controller;

1.The error is multiplied by a negative (for reverse action) proportional constant P, and added to the current output.
2.The error is integrated (averaged) over a period of time, and then divided by a constant I, and added to the current control output.
3.The rate of change of the error is calculated with respect to time multiplied by another constant D, and added to the output.

Which of the above statements are correct?
(a) 1, 2 and 3 (b) 1 and 3 only
(c) 1 and 2 only (d) 2 and 3 only

**6. Statement (I)**: Stability of a system deteriorates when integral control is incorporated into it.
**Statement (II)**: With integral control action, the order of a system increases and higher the order of the system, more the system tends to become unstable.

**[EE ESE - 2016]**

(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (II)
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

**7.** The transfer function $G(s) = \frac{10(s-1)}{(s+10)}$ represents

**[EC ESE - 2016]**

(a) Unstable system
(b) Minimum phase system
(c) Non-minimum phase system
(d) PID controller system

**8.** Consider the transfer function (0.1 + 0.1s) for a PD controller. What is the frequency at which the magnitude is 20 dB (by using asymptotic Bode's plot)?

**[EC ESE - 2016]**

(a) 2000 r/s (b) 1000 r/s
(c) 200 r/s (d) 100 r/s

**9.** The controller which is highly sensitive to noise is

(a) $P\ell$

(b) PD

(c) Both $P\ell$ and PD

(d) Neither $P\ell$ nor PD

**10.** In order to improve the system response transient behaviour, the type of controller used is

**[EC ESE - 2015]**

(a) Phase lead controller
(b) Phase lag controller
(c) PI controller
(d) P controller

**11.** A proportional plus derivative controller
1. Has high sensitivity.
2. Increases the stability of the system
3. Improves the steady-state accuracy.
Which of the above statements are correct?

**[EC ESE - 2014]**

(a) 1, 2 and 3 (b) 1 and 2 only
(c) 1 and 3 only (d) 2 and 3 only

**12.** In industrial control system, which one of the following methods is most commonly used in designing a system for meeting performance specifications?

**[EC ESE - 2014]**

(a) The transfer function is first determined then either a lead compensation or lag compensation is implemented.
(b) The transfer function is first determined and PID controllers are implemented by mathematically determining PID constants.
(c) PID controllers are implemented without the knowledge of the system parameters using Ziegler-Nichols method.
(d) PID controllers are implemented using Ziegler-Nichols method after determining the system transfer function.

**13.** Which one of the following is the transfer function of the PI-controller?

**[EC ESE - 2014]**

(a) $G(s)=\frac{(k_1s+k_2)}{k_3}$

(b) $G(s)=\frac{(k_1s+k_2s+k_3)}{k_4s}$

(c) $G(s)=\frac{(k_1s+k_2)}{k_3s}$

(d) $G(s)=\frac{k_1s}{k_2s}$

**14.** A plant is controlled by a proportion controller. If a time delay element introduced in the loop, its

**[EC ESE - 2014]**

(a) Phase margin remains the same
(b) Phase margin increases
(c) Phase margin decreases
(d) Gain margin increases

**15. Statement (I)**: A derivative controller produces a control action for constant error only.
**Statement (II)**: The PD controller increases the damping ratio and reduces the peak overshoot.

**[EC ESE - 2013]**

(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (II)
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

**16. Statement (I)**: A PI controller increases the order of a system by units but reduces the steady state error.

**Statement (II)**: A PI controller introduces a pole at either the origin or at a desired point on negative real axis.

**[EC ESE - 2013]**

(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (II)
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

**17.** Match List-I with List-II and select the correct answer using the code given below the lists:

**List-I**
A. PI control
B. PD control
C. PID control
D. On-off control

**List-II**
(i) Relay controller
(ii) Lead lag compensator
(iii) Lead compensator
(iv) Lag compensator

**[EC ESE - 2012]**

**Codes**:
(a) A-iv, B-ii, C-iii, D-i
(b) A-i, B-ii, C-iii, D-iv
(c) A-iv, B-iii, C-ii, D-i
(d) A-i, B-iii, C-ii, D-iv

**18.** A liquid controller linearly converts a displacement of 2 m to 3 m into 4–20 mA control signal. A relay serves as two position controller to open and close an inlet valve. Relay closes at 12 mA and opens at 10 mA. The hysteresis zone is

**[EC ESE - 2012]**

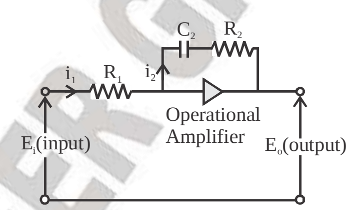
(a) 0.1 m (b) 0.125 m
(c) 0.15m (d) 0.2 m

**19.** A proportional integral (PI) controller results in which of the following?

**[EC ESE - 2012]**

(a) Improves the transient response without affecting steady state response.
(b) Improves the steady state response with out affecting transient response.
(c) Improves both transient response and steady state response.
(d) Improves the steady state response while marginally affecting transient response, for well designed control parameters.

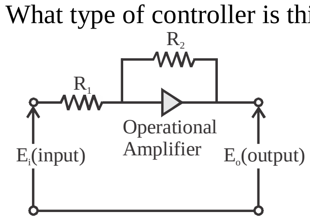
**20.** The circuit diagram of a controller is given in figure. What type of controller is this?




**[EC ESE - 2011]**

(a) Proportional
(b) Proportional + Derivative
(c) Integral
(d) Proportional + Integral

**21.** The circuit diagram of a controller is given in figure. What type of controller is this?




**[EC ESE - 2011]**

(a) Derivative
(b) Integral
(c) Proportional
(d) Proportional + Integral

**22. Assertion (A):** Integral windup effect in controller causes excessive overshoot.

**Reason (R):** Presence of saturation in controller and actuator deteriorates the PID control.

**[EC ESE - 2010]**

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

**23.** The transfer function of a controller is given as $K_p + K_d.s + \frac{K_i}{s}$ where $K_p$, $K_d$ and $K_i$ are constant. What type of controller is this?

**[EC ESE - 2009]**

(a) Proportional
(b) Proportional plus derivative
(c) Proportional plus integral
(d) Proportional plus integral plus derivative

**24.** The transfer function of a controller is given as $K_p$ + $K_{d.}$s where $K_p$ and $K_d$ are constant. What type of controller is this?

**[EC ESE - 2009]**

(a) Proportional
(b) Proportional plus integral
(c) Proportional plus derivative
(d) Integral plus derivative

**25.** Which of the following can be used as tachogenerator in control system?

**[EC ESE - 2009]**

(a) Microsyn
(b) DC servomotor
(c) AC servomotor
(d) Magnetic amplifier

**26.** The input to a controller is

**[EC ESE - 2008]**

(a) Sensed signal
(b) Error signal
(c) Desired variable value
(d) Signal of fixed amplitude not dependent on desired variable value.

**27.** A process is controlled by PID controller. The sensor has high measurement noise. How can this effect be reduced?

**[EC ESE - 2007]**

(a) By use of a bandwidth and derivative term
(b) By use of proportional and derivative terms in the forward path.
(c) By use of high proportional band.
(d) By use of low integral gain.

**28.** Match List-I (Components) with List-II (Functions) and select the correct answer using the code given below the lists:

**List-I**
A. Servomotor
B. Amplidyne
C. Potentiometer
D. Flapper valve

**List-II**
(i) Error detector
(ii) Transducer
(iii) Actuator
(iv) Power amplifier

**[EC ESE - 2007]**

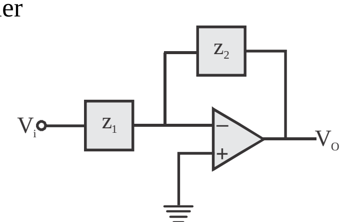
**Codes**
(a) A-ii, B-iv, C-i, D-iii
(b) A-iii, B-i, C-iv, D-ii
(c) A-ii, B-i, C-iv, D-iii
(d) A-iii, B-iv, C-i, D-ii

**29.** Which one of the following is an advantage of a PD controller in terms of damping (ξ) and natural frequency ($\omega_n$)?

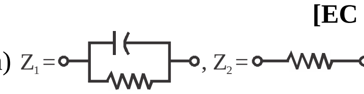
**[EC ESE - 2005]**

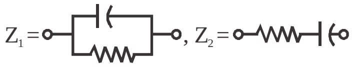
(a) ξ remains fixed but $\omega_n$ increases
(b) ξ remains fixed but $\omega_n$ decreases
(c) $\omega_n$ remains fixed but ξ increases
(d) $\omega_n$ remains fixed but ξ decreases

**30.** For which one of the following, given physical realization corresponds to PD controller

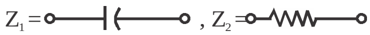



**[EC ESE - 2005]**

(a) 

(b) 

(c) 

(d) 

**31.** How does cascading an integral controller in the forwards path of a control system affect the relative stability (RS) and the steady-state error (SSE) of that system?

**[EC ESE-2004]**

(a) Both are increased
(b) RS is reduced but SSE is increased
(c) RS is increased but SSE is reduced
(d) Both are reduced

**32.** The maximum value of a controller output is 100 V and is obtained when the input error is 1 V. If the controller is working at 20% proportional band, the error and output will be respectively.

**[EC ESE - 2003]**

(a) 0.2 V and 100 V  (b) 1 V and 20 V
(c) 1 V and 120 V  (d) 0.2 V and 120 V

**33. Assertion (A)**: The bandwidth of a control system indicates the noise filtering characteristic of the system.
**Reason (R)**: The bandwidth is a measure of ability of a control system to reproduce the input signal.

**[EC ESE - 2002]**

(a)Both A and R are true and R is the correct explanation of A
(b)Both A and R are true but R is NOT the correct explanation of A
(c)A is true but R is false
(d)A is false but R is true.

**34.** Match List-I (Type of controller) with List-II (Operation) and select the correct answer using the codes given below the lists:
**List-I**
A. Pneumatic controller
B. Hydraulic controller
C. Electronic controller
**List-II**
(i) Flexible operation
(ii) High torque high speed operation
(iii) Fire and explosion proof operation

**[EC ESE - 2002]**

**Codes**:
(a) A-i, B-iii, C-ii
(b) A-i, B-ii, C-iii
(c) A-iii, B-i, C-ii
(d) A-iii, B-ii, C-i

**35.** In industrial control systems, which one of the following methods is most commonly used in designing a system for meeting performance specifications?

**[EC ESE - 2001]**

(a)The transfer function is first determined and then either a lead compensation or lag compensation is implemented.
(b)The transfer function is first determined and PID controllers are implemented by mathematically determining PID constants.
(c)PID controllers are implemented without the knowledge of the system parameters using Ziegler Nichols method.
(d)PID controllers are implemented using Ziegler Nichols method after determining the system transfer function.

**36.** Consider the following statements:
A proportional plus derivative controller
1. Has high sensitivity
2. Increases the Stability of the system
3. Improves the steady-state accuracy
Which of these statements are correct?

**[EC ESE - 2000]**

(a) 1, 2 and 3  (b) 1 and 2
(c) 2 and 3  (d) 1 and 3

**37. Assertion (A)**: Feedback control systems offer more accurate control over open-loop systems.
**Reason (R)**: The feedback path establishes a link for input and output comparison and subsequent error correction.

**[EC ESE - 2000]**

(a)Both A and R are true and r is the correct explanation of A
(b)Both A and R are true but R is NOT the correct explanation of A
(c)A is true but R is false

(d)A is false but R is true.

**38.** PID control for a plant is shown in Figures I and II.

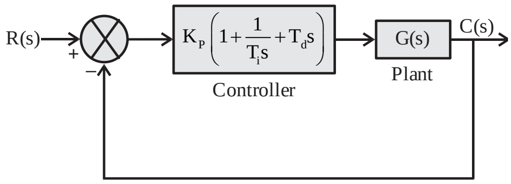



**Figure-I**

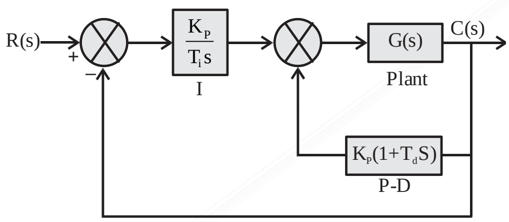



**Figure-II**

**Assertion (A)**: Figure-II is preferred over Figure-I as it avoids large changes in control signal for a sudden change in reference input.

**Reason (R)**: Placement of P-D action in the feedback path and larger values of $K_p$ and $T_d$ can be chosen in Figure-II.

**[EC ESE - 2000]**

(a) Both A and R are true and r is the correct explanation of A

(b) Both A and R are true but R is NOT the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true.

**39.** The industrial controller having the best steady-state accuracy is

**[EC ESE - 1999]**

(a) A derivative controller

(b) An integral controller

(c) A rate feedback controller

(d) A proportional controller

**40.** The transfer function G(s) of PID controller is

**[EC ESE - 1999]**

(a) $K\left[1+\frac{1}{T_i s}+\frac{1}{T_d s}\right]$ (b) $K[1 + T_i s + T_d s]$

(c) $K\left[1+\frac{1}{T_i s}+\frac{1}{T_d s}\right]$ (d) $K\left[1+T_i s+\frac{1}{T_d s}\right]$

# SOLUTIONS

**Sol.1. (b)**

**Sol.2. (b)**
From statement 2.

Output of controller = $\frac{Kde(t)}{dt}$

From statement 4, K is positive

From statement 3, if $\frac{de(t)}{dt} = 1\%$ then

Change in output of controller is |K|%
Hence the mode of controller is derivative controller.

**Sol.3. (a)**
P1D controllers are most popular controller and it is an essential part of any control loop in process industry.
The statement II is also correct and correct explanation of statement I.

**Sol.4. (a)**
For a derivative control action, the actuating signal consists of proportional error signal added with derivative of the error signal. Therefore, the actuating signal for derivative control actions given by

$$e_a(t) = e(t) + T_d \frac{de(t)}{dt}$$

Where, $T_d$ is a constant

**Sol.5. (a)**
**Proportional (Gain)**
For a heater, a controller with a proportional band of 10 deg C and a setpoint of 100 deg C would have an output of 100% upto 90 deg C, 50% at 95 Deg C and 10% at 99 deg C. if the temperature overshoots the setpoint value, the heating power would be cut back further. Proportional only control can provide a stable process temperature but there will always be an error between the required setpoint and the actual process temperature.
**Integral (Reset)**
I represents the steady state error of the system and will remove setpoint/measured value errors. For many applications proportional + Integral control will be satisfactory with good stability and at the desired setpoint.
**Derivative (Rate)**
The derivative term is use to determine a controller's response to a change or disturbance of the process temperature (e.g., opening an oven door). The larger the derivative term the more rapidly the controller with respond to changes in the process value.

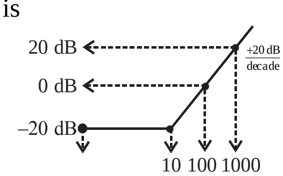
**Sol.6. (a)**

**Sol.7. (c)**

$$GH(s) = \frac{10(s-1)}{S+10}$$

If a system has at least a zero (or) a pole in right side of S plane then it is called Non-minimum phase system.

**Sol.8. (b)**
GH(s) = [0.1 + 0.01s] = 0.1 [1 + 0.1 s]
Bode plot is




**Sol.9. (b)**
PD controller increases system bandwidth since it is analogous to high pass filter. Hence it is highly sensitive to noise because Bandwidth $\propto$ Noise.

**Sol.10. (a)**

**Sol.11. (b)**

**Sol.12. (c)**
In Ziegler Nicholas method by giving a step input first we one obtained the response from

response curve by taking some parameter $k_p$, $k_j$, $k_d$ values one obtained. So here we need the output response curve we don't need any information about system parameter.

**Sol.13. (c)**
For a PI controller,

$$G(s) = K_p + \frac{K_1}{s} = \frac{sK_p}{s} = \frac{sK_p + K_1}{s}$$

**Sol.14. (c)**
Let the transfer function of the plant be

$$G(s) = \frac{K}{(s+a)}$$

When we introduce a delay now it becomes

$$G(s) = \frac{Ke^{-\tau_d s}}{(s+a)}$$

From the polar of $G_1(s)$ and $G_2(s)$ it can be shown that stability of $G_s(s)$ is less than $G_1(s)$ and hence phase margin decrease.

**Sol.15. (*)**

**Sol.16. (*)**

**Sol.17. (c)**

**Sol.18. (b)**
Since the liquid level controller linearly converts a displacement of 2 m to 3 m into 4-20 mA control signal. It can be represented as
I = 4 + K(x – 2) mA
i.e. when x = 2m, I = 4 mA
SO when x = 3m, then I = 20 mA
20 = 4 + K(3 – 2)
⇒ 16 = K
∴ I = 4 + 16 (x – 2)
When the relay closed at I = 12 mA, then
12 = 4 + 16 ($x_2$ – 2)⇒ 8 = 16 ($x_2$ – 2)

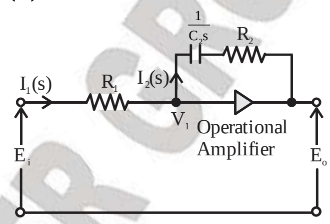
$$x_2 = \frac{5}{2} = 2.5\,\text{m}$$

When the relay opens at I = 10 mA, then
10 = 4 + 16 ($x_1$ – 2)
⇒ 6 = 16 ($x_1$ – 2)
$x_1$ = 2.375 m

So hysteresis zone = $x_2 - x_1$
= 0.125 m

**Sol.19. (d)**
(P + I) controller improves the steady state error due to integral action but proportional action improves the transient response marginally by speeding up the transients.

**Sol.20. (d)**




Applying KCL at node $V_1$ we have

$$\frac{E_i(s) - V_1}{R_1} = \frac{V_1 - E_0(s)}{\frac{R_2 C_2 s + 1}{C_2 s}}$$

But $V_1 = 0$; so

$$\frac{E_i(s)}{R_1} = \frac{-E_0(s) C_2 s}{R_2 C_2 s + 1}$$

$$\Rightarrow -E_0(s) = E_i(s)\left(\frac{R_2 C_2 s + 1}{C_2 s}\right)\frac{1}{R_1}$$

$$= \left(\frac{R_2 C_2 s}{C_2 s R_1} + \frac{1}{R_1 C_2 s}\right) E_i(s)$$

$$\Rightarrow -E_0(s)\ \ -E_0(s) = \frac{R_2}{R_1} E_i(s) + \frac{1}{R_1 C_1 s} E_i(s)$$

Taking inverse Laplace transform; we have

$$-E_0(t) = \frac{R_2}{R_i} E_i(t) + \frac{1}{R_1 C_1}\int E_i(t)$$

Hence proportional + Integral controller

**Sol.21. (c)**

$$E_0 = -\frac{R_2}{R_1} E_i$$

⇒ $E_0 \propto E_i$
Hence proportional controller.

**Sol.22. (b)**

**Sol.23. (d)**
$K_p$ is for proportional controller, $K_d$ s is for derivative controller and $\frac{K_i}{s}$ is for integral controller. Therefore, it is proportional plus derivative plus integral controller.

**Sol.24. (c)**
$K_p$ is for proportional controller and $K_d s$ is for derivative controller.
Therefore, it is proportional plus derivative controller.

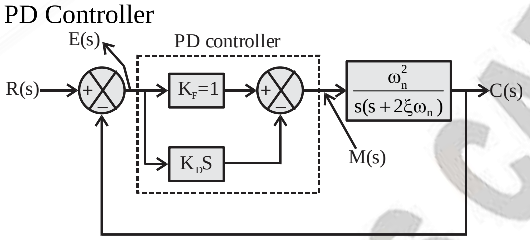
**Sol.25. (c)**

**Sol.26. (b)**

**Sol.27. (b)**

**Sol.28. (d)**

**Sol.29. (c)**
PD Controller

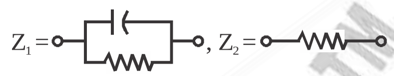



$$\frac{M(s)}{E(s)} = K_P + K_D S$$

$$\frac{C(s)}{R(s)} = \frac{(K_P + K_D s)\omega_n^2}{s^2 + (2\xi\omega_n + K_D\,\omega_n^2)s + K_P\,\omega_n^2}$$

Characteristic equation is
$s^2 + (2\xi\omega_n + K_D\omega_n^2)s + \omega_n^2 = 0$
$\because K_P = 1$
Comparing with $s^2 + 2\xi'\,\omega_n'S + \omega_n'^2 = 0$
$\omega_n' = \omega_n$; $\quad \xi' = \xi + \frac{K_D\omega_n}{2}$
Thus $\omega_n$ remains fixed but $\xi$ increases.

**Sol.30. (a)**
PD controller behaves like differentiator. So, $Z_1$ should be capacitive and $Z_2$ should be resistive.
Physical realization is possible with




Because capacitor alone is not physically realizable.

**Sol.31. (d)**
Integral controller acts like a low pass filter. It reduces the stability as well as steady state error.

**Sol.32. (d)**

**Sol.33. (d)**

**Sol.34. (d)**

**Sol.35. (c)**

**Sol.36. (b)**
A proportional plus derivative controller has the following features.
(i) It adds an open loop zero on negative real axis.
(ii) Undamped natural frequency remains same and damping ratio increases.
(iii) Peak overshoot decreases.
(iv) Bandwidth increases.
(v) Rise time decreases.
(vi) Effect of external noise increases.
(vii)Settling time decreases, i.e. response becomes faster.
(viii) Stability improves.

**Sol.37. (a)**

**Sol.38. (a)**

**Sol.39. (b)**
(i) Integral controller improves the steady state response.
(ii) Derivative controller improves the transient response.

**Sol.40. (a)**
G(s) of PID controller is

$$Q_0 = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix} \Rightarrow = K_P\left[1 + \frac{K_i}{K_P s} + \frac{K_D S}{K_P}\right]$$

# CHAPTER - 8
# *FREQUENCY RESPONSE ANALYSIS*

## 8.1 INTRODUCTION

### 8.1.1 The various Frequency Response Analysis Techniques are

1. Polar plot
2. Nyquist plot
3. Bode plot
4. M & N circles
5. Nicholas chart

### 8.1.1 Polar Plot

The sinusoidal transfer function $G(j\omega)$ is a complex function and is given by

$G(j\omega) = \text{Re}\, G(j\omega) + j\, I_m\, G(j\omega)$

Or $G(j\omega) = |G(j\omega)|\sqrt{G(j\omega)} = M\sqrt{\phi}$

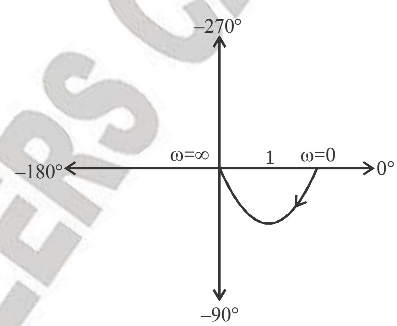
from above equation, it is seen that $G(j\omega)$ may be represented as a phasor of magnitude M and phase angle $\phi$. As the input frequency $\omega$ is varied from 0 to$\infty$, the magnitude M and phase angle $\phi$ change and hence the tip of the phasor $G(j\omega)$ traces a locus in the complex plane.

The locus thus obtained is known as polar plot.

When a transfer function consists of 'p' poles and 'Z' zeros, and it doesn't consist poles at origin then the polar plot start from $0^0$ with some magnitude and terminates at $-90^0 \times (P - Z)$ with zero magnitude.

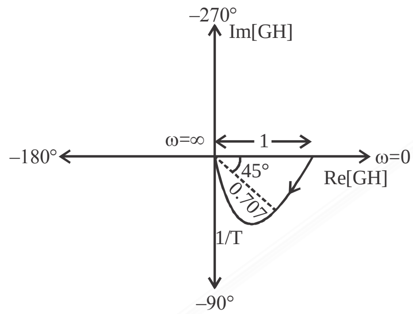
When a transfer consists of poles at origin, then the polar plot starts from $-90^0 \times$ no. of poles at origin with '$\infty$' magnitude and ends at $-90^0 \times (P - Z)$ with zero magnitude

Polar coordinates ($|GH|\angle GH$)

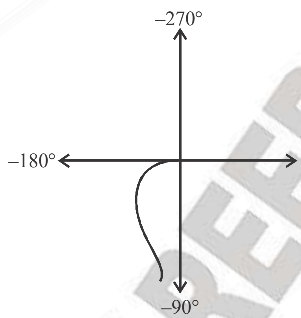



**Example.** Draw the polar plot for the following transfer function:

$$GH = \frac{1}{j\omega+1} \quad |GH| = \frac{1}{\sqrt{\omega^2+1}}, \angle GH = -\tan^{-1}\omega$$




| ω | \|GH\| | ∠GH |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 0.707 | $-45^0$ |
| ∞ | 0 | $-90^0$ |




**Example.** If $G(s)H(s)=\dfrac{1}{s(1+sT)}$

$$G(j\omega)H(j\omega)=\frac{1}{j\omega(1+j\omega T)}$$

$$|G(j\omega)H(j\omega)|=\frac{1}{\omega\sqrt{1+\omega^2T^2}}$$

$$\angle GH=-90^0 \quad -\tan^{-1}\omega T$$

| ω | ∠GH | \|GH\| |
|---|---|---|
| 0 | $-90^0$ | ∞ |
| ∞ | $-180^0$ | 0 |

> When a pole at origin is added, the head and tail of the polar plot shift by 90° in clockwise direction.

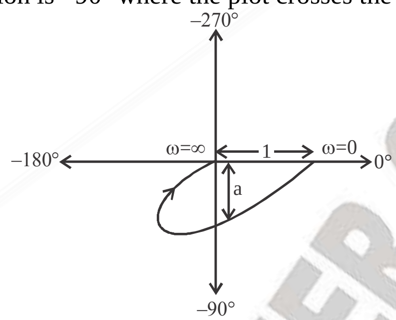
**Example.** $G(s)H(s)=\dfrac{1}{(1+sT_1)(1+sT_2)}$

$$G(j\omega)\,H(j\omega) = \frac{1}{\sqrt{1+\omega^2T_1^2}\,.\sqrt{1+\omega^2T_2^2}}$$

$\angle G(j\omega)H(j\omega) = -\tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2)$

| ω | \|G(s)H(s)\| | ∠G(jω)H (jω) |
|---|---|---|
| 0 | 1 | $0^0$ |
| ∞ | 0 | $-180^0$ |

The angle of the transfer function is $-90^0$ where the plot crosses the imaginary axis:




$-\tan^{-1}(\omega T_1) - \tan^{-1}(\omega T_2) = -90^0$

$$\tan^{-1}\left(\frac{\omega T_1 + \omega T_2}{1-\omega T_1\, T_2}\right) = 90°$$

$$\therefore \omega^2 = \frac{1}{T_1T_2} \qquad\qquad \therefore \omega^2 = \frac{1}{T_1T_2}$$

$$\therefore \omega = \frac{1}{\sqrt{T_1T_2}}$$

$$|\,G(j\omega)\,H(j\omega)\,|\ \text{at}\ \omega = \frac{1}{\sqrt{T_1T_2}}$$

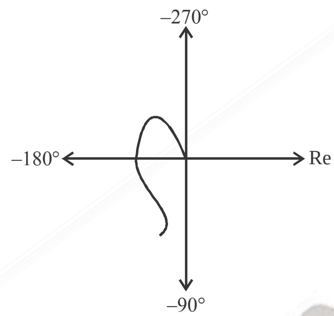
$$= \frac{1}{\sqrt{1+\omega^2T_1^2}\,\sqrt{1+\omega^2T_2^2}}$$

$$\therefore a = \left(\frac{\sqrt{T_1T_2}}{T_1+T_2}\right)$$

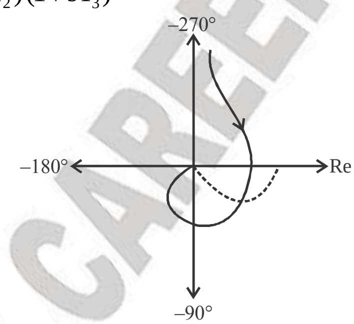
When a pole on real axis is added, head of polar plot shift by $90^0$ in clockwise direction and tail remains the same.

**Example.**

$$G(s)H(s) = \frac{1}{s(1+sT_1)(s1+sT_2)}$$

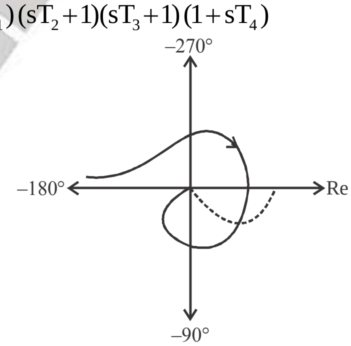


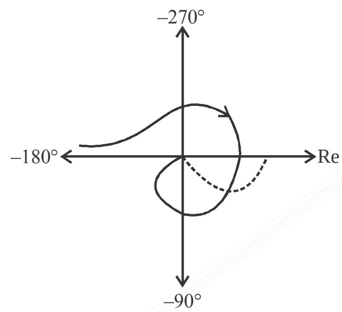

**Example.** $G(s)H(s) = \frac{1}{s^2(1+sT_1)(1+sT_2)(1+sT_3)}$




Shift of
Head $\rightarrow 5 \times 90^0 = 450^0$
Tail $\rightarrow 3 \times 90^0 = 270^0 \rightarrow$due to poles at origin

**Example .** $G(s)H(s) = \frac{1}{s^2(1+sT_1)(sT_2+1)(sT_3+1)(1+sT_4)}$






Head → $5 \times 90^0 = 450^0$
Tail → $2 \times 90^0 = 180^0$

### 8.1.2 Nyquist Stability Criteria

It is used to determine the stability of a closed – loop system using polar plots.

Let $G(s) = \dfrac{(s+Z_1)(s+Z_2)}{(s+P_1)(s+P_2)}$ …(i)

Characteristic equation i.e. 1 + G(s) = 0

$$1+G(s) = 1+\frac{(s+Z_1)(s+Z_2)}{(s+P_1)(s+P_2)}$$

$$= \frac{(s+P_1)(s+P_2)+(s+Z_1)(s+Z_2)}{(s+P_1)(s+P_2)} \quad \text{…(ii)}$$

From (i) and (ii), the open loop poles and CE poles are same.

$$C.E = \frac{(s+Z_1)\ (s+Z_2)}{(s+P_1)(s+P_2)} = 0 \quad \text{… (iii)}$$

Overall transfer function $= \dfrac{G(s)}{1+G(s)} = \dfrac{(s+Z_1)(s+Z_2)}{(s+Z_1^1)(s+Z_2^1)}$ … (iv)

From (iii) and (iv), the C.E. zeros and closed – loop poles are same.

For the closed-loop system to be stable, the zeros of the C.E should not be located on the right half of the s-plane.

Nyquist criterion can be expressed as:

$$N = P - Z$$

Where P is number of poles G(s) in the right-half s-plane or open loop poles in the right half s-plane.

Z is no. of zeros of 1+G(s) i.e. closed loop system in the right-half s-plane

N is no. of counter clockwise encirclements of (-1 + j0) point.

In examining the stability of linear control system using the Nyquist stability criterion, following possibilities can occur:

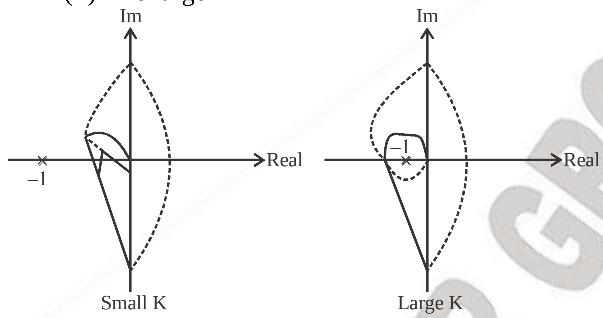
1. There is no encirclement of the (–1 + j0) point. This implies that the system is stable if there are no poles of G(s) i.e. open loop poles (P = 0)
2. There is a counter clockwise encirclement of (–1 + j0) point. In this case, the system is stable if N = P. In that case, Z will be 0.

**Example.** Consider the system with the following open-loop transfer function:

$$G(s) = \frac{k}{s(T_1 s + 1)(T_2 s + 1)}$$

Determine the stability of the system when :

(i) K is small (ii) K is large




**Solution.**

The Nyquist plots are shown below :

⇒ For small values of K, there is no encirclement of the -1 + j0 point . i.e. N = 0. Open loop poles of G(s) in the right – half s-plane, i.e. P = O ⇒ Z = O

Hence, the system is stable for small values of K as there are no zeros of closed loop system in the right-half s-plane

⇒ For large values of K, (-1 + j0) point is encircled twice in the clockwise direction

∴ N = -2, and P = 0

⇒N = P – Z

Z = 2

It indicates 2 closed loop poles in the right half. So, the system is unstable.

**Example.** Comment on the stability of the system whose open loop transfer function :

$$G(s) = \frac{1}{s(1+s)(1+2s)}$$

Also find Gain & Phase Margin.

$$|G(j\omega)| = \frac{1}{\omega\sqrt{1+\omega^2}\sqrt{1+4\omega^2}} = M$$

$\phi = -90° - \tan^{-1}\omega - \tan^{-1}2\omega$

When $\omega = 0$, $M = \infty$, $\phi = -90°$

$\omega = \infty$, $M = 0$, $\phi = -270°$

$$\text{Now, } G(j\omega) = \frac{1}{j\omega(1+2j\omega)(1+j\omega)}$$

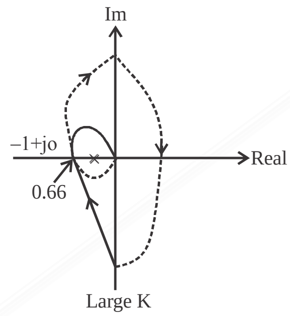
$$= \frac{1}{3\omega^2 + j\omega[1-2\omega^2]} \times \frac{-3\omega^2 - j\omega(1-2\omega^2)}{-3\omega^2 - j\omega(1-2\omega^2)}$$

Equating imaginary part to zero:

$(1 - 2\omega^2) = 0$

$$\omega = \frac{1}{\sqrt{2}} = 0.707$$

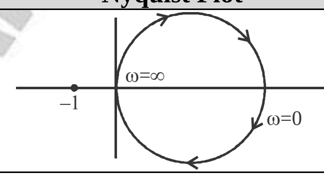
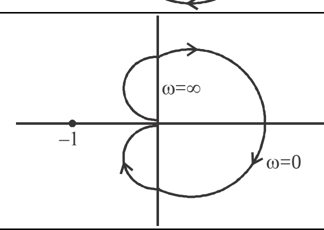
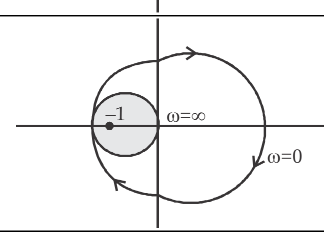
⇒ Phase crossover frequency
∴ $M_{at}$ ω = 0.707 = 0.66
The Nyquist plot is:

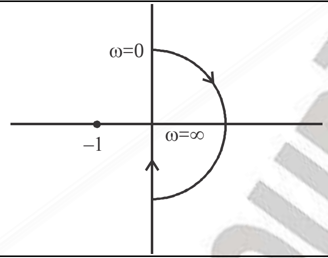
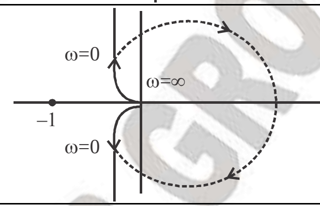
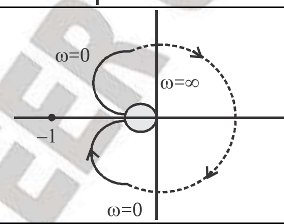
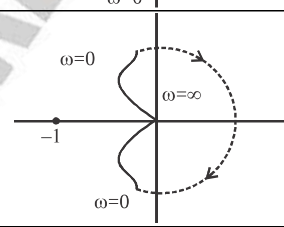



So, point (-1 + j0) is not encircled.
∴ N = 0
Also P = 0 (i.e. open loop poles on the right half of s-plane)
∴ Z = 0 and hence the system is stable.

Gain Margin $= 20\log\frac{1}{a} = 20\ \log\frac{1}{0.66}$ = 3.61 db

**Nyquist Plots for Typical Transfer Functions**

| Sr. No | G(s) | Nyquist Plot |
|---|---|---|
| **1.** | $\frac{K}{sT_1+1}$ |   |
| **2.** | $\frac{K}{(sT_1+1)(sT_2+1)}$ |   |
| **3.** | $\frac{K}{(sT_1+1)(sT_2+1)(sT_3+1)}$ |   |

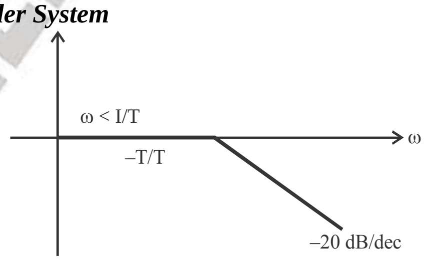
| | | |
|---|---|---|
| **4.** | $\frac{K}{s}$ |   |
| **5.** | $\frac{K}{s(sT_1+1)}$ |   |
| **6.** | $\frac{K}{s(sT_1+1)(sT_2+1)}$ |   |
| **7.** | $\frac{K(sT_a+1)}{s(sT_1+1)(sT_2+1)}$ |   |

### 8.1.3 BODE PLOTS

It is used to sketch the frequency response of a closed – loop system.

The representation of the logarithm of $|G(j\omega)|$ and phase angle of $G(j\omega)$, both plotted against frequency in logarithmic scale. These plots are called Bode plots.

#### *8.1.3.1 Bode Plot of first Order System*




Let the Transfer Function $= \frac{1}{1+Ts}$

Subs. S = jω

$$\text{T.F.} = \frac{1}{1+j\omega T}$$

$$M = \frac{1}{\sqrt{1+(\omega T)^2}}; \phi = -\tan^{-1}(\omega T)$$

$$M = 20\log\frac{1}{\sqrt{1+(\omega T)^2}} = -10\log\left[1+(\omega T)^2\right] M \qquad \omega << 1/T \qquad \omega << 1/T$$

$M_{dB} \cong 10 \log 1$ $\qquad$ $M_{dB} = -10 \log (\omega T)^2$

$\cong 0$ $\qquad$ $= -20 \log \omega T$

Therefore, the error at the corner frequency $\omega = 1/T$ is

$-10 \log 2 + 10 \log 1 = -1\text{dB}$

### *8.1.3.2 Basic Terms that Appear in a Transfer Function & Method of Plotting Bode Plot*

***(i) Constant term "K"***

It gives a constant magnitude of 20log K.. It does not give any phase shift. It is represented by a line parallel to 0 db line & starts from a point having a magnitude 20 log K.

***(ii) $\frac{1}{s}$ factor (i.e. a pole at the origin)***

Its magnitude is 20 log w. It is a straight line having a slope of -20 db/dec or -6db/octane. It passes through w = 1 rad/sec where its magnitude is 0db. Phase angle is constant and equal to -90°

***(iii) s factor (i.e. a zero at the origin)***

It magnitude is 20 log w. It is a straight line having a slope of 20 db/dec or 6 db/octane, Phase angle is +90°

> If the transfer function contains the factor $\left(\frac{1}{s}\right)^n$ or $(s)^n$, then the slopes will be $-20n$ db/decade and 20n db/dec respectively. The phase angle of $\left(\frac{1}{s}\right)^n$ is $-90° \times n$ and that of $(s)^n$ is $90° \times n$

***(iv) $(1 \pm sT)^{\pm n}$***

Magnitude is given by $\pm n \times 20\log\sqrt{1+\omega^2T^2}$ having a slope of ± n × 20 db/dec. Asymptotes are approximated by

(a) If $\omega << \frac{1}{T}$, magnitude = 0 db

(a) $\omega >> 1/T$, magnitude is $\pm n \, 20 \log \omega T$. It has a slope of ± n × 20 db/dec. Asymptotes meet at a point where:

$20 \log \omega T = 0$ i.e. $\omega T = 1$

Or $\omega = \dfrac{1}{T}$

which is called the corner frequency.

**Example**

Sketch the bode plot and determine:

(i) The phase-crossover frequency

(ii) The gain crossover frequency.

(iii) Gain margin

(iv) Phase margin

$$G(s) = \frac{10}{s+(1+0.5s)(1+0.1s)}$$

**Solution.**

$$G(s) = \frac{10}{s(1+0.5s)(1+0.1s)} = \frac{10}{s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)}$$

Corner frequencies are $\omega_1 = 2$ rad/sec $\omega_2 = 10$ rad/sec

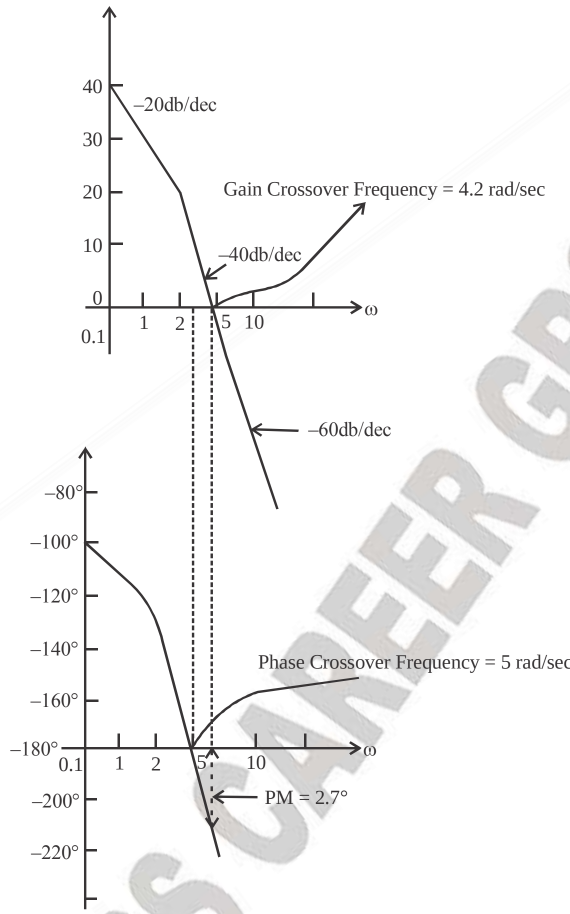
***8.1.3.4 Magnitude plot***

| S. No | Factor | Corner Frequency | Asymptotic log-magnitude characteristic |
|---|---|---|---|
| 1. | $\frac{1}{s}$ | None | Straight line of constant slope -20 db/dec passing through w = 1 rad/sec |
| 2. | $\frac{1}{(1+0.5s)}$ | $\omega_1 = 2$ | Straight line of constant slope -20 db/dec origination from $\omega_1 =$ 2 rad/sec |
| 3. | $\frac{1}{(1+0.1s)}$ | $\omega_2 = 10$ | Straight line of constant slope -20db/dec origination from $\omega_2 = 10$ rad/sec |
| 4. | 10 | None | Straight line of constant slope of 0 db/dec starting from 20 log 10 = 20 db point. |

***8.1.3.5 Phase plot***

$\phi = -90° - \tan^{-1} 0.5\omega - \tan^{-1} 0.1\omega$

| $\omega$ | 0 | 0.1 | 1 | 2 | 5 | 5 | 10 |
|---|---|---|---|---|---|---|---|
| $\phi$ | $-90^o$ | $-93.43^o$ | $-122^o$ | $-146^o$ | $-184.7^o$ | $-184.7^o$ | $-213.7^o$ |

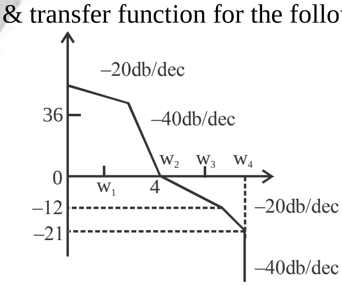



From the gain cross over frequency, draw a perpendicular at the phase plot. Measure the angle from -180° line & where it meets the phase plot.

That angle will be phase margin. In this case, PM = 2.7°

⇒ From the phase cross over frequency, draw a perpendicular at the magnitude plot. Measure the gain from 0 db line & where it meets the magnitude plot. In this case, GM = 3.57 db.

**Example .** Find the frequencies & transfer function for the following bode plot:



**Solution.**

Between $w_1$ & w = 4 rad/sec, there is a decrease of 36 db.

So, to find the corner frequencies, the formula is:

Change in magnitude = slope between that two frequencies [ log $w_2$ – log $w_1$ ]

$\Rightarrow$ -36 = -40 [log 4 – log$w_1$]

$W_1$ = 0.5 rad/sec

Calculations for $w_3$ : -12 = -40[log$w_3$ – log 4] Or $w_3$ = 8 rad/sec

Calculation for $w_4$ : –21 + 12 = -20 [log $w_4$ – log 8] Or $w_4$ = 22.5 rad/sec

Calculation of K :- 20 log K = 36 + 20 log 0.5

K = 31.62

First line has a slope of -20 db/dec indicating a term $\frac{1}{s}$ & since it is not passing through w = 1 rad/sec, the term is $\frac{K}{s}$ or $\frac{31.62}{s}$

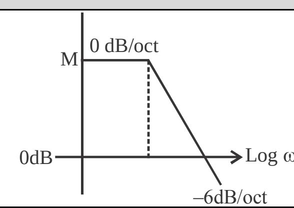
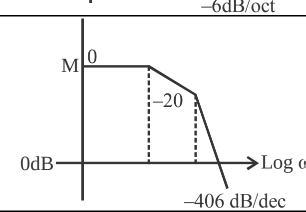
At $w_1$ = 0.5 rad/sec, slope changes to -40 db/dec indicating a term $\frac{1}{\left(1+\frac{s}{0.5}\right)}$ or $\frac{1}{1+2s}$

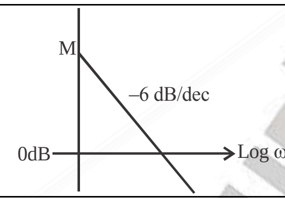
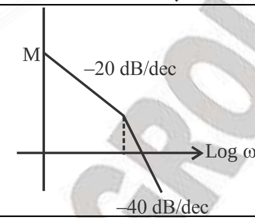
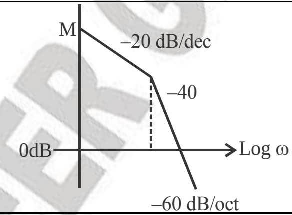
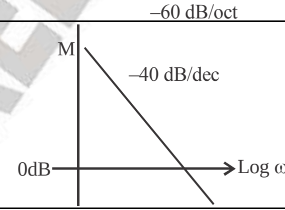
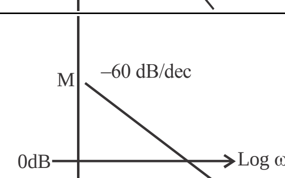
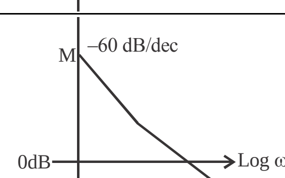
At $w_3$ = 8 rad/sec, slope changes to -20 db/dec indicating a term $\left[1+\frac{s}{8}\right]$ or$(1+0.125s)$

At $w_4$ = 2.5 rad/sec, slope changes to -40 db/dec indicating a term $\left[\frac{1}{1+\frac{s}{22.5}}\right]$ or $\left(\frac{1}{1+0.004s}\right)$

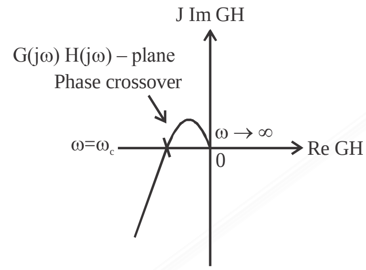
Combining all the terms G(s) =

**Bode Plots for Typical Transfer Functions**

| Sr. No. | G(s) | Bode Plot |
|---|---|---|
| **1.** | $\frac{K}{sT_1+1}$ |  |
| **2.** | $\frac{K}{(sT_1+1)(sT_2+1)}$ |  |

| | | |
|---|---|---|
| **3.** | $\frac{K}{s}$ |  |
| **4.** | $\frac{K}{s(sT_1+1)}$ |  |
| **5.** | $\frac{K}{s(sT_1+1)(sT_2+1)}$ |  |
| **6.** | $\frac{K}{s^2}$ |  |
| **7.** | $\frac{K}{s^3}$ |  |
| **8.** | $\frac{K(sT_a+1)}{s}$ |  |

## 8.2 GAIN MARGIN




The gain margin is a factor by which the gain of a stable system is allowed to increase driving the system to the verge of instability.

GM = $\frac{1}{a}$, where a is magnitude, M at $\omega_c$

The phase cross - over frequency is denoted by $\omega_c$, and the magnitude of G(jω) H(jω) at $\omega = \omega_c$ is designated by $|G(j\omega_c)\ H(j\omega_c)|$. In decibel, the gain margin is given by

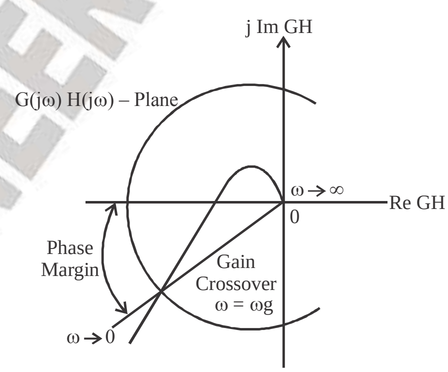
G.M.=20$\log_{10}$

### 8.2.1 Procedure to Calculate Gain Margin

1. Calculate phase crossover frequency
(a) By equating phase equation to $180^0$ or
(b) By equating imaginary part to zero
2. Calculate the magnitude at phase crossover frequency and is equal to 'a'.
3. Gain margin is equal to 20 log (1/a).

For stable systems as $|G(j\omega_c)\ H(j\omega_c)| < 1$, the gain margin in dB is positive.
For marginally stable systems as $|G(j\omega_c)\ H(j\omega_c)| = 1$, the gain margin in dB is zero.
For unstable systems as $|G(j\omega_c)\ H(j\omega_c)| > 1$, the gain margin in dB is negative an the gain is to be reduced to make the system stable.

## 8.3 PHASE MARGIN




The phase margin of a stable system is the amount of additional phase lag at gain cross over frequency required to bring the system to the point of instability.
The phase margin is given by P.M. = $180^0 + \angle G(s)\ H(s)$

### 8.3.1 Procedure for Calculation of P.M

1. Calculate '$\omega_G$' by equating magnitude equation to '1'
2. Calculate the phase at $\omega = \omega_G$
3. P.M. is positive, the system is stable
4. P.M. is negative, the system is unstable
5. P.M. is zero, the system is marginally stable.

**Example.** Find PM for a system whose open loop transfer function is $G(s) = \dfrac{2\sqrt{3}}{s(s+1)}$

**Solution.**

Gain crossover frequency where gain is 1 is

$$\left|\frac{2\sqrt{3}}{jw(1+jw)}\right| = 1$$

Hence $w_g = \sqrt{3}\,\text{rad/sec}$

$$\angle G(jw) = 90° - \tan^{-1}\sqrt{3} = 150°$$

$$\frac{2\sqrt{3}}{w\sqrt{1+w^2}} = 1 \Rightarrow w = \sqrt{3}$$

wg. $\therefore$ PM = 180°-150°=30°

| |
|---|
| At point (–1 + jo), GM is 0db & PM is 0° |

**Example.** Find gain margin for a system whose open loop transfer function is

$$G(s) = \frac{1}{s\left(s^2+s+1\right)} = \frac{1}{s^2+s^2+s}$$

Equating imaginary part to 0.

i.e. $-w(1-w^2) = 0 \Rightarrow w$ = phase cross over frequency = 1 rad/sec

$|G(j\omega)|$ = a = 1

GM = -20log a

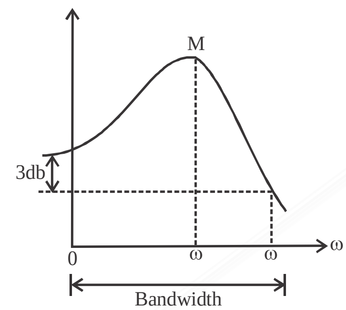
= –20 log1 = 0 dB

### 8.3.2 Cut off frequency and Bandwidth

The closed – loop frequency response of a system is shown in the fig. The response falls by 3 dB from its low frequency value $\omega_c$. The frequency $\omega_c$ is called cut Off frequency and the frequency range 0 to $\omega_c$ is called the bandwidth of the system. The resonant Peak $M_r$ occurs at resonance frequency $\omega_r$.

The bandwidth is defined as the frequency at which The magnitude gain of the frequency response plot reduces to $\frac{1}{\sqrt{2}} = 0.707$ i.e. 3db of its low frequency value.

For a second order system




$M(s)=\dfrac{\omega_n^2}{s^2+2\varsigma\omega_n s+\omega_n^2}$

$\therefore\ M(j\omega)=\dfrac{\omega_n^2}{\left(\omega_n^2-\omega^2\right)+j2\varsigma\omega_n}$

$\therefore\ \left|M\left(j\omega\right)\right|=\dfrac{\omega_n^2}{\left(\omega_n^2-\omega^2\right)+j2\varsigma\omega_n}$

Let $\omega_c$ be the frequency at which the magnitude gain $|M(j\omega)|$ is reduced to $1/\sqrt{2}$ time of its low frequency value.

$\therefore\ \dfrac{1}{\sqrt{2}}=\dfrac{\omega_n^2}{\sqrt{\left(\omega_n^2-\omega^2\right)+4\varsigma^2\omega_n^2\omega_c^2}}$

Rearranging the above equation $\dfrac{\omega_n^2}{\sqrt{\left(\omega_n^2-\omega^2\right)+4\zeta^2\omega_n^2\omega_c^2}}$

Or $(\omega_n^2-\omega_c^2)^2+4\varsigma^2\omega_n^2\,\omega_c^2=2\omega_n^4$ Or $\omega_n^4-2\omega_n^2\,\omega_c^2+\omega_c^4+4\varsigma^2\omega_n^2\,\omega_c^2=2\omega_n^4$

Or $\omega_c^4+2\omega_n^2\,(2\varsigma^2-1)\,\omega_c^2-\omega_n^4=0$

Solving for $\omega_c^2$ :

$\omega_c^2=\dfrac{-2\omega_n^2\left(2\varsigma^2-1\right)+\sqrt{4\omega_n^4\left(2\varsigma^2-1\right)+4\omega_n^4}}{2}\ \omega_c=\omega_n\left(1-2\varsigma^2+\sqrt{4\varsigma^4-4\varsigma^2+2}\right)^{1/2}$

The bandwidth of a second order system having non – zero magnitude at $\omega = 0$ is given by

B.W. = $\omega_n\left(1-2\varsigma^2+\sqrt{4\varsigma^4-4\varsigma^2+2}\right)^{1/2}$

The frequency at which the maximum value of magnitude is attained, is called resonant frequency and denoted by $\omega_r$. The resonant frequency is given by

$\omega_r=\omega_n\sqrt{1-2\varsigma^2}$

At resonance frequency $\omega_r$ the magnitude attains maximum value and is known as resonant peak denoted by $M_r$. The resonant peak $M_r$ is calculated below in terms of damping ratio.

$M_r=\dfrac{1}{\sqrt{\left[\left(1-\omega_r^2/\omega_n^2\right)^2+\left(2\varsigma\omega_r/\omega_n\right)^2\right]}}$

Since $\omega_r = \omega_n \sqrt{1-2\varsigma^2}$ for $\varsigma < 0.707$ or $M = \frac{1}{2\varsigma\sqrt{(1-\varsigma)^2}}$

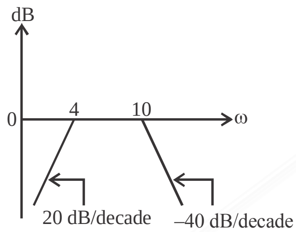
**(i) Minimum phase transfer function**
Transfer function have no poles and zeros in the RHS of s-plane.

**(ii) Non-minimum phase transfer function**
Transfer function having at least one pole or zero in the RHS of s – plane.

### 8.3.3 All Pass Transfer Function
Transfer function have symmetric pole and zero about the imaginary axis in s-plane.

# ASSIGNMENT

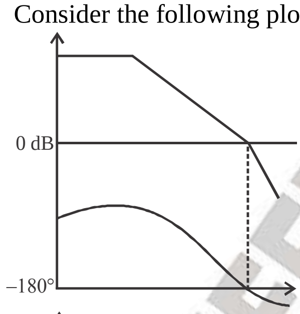
**1.** Straight line asymptotic Bode magnetic plot for a certain system is shown in the given figure. What will be its transfer function?

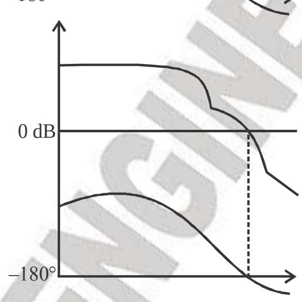


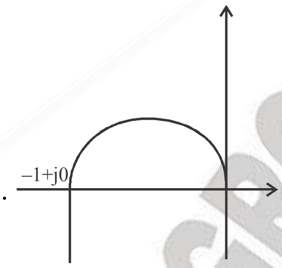

(a) $\dfrac{4s}{\left(1+\dfrac{s}{4}\right)\left(1+\dfrac{s}{10}\right)}$  (b) $\dfrac{4\left(1+\dfrac{s}{4}\right)}{\left(1+\dfrac{s}{10}\right)^2}$

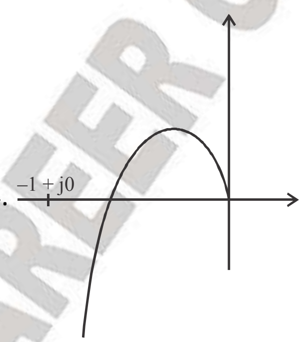
(c) $\dfrac{0.25}{\left(1+\dfrac{s}{4}\right)\left(1+\dfrac{s}{10}\right)}$  (d) $\dfrac{0.25s}{\left(1+\dfrac{s}{4}\right)\left(1+\dfrac{s}{10}\right)^2}$

**2.** Consider the following plots.

1.



2.


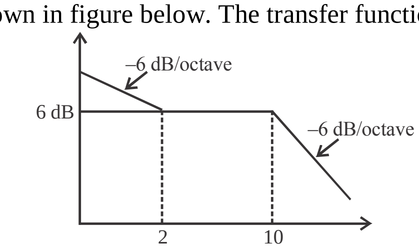

3.



4.



The plots which represents marginal stability would include

(a) 1 and 2  (b) 3 and 4
(c) 1 and 3  (d) 2 and 4

**3.** The magnitude plot of a transfer function is shown in figure below. The transfer function is




(a) $\dfrac{4\left(1+\dfrac{s}{2}\right)}{s\left(1+\dfrac{s}{10}\right)}$  (b) $\dfrac{4\left(1+\dfrac{s}{2}\right)}{\left(1+\dfrac{s}{10}\right)}$

(c) $\dfrac{4(1+2s)}{s(1+10s)}$  (d) $\dfrac{4s(1+2s)}{(1+10s)}$

**4.** The open-loop transfer function of a system is $G(s)H(s)=\dfrac{k}{(1+s)(1+2s)(1+3s)}$ the phase crossover frequency $\omega_c$ is

(a) $\sqrt{2}$ (b) 1

(c) Zero (d) $\sqrt{3}$

**5.** The plane margin (in degrees) of a system having the loop transfer function $G(s)H(s)=\dfrac{2\sqrt{3}}{s(s+1)}$ is

is

(a) 45° (b) –30°

(c) 60° (d) 30°

**6.** In the above question the gain margin in dB is

(a) $-\infty$ (b) Zero

(c) $+\infty$ (d) 1

**7.** In the Bode-plot of a unity feedback control system, the value of phase of $G(j\omega)$ at the gain cross-over frequency is –125°. The phase margin of the system is

(a) –125° (b) –55°

(c) 55° (d) 125°

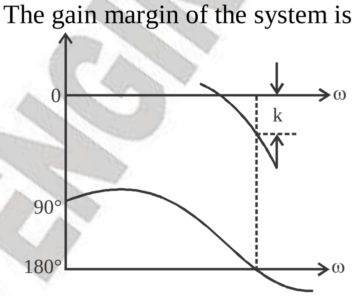
**8.** The open-loop transfer function of a feedback system is $G(s)H(s)=\dfrac{1}{(s+1)^3}$

The gain margin of the system is

(a) 2 (b) 4

(c) 8 (d) 16

**9.** Bode plots of an open-loop transfer function of a control system are shown in the given figure. The gain margin of the system is




(a) k (b) – k

(c) $\dfrac{-1}{k}$ (d) $\dfrac{1}{k}$

**10.** The open loop transfer function of a feedback system is

$G(s)=\dfrac{1}{s+2}$

The corner frequency of the system is

(a) 0.5 rad/sec (b) 2 rad/sec

(c)1 rad/sec (d) none

**11.** If X= ReG(jω) and y = imG(jω) then for $\omega \to 0$ the Nyquist plot for $G(s)=\dfrac{1}{s(s+1)(s+2)}$

(a) X = 0 (b) $X=-\dfrac{3}{4}$

(c) $X=y-\dfrac{1}{6}$ (d) $X=\dfrac{y}{\sqrt{3}}$

**12.** For a second order system with unity feedback & $G(s)=\dfrac{225}{s(s+6)}$ . The B.W. of the system is

(a) 15 rad/sec (b) 6 rad/sec

(c) 22.64 rad/sec (d) 14.39 rad/sec

**13.** In the above question the peak resonant is

(a) 2.55 (b) 14.39

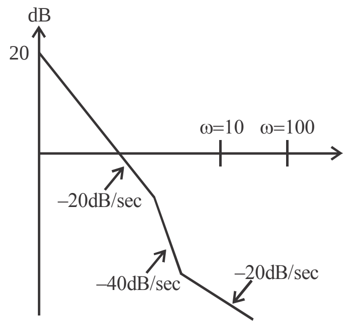
(c) 6 (d) 22.64

**14.** For $\xi>\dfrac{1}{\sqrt{2}}$ ,M is equal to

(a) $\dfrac{1}{2}$ (b) 1

(c) $\dfrac{1}{3}$ (d) Zero

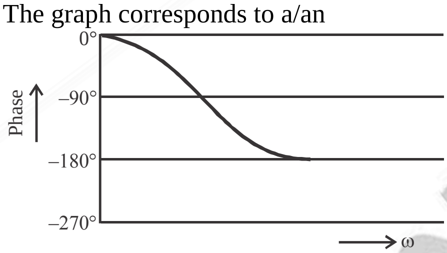
**15.** Find the transfer function of a system having the Bode plot (magnitude) shown in below:




(a) $\frac{100(s+100)}{s(s+10)}$ (b) $\frac{s+100}{s+10}$

(c) $\frac{10(s+100)}{s(s+10)}$ (d) $\frac{s+100}{10s(s+10)}$

**16.** The graph corresponds to a/an




(a) Minimum phase function
(b) All pass function
(c) Non-minimum phase function
(d) None

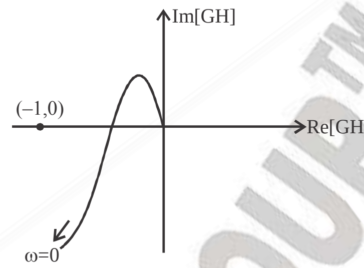
**17.** The slope of a bode plot with the region 5 < ω < 20 for the open-loop system function

$$G(s)=\frac{1}{s(s+5)(s+20)}$$ is

(a) –6 dB/octave (b) – 12 dB/octave
(c) 6 dB/octave (d) 12 dB/octave

**18.** The Nyquist plot for the function is

$$G(s)H(s)=\frac{k}{s(1+T_1s)(1+T_2s)}$$




The condition for stability is given by

(a) $\frac{k(T_1+T_2)}{T_1T_2}<1$ (b) $\frac{k(T_1+T_2)}{T_1T_2(T_1-T_2)}<1$

(c) $\frac{T_1+T_2}{k(T_1-T_2)}<1$ (d) $\frac{T_1-T_2}{k(T_1+T_2)}<1$

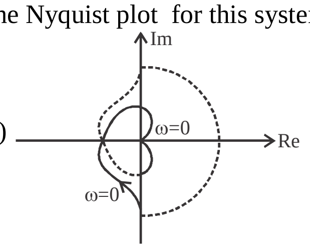
**19.** If the gain margin of a certain feedback system is given as 40 dB, the Nyquist plot will cross the negative real axis at the point

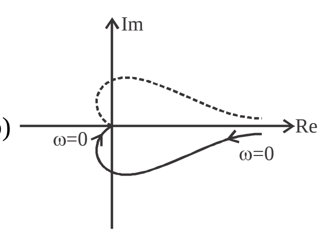
(a) –0.1 (b) –0.01
(c) –0.04 (d) –0.2

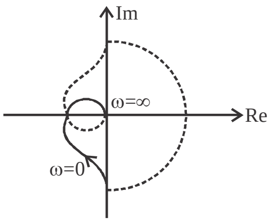
**20.** Consider a ufb system whose open-loop transfer function is

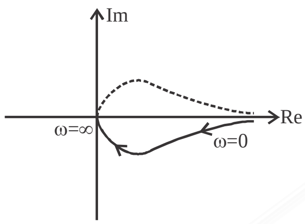
$$G(s)=\frac{k}{s(s+2)(2s+1)(3s+1)}$$

The Nyquist plot for this system is

(a)



(b)


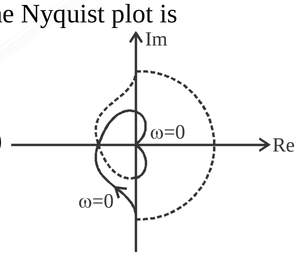
(c)


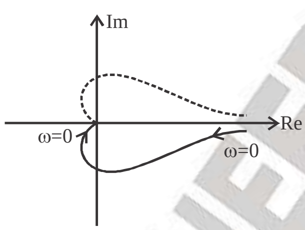

(d)


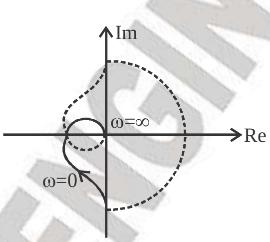

**21.** For the certain unity feedback system

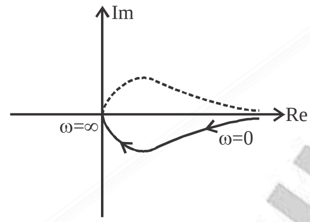
$$G(s)H(s)=\frac{k}{s(s+1)(s+2)}$$

The Nyquist plot is

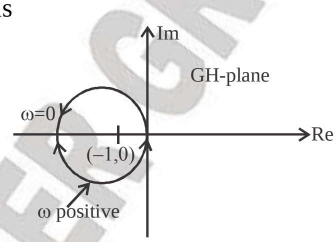
(a)



(b)



(c)



(d)



**22.** In the figure, the Nyquist plot of the open loop transfer function G(s)H(s) of a system. If G(s)H(s) has 2 right-hand pole, the closed loop system is




(a) Always stable
(b) Unstable with one closed-loop right and pole
(c) Unstable with one zero right hand plane
(d) Unstable with two zero in right hand plane.

**23.** The Nyquist plot of G(jω)H(jω) for a closed loop control system is enclosed the unity circle. The gain margin of the system in dB is equal to
(a) Infinite
(b) Greater than zero
(c) Less than zero
(d) Zero

**24.** Consider a feedback system having the characteristic equation

$$1+\left(\frac{3}{s+15}\right)\left(\frac{15}{s+1}\right)=0$$

It is desired that all roots of the characteristic equation have real part less than –1. Find the largest value of k satisfying this condition.

(a) 0.75 (b) 1.33
(c) 0.5 (d) 0.95

**25.** The open-loop transfer function of a feedback system is

$$G(s)H(s)=\frac{k}{s(s+1)(s+2)}$$

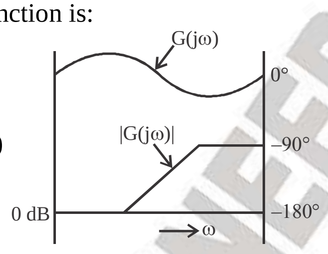
Find the value of gain k for obtaining a gain margin of 3 dB for stable system.
(a) 6 (b) 1.41
(c) 3.25 (d) 4.25

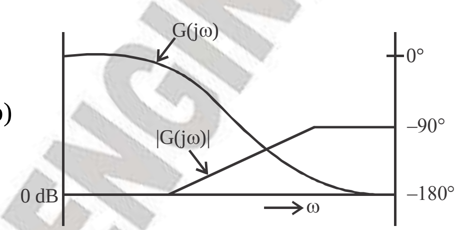
**26.** The open-loop transfer function of a feedback system is $G(s)H(s) = \dfrac{k}{s(s+1)(s+2)}$.
The phase cross over frequency in rad/sec and gain margin in dB for k = 1 are
(a) 1.42 and 6 (b) 15.5 and 1.42
(c) 1.42 and 15.5 (d) 2 and 15.5

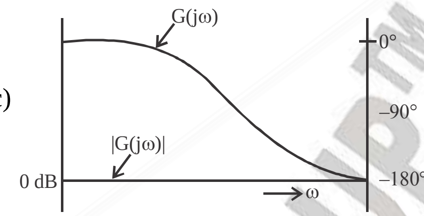
**27.** The open-loop transfer function of unity feedback system is $G(s) = \dfrac{k}{s(s+2)(s+10)}$ .The range of k for which closed loop system is
(a) k < 240 (b) k > 240
(c) k < 340 (d) k > 340

**28.** The open-loop transfer function of a system is given by $G(s) = \dfrac{30}{s(1+0.5s)(1+0.08s)}$ . The corner frequencies in rad/sec are:
(a) 0.5 and 0.08 (b) 2 and 0.08
(c) 0.5 and 12.5 (d) 2 and 12.5

**29.** The Bode plot for minimum phase transfer function is:

(a)



(b)



(c)


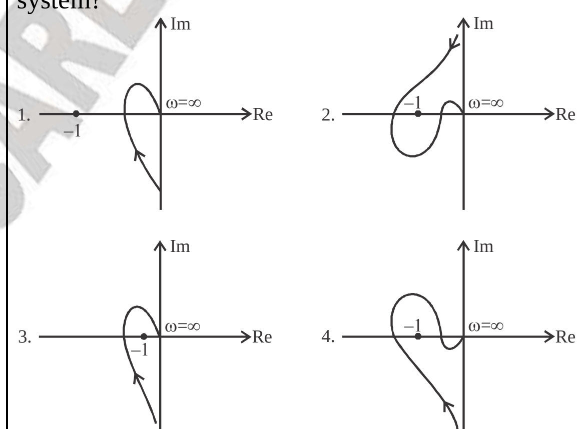

(d) None

**30.** The frequency response of a linear system G(jω) is provided in the tabular form below:

| **\|G(jω)\|** | 1.3 | 1.2 | 1.0 | 0.8 | 0.5 | 0.3 |
|---|---|---|---|---|---|---|
| **∠G(jω)** | -130° | -140° | -150° | -160° | -180° | -200° |

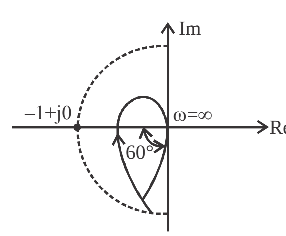
The gain margin and phase margin of the system are
(a) –6dB and 30° (b) –6dB and −30°
(c) 6 dB and 30° (d) 6 dB and −30°

**31.** Consider the following Nyquist plots of loop transfer function over ω = 0 to ω = ∞.. Which of these plots represents a stable closed loop system?



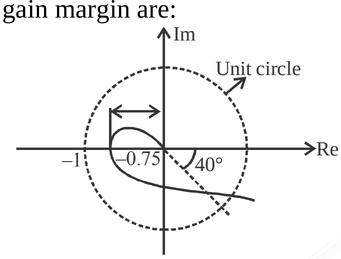

(a) 1 only (b) All
(c) All, except (3) (d) (1) and (2) only

**32.** In the figure given below the phase margin is



(a) $60^o$ (b) $120^o$
(c) $180^o$ (d) $240^o$

**33.** In the figure given below the phase margin and the gain margin are:

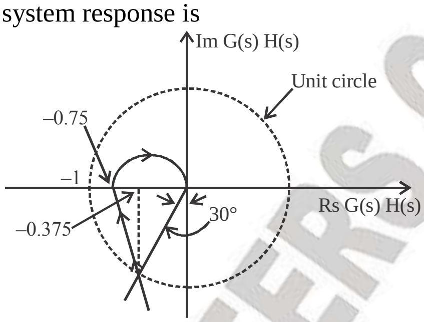



(a) $\frac{2}{3}$ and 140° (b) 140° and $\frac{4}{3}$
(c) 40° and $\frac{4}{3}$ (d) 24

**34.** The response c(t) of a system is described by the differential equation

$$\frac{d^2c(t)}{dt^2}+4\frac{dc(t)}{dt}+5c(t)=0$$

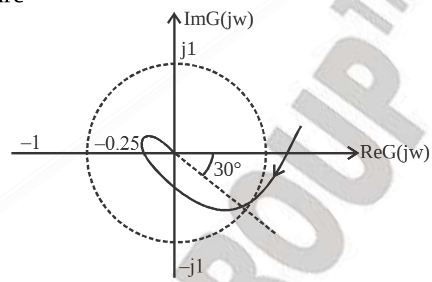
The system response is




(a) Undamped (b) Underdamped
(c) Critically damped (d) Oscillatory

**35.** A portion of the polar plot of an open-**loop** transfer function is shown in the given figure.
The phase margin and gain margin will be respectively.
(a) 30° and 0.75 (b) 60° and 0.375
(c) 60° and 0.75 (d) 60° and 1/0.75

**36.** The polar plot (for positive frequencies) for the open-loop transfer function of a unity feedback control system is shown in the given figure




The phase margin and the gain margin of the system are respectively.
(a) 150° and 4 (b) 150° and 3/4
(c) 30° and 4 (d) 30° and ¾

**37.** Consider the unity feedback system with $G(s)=\frac{2}{s(s+1)(2s+1)}$. What is the gain margin of the system?
(a) 3/4 (b)4/3
(c) 1/2 (d)3/5

**38.** Match List-I (Shape of Nyquist plot) with List-II (Gain Margin) and select the correct answer using the codes given below the lists:

**List-I**
A.The plots does not intersect negative real axis
B.The plot intersects negative real axis between 0 and (-1, j0)
C.The plot passes through the point (-1, j0)
D.The plot encloses the point (-1, j0)

**List-II**
(i) < 0 dB
(ii) dB
(iii) dB
(iv) ∞ dB

**Codes:**
(a) A-iii, B-iv, C-i, D-ii
(b) A-iv, B-iii, C-ii, D-i
(c) A-iii, B-iv, C-ii, D-i
(d) A-iv, B-iii, C-i, D-ii

**39.** If the Nyquist plot cuts the negative real axis at a distance of 0.4. the gain margin of the system is

(a) 0.4 (b) –0.4
(c) 4% (d) 2.5

**40.** A minimum phase unity feedback system has a Bode plot with a constant slope of –20 db/decade for all frequencies. What is the value of the maximum phase margin for the system
(a) $0^0$ (b) $90^0$
(c) $-90^0$ (d) $180^0$

**41.** The initial slope of the bode – plot gives an indication of
(a) Type of the system
(b) Nature of the system
(c) System stability
(d) Gain margin

**42.** If the magnitude of the polar plot at phase cross over is 'a', the gain margin is
(a) –a (b) 0
(c) a (d) 1/a

**43.** For the transfer function

$$G(s).H(s) = \frac{1}{s(s+1)(s+0.5)}$$

the phase crossover frequency is
(a) 0.5 rad/sec (b) 0.707 rad/sec
(c) 1.732 rad/sec (d) 2 rad/sec

**44.** In the bode- plot of a unity feedback control system, the value of phase of G(jω) at the gain cross over frequency is $-125^0$. The phase margin of the system is
(a) $-125^0$ (b) $-55^0$
(c) $+55^0$ (d) $+125^0$

**45.** Nichol's chart is useful for detailed study and analysis of
(a) Closed loop frequency response
(b) Open loop frequency response
(c) Close loop and open loop frequency responses
(d) None of the above

**46.** The open loop transfer function of a unity feedback control system is given as G(s).H(s) = $\frac{1}{s(s+1)(s+0.5)}$. The phase crossover frequency and the gain margin are respectively.

(a) $\frac{1}{\sqrt{T_1T_2}}$ and $\frac{T_1+T_2}{T_1T_2}$

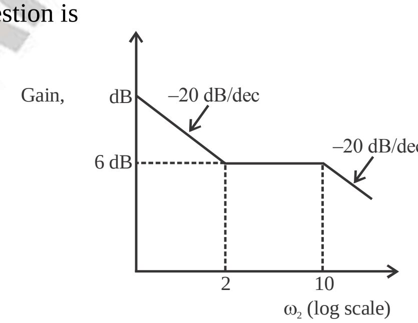
(b) $\sqrt{T_1T_2}$ and $\frac{T_1+T_2}{T_1T_2}$

(c) $\frac{1}{\sqrt{T_1T_2}}$ and $\frac{T_1T_2}{T_1+T_2}$

(d) $\sqrt{T_1T_2}$ and $\frac{T_1T_2}{T_1+T_2}$

**47.** The polar plot of a transfer function passes through the critical
point (–1, 0). Gain margin is
(a) Zero (b) –1 dB
(c) 1 dB (d) infinity

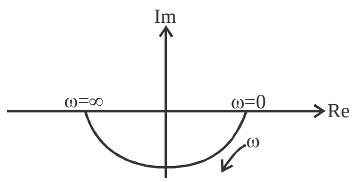
**48.** The magnitude plot of a transfer function is shown in the figure. The transfer function is question is




(a) $\frac{4\left(1+\frac{s}{2}\right)}{s\left(1+\frac{s}{10}\right)}$ (b) $\frac{4s\left(1+\frac{s}{2}\right)}{\left(1+\frac{s}{10}\right)}$

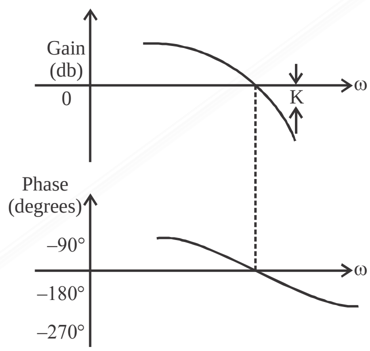
(c) $\frac{4(1+2s)}{s(1+10s)}$ (d) $\frac{4s(1+2s)}{s(1+10s)}$

**49.** A Nyquist plot of a system is shown in the figure. What is the type of this system?




(a) 0 (b) 1
(c) 2 (d) 3

**50.** Bode plots of an open-loop transfer function of a control system are shown in the given figure:




The gain margin is ………
(a) K (b) –K
(c) $\frac{1}{K}$ (d) $-\frac{1}{K}$

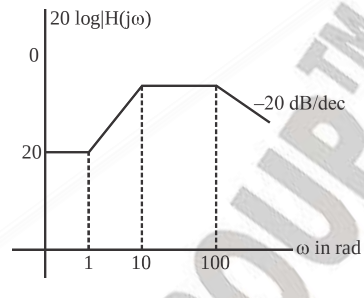
**51.** The polar plots of the open-loop transfer function of a feedback control system intersects the real axis at -2. The gain margin of the system is
(a) -5 dB (b) 0 dB
(c) -6 dB (d) 40 dB

**52.** The corner frequencies are

$$G(s)=\frac{1+s}{s(1+0.5s)}$$

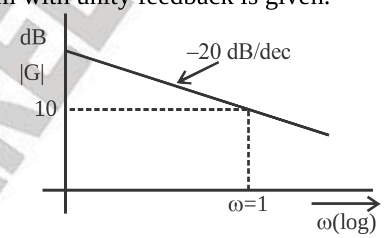
(a) 0 and 1 (b) 0 and 2
(c) 0 and -1 (d) 1 and 2

**53.** Consider the Bode magnitude plot shown in fig. The transfer function H(s) is




(a) $\frac{(s+10)}{(s+1)(s+100)}$ (b) $\frac{10(s+1)}{(s+10)(s+100)}$
(c) $\frac{10^2(s+1)}{(s+10)(s+100)}$ (d) $\frac{10^2(s+100)}{(s+1)(s+10)}$

**54.** A Bode plot of the low frequency magnitude of the forward transfer function of an open loop system with unity feedback is given.



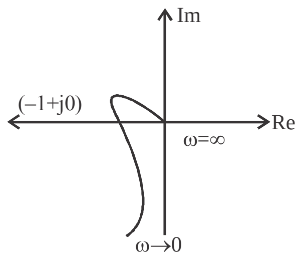

1. This is a type 1 system.
2. The open loop gain K=
3. $K_p$ = the position error coefficient =
4. Of these, the correct statements are

(a) 1, 2, 3 (b) 1, 2
(c) 2, 3 (d) 1, 3

**55.** Consider a system with an open loop transfer function.

$$G(s)H(s)=\frac{(4s+1)}{s^2(s+1)(2s+1)}$$

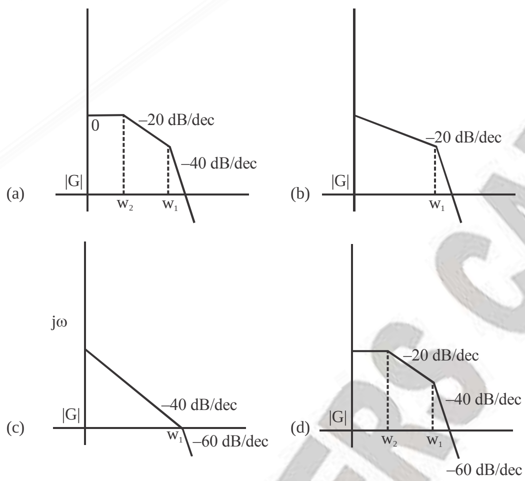
whose nyquist plot is as shown below.




**Nyquist plot**

(a) The system is stable
(b) The system is unstable
(c) Nothing can be said
(d) The system is marginally stable

**56.** The Nyquist plot for control system is shown in figure. The Bode plot for the system will be as in




**57.** The phase margin (in degrees) of a system having the loop transfer function

G(s) H(s) is $\frac{2\sqrt{3}}{s(s+1)}$

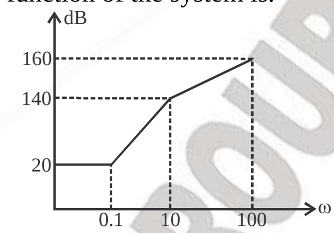
(a) 45° (b) -30°
(c) 60° (d) 30°

**58.** The phase margin of a system with the open-loop transfer function

G(s)H(s) is = $\frac{(1-s)}{(1-s)(2+s)}$

(a) 0° (b) 63.4°
(c) 90° (d) $\infty$

**59.** The approximate Bode magnitude plot of a minimum-phase system is shown in fig. the transfer function of the system is.




(a) $10^7 \frac{(s+0.1)^2}{(s+10)^2(s+100)}$

(b) $10^5 \frac{(s+0.1)^2}{(s+10)^2(s+100)}$

(c) $10^7 \frac{(s+0.1)^2}{(s+10)^2(s+100)}$

(d) $10^9 = \frac{(s+0.1)^2}{(s+10)^2(s+100)^2}$

**60.** Consider a unity feedback system whose open loop transfer function is:

$G(s) = \frac{as+1}{s^2}$

The value of 'a ' so that the system has a phase-margin equal to $\frac{\pi}{4}$ is approximately equal to.

(a) 2.40 (b) 1.40
(c) 0.84 (d) 0.74

**61.** For the open loop transfer function given in Q99, With the value of "a" set for phase-margin of $\frac{\pi}{4}$, the value of unit-impulse response of the open-loop system at t = 1 second is equal to.

(a) 3.40 (b) 2.40
(c) 1.84 (d) 1.74

# SOLUTIONS

**Sol.1.**

Initially plot has a slope of +20 dB/dec. So there must be zero at origin. At $\omega$ = 4rad/sec slope change to 0 dB/dec., so plot at $\omega$ =4. Again slope at $\omega$ = 10 change to –40dB/dec . So there are two poles at $\omega = 10$. Thus transfer function will be

To find the value of k

y = mx + cy

0 = 20 log4 + log k

k = 0.25

Transfer function = G(s) = $\dfrac{0.25s}{\left(1+\frac{s}{4}\right)\left(1+\frac{s}{10}\right)^2}$

**Sol.2. (c)**

**Sol.3. (a)**

Initially slope is $-6$ dB/octave i.e. $-20$dB/dec. So there must be a pole at origin. At $\omega$ = 2rad/sec slope change to 0 dB/dec. so there is a zero at $\omega$ = 2 and at $\omega$ = 10 rad/sec, slope change to $-20$ dB/sec. so there is a pole at $\omega$ = 10.

Transfer function = $\dfrac{k\left(1+\frac{s}{2}\right)}{s\left(1+\frac{s}{10}\right)}$

y = mx + Cy

6 = –20 log (2) + 20logk

k = 4

Transfer function = $\dfrac{4\left(1+\frac{s}{2}\right)}{s\left(1+\frac{s}{10}\right)}$

**Sol.4. (b)**

G(j$\omega$)H(j$\omega$)

$= \dfrac{k}{(1+j\omega)(t+2j\omega)(1+3j\omega)}$ G(j$\omega$)H(j$\omega$)

$= -\tan^{-1}\omega - \tan^{-1}2\omega - \tan^{-1}3\omega$

$-180^o = -(\tan^{-1}\omega + \tan^{-1}2\omega + \tan^{-1}3\omega)$

$180^o = \tan^{-1}\left\{\dfrac{3\omega}{1-2\omega}\right\} + \tan^{-1}3\omega$

$0 = \dfrac{3\omega}{1-2\omega^2} + 3\omega$

$\omega^2 = 1$

$\omega$ = 1 rad/sec

**Sol.5. (d)**

$G(j\omega)H(j\omega) = \dfrac{2\sqrt{3}}{j\omega(1+j\omega)}$

$1 = \dfrac{2\sqrt{3}}{\omega\sqrt{(1+\omega)^2}}$

$\omega^4 + \omega^2 - 12 = 0$

$\omega^2 = y$

$y^2 + y - 12 = 0$

$y^2 + 4y - 3y - 12 = 0$

$y(y + 4) - 3(y + 4) = 0$

$(y + 4)(y - 3) = 0$

y = –4, 3

Ignore the –ve value i.e. y = 3

$\omega^2 = 3$

$\omega = \sqrt{3}$rad / sec

$\omega = \omega_{gc} = \sqrt{3}$ rad / sec

$|G(j\omega)H(j\omega)|_{\omega gc} = -90° - \tan^{-1}\sqrt{3}$

$\phi = -150^o$

PM = $180^o + \phi = 180^o - 150^o = 30^o$

**Sol.6. (c)**

$G(j\omega)H(j\omega)\big|_{\omega ge} = -90° - \tan^{-1}\sqrt{3}$

$-180^o = -90^o - \tan^{-1}\omega$

$\tan^{-1}\omega = 90^o$

$\omega = \tan 90^o = \infty$

$\omega = \omega_{pc} = \infty$

$$\left|G(j\omega)H(\omega)_{\omega pc}\right. = X = \frac{2\sqrt{3}}{\omega\sqrt{1+\omega^2}}$$

X = 0

$$G.M. = \begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 3 & k \\ s^1 & 6-k & \\ s^0 & k^3 & \end{array}$$

$$(GM)_{dB} = 20lag\frac{1}{X} = 20lag\infty = +\infty dB$$

**Sol.7. (c)**

$\phi at \omega C = -125°$

Phase-margin (P.M.) = $-180^o + \phi$

$= 180^o - 125^o = 55^o$

**Sol.8. ()**

$$G(j\omega)\ H(j\omega) = \frac{1}{(1+j\omega)^3}$$

$$\left|G(j\omega)H(j\omega)\right| = 3\tan^{-1}\omega$$

$-18° = -3\tan^{-1}\omega$

$\omega = \omega_{gc}\sqrt{3} = \text{rad/sec}$

$$\left|G(j\omega)H(j\omega)\right| = X = \frac{1}{\left(\sqrt{1+\omega}\right)^3}$$

$\omega = \sqrt{3}\text{rad/sec}$

$X = \frac{1}{8}$

$G.M. = \frac{1}{k}$

**Sol.9. (a)**

$G.M. = \frac{1}{X}$

X = k

$G.M. = \frac{1}{X}$

**Sol.10. (b)**

$$G(s) = \frac{1}{2\left(1+\frac{s}{2}\right)}$$

Here, $T = \frac{1}{2}$ second

Corner frequency, $\omega_{cf} = \omega = \frac{1}{T} = 2\text{rad/sec}$

**Sol.11. (b)**

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(2+j\omega)}$$

$$= \frac{(j\omega-1)(j\omega-2)}{j\omega(\omega^2+1)(\omega^2+4)} = \frac{\omega^2-3j\omega+2}{j\omega(\omega^2+1)(\omega^2+4)}$$

$$= = \frac{(2-\omega^2)-3j\omega}{j\omega(\omega^2+1)(\omega^2+4)}$$

In imaginary part $\frac{-3j\omega}{j\omega(\omega^2+1)(\omega^2+4)} = -\frac{3}{4}$

$X = -\frac{3}{4}$

**Sol.12. (c)**

The characteristics equation

1 +G(s) H(s) = 0

$$1+\frac{225}{s(s+6)} = 0$$

$s^2+6s+225=0$

$\omega_n = \sqrt{225} = 15\text{rad/sec}$

$2\xi\omega_n = 6$

$\xi = \frac{6}{2\times15} = 0.2$

$$B.W = \omega_n \left\{\left(1-2\xi^2\right)+\sqrt{4\xi^4-4\xi+2}\right\}^{1/2}$$

$$= 15\left\{\left(1-2\xi^2\right)+\sqrt{4\times2^4-4\times2^2+2}\right\}^{1/2}$$

$= \{0.92+1.359\}^{1/2}$ = 22.64 rad/sec

**Sol.13. (a)**

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = \frac{1}{2\times0.2\sqrt{1-0.04}} = 2.55$$

**Sol.14. (d)**

**Sol.15. (d)**

Initially slope = –20dB/dec

$$\text{T.F.} = \frac{k(1+T_2 s)}{s(1+T_1 s)}$$

To find k:
y = mx + C
0 = –20 log (1) + 20 log k
k = 1

$$\text{T.F.} = \frac{1\left(1+\frac{s}{100}\right)}{s+\left(1+\frac{s}{10}\right)} = \frac{(s+100)}{100s(s+10)} = \frac{s+100}{10s(s+10)}$$

**Sol.16. (b)**

**Sol.17. (d)**
20dB/dec = 6dB/octave
40dB/dec = 12 dB/octave

**Sol.18. (a)**

**Sol.19. (b)**

GM in dB = $20\log\frac{1}{|G(j\omega)|}$

$$\frac{40}{20} = \log\frac{1}{|G(j\omega)|}$$

$$\frac{1}{|G(j\omega)|} = 100$$

$$|G(j\omega)| = \frac{1}{100} = 0.01$$

So it will cross at s = – 0.01

**Sol.20. (c)**
The given open loop transfer function is Type 1 and order 3 and in only (c) option is satisfied.

**Sol.21. (*)**
It is type-1, order-4 system.

**Sol.22. (*)**
P = 2
N = +1
N = P – Z
+1 = 2 – Z
Z = i.e. system is unstable.
Where Z = number of zero in R.H.S. of s-plane.

**Sol.23. (*)**

$$\text{G.M.} = \frac{1}{X} = +\text{ve}$$

(G.M.(dB= $20\log\frac{1}{X} = +\text{ve}$

So G.M. is greater than zero.

**Sol.24. (a)**

$$G(s) = \frac{k}{(1+s)(1.5+s)(2+s)}$$

$$G(s) = \frac{k}{(1+s)(1.5+s)(2+s)}$$

s = –1 + jω

$$G(-1+j\omega) = \frac{k}{j\omega(0.5+j\omega)(1+j\omega)}$$

$$|G(-1-j\omega) = -90° - \tan^{-1}\omega - \tan^{-1}2\omega$$

$|G(-1+j\omega) = -90^o - \tan^{-1}\omega - \tan^{-1}2\omega$

$-180^o = -90^o - \tan^{-1}\omega - \tan^{-1}2\omega$

$\tan^{-1}(2\omega) + \tan^{-1}\omega = 90^o$

$$\frac{2\omega+\omega}{1-2\omega} = \infty = \frac{1}{0}$$

$1 - 2\omega^2 = 0$

$$\omega^2 = \frac{1}{2}$$

$$\omega = \frac{1}{\sqrt{2}} = 0.707$$

Gain cross over frequency, $\omega_{gc}$ = 707rad/sec $\omega_{gc}$

$$|G(-1+j\omega)|_{\omega ge} = \frac{2k}{0.707\sqrt{1+4\times707^2}\sqrt{1+707^2}}$$

= 1.33k
For stability, 1.33 k < 1

$k < \frac{1}{1.33}$ ∴largest value of k = 0.75

$k < \frac{1}{1.33}$ ∴ largest value of k = 0.75

∴ Largest value of k = 0.75
K<0.75

**Sol.25. (d)**

$$G(j\omega)H(j\omega) = \frac{k}{j\omega(1+j\omega)(2+j\omega)}$$

$$G(j\omega)H(j\omega) = \frac{k}{\omega\sqrt{1+\omega^2}.\sqrt{4+\omega^2}}$$

$|G(j\omega)H(j\omega)| = -90^o - \tan^{-1}\omega - \tan^{-1}\omega/2$

$-180^o = -90^o - \tan^{-1}\omega - \tan^{-1}\omega/2$

$\tan^{-1}\omega + \tan^{-1}\omega/2 = 90^o$

$$\frac{\omega\frac{\omega}{2}}{1-\frac{\omega^2}{2}} = \infty = \frac{1}{0}$$

$$1-\frac{\omega^2}{2} = 0$$

$\omega_{pc} = \sqrt{2}$

$\omega = \sqrt{2}\text{rad/sec}$

$$\left|G(j\omega)H(j\omega)\right|_{ope} = \frac{k}{\sqrt{2}\sqrt{3}\sqrt{6}} = \frac{k}{6}$$

$$A = \frac{k}{6}$$

$$(G.M.)_{dB} = 20\log\frac{1}{a}$$

$$3 = 20\log\frac{6}{k}$$

$$\frac{6}{k} = 1.41$$

$$k = \frac{6}{1.41} = 4.25$$

**Sol.26. (c)**

$$G(s) = \frac{k}{s(1+s)(2+s)}$$

$$= \frac{1}{s(1+s)(2+s)} = \frac{k}{2s(1+s)(s+T_2)}$$

$T_1$ = 1 sec

$T_2$ = 0.5 sec

$$\omega_{pc} = \frac{1}{\sqrt{T_1T_2}} = \frac{1}{\sqrt{1\times0.5}} = 1.72\,\text{rad/sec}$$

= 1.42 rad/sec

$$a = \frac{K}{2}\left(\frac{T_1T_2}{T_1+T_2}\right) = \frac{k}{2}\left(\frac{1\times0.5}{1.5}\right) = 0.167k$$

k= 1

a = 0.167

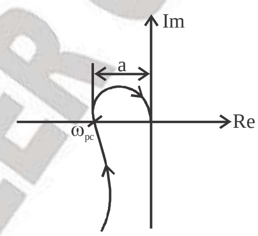
$$(G.M)_{db} = 20\log\frac{1}{a} = 201\log\frac{1}{0.167} = 15.5\text{db}$$

**Sol.27. (a)**

$$G(s) = \frac{k}{20s\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)}$$

$$\frac{1}{s(i+sT_1)(1+sT_2)}$$

It is type-1 and order-3 system so the Nyquist plot is




$$T_1 = \frac{1}{2} = 0.5\text{sec}$$

$$T_2 = \frac{1}{10} = 0.1\text{sec}$$

$$a = \frac{kT_1T_2}{2(T_1+T_2)} = \frac{k\times0.5\times0.1}{20\times0.6} = \frac{k}{240}$$

For stability: $\frac{k}{240} < 1$

k < 240

**Sol.28. (d)**

The corner frequencies are:

$$\omega_1 = \frac{1}{0.5} = 2\text{rad/sec}$$

$$\omega_2 = \frac{1}{0.08} = 12.5\text{rad/sec}$$

**Sol.29. (*)**

(a) In option (a) Bode plot represents-Minimum phase transfer function

(b) In option (a) Bode plot represents-Minimum phase transfer function

(c) In option (b) Bode plot represents-Non-minimum phase transfer function
(d) In option (c) Bode plot represents-All pass transfer function

**Sol.30. (c)**
A $\left|G\left(j\omega\right)\right|=1$ the phase angle,
$\phi = -150^o$
Phase margin = $180^o + \phi = 180^o - 150^o = 30^o$

And at $\underline{|G(j\omega)}$ = $-180^o$ the gain X = 0.5
$(G.M.)_{dB} = 20\log\frac{1}{X} = -20\log X$
$= -20 \log 0.5 \approx 6dB$

**Sol.31. ()**
In polar plot if the critical point (–1 + j0) is not enclosed then the system is said to be stable. A point is said to be enclosed by contour if lie to the right side of the direction of the contour.

**Sol.32. (a)**

**Sol.33.**
P.M. = $180^o - 40^o = 140^o$
G.M. = $\frac{1}{0.75} = \frac{4}{3}$

**Sol. 34. (b)**
$\omega_n = \sqrt{5}\text{rad/s}$
$\Rightarrow$ System response is underdamped.

**Sol. 35. (d)**
Phase margin = 90° - 30° = 60°
Gain margin = 1/0.75

**Sol. 36. (a)**
Phase margin = 180° - 30° = 150°
Gain margin = $\frac{1}{0.25} = 4$

**Sol. 37. (a)**
$\angle G(j\omega) = -90° - \tan^{-1}\omega - \tan^{-1}2\omega = -180°$
$$\Rightarrow \tan^{-1}\left(\frac{\omega - 2\omega}{1-2\omega^2}\right) = 90°$$
$\Rightarrow 1 - 2\omega^2 = 0$
$$\left|G(j\omega)_{\omega}\right| = \frac{1}{\sqrt{2}} = \frac{2}{\frac{1}{\sqrt{2}}\sqrt{\frac{1}{2}+1}\sqrt{\frac{4}{2}+1}}$$
$$\frac{2\sqrt{2}}{\frac{\sqrt{3}}{\sqrt{2}}.\frac{\sqrt{6}}{\sqrt{2}}} = \frac{4}{3}$$
$$\text{Gain margin} = \frac{1}{|G(j\omega)|}\Bigg|_{\omega=\omega_{pe}} = \frac{3}{4}$$

**Sol. 38. (b)**
GM = 20 log (1/a)
For a < 1’ GM = 0 dB
For a = 1, GM = 0 dB
For a > 1, GM < 0 dB

**Sol. 39. (d)**
GM = $\frac{1}{0.4} = 2.5$

**Sol. 40. (b)**
Phase margin = 180° + LG(jω) H(jω)
Where <G(jω)H(jω) = -90° for – 20 db/decade slope
$\therefore$ Pm = 180° - 90° = 90°

**Sol. 41. (a)**

**Sol. 42. (d)**

**Sol. 43. (b)**
$$G(s)H(s) = \frac{1}{s(s+1)(s+0.5)}$$
$$\left|G\left(j\omega\right)H\left(j\omega\right)\right| = \frac{1}{j^3\omega^3 + 1.5j^2\omega^2 + 0.5j\omega}$$
$$= \frac{1}{0.5j\omega - 1.5\omega^2 - jw^3} \quad \frac{1}{-1.5\omega^2 + j\left[0.5\omega - \omega^3\right]} =$$
$$\frac{1}{-1.5\omega^2 + j\left(0.5\omega - \omega^3\right)} \times \frac{-1.5w^2 - j\left(0.5\omega - \omega^3\right)}{-1.5\omega^2 - j\left(0.5\omega - \omega^3\right)}$$
Put imagninary part equal to 0

i.e. $0.5\omega-\omega^3 = 0$

or $0.5 = \omega^2$ or $\omega = \sqrt{0.5}$

$\omega = 0.707$ rad/sec

This is the phase cross over frequency

**Sol. 44. (c)**

$PM = 180^0 - 125^0 = 55^0$

**Sol. 45. (a)**

**Sol. 46. (b)**

**Sol. 47. (a)**

**Sol. 48. (a)**

$$G(s) = \frac{4\left(1+\frac{s}{2}\right)}{s\left(1+\frac{s}{10}\right)}$$

Calculation for K:

6 = 20 log K – 20 log 2

So K = 4

**Sol. 49. (a)**

**Sol. 50. (a)**

**Sol. 51. (c)**

**Sol. 52. (d)**

**Sol. 53. (c)**

**Sol. 54. (a)**

**Sol. 55. (b)**

**Sol. 56. (d)**

Nyquist plot shown corresponds to a function of the type of

$$G(s)H(s) = \frac{K}{s(1+sT_1)(1+sT_2)}$$

The bode plot for such a function is of type shown at (d)

**Sol. 57. (d)**

Gain crossover frequency where gain is 1 is$\omega_g$

1 is $\omega_g$ $\left|\frac{2\sqrt{3}}{j\omega(1+j\omega)}\right| = 1$

$$\frac{2\sqrt{3}}{\omega\sqrt{1+j\omega^2}} = 1$$

$\Rightarrow 2\sqrt{3} = \omega\sqrt{1+\omega^2}$

$\Rightarrow \omega = \sqrt{2}$

$\angle G(j\omega)H(j\omega) = -90° - \tan^{-1}\omega$

$= -90° - \tan^{-1}\sqrt{3}$

$\therefore$ PM = 180° - 150° = - 150°

$\therefore$ PM = 180° -150° = 30°

**Sol. 58. (d)**

$\omega_g$ where$|G(s)H(s)| = 1$

$$\frac{1-s}{(1+s)(2+s)} = \frac{\sqrt{1+\omega^2}}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}} = 1$$

$\sqrt{4+\omega^2} = 1$

$\Rightarrow 4 + \omega^2 = 1$

$\Rightarrow \omega^2 = -3$ (imaginary)

So n o gain crossover frequency

$\therefore$ PM = $\infty$.

**Sol 59. (a)**

**Sol. 60. (c)**

**Sol. 61. (c)**

$PM = 180^0 + \tan^{-1}(a\omega) - 180^0$

$45^0 = \tan^{-1}(a\omega)$

$1 = a\omega$

W= (1/a) rad/sec

$M = \frac{\sqrt{a^2W^2+1}}{W^2} = 1 \quad \Rightarrow a = 0.841$

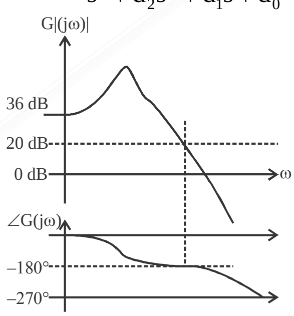
# GATE QUESTIONS

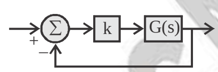
**1.** For a unity feedback control system with the forward path transfer function $G(s)=\dfrac{K}{s(s+2)}$. The peak resonant magnitude $M_r$ of the closed-loop frequency response is 2. The corresponding value of the gain K (correct to two decimal places) is _______

**[GATE - 2018]**

**2.** The figure below shows the Bode magnitude and phase plots of a stable transfer function

$$G(s)=\frac{n_0}{s^3+d_2s^2+d_1s+d_0}$$



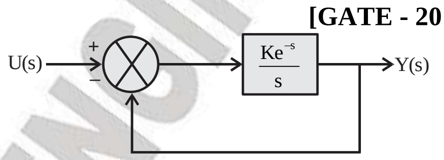




Consider the negative unity feedback configuration with gain k in the feedforward path. The closed loop is stable for $K < k_0$. The maximum value of $k_0$ is ________.

**[GATE - 2018]**

**3.** Consider the unity feedback control system shown. The value of K that results in a phase margin of the system to be 30° is _______. (Give the answer up to two decimal places).

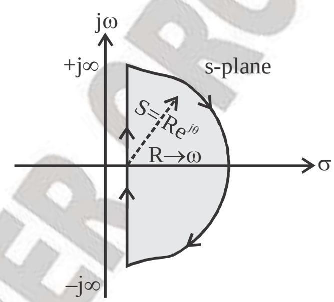
**[GATE - 2017]**



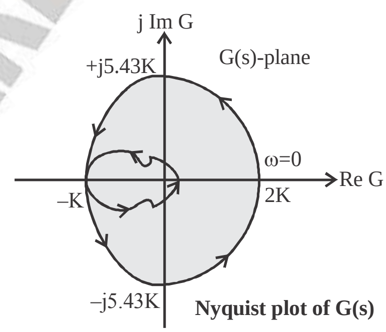

**4.** A unity feedback control system is characteriszed by the open loop transfer function

$$G(s)=\frac{10K(s+2)}{s^3+3s^2+10}$$

The Nyquist path and the corresponding Nyquist plot of G(s) are shown in the figures below.




**Nyquist plot of G(s)**




**Nyquist plot of G(s)**

If 0 < K < 1, then number of poles of the closed loop transfer function that lie in the right half of the s-plane is

**[GATE - 2017]**

(a) 0 (b) 1
(c) 2 (d) 3

**5.** The Nyquist plot of the transfer function

$$G(S)=\frac{K}{(s^2+2s+2)(s+2)}$$

Does not encircle the point (–1+j0) for K = 10 but does encircle the point (–1 + j0) for

K = 100. Then the closed loop system (having unity gain feedback) is

**[GATE - 2017]**

(a) Stable for K = 10 and stable for k = 100
(b) Stable for K = 10 and unstable for K = 100
(c) Unstable for K = 10 and stable for K = 100
(d) Unstable for K = 10 and unstable for K = 100

**6.** For a first order low pass filter with unity d.c. gain and –3dB corner frequency of 2000π rad/s, the transfer function H(jω) is

**[GATE - 2017]**

(a) $\frac{1}{1000 + j\omega}$ (b) $\frac{1}{1 + j1000\omega}$

(c) $\frac{2000\pi}{2000\pi + j\omega}$ (d) $\frac{1000\pi}{1 + j1000\omega}$

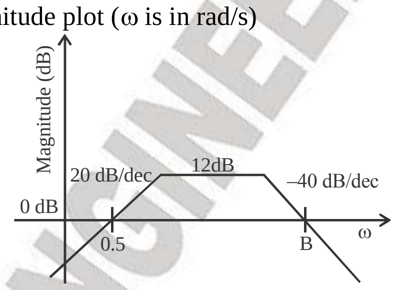
**7.** Loop transfer function of a feedback system is $G(s)H(s) = \frac{s+3}{s^2(s-3)}$.

Take the Nyquist contour in the clockwise direction. Then the Nyquist plot of G(s) encircles -1 + j0

**[GATE - 2016]**

(a) Once in clockwise direction
(b) Twice in clockwise direction
(c) Once in anti clockwise direction
(d) Twice in anti clockwise direction

**8.** Consider the following asymptotic Bode magnitude plot (ω is in rad/s)




**[GATE - 2016]**

(a) $\frac{2s}{(1+0.5s)(1+0.25s)^2}$

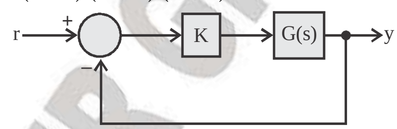
(b) $\frac{4(1+0.5s)}{s(1+0.25s)}$

(c) $\frac{2s}{(1+2s)(1+4s)}$

(d) $\frac{4s}{(1+2s)(1+4s)^2}$

**9.** In the feedback system shown below

$G(s) = \frac{1}{(s+1)(s+2)(s+3)}$



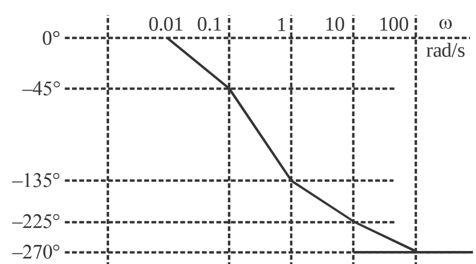

The positive value of k for which the gain margin of the loop is exactly 0 dB and the phase margin of the loop is exactly zero degree is ________.

**[GATE - 2016]**

**10.** The asymptotic Bode phase plot of

$G(S) = \frac{k}{(s+0.1)(s+10)(s+p_1)}$

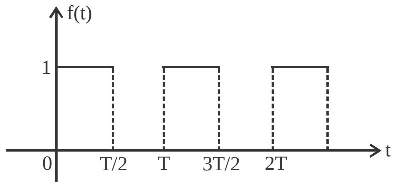
with k and $p_1$ both positive, is shown below.




The value of $p_1$ is ______.

**[GATE - 2016]**

**11.** The Laplace transform of the causal periodic square wave of period T shown in the figure below is

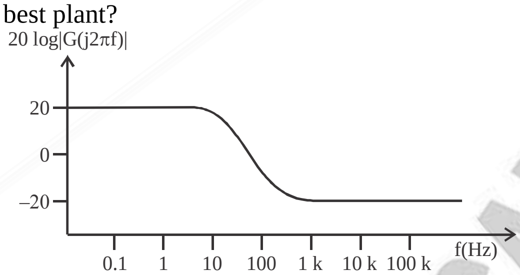



**[GATE - 2016]**

(a) $F(s)=\dfrac{1}{1+e^{-sT/2}}$  (b) $F(s)=\dfrac{1}{s(1+e^{sT/2})}$

(c) $F(s) = \dfrac{1}{s(1+e^{sT/2})}$  (d) $F(s)= \dfrac{1}{1-e^{-sT}}$

**12.** A Bode magnitude plot for the transfer function G(s) of a plant is shown in the figure. Which one of the following transfer functions best plant?

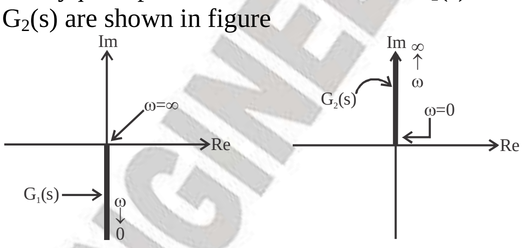



**[GATE - 2015]**

(a) $\dfrac{1000(s+10)}{s+1000}$  (b) $\dfrac{10(s+10)}{s(s+1000)}$

(c) $\dfrac{s+1000}{10s(s+10)}$  (d) $\dfrac{s+1000}{10(s+10)}$

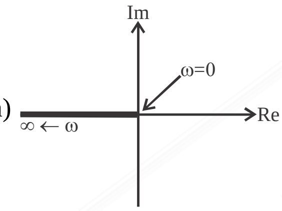
**13.** Nyquist plots of two functions $G_1(s)$ and $G_2(s)$ are shown in figure



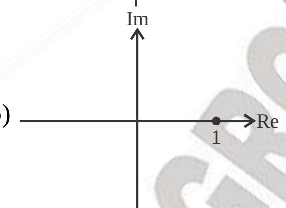

Nyquist plot of the product of $G_1(s)$ and $G_2(s)$ is

**[GATE - 2015]**

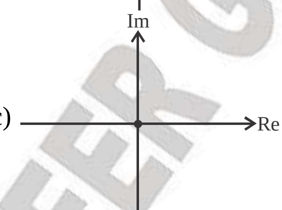
(a)



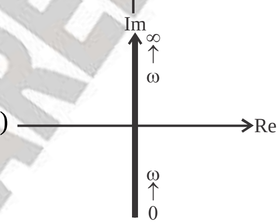

(b)




(c)




(d)




**14.** The polar plot of the transfer function $G(s)=\dfrac{10}{s(s+10)}$ is..... will be in the

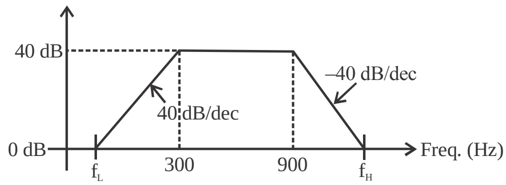
**[GATE - 2015]**

(a) First quadrant  (b) Second quadrant
(c) Third quadrant  (d) Fourth quadrant

**15.** The phase margin (in degrees) of the system $G(s)=\dfrac{10}{s(s+10)}$ is ___

**[GATE - 2015]**

**16.** Consider the Bode plot shown in the figure. Assume that the poles and zeros are real-valued.




The value of $f_H - f_L$ (in Hz) is ____

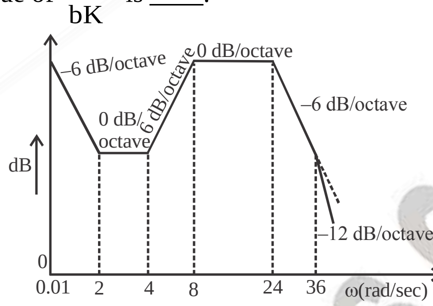
**[GATE - 2015]**

**17.** The Bode magnitude plot of the transfer function

$$G(s) = \frac{K(1+0.5s)(1+as)}{s\left(1+\frac{s}{8}\right)(1+bs)\left(1+\frac{s}{36}\right)}$$

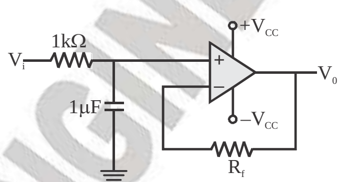
is shown below :

Note that –6dB/octave = –20dB/decade. The value of $\frac{a}{bK}$ is ____.



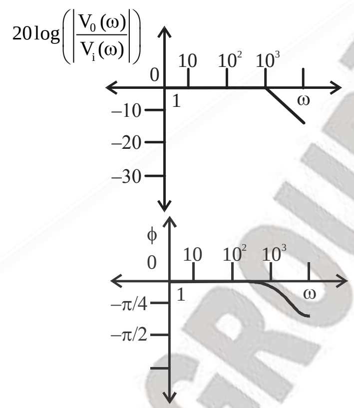

**[GATE - 2014]**

**18.** In the figure shown, assume the op-amp to be ideal, which of the alternatives gives the correct Bode plot for the transfer function $\frac{v_1\omega}{v_i\omega}$?

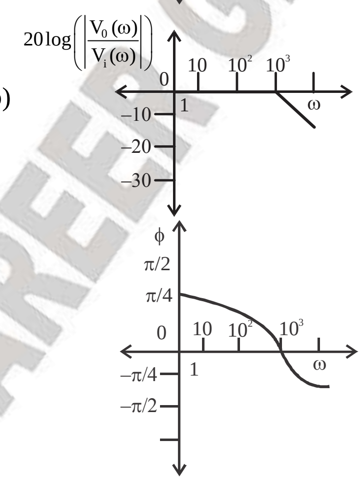



**[GATE - 2014]**

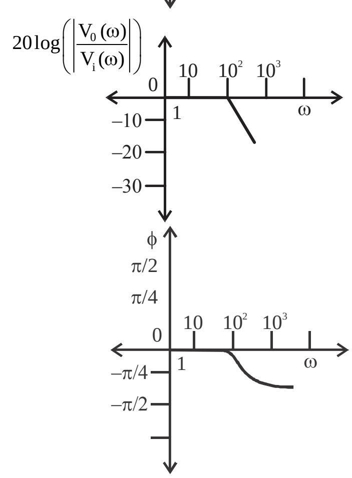
(a)



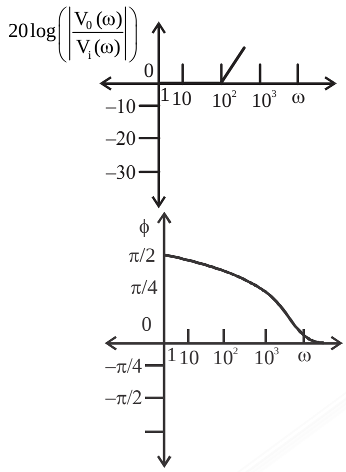

(b)




(c)



(d)

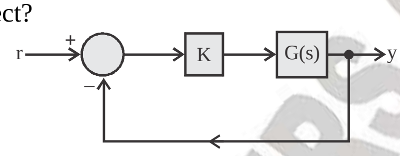


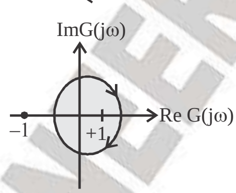

**19.** In a bode magnitude plot, which one of the following slopes would be exhibited at high frequencies by a $4^{th}$ order all – pole system?

**[GATE - 2014]**

(a) –80 dB/decade (b) –40 dB/decade
(c) +40dB/decade (d) +80 dB decade

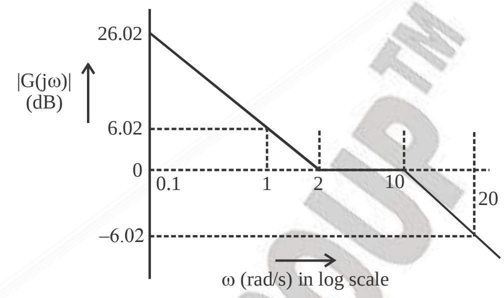
**20.** Consider the feedback system shown in the figure. The Nyquist plot of G(s) is also shown. Which one of the following conclusions is correct?




**[GATE - 2014]**

**21.** The Bode asymptotic magnitude plot of a minimum phase system is shown in the figure.




If the system is connected in a unity negative feedback configuration, the steady state error of the closed loop system, to a unit ramp input, is _______.

**[GATE - 2014]**

**22.** For the transfer function

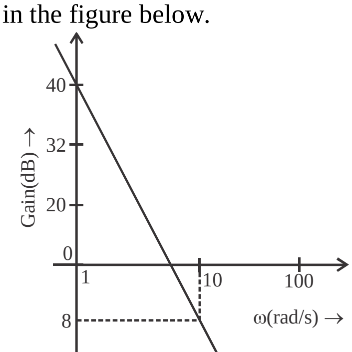
$$G(s) = \frac{5(s+4)}{s(s+0.25)(s^2+4s+2.5)}$$

The values of the constant gain term and the highest corner frequency of the Bode plot respectively are

**[GATE - 2014]**

(a) 3.2, 5.0 (b) 16.0, 4.0
(c) 3.2, 4.0 (d) 16.0, 5.0

**23.** The Bode plot of a transfer function G(s) is shown in the figure below.




The gain (20 log |G(s)|) is 32 dB and –8dB at 1 rad/s and 10 rad/s respectively. The phase is negative for all ω. Then G(s) is

**[GATE - 2013]**

(a) $\frac{39.8}{s}$ (b) $\frac{39.8}{s^2}$

(c) $\frac{32}{s}$ (d) $\frac{32}{s^2}$

**24.** The frequency response of a linear system G(jω) is provided in the tubular form below

| \|G(j ω)\| | ∠G(jω) |
|---|---|
| 1.3 | $-130^o$ |
| 1.2 | $-140^o$ |
| 1.0 | $-150^o$ |
| 0.8 | $-160^o$ |
| 0.5 | $-180^o$ |
| 0.3 | $-200^o$ |

**[GATE - 2011]**

(a) 6dB and $30^o$ (b) 6 dB and $-30^o$
(c) –6dB and $30^o$ (d) –6dB and $-30^o$

**25.** An open loop system represented by the transfer function

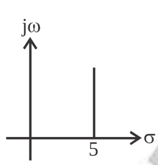
$$G(s) = \frac{(s-1)}{(s+2)(s+3)} \text{ is}$$

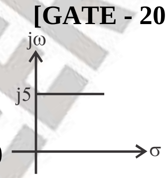
**[GATE - 2011]**

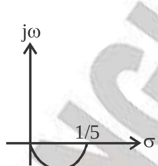
(a) Stable and of the minimum phase type
(b) Stable and of the non – minimum phase type
(c) Unstable and of the minimum phase type
(d) Unstable and of non – minimum phase type

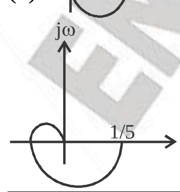
**26.** For the transfer function (jω) = 5 + jω, the corresponding Nyquist plot for positive frequency has the form

**[GATE - 2011]**

(a) 


(b) 

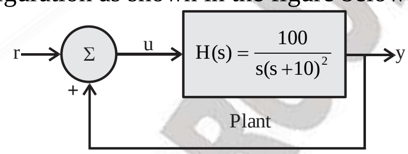

(c) 


(d) 


**Common Data for Q. 25 and Q. 26**

The input-output transfer function of a plant $H(s) = \frac{100}{s(s+10)^2}$

The plant is placed in a unity negative feedback configuration as shown in the figure below.



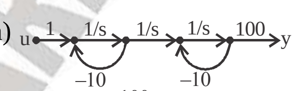

**27.** The gain margin of the system under closed loop unity negative feedback is

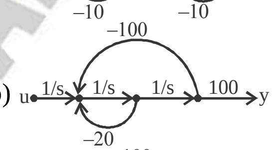
**[GATE - 2011]**

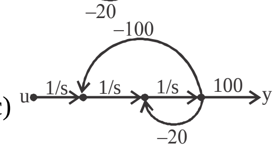
(a) 0 dB (b) 20 dB
(c) 26 dB (d) 46 dB

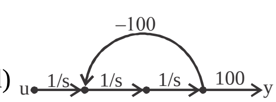
**28.** The signal flow graph that DOES NOT model the plant transfer function H(s) is

**[GATE - 2011]**

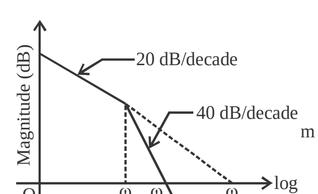
(a) 


(b) 


(c) 


(d) 


**29.** For the asymptotic Bode magnitude plot shown below, the system transfer function can be



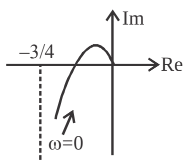

**[GATE - 2010]**

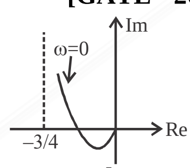
(a) $\frac{10s+1}{0.1s+1}$ (b) $\frac{100s+1}{0.1s+1}$

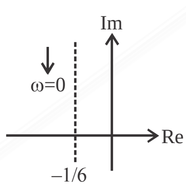
(c) $\frac{100s}{10s+1}$ (d) $\frac{0.1s+1}{10s+1}$

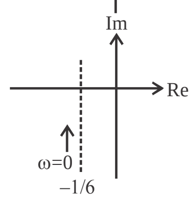
**30.** The frequency response of G(s) = $\frac{1}{s(s+1)(s+2)}$ plotted in the complex G(jω) plane(for $0 < \omega < \infty$) is

**[GATE - 2010]**

(a)


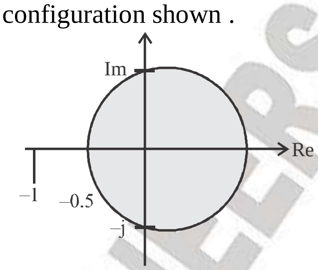

(b)



(c)



(d)



**Common Data for Q. 31 and Q. 32**

The Nyquist plot of a stable transfer function G(s) is shown in the figure are interested in the stability of the closed loop system in the feedback configuration shown .





**31.** Which of the following statements is true?

**[GATE - 2009]**

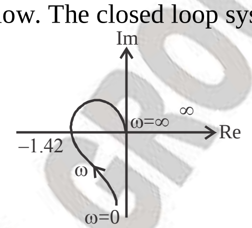
(a) G(s) is an all – pass filter
(b) G(s) has a zero in the right – half plane
(c) G(s) is the impedance of a passive network
(d) G(s) is marginally stable

**32.** The gain and phase margins of G(s) for closed loop stability are

**[GATE - 2009]**

(a) 6 dB and $180^o$ (b) 3 dB and $180^o$
(c) 6 dB and $90^o$ (d) 3 dB and $90^o$

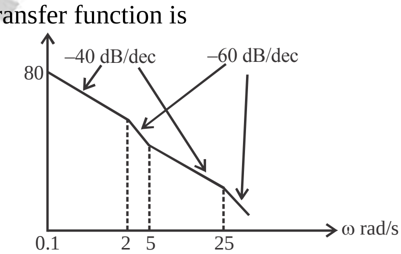
**33.** The polar plot of an open loop stable system is shown below. The closed loop system is




**[GATE - 2009]**

(a) Always stable
(b) Marginally stable
(c) Unstable with one pole on the RH s-plane
(d) unstable with two poles on the RH s-plane

**34.** The asymptotic approximation of the log – magnitude v/s frequency plot of a system containing only real poles and zeros is shown. Its transfer function is




**[GATE - 2009]**

(a) $\frac{10(s+5)}{s(s+2)(s+25)}$ (b) $\frac{1000(s+5)}{s^2(s+2)(s+25)}$

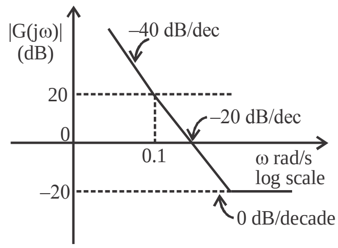
(c) $\frac{100(s+5)}{s(s+2)(s+25)}$ (d) $\frac{80(s+5)}{s^2(s+2)(s+25)}$

**35.** The open loop transfer function of a unity feedback system is given by G(s) = $(e^{-0.1s})/s$. The gain margin of the is system is

**[GATE - 2009]**

(a) 11.95 dB (b) 17.67 dB
(c) 21.33 dB (d) 23.9 dB

**36.** The asymptotic Bode magnitude plot of a minimum phase transfer function is shown in the figure.

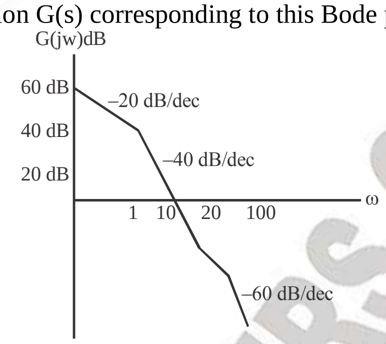



**[GATE - 2008]**

This transfer function has

(a) Three poles and one zero
(b) Two poles and one zero
(c) Two poles and two zero
(d) one pole and two zeros

**37.** The asymptotic Bode plot of a transfer function is as shown in the figure. The transfer function G(s) corresponding to this Bode plot is



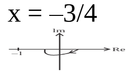

**[GATE - 2007]**

(a) $\frac{1}{(s+1)(s+20)}$ (b) $\frac{1}{s(s+1)(s+20)}$

(c) $\frac{100}{s(s+1)(s+20)}$ (d) $\frac{100}{s(s+1)(1+0.05s)}$

**38.** If x = Re[G(jω)], and y=Im[G(jω)] then for $\omega \to 0^+$, the Nyquist plot for G(s) = 1/s(s+1)(s+2) is

**[GATE - 2007]**

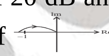
(a) x = 0 (b) x = –3/4
(c) x = y – 1/6 (d) 

**39.** The system 900/s(s+1)(s+9) is to be such that its gain-crossover frequency becomes same as its uncompensated phase crossover frequency and provides a 45° phase margin. To achieve this, one may use

**[GATE - 2007]**

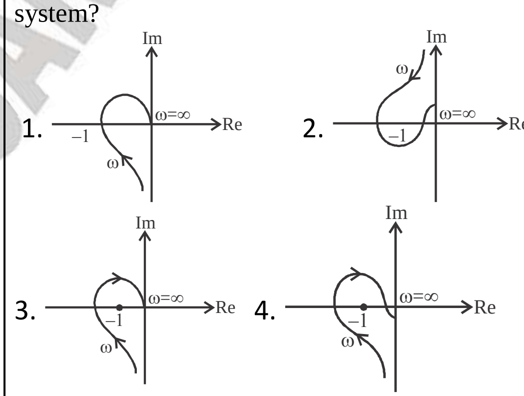
(a) A lag compensator that provides and attenuation of 20 dB and a phase lag of 45° at the frequency of $3\sqrt{3}$ rad/s

(b) A lead compensator that provides and amplification of 20 dB and a phase lead of 45° at the frequency of 3 rad/s

(c) A lag – lead compensator that provides an amplification of 20 dB and a phase lag of 45° at the frequency of 

(d) A lag – lead compensator that provides an attenuation of 20 dB and phase lead of 45° at the frequency of 3 rad/s

**40.** Consider the following Nyquist plots of loop transfer functions over ω = 0 to ω = ∞. Which of these plots represent a stable closed loop system?



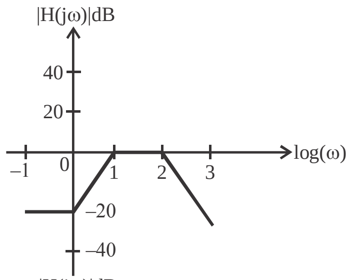

**[GATE - 2006]**

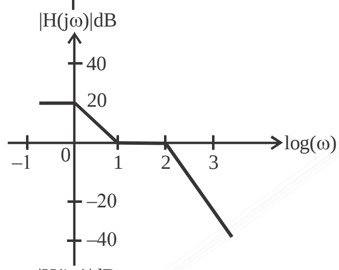
(a) 1 only (b) All, except 1
(c) All, except 3 (d) 1 and 2 only

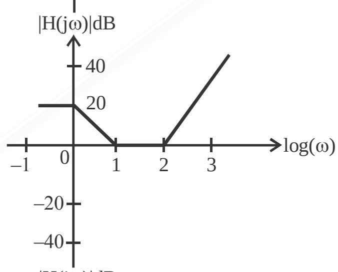
**41.** The Bode magnitude plot

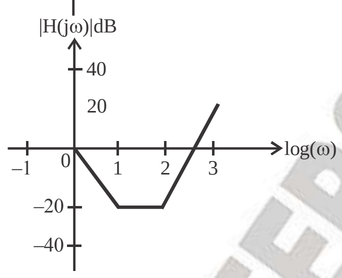
$$H(j\omega) = \frac{10^4(1+j\omega)}{(10+j\omega)(100+j\omega)^2} \text{ is}$$

**[GATE - 2006]**

(a)



(b)



(c)



(d)



**42.** The open loop function of a unity – gain feedback control system is given by

$$G(s) = \frac{K}{(s+1)(s+2)}$$

The gain margin of the system is dB is given by

**[GATE - 2006]**

(a) 0 (b) 1

(c) 20 (d) ∞

**43.** The Nyquist plot of G(jω)H(jω) for a closed loop control system, passes through (–1, j0) point in the GH plane. The gain margin of the system in dB is equal to

**[GATE - 2006]**

(a) Infinite (b) Greater than zero

(c) Less than zero (d) Zero

**Common data for Q. 44 and Q. 45**

Consider a unity – gain feedback control system whose open – loop transfer function is: $G(s) = \frac{as+1}{s^2}$

**44.** The value of a so that the system has a phase-margin equal to π/4 is approximately equal to

**[GATE - 2006]**

(a) 2.40 (b) 1.40

(c) 0.84 (d) 0.74

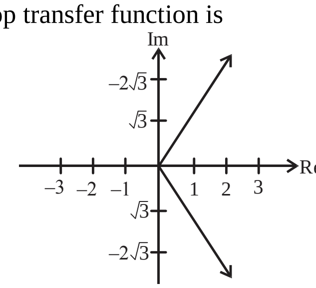
**45.** With the value of a set for a phase – margin of π/4, the value of unity- impulse response of the open – loop system at t = 1 second is equal to

**[GATE - 2006]**

(a) 3.40 (b) 2.40

(c) 1.84 (d) 1.74

**46.** Figure shows the root locus plot (location of poles not given) of a third order system whose open loop transfer function is




**[GATE - 2005]**

(a) $\frac{K}{s^3}$ (b) $\frac{K}{s^2(s+1)}$

(c) $\frac{K}{s(s^2+1)}$ (d) $\frac{K}{s(s^2-1)}$

**47.** The gain margin of a unity feed back control system with the open loop transfer function $G(s) = \frac{(s+1)}{s^2}$ is

**[GATE - 2005]**

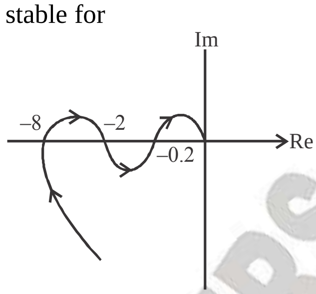
(a) 0 (b) $\frac{1}{\sqrt{2}}$
(c) $\sqrt{2}$ (d) $\infty$

**48.** In the G(s) H(s) – plane the Nyquist plot of the loop transfer function G(s)H(s)= $= \frac{\pi e^{-0.25r}}{s}$ passes through the negative real axis at the point

**[GATE - 2005]**

(a)(–0.25, j0) (b) (–0.5 j0)
(c) 0 (d) 0.5

**49.** The polar diagram of a conditionally stable system for open loop gain K = 1 is shown in the figure. The open loop transfer function of the system is known to be stable. The closed loop system is stable for




**[GATE - 2005]**

(a) K<5 and $\frac{1}{2} < K < \frac{1}{8}$

(b) $K < \frac{1}{8}$ and $\frac{1}{2} < K < 5$

(c) $K < \frac{1}{8}$ and $5 < K$

(d) $K > \frac{1}{8}$ and $5 > K$

**Common data for Q. 50 and Q. 51**
The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{3e^{-2s}}{s(s+2)}$$

**50.** The gain and phase crossover frequencies in rad/sec are, respectively

**[GATE - 2005]**

(a) 0.632 and 1.26 (b) 0.632 and 0.485
(c) 0.485 and 0.632 (d) 1.26 and 0.632

**51.** Based on the above results, the gain and phase margins of the system will be

**[GATE - 2005]**

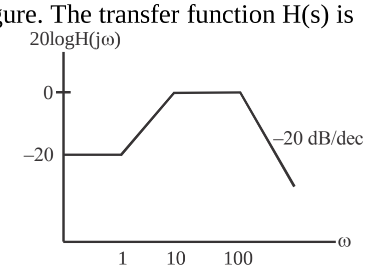
(a) –7.09 dB and 87.5°
(b) 7.09 dB and 87.5°
(c) 7.09 db and –87.5°
(d) –7.09 and –87.5°

**52.** The gain margin for the system with open-loop transfer function $G(s)H(s) = \frac{2(1+s)}{s^2}$ is

**[GATE - 2004]**

(a) $\infty$ (b) 0
(c) 1 (d) $-\infty$

**53.** Consider the Bode magnitude plot shown in the figure. The transfer function H(s) is




**[GATE - 2004]**

(a) $\frac{(s+10)}{(s+1)(s+100)}$ (b) $\frac{10(s+1)}{(s+10)(s+100)}$

(c) $\frac{10^2(s+1)}{(s+10)(s+100)}$ (d) $\frac{10^3(s+100)}{(s+1)(s+10)}$

**54.** A system has poles at 0.1 Hz, 1 Hz and 80 Hz; zeros at 5Hz, 100 Hz and 200 Hz . The approximate phase of the system response at 20 Hz is

**[GATE - 2004]**

(a) $-90^o$ (b) $0^o$
(c) $90^o$ (d) $-180^o$

**55.** The Nyquist plot of loop transfer function G(s)H(s) of a closed loop control system passes through the point (–1 j 0) in the G(s) H(s) plane. The phase margin of the system is

**[GATE - 2004]**

(a) $0^o$ (b) $45^o$
(c) $90^o$ (d) $180^o$

**56.** The open loop transfer function of a unity feedback control system is given as

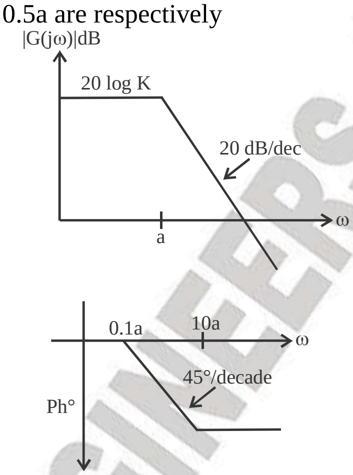
$$G(s) = \frac{as+1}{s^2}$$

The value of 'a' to give a phase margin of $45^o$ is equal to

**[GATE - 2004]**

(a) 0.141 (b) 0.441
(c) 0.841 (d) 1.141

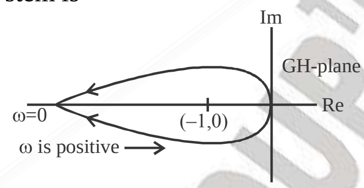
**57.** The asymptotic Bode plot of the transfer function K/[1 + (s/a)] is given in figure. The error in phase angle and dB gain at a frequency of ω = 0.5a are respectively




**[GATE - 2003]**

(a) $4.9^o$, 0.97 dB (b) $5.7^o$, 3dB
(c) $4.9^o$, 3dB (d) $5.7^o$, 0.97 dB

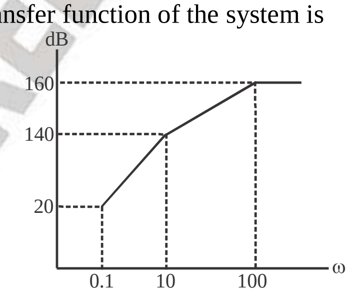
**58.** Fig shows the Nyquist plot of the open – loop transfer function G(s) H(s) of a system. If G(s) H(s) has one right – hand pole, the closed – loop system is




**[GATE - 2003]**

(a)Always stable
(b)Unstable with one closed-loop right hand pole
(c)Unstable with two closed-loop right hand pole
(d)Unstable with three closed- loop right hand poles

**59.** The approximate Bode magnitude plot of a minimum phase system is shown in fig. below. The transfer function of the system is




**[GATE - 2003]**

(a) $10^8 \dfrac{(s+0.1)^3}{(s+10^2)(s+100)}$

(b) $10^7 \dfrac{(s+0.1)^3}{(s+10)(s+100)}$

(c) $\dfrac{(s+0.1)^2}{(s+10)^2+(s+100)}$

(d) $\dfrac{(s+0.1)^3}{(s+10)(s+100)^2}$

**60.** The gain margin and the phase margin of feedback system with $\dfrac{8}{(s+100)^3}$ are

**[GATE - 2003]**

(a) dB, $0^o$ (b) $\infty$, $\infty$
(c) $\infty$, $0^o$ (d) 88.5 dB, $\infty$

**61.** The phase margin of a system with the open = loop transfer function

$$G(s)H(s) = \frac{(1-s)}{(1+s)(2+s)}$$

**[GATE - 2002]**

(a) $0^o$ (b) $63.4^o$
(c) $90^o$ (d) $\infty$

**62.** T he system with the open loop transfer function $G(s)H(s) = \frac{1}{s(s^2+s+1)}$ has a gain margin of

**[GATE - 2002]**

(a) –6 db (b) 0 db
(c) 35 db (d) 6 db

**63.** The Nyquist plot for the open – loop transfer function G(s) of a unity negative feedback system is shown in the figure, if G(s) has no pole in the right – half of s-plane, the number of roots of the system characteristic equation in the right – half of s-plane is

**[GATE - 2001]**

(a) 0 (b) 1
(c) 2 (d) 3

# SOLUTIONS

**Sol.1. (14.928)**

Maximum resonant peak,

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 2$$

$$2\xi\sqrt{1-\xi^2} = \frac{1}{2}$$

$$\xi^4 - \xi^2 + \frac{1}{16} = 0$$

$$\xi^2 = \frac{1}{2} \pm \sqrt{\frac{1-\frac{1}{4}}{4}} = \frac{1}{2} \pm \frac{\sqrt{3}}{4}$$

As Mr = 2 > 1, $\xi < \frac{1}{\sqrt{2}}$ and $\xi^2 < \frac{1}{2}$

So, $\xi^2 = \frac{1}{2} - \frac{\sqrt{3}}{4}$

Given, $\omega_n = \sqrt{K}$

$$2\xi\sqrt{K} = 2$$

$$\sqrt{K} = \frac{1}{\xi}$$

$$K = \frac{1}{\xi^2} = \frac{1}{\left(\frac{1}{2} - \frac{\sqrt{3}}{4}\right)}$$

$$= \frac{4}{2-\sqrt{3}} = 14.928$$

**Sol.2. (0.10)**

For G(s), $M_{dB}(\omega_{pc}) = 20$ dB

When cascaded with k,

$GM_{dB} = -20 \text{ dB} - 20\log_{10}(k) > 0 \text{ dB}$

$20 + 20\log_{10}(0) < 0$

$20\log_{10}(0) < -20$

$k < 10^{-1} = 0.10$

So, $k_0 = 0.10$

**Sol.3. (1.05)**

$$PM = 180° + \angle\frac{Ke^{-j\omega_{gc}}}{j\omega_{gc}} = 30°$$

and $\left|\frac{Ke^{-j\omega_{gc}}}{j\omega_{gc}}\right| = 1$

$\Rightarrow \frac{K}{\omega_{gc}} = 1$

$\Rightarrow \omega_{gc} = K$

PM = 180° – $\omega_{gc}$ – 90° = 30°

$\Rightarrow \omega_{gc} = 60° = \frac{\pi}{3} = 1.05$

**Sol.4. (c)**

$$G(s) = \frac{10k(S+2)}{S^3 + 3S^2 + 10}$$

$$G(s) = \frac{10k(S+2)}{(S+3.72)(S-0.36+j1.5)(S-0.36-j1.5)}$$

P = 2 (Two poles in the RHS)

If K < 1, the number of encirclements about (–1, j0) is 0

N = P – Z

N = 2 – 0 = 2

⇒ 2CL poles lies in the RHS – plane.

**Sol.5. (b)**

For given system, P = 0

For stability N = 0

For k = 10, no encirclements about 9–1, j0). Hence the system is stable

For K = 100, encircles the point (–1, j0). Hence the system is unstable.

**Sol.6. (c)**

For –3dB corner frequency

We equate

$$\left.\frac{2000\pi}{\sqrt{(2000\pi)^2 + \omega^2}}\right|_{\omega=2000\pi} = \frac{1}{\sqrt{2}} \quad \ldots\ldots \text{(i)}$$

$$\text{LHS} = \frac{2000\pi}{\sqrt{(2000\pi)^2 + (2000\pi)^2}} = \frac{1}{\sqrt{2}}$$

RHS = LHS

Hence option (c) is the required LPF

**Sol.7. (a)**

$$CE = 1 + \frac{s+3}{s^3 - 3s^2} = 0$$

$s^3 + 3s^2 + s + 3 = 0$

| | | |
|---|---|---|
| $S^3$ | 1 | 1 |
| $S^2$ | −3 | 3 |
| $S^1$ | 2 | |
| $S^0$ | 3 | |

Unstable with two right half of s-plane poles

$\therefore Z = 2, P = 1$

N = P - Z

N = 1 - 2 = -1 once in the cw direction

**Sol.8. (a)**

From the given Bode plot the corner frequencies are 2 rad/sec and 4 rad/sec

$$TF = \frac{Ks}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{4}\right)^2}$$

$20\log K + 20\log\omega = 0$ dB at $\omega = 0.5$

K = 2

$$\therefore\ TF = \frac{2s}{(1+0.5s)(1+0.25s)^2}$$

**Sol.9. (60)**

Given Forward path $TF = \dfrac{1}{(s+1)(s+2)(s+3)}$

Given GM = 0dB, PM = $0^0$ That Means Given System is Marginal Stable

1+KG(s) = 0

$\Rightarrow CE = s^3 + 11s^2 + 6s + 6 + K = 0$

| | | |
|---|---|---|
| $S^3$ | 1 | 6 |
| $S^2$ | 11 | 6+K |
| $S^1$ | $\left(\frac{66-6-K}{11}\right) = 0$ | |
| $S^0$ | (6+K) | |

$\Rightarrow$K = 60 For Marginal Stable

**Sol.10. (1)**

From the Bode Diagram at $\omega = 1$, the phase Angle is – 135°

$$-135°\big|_{\omega=1}$$

$$= -\tan^{-1}\left(\frac{\omega}{0.1}\right) - \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{p_1}\right)$$

$-135° = -84.28 - 5.71 - \tan^{-1}(1/p_1)$

$$45° = \tan^{-1}\left(\frac{1}{p_1}\right) \Rightarrow 1 = \frac{1}{p_1} \Rightarrow p_1 = 1$$

**Sol.11. (b)**

One period of signal $x_1(t) = u(t) - u(t-T/2)$

$$X_1(s) = \frac{1}{s} - \frac{e^{sT/2}}{s} = \frac{1-e^{-sT/2}}{s}$$

$$X(s) = \frac{1}{1-e^{-sT}} X_1(s) = \frac{1-e-sT}{s\left(1-e^{-sT}\right)} = \frac{1}{s\left(1+e^{-sT/2}\right)}$$

**Sol.12. (d)**

20logk=20

k = 10

It has a pole at 10 & zero at 1000 approximately. So $G(s) = \dfrac{s+1000}{10(s+10)}$ is the best describe transfer function;

$$G(s) = \frac{k\left(\frac{s}{1000}+1\right)}{\left(\frac{s}{10}+1\right)} = \frac{10(5+1000)}{100(s+10)}$$

$$= \frac{(s+1000)}{10(s+10)}$$

**Sol.13. (b)**

$G_1(s) = \dfrac{1}{s}$

$G_2(s) = s$

$G_1(s) . G_2(s) = 1$

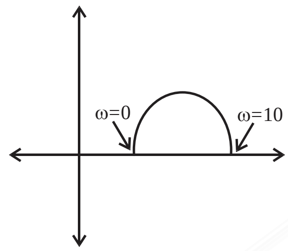
**Sol.14. (a)**

First quadrant

$$G(S) = \frac{10(S+1)}{(S+10)};\ G(j\omega) = \frac{10(j\omega+1)}{(j\omega+10)}$$

$$\frac{10\sqrt{1+\omega^2}}{\sqrt{\omega^2+100}} = \left|G(j\omega)\right|$$

$$\angle G(j\omega) = \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{10}\right)$$




| ω | \|G(jω)\| | ∠G(jω) |
|---|---|---|
| 0 | 1 | 0 |
| ∞ | 10 | 0 |

**Sol.15. (84.29)**

$$G(s) = \frac{10}{s(s+10)} = \frac{1}{s(10^2+1)}$$

$$G(jw).\frac{10}{jw(jw+10)}$$

$$\left|G(j\omega)\right| = \left|\frac{10}{w\sqrt{w^2+100}}\right|_{w=wge} = 1$$

$10 = w^2(w^2 + 100)$

$W^4 + 100w^2 + 100 = 0$

$$w^2 = \frac{-100 \pm \sqrt{(10)^4\,400}}{2}$$

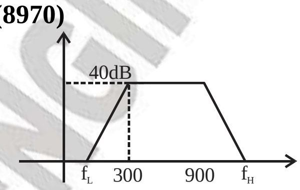
$w = \sqrt{0.9902} = 0.99508$

$\angle G(jw) = -90 - \tan^{-1}(w/10)$

$\phi = -90 - \tan^{-1}(0.1) = -90 - 5.7106$

$PM = 180 + \phi = 84.2894$

**Sol.16. (8970)**




$m = 90db$

$$\frac{40-0}{\log_{10}300 - \log_{10}f_L} = 40$$

$$\frac{0.40}{\log_{10}f_H - \log_{10}900} = -40$$

$$\frac{40}{40} = \log_{10}300 - \log_{10}f_L$$

$$\log_{10}\left(\frac{f_H}{900}\right) = 1$$

$$\log_{10}\left(\frac{300}{f_L}\right) = 1$$

$f_H = 9000$

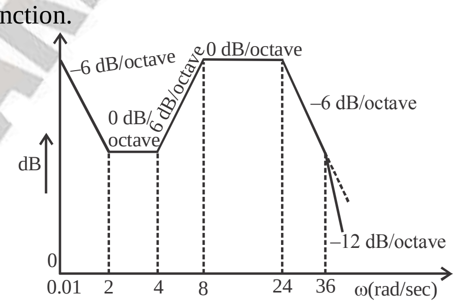
$\log_a x = M$

$$\frac{300}{f_L} = 10 f_L = 30$$

$F_H - f_L = 9000 - 30 = 8970$

**Sol.17. (0.75)**

Given the Bode magnitude plot of the transfer function.




Also from the given transfer function, we have

$$G(s) = \frac{K(1+0.55)(1+as)}{s\left(1+\frac{s}{8}\right)(1+bs)\left(1+\frac{s}{36}\right)}$$

$$= \frac{K(1-s/2)\left(1+\frac{s}{1/a}\right)}{s\left(1+\frac{s}{8}\right)\left(1+\frac{s}{1/b}\right)\left(1+\frac{s}{36}\right)}$$

The first slope – dB/octave is due to one pole that is 1/s

Then, slope 0 dB/octave is due to addition of a zero in T.F. (1 + s/2).

Again, +6dB/octave slope is due to one zero at corner frequency $\omega_C = 4$.

Comparing it to the transfer function, we get
(1 + as) = (1 + s/4)
Or a = ¼
Similarly , at $\omega_c = 24$, there is an addition of a pole (–6dB/octave). So, we get

$(1 + bs) = (1 + s/24)$ or $b = \frac{1}{24}$

From the shown Bode plot, we observe that if we extended the slope –6dB/octave , it meets the frequency axis at $\omega_C = 8$. So we have

$$0 = 20\log\left|\frac{KX}{s}\right|_{\omega c=8}$$

or $1 = \frac{K}{8}$

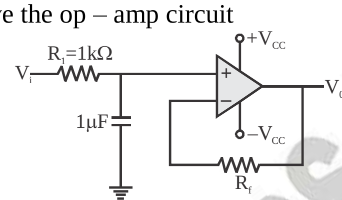
or K = 8

Therefore, we obtain the desired value as

$$\frac{a}{bK} = \frac{1/4}{\frac{1}{24}\times 8} = \frac{24}{4\times 8} = 0.75$$

**Sol.18. (a)**

We have the op – amp circuit




Let the voltage at inverting terminal of op-amp be X. So, we have

$$\frac{X(s)-V_0(s)}{R_f}$$

Or X(s) = $V_0(s)$ ... (i)

Applying KCL at non – inverting terminal of op-amp, we get

$$[X(s)-0]Cs + \frac{X(s)-V_i(s)}{R_1} = 0$$

or $X(s)\left[Cs + \frac{1}{R_1}\right] = \frac{V_1(s)}{R_1}$ [From equation (i)]

So, $\frac{V_0(s)}{V_i(s)} = \frac{1}{1+R_1Cs} = \frac{1}{1+10^{-3}}$

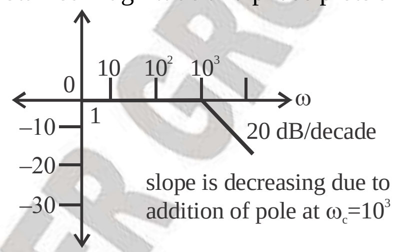
Therefore, the corner frequency for the transfer function is

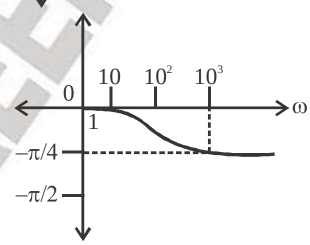
$$\omega_C = \frac{1}{10^{-3}} = 10^3$$

Hence, we draw the Bode plot for the function (in decibel).

$$20\log\left|\frac{V_0(\omega)}{V_i(\omega)}\right| = 20\log\left(\frac{1}{1+\frac{j\omega}{10^3}}\right)$$

The obtained magnitude and phase plots are




**Sol.19. (a)**

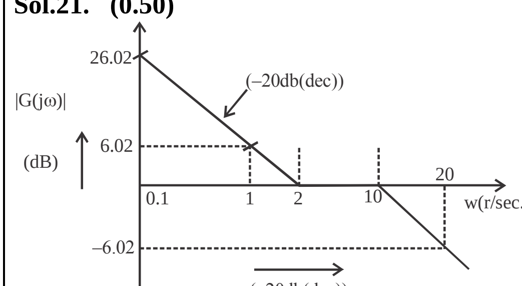
In a BODE diagram ,in plotting the magnitude with respect to frequency, a pole introduce a line 4 slope –20dB/dc

If 4th order all – pole system means gives a slope of (–20)*4dB/dec i.e. –80dB/dec

**Sol.20. (d)**

For larger values of K, it will encircle the critical point (–1 + j0), which makes closed-loop system unstable.

**Sol.21. (0.50)**



→ Due to initial slope, it is a type – 1 system, and it has non zero velocity error coefficient ($K_v$)
→ The magnitude plot is giving 0dB at 2r/sec Which gives $k_v$
$\therefore k_v = 2$

The steady state error $e_{ss} = \frac{A}{k_v}$

Given unit ramp input; A = 1

$$e_{ss} = \frac{1}{2}$$

$e_{ss} = 0.50$

**Sol.22. (a)**

$$G(s) = \frac{5(s+4)}{(s+0.25)(s^2+4s+25)}$$

$$= \frac{5\times 4}{0.25\times 25}\frac{(1+s/4)}{\left(1+\frac{s}{0.25}\right)\left(1+\frac{4s}{25}+\frac{s^2}{25}\right)}$$

$$= 3.2\times\frac{(1+s/4)}{\left(1+\frac{s}{0.25}\right)\left(1+\frac{4s}{25}+\frac{s^2}{25}\right)}$$

So constant gain terms, K = 3.2
Highest corner frequency ωH = 5.

**Sol.23. (b)**

From the given plot, we obtain the slop as

$$\text{Slope} = \frac{20\log G_2 - 20\log G_1}{\log w_2 - \log w_1}$$

From the figure.
$20\log G_2 = -8\text{dB}$
$20\log G_1 = 32$ dB
And $\omega_1 = 1$ rad/s
So the slope is

$$\text{Slope} = \frac{-8-32}{\log_{10}-\log_1} = -40 \text{ dB/decade}$$

Therefore, the transfer function can be given as
At ω = 1

$$|G(j\omega)| = \frac{k}{|w|^2} = k$$

In decibel,

$20\log|G(j\omega)| = 20\log k = 32$

Or, $k = 10^{32/20}$

Hence , the Transfer function is

$$G(s) = \frac{k}{s^2} = \frac{39.8}{s^2}$$

**Sol.24. (a)**

Gain margin is simply equal to the gain at phase cross over frequency ($\omega_P$). Phase cross over frequency is the frequency at which phase angle is equal to $-180^o$.
From the table we can see that $\angle G(j\omega_p) = -180^o$, at which gain is 0.5.

$$GM = 20\log_{10}\left(\frac{1}{|G(j\omega)|}\right)$$

$$= 20\log\left(\frac{1}{0.5}\right) = 6\text{dB}$$

Phase Margin is equal to $180^o$ plus the phase angle $\phi_g$ at the gain cross over frequency ($\omega_g$). Gain cross over frequency is the frequency at which gain is unity.
From the table it is clear that $|G(j\omega_g)| = 1$, at which phase angle is $-150^o$.
$\phi_{PM} = 180^o + \angle G(j\omega_g) = 180 - 150 = 30^o$

**Sol.25. (b)**

Transfer function having at least one zero or pole in RHS of s-plane is called non – minimum phase transfer function

$$G(s) = \frac{s-1}{(s+2)(s+3)}$$

1.In the given transfer function one zero is located at s = 1 (RHS), so this is non – minimum phase system.
2.Poles –2, –3, are in left side of the complex plane, so the system is stable.

**Sol.26. (a)**

We have $G(j\omega) = 5 + j\omega$
Here σ = 5. Thus $G(j\omega)$ is a straight line parallel to jω axis.

**Sol.27. (c)**

WE have $G(s)H(s) = \frac{100}{s(s+10)^2}$

Now $G(j\omega)H(j\omega) = \frac{100}{j\omega(j\omega+10)^2}$

If $\omega_p$ is phase cross over frequency $\angle G(j\omega) H(j\omega) = 180^o$ . Thus

$$-180° = 90 - 2\tan^{-1} 0 - \tan^{-1} \infty - 2\tan^{-1}\left(\frac{\omega_p}{10}\right)$$

Or $-180^o = 90 - 2\tan^{-1}(0.1\omega_P)$
Or $45^o = \tan^{-1}(0.1\omega_p)$
Or $\tan 45^o\ 0.1\ \omega_p = 1$
Or $\omega_p = 10$ rad/sec

Now $\left|G(j\omega)H(j\omega)\right| = \frac{100}{\omega(\omega^2+100)}$

At $\omega = \omega_p$

$$\left|G(j\omega)H(j\omega)\right| = \frac{100}{100(100+100)} = \frac{1}{20}$$

Gain Margin = $-20 \log_{10}|G(j\omega)H(j\omega)|$

$$= -20\log_{10}\left(\frac{1}{20}\right) = 26 \text{ db}$$

**Sol.28. (d)**
From (D) TF = H(s)

$$= \frac{100}{s(s^2+100)} \neq \frac{100}{s(s+10^2)}$$

**Sol.29. (a)**
Initial slope is zero, so K = 1
At corner frequency $\omega_1$ = 0.5 rad/sec, slope increases by +20 dB/decade, so there is a zero in the transfer function at $\omega_1$
At corner frequency $\omega_2$ = 10 rad/sec, slope decrease by –20 dB/decade and becomes zero, so there is a pole in transfer function at $\omega_2$

Transfer function $\frac{K\left(1+\frac{s}{\omega_1}\right)}{\left(1+\frac{s}{\omega_2}\right)}$

$$= \frac{1\left(1+\frac{s}{0.1}\right)}{\left(1+\frac{s}{0.1}\right)} = \frac{(1+10s)}{(1+0.1s)}$$

**Sol.30. (a)**

Given $G(s) = \frac{1}{s(s+1)(s+2)}$

$G(j\omega) = \frac{1}{j\omega(j\omega+1)(j\omega+2)}$

$$\left|G(j\omega)\right|\frac{1}{\omega\sqrt{\omega^2+1\sqrt{\omega^2+4}}}$$

$\angle G(j\omega) = -90^o - \tan^{-1}(\omega) - \tan^{-1}(\omega/2)$
In nyquist plot
For $\omega = 0$, $|G(j\omega)| = \infty$
$\angle G(j\omega) = -90^o$
For $\omega = \infty$, $|G(j\omega)| = 0$
$\angle G(j\omega) = -90^o - 90^o - 90^o = -270^o$
Intersection at real axis

$$G(j\omega) = \frac{1}{j\omega(j\omega+1)(j\omega+2)}$$

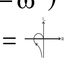
$$= \frac{1}{j\omega(-\omega^2 + j3\omega + 2)}$$

$$= \frac{1}{-3\omega^2 + j\omega(2-\omega^2)} \times \frac{-3\omega^2 - j\omega(2-\omega^2)}{-3\omega^2 - j\omega(2-\omega^2)}$$

$$\frac{-3\omega^2 - j\omega(2-\omega^2)}{9\omega^2+\omega^2(2-\omega^2)^2} - \frac{j\omega(2-\omega^2)}{9\omega^4+\omega^2(2-\omega^2)^2}$$

At real axis Im[G(jω)] = 0

So, $\frac{\omega(2-\omega^2)}{9\omega^2+\omega^2(2-\omega^2)} = 0$

$2 - \omega^2 = 0 \Rightarrow \omega =$  rad/sec

At $\omega = \sqrt{2}$ rad/sec, magnitude response is

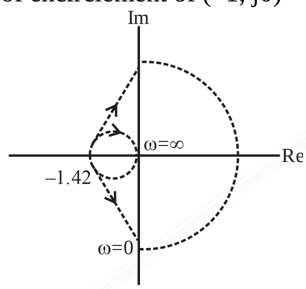
$$\left|G(j\omega)\right|_{at\ \omega=\sqrt{2}} = \frac{1}{\sqrt{2}\sqrt{2+1/\sqrt{2+4}}} = \frac{1}{6} < \frac{3}{4}$$

**Sol.31. (b)**
The plot has one encirclement of origin in clockwise direction. Thus G(s) has a zero is in RHP.

**Sol.32. (c)**
The Nyzuist plot intersect the real axis ate – 0.5.
Thus,
G.M. = –20 log x = –20 log 0.5 = 6.020 dB

And its phase margin is $90^o$

**Sol.33. (d)**
From Nyquist stability criteria, no. of closed loop poles in right half of s-plane is given as
Z = P – N
P $\rightarrow$ No. of open loop poles in right half s-plane
N $\rightarrow$ No. of encirclement of (–1, j0)




Here, N = –2 ($\because$ encirclement is in clockwise direction)
P = 0 ($\because$ system is stable)
So, Z = 0 – (–2)
Z = 2, System is unstable with 2-poles on RH of s-plane.

**Sol.34. (b)**
Since initial slope of the bode plot is –40dB/decade, so no. of poles at origin is 2. Transfer function can be written in following steps:
1. Slope changes from –40dB/dec. to –60 dB/dec. at $\omega_1$ = 2 rad/sec. so at $\omega_1$ there is a pole in the transfer function.
2. Slope changes from –60 dB/dec to –40 dB/dec at $\omega_2$ = 5 rad/sec., so at this frequency there is a zero lying in the system function.
3. The slope changes from –40dB /dec to –60 dB/dec at $\omega_3$ = 25 rad/sec , so there is a pole in the system at this frequency.
Transfer function

$$T(s)=\frac{K(s+5)}{s^2(s+2)(s+25)}$$

Constant term can be obtained as.
$T(j\omega)|_{at\ \omega=0.1} = 80$

So, $80 = 20\log \dfrac{K(5)}{(0.1)^2\times 50}$

K = 1000
Therefore, the transfer function is

$$T(s)=\frac{1000(s+5)}{s^2(s+2)(s+25)}$$

**Sol.35. (d)**
Open loop transfer function of the figure is given by

$G(s) = \dfrac{e-0}{s}$ is; $G(j\omega)=\dfrac{e^{-j0.1\omega}}{j\omega}$

Phase cross over frequency can be calculated as
$\angle G(j\omega_P) = -180^o$

$$\left(-0.1\omega_p\times\frac{180}{\pi}\right)=90°=-180°$$

$$0.1\omega_p\times\frac{180}{\pi}=90°$$

$$0.1\omega_p=\frac{90°\times\pi}{180°}$$

$\omega_P$ = 15.7 rad/sec
So the gain margin (dB)

$$=20\log\left(\frac{1}{|G(j\omega_p)|}\right)=20\log\left[\frac{1}{\left(\frac{1}{15.7}\right)}\right]$$

= 20 log 15.7 = 23.9 dB

**Sol.36. (c)**
From the given bode plot we can analyze that
1. Slope – 40 dB/decade : 2 poles
2. Slope –20dB /decade (Slope changes by +20 dB/decade) : 1 Zero
3. Slope 0 dB/decade (Slope changes by + 20 dB/decade) : 1 zero
So there are 2 poles and 2 zeroes in the transfer function.

**Sol.37. (d)**
At every corner frequency there is change of –20db/decade in slope which indicate pole at every corner frequency. Thus

$$G(s)=\frac{K}{s(1+s)\left(1+\frac{s}{20}\right)}$$

Bode plot is in (1 + ST) form

$$20\log\frac{K}{\omega}\bigg|_{\omega-0.1} = 60\,\text{dB} = 1000$$

Thus K = 5

Hence $G(s) = \dfrac{100}{s(s+1)(1+0.5s)}$

**Sol.38. (b)**

Given function is

$$G(s) = \frac{1}{s(s+1)(s+2)}$$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)(2+j\omega)}$$

By simplifying

$$G(j\omega) = \left(\frac{1}{j\omega}\times\frac{-j\omega}{-j\omega}\right)\left(\frac{1}{1+j\omega}\times\frac{1-j\omega}{1-j\omega}\right)\left(\frac{1}{2+j\omega}\times\frac{2-j\omega}{2-j\omega}\right)$$

$$= \left(\frac{j\omega}{\omega^2}\right)\left(\frac{1-j\omega}{1+\omega^2}\right)\left(\frac{2-j\omega}{4+\omega^2}\right) = \frac{-j\omega(\omega^2-2)}{\omega^2(1+\omega^2)(4+\omega^2)}$$

$$= \frac{-3\omega^2}{\omega^2(1+\omega^2)(4+\omega^2)} + \frac{j\omega(\omega^2-2)}{\omega^2(1+\omega^2)(4+\omega^2)}$$

G(jω) = x + iy

$$Xx = \text{Re}[G(j\omega)]\big|_{\omega\to 0+} = \frac{-3}{1\times 4} = -\frac{3}{4}$$

**Sol.39. (d)**

Let response of the un-compensated system is

$$H_{UC}(s) = \frac{900}{s(s+1)(s+9)}$$

Response of compensated system.

$$H_C(s) = \frac{900}{s(s+1)(s+9)}G_C(s)$$

Where $G_C(s)$ is Response of compensator

Given that gain – crossover frequency of compensated system is same as phase crossover frequency of un-compensated system

So, $(\omega_g)_{\text{compensated}} = (\omega_p)_{\text{uncompensated}}$

$-180^o = \angle H_{UC}(j\omega_P)$

$$-180° = -90° - \tan^{-1}\left[\frac{\omega_p + \frac{\omega_p}{9}}{1-\frac{\omega_p}{9}}\right]$$

$$1 - \frac{\omega_p^2}{9} = 0$$

$\omega_P$ = 3 rad/sec

So, $(\omega_g)_{\text{compensated}}$ = 3rad/sec

At this frequency phase margin of compensated system is

$\phi_{PM} = 180^o + \angle H_C(j\omega_g)$

$45^o = 180^o - 90^o - \tan^{-1}(\omega_g/9) + \angle G_C(j\omega_g)$

$45^o = 180^o - 90^o - \tan^{-1}(3) - \tan^{-1}(1/3j) + \angle G_C(j\omega_g)$

$$45° = 90° - \tan^{-1}\left[\frac{3+\frac{1}{3}}{1-3\left(\frac{1}{3}\right)}\right] + \angle G_C(j\omega_g)$$

$45° = 90° - 90° + \angle G_C(j\omega_g)$

$\angle G_C(j\omega_g) = 45^o$

The gain cross over frequency of compensated system is lower than un-compensated system, so we may use lag – lead compensator.

At gain cross over frequency gain of compensated system is unity so.

$|HC(j\omega_g)| = 1$

$$\frac{900|G_C(j\omega)|}{\omega_g\sqrt{\omega_g^2+1}\sqrt{\omega_g^2+81}} = 1$$

$$|G_C(j\omega_g)| = \frac{3\sqrt{9+1}\sqrt{9+81}}{900} = \frac{3\times 30}{900} = \frac{1}{10}$$

$$\text{In dB}|G_C(\omega_g)| = 20\log\left(\frac{1}{10}\right)$$

= –20 dB (attenuation)

**Sol.40. (a)**

In the given options only in option (a) the nyquist plot does not enclosed the unit circle (–1, j0) , so this is stable.

**Sol.41. (a)**

Given function is

$$H(j\omega) = \frac{10^4(1+j\omega)}{(10+j\omega)(100+j\omega)^2}$$

Function can be rewritten as,

$$H(j\omega) = \frac{10^4(1+j\omega)}{10\left[1+j\frac{\omega}{10}\right]10^4\left[1+j\frac{\omega}{100}\right]^2}$$

$$= \frac{0.1(1+j\omega)}{\left(1+j\frac{\omega}{10}\right)\left(1+\frac{j\omega}{100}\right)^2}$$

The system is type 0, so, Initial slope of the bode plot is 0 dB/decade.

Corner frequencies are

$\omega_1$ = 1 rad/sec

$\omega_2$ = 10 rad/sec

$\omega_3$ = 100 rad/sec

As the intial slope of bode plot is 0 dB/decade and corner frequency $\omega_1$ = 1 rad/sec, the Slope after $\omega$ = 1 rad/sec or log $\omega$ = 0 is (0 + 20) = +20 dB/dec.

After corner frequency $\omega_2$ = 10 rad/sec or log $\omega_2$ = 1, the Slope is (+20 – 20) = 0 dB/dec

Similarly after $\omega_3$ = 100 rad/sec or log $\omega$ = 2, the Slope is (0 – 20 × 2) = - 40 dB/dec.

Hence (A) is correct option.

**Sol.42. (d)**

Given system is 2nd order and for 2nd order system G.M. is infinite.

**Sol.43. (d)**

If the Nyquist polt of $G(j\omega)H(j\omega)$ for a closed loop system pass through (–1, j0) point, the gain margin is 1 and 1 in dB

GM = –20 log 1= 0 dB

**Sol.44. (c)**

We have $G(s) = \frac{as+1}{2}$

$\angle G(j\omega) = \tan^{-1}(\omega a) - \pi$

Since PM is $\frac{\pi}{4}$ i.e. 45°, thus

$\frac{\pi}{4} = \pi + \angle G(j\omega_g)\omega g \rightarrow$ Gain cross over Frequency

Or $\frac{\pi}{4}\frac{\pi}{4} = \pi + \tan^{-1}(\omega_g a) - \pi$

Or $\frac{\pi}{4} = \tan^{-1}(\omega_g a)$ or $a\omega_g = 1$

Or $a\omega_g = 1$

At gain cross over frequency $|G(j\omega_g)| = 1$

Thus $\frac{\sqrt{1+a^2\omega_g^2}}{\omega_g^2} = 1$

Or $\sqrt{1+1} = \omega_g^2$ (as $a\omega_g = 1$)

Or $\omega g = (2)^{\frac{1}{4}}$

**Sol.45. (c)**

For a = 0.84 we have

$$G(s) = \frac{0.84s+1}{s^2}$$

Due to ufb system H(s) = 1 and due to unit impulse response R(s) = 1, thus

C(s) = G(s) R(s) = G(s)

$$= \frac{0.84s+1}{s^2} = \frac{1}{s^2} + \frac{0.84}{s}$$

Taking inverse Laplace transform

C(t) = (t + 0.84) u(t)

At t = 1 c(1 sec) = 1 + 0.84 = 1.84

**Sol.46. (a)**

From the given plot we can see that centroid C (point of intersection) where asymptotes intersect on real axis) is 0

So for option (a)

$$G(s)=\frac{K}{s^3}$$

$$\text{Centroid} = \frac{\sum \text{Poles} - \sum \text{Zeros}}{n-m} = \frac{0-0}{3-0} = 0$$

**Sol.47. (a)**

Open loop transfer function is.

$$G(s) = \frac{(s+1)}{s^2}$$

$$G(j\omega) = \frac{j\omega+1}{-\omega^2}$$

Phase crossover frequency can be calculated as.

$\angle G(j\omega_p) = -180^o$

$\tan^{-1}(\omega_P) = -180^o$

$\omega_p = 0$

Gain margin of the system is

$$G.M. = \frac{1}{|G(j\omega_p)|} = \frac{1}{\sqrt{\frac{\omega_p^2}{\omega_p^2}}} = \frac{\omega_p^2}{\sqrt{\omega_p^2+1}} = 0$$

**Sol.48. (b)**

When it passes through negative real axis at that point phase angle is –180° so $\angle G(j\omega)\ H(j\omega) = 180^o$

$$-0.25j\omega - \frac{\pi}{2} = -\pi$$

$$-0.25j\omega = -\frac{\pi}{2}$$

$$j0.25\omega = \frac{\pi}{2}$$

$$j\omega = \frac{\pi}{2\times 0.25}$$

$s = j\omega = 2\pi$

Put $s = 2\pi$ in given open loop transfer function we get

$$G(s)H(s)\big|_{s=2\pi} = \frac{\pi e^{-0.25\times 2\pi}}{2\pi} = -0.5$$

So it passes through $(-0.5, j_0)$

**Sol.49. (b)**

**Sol.50. (d)**

$$G(s) = \frac{3e^{-2s}}{s(s+2)}$$

Or $G(j\omega) = \dfrac{3e^{-2j\omega}}{j\omega(j\omega+2)}$

$$|G(j\omega)| = \frac{3}{\omega\sqrt{\omega^2+4}}$$

Let at frequency $\omega_g$ the gain is 1. Thus

$$\frac{3}{\omega_g\sqrt{\omega_g^2+4}} = 1$$

Or $\omega_g^2 + 4\omega_g^2 - 9 = 0$

Or $\omega_g^2 = 1.606$

Or $\omega_g = 1.26$ rad/sec

Now $\angle G(j\omega) = -2\omega - \dfrac{\pi}{2} - \tan^{-1}\dfrac{\omega_\phi}{2}$

Or $2\omega_\phi + \left(\dfrac{\omega_\phi}{2} - \dfrac{1}{3}\left(\dfrac{\omega_\phi}{2}\right)^3\right) = \dfrac{\pi}{2}$

Or $\dfrac{5\omega\phi}{2} - \dfrac{\omega_\phi^3}{24} = \dfrac{\pi}{2}$

Or $\dfrac{5\omega\phi}{2} \approx \dfrac{\pi}{2}$

Or $\omega_\phi = 0.63$ rad

The gain at phase cross over frequency $\omega_\phi$ is

$$|G(j\omega_g)| = \frac{3}{\omega\phi\sqrt{(\omega_\phi^2+4)}} = \frac{3}{0.63(0.63^2+4)^{1/2}}$$

or $|G(j\omega_g)| = 2.27$

G.M. = $-20 \log |G(j\omega_g)|$

$-20 \log 2.26 = -7.08$ dB

since G.M. is negative system is unstable.

The phase at gain cross over frequency is

$$\angle G(j\omega_g) = -2\omega - \frac{\pi}{2} - \tan^{-1}\frac{\omega_g}{2}$$

$$= -2\times 1.26 - \frac{\pi}{2} - \tan^{-1}\frac{1.26}{2}$$

Or = –4.65 rad or – 2.66.5°

PM = $180^o + \angle G(j\omega_g)$

= $180^o - 266.5^o = -86.5^o$

**Sol.51. (d)**

**Sol.52. (d)**

The open loop transfer function is

$$G(s)H(s) = \frac{2(1+s)}{s^2}$$

Substituting $s = j\omega$ we have

$G(j\omega)H(j\omega) = -180° + \tan^{-1}\omega$

The frequency at which phase becomes –180°, is called phase crossover frequency.

Thus $-180 = -180^o + \tan^{-1}\omega_\phi$

Or $\tan^{-1}\omega_\phi = 0$

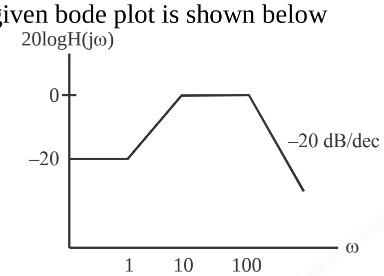
Or $\omega_\phi = 0$

The gain at $\omega_\phi = 0$ is

$$|G(j\omega)H(j\omega)| = \frac{2\sqrt{1+\omega^2}}{\omega^2} = \infty$$

Thus gain margin is = $\frac{1}{\infty}$ and in dB this is $-\infty$.

**Sol.53. (c)**
The given bode plot is shown below

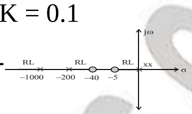



At $\omega = 1$ change in slope is + 20dB → 1 zero at $\omega = 1$
At $\omega = 10$ change in slope is –20dB → 1 poles at $\omega = 10$
At $\omega = 100$ change in slope is –20dB → 1 poles at $\omega = 100$

Thus $T(s) = \dfrac{K(s+1)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)}$

Now $20 \log_{10} K = -20 \rightarrow K = 0.1$

Thus $\dfrac{0.1(s+1)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)}$ +



**Sol.54. (a)**
Approximate (comparable to $90^o$) phase shift are
Due to pole at 0.01 Hz : $-90^o$
Due to pole at 80Hz : $-90^o$
Due to pole at 80 Hz : –0
Due to zero at 5 Hz : $90^o$
Due to zero at 100 Hz : 0
Due to zero at 200 Hz : 0
Thus approximate total : $90^o$ phase shift is provided.

**Sol.55. (a)**
Phase margin of a system is the amount of additional phase lag required to bring the system to the point of instability or (–1, j0)
So here phase margin = $0^o$

**Sol.56. (c)**
Given open loop transfer function

$$G(j\omega) = \frac{ja\omega+1}{(j\omega)^2}$$

Gain cross over frequency ($\omega_g$) for the system
$|G(j\omega_g)| = 1$

$$\frac{\sqrt{a^2\omega_g^2+1}}{-\omega_g^2} = 1$$

$$a^2\omega_g^2 + 1 = \omega_g^4 \qquad \dots (i)$$

$$\omega_g^4 - a^2\omega_g^2 - 1 = 0$$

Phase margin of the system is
$\phi_{PM} = 45^o = 180^o + \angle G(j\omega_g)$
$45^o = 180^o + \tan^{-1}(\omega_g a) - 180^o$
$\tan^{-1}(\omega_g a) = 45^o$
$\omega_g a = 1$
From equation (1) and (2)

$$\frac{1}{a^4} - 1 - 1 = 0$$

$$a^4 = \frac{1}{2} \Rightarrow a = 0.841$$

**Sol.57. (a)**
The maximum error between the exact and asymptotic plot occurs at corner frequency
Here exact gain (dB) at $\omega = 0.5a$ is given by

$$\text{gain}|dB|_{\omega=0.5a} = 20\log K - 20\log\sqrt{1+\frac{\omega^2}{a^2}}$$

$$= 20\log K - 20\log\left[1+\frac{(0.5a)^2}{a^2}\right]^{1/2}$$

= 20 log K – 0.96
Gain (dB) calculated from asymptotic plot at $\omega = 0.5a$ is
= 20 log K
Error in gain (dB) = 20 log K – (20 log K – 0.96) dB = 0.96 dB
Similarly exact phase angle at $\omega = 0.5a$ is.

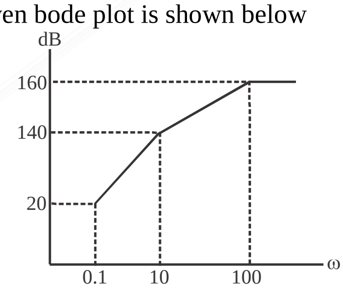
$$\theta_h(\omega)_{\omega=0.5a} = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

$$= -\tan^{-1}\left(\frac{0.5a}{a}\right) = -26.56°$$

Phase angle calculated from asymptotic plot at ($\omega$ = 0.5a) is $-22.5^o$
Error in phase angle = $-22.5 - (-26.56^o) = 4.9^o$

**Sol.58. (a)**
Z = P – N
N $\rightarrow$ Net encirclement of (–1 + j0) by Nyquist plot,
P $\rightarrow$ Number of open loop poles in right hand side of s – plane
Z $\rightarrow$ Number of closed loop poles in right hand side of s - plane
Here N = 1 and P = 1
Thus Z = 0
Hence there are no roots on RH of s – plane and system is always stable.

**Sol.59. (a)**
The given bode plot is shown below




At $\omega$ = 0.1 change in slope is + 60dB : 3 zeroes at $\omega$ = 0.1
At $\omega$ = 10 change in slope is –40dB: 2 poles at $\omega$ = 10
At $\omega$ = 100 change in slope is –20dB : 1 poles at $\omega$ = 100

$$\text{Thus } T(s) = \frac{K\left(\frac{s}{0.1}+1\right)^3}{\left(\frac{s}{10}+1\right)^2\left(\frac{s}{100}+1\right)}$$

Now 20 log $_{10}$ K = 20 Or K = 10

$$\text{Thus } T(s) = \frac{10\left(\frac{s}{0.1}+1\right)^3}{\left(\frac{s}{10}+1\right)^2\left(\frac{s}{100}+1\right)}$$

$$= \frac{10^8(s+0.1)^3}{(s+10)^2(s+100)}$$

**Sol.60. (b)**

**Sol.61. (d)**
From the expression of OLTF it may be easily see that the maximum magnitude is 0.5 and does not become 1 at any frequency. Thus gain cross over frequency does not exist. When gain cross over frequency does not exist, the phase margin in infinite.

**Sol.62. (b)**
The open loop transfer function is

$$G(s)H(s) = \frac{1}{s(s^2+s+1)}$$

Substituting s = j$\omega$ we have

$$G(j\omega)H(j\omega) = \frac{1}{j\omega(-\omega^2+j\omega+1)}$$

$$\angle G(j\omega)H(j\omega) = -\frac{\pi}{2} - \tan^{-1}\frac{\omega}{(1-\omega^2)}$$

The frequency at which phase becomes $-180^o$, is called phase crossover frequency

$$\text{Thus } -180 = -90 - \tan^{-1}\frac{\omega_\phi}{1-\omega_\phi^2}$$

Or $1-\omega_\phi^2 = 0$

$\omega_\phi$ = 1 rad/sec
The gain margin at this frequency $\omega_\phi$ = 1 is
GM = $-20\log_{10}|G(j\omega_\phi)H(j\omega_\phi)|$

$$= 20\log_{10}\left(\omega_\phi\sqrt{\left(1-\omega_\phi^2\right)^2+\omega_\phi^2}\right) = -20\log 1 = 0$$

**Sol.63. (a)**
Z = P – N
N is Net encirclement of (–1 + j0) by Nyquist plot,
P is Number of open loop poles in right hand side of s –plane
Z is Number of closed loop poles in right hand side of s – plane
Here N = 0 (1 encirclement in CW direction and other in CCW) and P = 0
Thus Z = 0 Hence there are no roots on RH of s– plane.

# ESE OBJ QUESTIONS

**1. Statement I.**
The transportation lag in a system can be easily handled by using Bode plot.
**Statement II.**
The magnitude plot is unaffected, and only the phase plot shifts by $-\omega T$ rad due to the presence of $e^{-st}$.

**[EE ESE - 2018]**

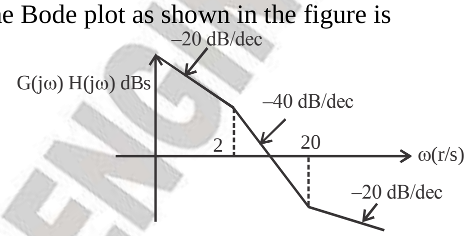
**Codes:**
(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

**2.** The open loop transfer function of a system has two poles on the imaginary axis, one in the left – half and the other in the right – half, together with a zero at the origin of coordinates and also two zeros in the left half of the s-plane. The closed – loop response for unity feedback will be stable if the encirclement of the critical point (–2, j0) is

**[EE ESE - 2018]**

(a) –1 (b) +1
(c) –2 (d) +2

**3.** The open – loop transfer function G(s)H(s) of the Bode plot as shown in the figure is

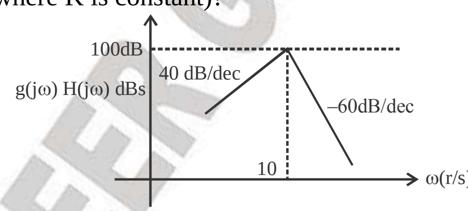



**[EE ESE - 2018]**

(a) $\frac{Ks(s+2)}{s+20}$ (b) $\frac{K(s+20)}{(s+2)}$

(c) $\frac{K(s+2)}{s(s+20)}$ (d) $\frac{Ks(s+20)}{s+2}$

**4.** Which one of the following transfer functions represents the Bode plot as shown in the figure (where K is constant)?




**[EE ESE - 2018]**

(a) $\frac{Ks^2}{\left(1+\frac{s}{10}\right)^3}$ (b) $\frac{Ks^2}{\left(1+\frac{s}{10}\right)^4}$

(c) $\frac{Ks^2}{\left(1+\frac{s}{10}\right)^5}$ (d) $\frac{Ks^2}{\left(1+\frac{s}{10}\right)^2}$

**5.** The low – frequency asymptote in the Bode plot of

$$G(s) = \frac{6(s^2+10s+100)}{s^2(50s^2+15s+11)}$$

has a slope of

**[EE ESE - 2018]**

(a) –10 dB /dec (b) –20 dB/dec
(c) –40 dB/dec (d) –60 dB/dec

**6. Statement (I):** Roots of closed-loop control system can be obtained from the Bode plot.
**Statement (II):** Nyquist criterion does not give direct value of corner frequencies.

**[EE ESE - 2017]**

(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

**7.** A system has 14 poles and 2 zeros in its open – loop transfer function. The slope of its highest frequency asymptote in its magnitude plot is

**[EE ESE - 2017]**

(a) –40 dB/dec (b) –240 db/dec
(c) +40 dB/dec (d) +240 dB/dec

**8.** Consider the following statements regarding 'relative stability':
It is defined
1. In terms of gain margin only
2.In terms of phase margin and certain and certain other parameters
3.In terms of gain margin, phase margin and location of poles in s-plane
4.In relation to another identified system
Which of the above statements are correct?

**[EC ESE - 2017]**

(a) 1 and 2 (b) 2 and 3
(c) 3 and 4 (d) 1 and 4

**9.** Consider the following statements with reference to the response of a control system:
1.A large resonant peak corresponds to a small overshoot in transient response.
2.A large bandwidth corresponds to slow response.
3.The cut-off rate indicates the ability of the system to distinguish the signal from noise
4.Resonant frequency is indicative of the speed of transient response.
Which of the above statements are correct?

**[EE ESE - 2016]**

(a) 1 and 2 (b) 2 and 3
(c) 1 and 4 (d) 3 and 4

**10.** The frequency of sustained oscillation for marginal stability, for a control system

$$G(s)H(s) = \frac{2K}{s(s+1)(s+5)}$$

And operating with negative feedback, is

**[EE ESE - 2016]**

(a) $\sqrt{5}$ r / s (b) $\sqrt{6}$ r / s
(c) 5 r/s (d) 6 r/s

**11.** Gain margin is the factor by which the system gain can be increased to drive it to

**[EE ESE - 2016]**

(a) Stability
(b) Oscillation
(c) The verge of instability
(d) Critically damped state

**12.** Nichols' chart is used to determine

**[EE ESE - 2016]**

(a) Transient response
(b) Closed-loop frequency response
(c) Open-loop frequency response
(d) Settling time due to step input

**13.** For a type-I system the intersection of the initial slope of the Bode plot with 0 dB axis gives

**[EE ESE - 2016]**

(a) Steady-state error
(b) Error constant
(c) Phase margin
(d) Cross-over frequency

**14.** For a unity feedback system with open-loop transfer function $\frac{25}{s(s+6)}$, the resonant peak at output $M_m$ and the corresponding resonant frequency $\omega_m$, are respectively.

**[EC ESE - 2016]**

(a) 2.6 and 2.67 r/s (b) 1.04 and 2.67 r/s
(c) 2.6 and 4.8 r/s (d) 1.04 and 4.8 r/s

**15.** Consider the following
1. Bode plot
2. Nyquist plot
3. Nichols plot

Which of the above frequency response plots are commonly employed in the analysis if control systems?

**[EC ESE - 2016]**

(a) 1 and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3

**16.** Consider the transfer function:

$$G(s)=\frac{5(s^2+10s+100)}{s^2(s^2+15s+1)}$$

The corner frequencies in Bode's plot for this transfer function are as

**[EC ESE - 2016]**

(a) 10 r/s and 10 r/s
(b) 100 r/s and 10 r/s
(c) 10 r/s and 1 r/s
(d) 100 r/s and 1 r/s

**17.** The open-loop transfer function of a unity feedback system is $G(s)=\frac{K}{s(s+5)}$. The gain K that results in a phase margin of 45° is

**[EC ESE - 2016]**

(a) 35 (b) 30
(c) 25 (d) 20

**18.** Consider the following statements:
The gain margin and Phase margin of an unstable system may respectively by
1. Positive, negative
2. Negative, positive
3. Negative, negative
Which of the above statements is/are correct?

**[EC ESE - 2016]**

(a) 3 Only (b) 1 and 2 only
(c) 2 and 3 only (d) 1, 2 and 3

**19.** The Bode plot of the open-loop transfer function of a system is described as follows:
Slope – 40 dB/decade $\omega < 0.1$ rad/s
Slope – 20 dB/decade $0.1 < \omega < 10$ rad/s
Slope 0 $\omega > 10$ rad/s
The system described will have

**[EC ESE - 2016]**

(a) 1 pole and 2 zeros
(b) 2 poles and 2 zeros
(c) 2 poles and 1 zeros
(d) 1 pole and 1 zero

**20.** From the Nichols chart, one can determine the following quantities pertaining to a closed-loop system.

**[EC ESE - 2016]**

(a) Magnitude, bandwidth and phase
(b) Bandwidth and phase only
(c) Magnitude and phase only
(d) Bandwidth only

**21.** Which of the following techniques are used to determine relative stability of a closed loop linear system?
1. Bode plot
2. Nyquist plot
3. Nichol's chart
4. Routh-Hurwitz criterion

**[EC ESE - 2015]**

(a) 1, 2 and 4 (b) 1, 3 and 4
(c) 1, 2 and 3 (d) 1, 2, 3 and 4

**22.** The Bode plots of the transfer function G(s) = s is
1. Constant magnitude
2. 20 dB/decade
3. Constant phase shift angle
4. Constant phase shift of $\pi/2$
Which of these are correct?

**[EC ESE - 2015]**

(a) 1 and 3 (b) 1 and 4
(c) 2 and 3 (d) 2 and 4

**23.** The transfer function of any stable system which has no zeros of poles in the right half of this s-plane is said to be

**[EC ESE - 2015]**

(a) Minimum phase transfer function
(b) Non-minimum phase transfer function
(c) Minimum frequency response function
(d) Minimum gain transfer function

**24. Statement (I):** If a ramp input is applied to a second-order system, the steady-state error of the response can be reduced by reducing damping and increasing natural frequency of oscillation.

**Statement (II):** In the frequency response of a second-order system, the change in slope at one of the corner frequencies is of ±40 dB decade.

**[EE ESE - 2015]**

**Codes:**

(a)Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).

(b)Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).

(c)Statement (I) is true but Statement (II) is false.

(d)Statement (I) is false but Statement (II) is true.

**25. Statement (I):** A large resonance peak in frequency response also corresponds to a large peak overshoot in transient response.

**Statement (II):** All the systems which exhibit overshoot in time response will also exhibit resonance.

**[EE ESE - 2014]**

**Codes:**

(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).

(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).

(c) Statement (I) is true but Statement (II) is false.

(d) Statement (I) is false but Statement (II) is true.

**26. Statement (I):** The polar plot has limitation for portraying the frequency response of a system.

**Statement (II):** The calculation of frequency response is tedious and does not indicate effect of the individual poles and zeros.

**[EE ESE - 2014]**

**Codes:**

(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).

(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).

(c) Statement (I) is true but Statement (II) is false.

(d) Statement (I) is false but Statement (II) is true.

**27.** In the Bode plot of a unity feedback control system, the value of phase of G(jω) at the gain cross–over frequency is– $125^0$ . The phase margin of the system is

**[EE ESE - 2014]**

(a) – $125^0$ (b) – $55^0$

(c) $55^0$ (d) $125^0$

**28.** By adding a pole at the origin of s-plane, the Nyquist plot of a system will rotate by

**[EE ESE - 2014]**

(a) $90^0$ in anti – clockwise direction

(b) $90^0$ in clsockwise direction

(c) $180^0$ in anti – clockwise direction

(d) $180^0$ in clockwise direction

**29.** What will be the gain margin in dB of a system having the following open-loop transfer function

$$G(s)H(s) = \frac{2}{s(s+1)}$$

**[EE ESE - 2014]**

(a) 0 (b) 2

(c) $\frac{1}{2}$ (d) ∞

**30.** For a $3^{rd}$ order system given below, what is the phase crossover frequency?

$$G(s)H(s) = \frac{K}{s^3 + 6s^2 + 11s + 6}$$

**[EC ESE - 2014]**

(a) $\sqrt{6}$ (b) $\sqrt{11}$

(c) $\pm\sqrt{11}$ (d) $\pm\sqrt{6}$

**31.** The transfer function of a system is $\frac{10}{1+s}$. At a frequency of 0.1 rad/sec, the straight line bode plot will have a magnitude of:

**[EC ESE - 2013]**

(a) 10 dB (b) 20 dB
(c) 0 dB (d) 40 dB

**32.** A second order system has $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$. Its frequency response will have maximum value at the frequency:

**[EC ESE - 2013]**

(a) $\omega_n\sqrt{1-\xi^2}$ (b) $\omega_n\xi$
(c) $\omega_n\sqrt{1-2\xi^2}$ (d) Zero

**33.** The critical value of gain for a system is 40. The system is operating at a gain of 20. The gain margin of the system is:

**[EC ESE - 2013]**

(a) 2 dB (b) 3 dB
(c) 6 dB (d) 4 dB

**34.** In a feedback control system, phase margin (PM) is:
1. Directly proportional to $\xi$
2. Inversely proportional to $\xi$
3. Independent of $\xi$
4. Zero when $\xi = 0$

Which of the above statements are correct?

**[EC ESE - 2013]**

(a) 1 and 2 (b) 2 and 3
(c) 3 and 4 (d) 1 and 4

**35.** The gain margin in dB's of a unity feedback control system whose open-loop transfer function, $G(s)H(s) = \frac{1}{s(s+1)}$ is

**[EC ESE - 2013]**

(a) 0 (b) 1
(c) –1 (d) ∞

**36. Statement (I)**: Nyquist plot is the locus of GH (jω) indicating the magnitude and phase angle on the GH (jω) plane.
**Statement (II)**: Given the values of |GH(jω)| and ∠GH (jω) using the Nichols chart $M_m$, $\omega_m$ and bandwidth can be determined.

**[EC ESE - 2013]**

(a) Both Statement (I) and Statement (II) are individually true and statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
(c) Statement (I) is true but Statement (II) is false
(d) Statement (I) is false but Statement (II) is true.

**37.** If the s-plane contour enclose 3-zeros and 2-poles of q(s), the corresponding q(s) plane contour will encircle the origin of q(s) plane

**[EE ESE - 2013]**

(a) Once in clockwise direction
(b) Once in counter clockwise direction
(c) Thrice in clockwise direction
(d) Twice in counter clockwise direction

**38.** The compensator $G_c(s) = \frac{5(1+0.3s)}{1+0.1s}$ would provide a maximum phase shift of

**[EE ESE - 2012]**

(a) $20^0$ (b) $30^0$
(c) $45^0$ (d) $60^0$

**39.** If the phase margin of a unity feedback control system is zero, then the Nyquist plot of the system passes through

**[EE ESE - 2012]**

(a) The origin in the GH plane
(b) Left-hand side of (– 1, j0) point in the GH plane.
(c) Exactly on (– 1, j0) point in the GH plane.
(d) In between origin and (– 1, j0) point in the GH plane.

**40.** A unity feedback system has an open – loop transfer function as

**[EE ESE - 2012]**

$$G(s) = \frac{K}{s(1+0.2s)(1+0.05s)}$$

The phase crossover frequency of the Nyquist plot is given by
(a) 5 rad/s (b) 10 rad/s

(c) 50 rad/s (d) 100 rad/s

**41.** The range of K for stability of a feedback system whose open-loop transfer function is $G(s)=\frac{K}{s(s+1)(s+2)}$ is

**[EC ESE - 2012]**

(a) 0 < K < 3 (b) 0 < K < 6
(c) K > 6 (d) 0 > K > 3

**42.** The sensitivity $S_T(K)$ of transfer function $T=\frac{(1+2K)}{(3+4K)}$ with respect to the parameter K is given by

**[EC ESE-2012]**

(a) $\frac{K}{3+K^2}$ (b) $\frac{3K}{2+4K+K^2}$
(c) $\frac{2K}{3+10K+8K^2}$ (d) $\frac{4K}{2+5K+7K^2}$

**43.** A system is described by the transfer function $G(s)=\frac{2s+5}{(s+5)(s+4)}$. The dc gain of the system is

**[EC ESE - 2012]**

(a) 0.25 (b) 0.5
(c) 1 (d) ∞

**44.** For a type 1 system, the low frequency asymptote of its Bode plot will have a slope of

**[EC ESE - 2012]**

(a) 0dB/decade (b) 6 dB/decade
(c) 20 dB/decade (d) –20 dB/decade

**45.** The gain cross-over frequency and phase margin of the transfer function $\frac{1}{s(s+1)}$ are

**[EC ESE - 2012]**

(a) 1 rad/s and 45° (b) 2 rad/s and 45°
(c) 2 rad/s and 135° (d) 1 rad/s and 135°

**46.** For a unity feedback control system, if its open-loop transfer function is given by $G(s)H(s)=\frac{10}{(s+5)^3}$, then its margin will

**[EC ESE - 2012]**

(a) 20 dB (b) 40 dB
(c) 60 dB (d) 80 dB

**47.** All the constant –N loci in G-plane intersect the real axis in points.

**[EC ESE - 2012]**

(a) –1 and origin (b) –0.5 and +0.5
(c) –1 and +1 (d) Origin and +1

**48.** The constant magnitude locus for M = 1, in G-plane is given by the following equation where x = Re [G(jω)] and y = lm [G(jω)]

**[EC ESE - 2012]**

(a) x = –0.5 (b) x = 0
(c) $x^2 + y^2 = 0.25$ (d) $x^2 + y^2 = 1$

**49. Statement (I)**: The phase angle plot is Bode diagram is not affected by the variation in open loop gain of the system.

**Statement (II)**: The variation in gain of the system has no effect on the phase margin.

**[EC ESE - 2012]**

(a) Both Statement (I) and Statement (II) are individually true and statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
(c) Statement (I) is true but Statement (II) is false
(d) Statement (I) is false but Statement (II) is true.

**50. Statement (I)**: In a prototype second order system the rise time $t_r$ and bandwidth are inversely proportional.

**Statement (II)**: Increasing $\omega_n$ increases bandwidth while $t_r$ reduces.

**[EC ESE - 2012]**

(a) Both Statement (I) and Statement (II) are individually true and statement (II) is the correct explanation of Statement (I).

(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
(c) Statement (I) is true but Statement (II) is false
(d) Statement (I) is false but Statement (II) is true.

**51. Statement (I)**: A second order system subjected to a unit impulse oscillates at its natural frequency.
**Statement (II)**: Impulse input contains frequencies from –∞ to +∞.
**[EC ESE - 2012]**
(a) Both Statement (I) and Statement (II) are individually true and statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
(c) Statement (I) is true but Statement (II) is false
(d) Statement (I) is false but Statement (II) is true.

**52. Statement (I)**: Nyquist criterion is a powerful tool to determine stability of a closed loop system using open loop transfer function.
**Statement (II)**: Nyquist criterion relates the locations of poles and zeros of the closed loop transfer function.
**[EC ESE - 2012]**
(a) Both Statement (I) and Statement (II) are individually true and statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
(c) Statement (I) is true but Statement (II) is false
(d) Statement (I) is false but Statement (II) is true.

**53.** A family of constant N circles has the center as
**[EC ESE - 2011]**
(a) X = 1and Y = 2 N
(b) $X = -\frac{1}{4}$ and $Y = 4N$
(c) $X = -\frac{1}{2}$ and $Y = \frac{1}{4N}$
(d) $X = -\frac{1}{2}$ and $Y = \frac{1}{2N}$

**54.** System is said to be marginally stable, if
**[EC ESE - 2011]**
(a) Gain crossover frequency > Phase crossover frequency
(b) Gain crossover frequency = Phase crossover frequency
(c) Gain crossover frequency < Phase crossover frequency
(d) Gain crossover frequency ≠ Phase crossover frequency

**55.** If the gain margin of a system in decibels is negative, the system is
**[EC ESE - 2011]**
(a) Stable
(b) Marginally stable
(c) Unstable
(d) Could be stable or unstable or marginally stable.

**56.** For the Bode plot of the system $G(s) = \frac{10}{0.66s^2 + 2.33s + 1}$ the corner frequencies are
**[EC ESE - 2011]**
(a) 0.66 and 0.33 (b) 0.22 and 2.00
(c) 0.30 and 2.33 (d) 0.50 and 3.00

**57.** An electrical system transfer function has a pole at s = –2 and a zero at s = –1 with system gain 10. For sinusoidal current excitation voltage response of the system
**[EC ESE - 2011]**
(a) Is zero
(b) Is in phase with the current
(c) Leads the current
(d) Lags behind the current

**58.** Match List-I with List-II and select the correct answer using the code given below the lists:

**[EE ESE - 2011]**

**List-I**

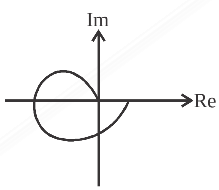
A. $G(s) = \dfrac{1+sT}{1+2sT}$

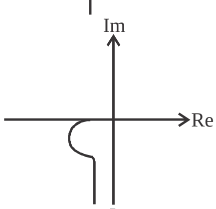
B. $G(s) = \dfrac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$

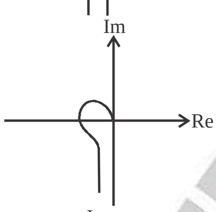
C. $G(s) = \dfrac{1+sT_1}{s(1+sT_2)(1+sT_3)}$

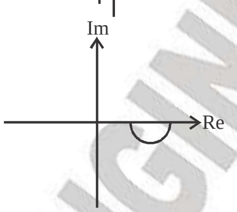
D. $G(s) = \dfrac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$

**List-II**

i. 


ii. 


iii. 


iv. 


**Codes:**
(a) A-iii, B-ii, C-i, D-iv
(b) A-iv, B-ii, C-i, D-iii
(c) A-iii, B-i, C-ii, D-iv
(d) A-iv, B-i, C-ii, D-iii

**59.** The transfer function of a linear control system is given by

$$G(s) = \frac{100(s+15)}{s(s+4)(s+10)}$$

In its Bode diagram, the value of gain for ω = 0.1 rad/sec is

**[EE ESE - 2011]**

(a) 20 dB (b) 40 dB
(c) 60 dB (d) 80 dB

**60. Assertion (A):** The effects of noise disturbance and parameter variations are relatively easy to visualize and access through frequency response.
**Reason (R):** Frequency response test is suitable for system with very large time constant.

**[EE ESE - 2010]**

(a) Both A and R are true and r is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

**61.** For the network function $T(s) = \dfrac{s}{s^2+2s+100}$, the resonant frequency and bandwidth are respectively.

**[EE ESE - 2010]**

(a) 10, 1 (b) 10, 2
(c) 100, 1 (d) 100, 2

**62.** For a parallel resonant circuit, circuit, if the damped frequency is $\sqrt{8}$ rad/s and the bandwidth is 2 rad/s, the resonant frequency of the circuit is

**[EE ESE - 2010]**

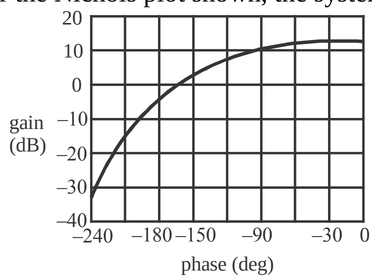
(a) 10 rad/s (b) 7 rad/s
(c) 3 rad/s (d) 2 rad/s

**63.** From the point of view of stability and response speed of a closed loop system, the appropriate range for the value of damping ratio lies between.

**[EC ESE - 2010]**

(a) 0 to 0.2 (b) 0.4 to 0.7
(c) 0.8 to 1.0 (d) 1.1 to 1.5

**64.** For the Nichols plot shown, the system is




**[EC ESE - 2010]**

(a) Unstable (b) Stable
(c) Overdamped (d) Critically stable

**65.** The Nyquist plot of loop transfer function G(s) H(s) of a closed loop control system passes through the point (–1, j0) in the G(s) H(s) plane. The phase margin of the system is

**[EC ESE - 2010]**

(a) 0° (b) 45°
(c) 90° (d) 180°

**66.** In the Bode plot of a unity feedback control system, the value of phase angle of G(jω) is –90° at the gain cross over frequency of the Bode plot, the phase margin of the system is

**[EC ESE - 2010]**

(a) – 180° (b) + 180°
(c) – 90° (d) + 90°

**67.** The addition of open loop zero pulls the root loci towards

**[EC ESE - 2010]**

(a) The left and therefore system becomes more stable
(b) The right and therefore system becomes unstable
(c) Imaginary axis and therefore system becomes marginally stable.
(d) The left and therefore system becomes unstable.

**68.** The open loop transfer function of a system has one pole in the right half of s – plane. If the system is to be closed loop stable, then (– 1 +j0) point should have how many encirclements in the GH – plane ?

**[EE ESE - 2009]**

(a) – 2 (b) – 1
(c) + 1 (d) +2

**69.** What is the initial slope of Bode magnitude plot of a type – 2 system ?

**[EE ESE - 2009]**

(a) – 20 db/decade (b) +20 db/decade
(c) – 40 db/decade (d) +40 db/decade

**70.** What is the slope of the line due to 1/ω factor in magnitude part of Bode plot ?

**[EE ESE - 2009]**

(a) – 20 db per octave
(b) – 10 db per octave
(c) – 6 db per octave
(d) – 2 db per octave

**71.** What is the error in magnitude at the corner frequency for an asymptotic Bode magnitude plot for the term $(1 + s\tau)^{\pm n}$ ?

**[EE ESE - 2009]**

(a) ± 20 n db (b) ± 6 n db
(c) ± 3 n db (d) ± 1 n db

**72.** Consider the following:
(i) Phase margin
(ii) Gain margin
(iii) Maximum overshoot
(iv) Bandwidth
Which of the above are the frequency domain specifications required to design a control system ?

**[EE ESE - 2009]**

(a) i and ii only (b) i and iii only
(c) i, iii and iv (d) i, ii and iv

**73.** The low frequency and high frequency asymptotes of Bode magnitude plot are respectively – 60 db/decade and – 40 db/decade. What is the type of the system ?

**[EE ESE - 2008]**

(a) Type – 0 (b) Type – 1
(c) Type – 2 (d) Type – 3

**74.** Which one of the following is correct ?

If the open – loop transfer function has one pole in the right half of s – plane, the closed loop system will be stable if the Nyquist plot of GH

**[EE ESE - 2008]**

(a) Does not encircle the (– 1 + j0) point
(b) Encircles the (– 1 + j0) point once in the counter – clockwise direction.
(c) Encircles the (– 1 + j0) point once in the clockwise direction.
(d) Encircles the origin once in the counter – clockwise direction.

**75.** Which one of the following is correct ?

**[EE ESE - 2008]**

(a) ± 40 db/decade (b) ± 12 db/octave
(c) ± 6 dn/octave (d) ± 3 db/octave

**76.** The Nyquist plot of system passes through (– 1, j0) point in the G(jω) H(jω) plane, the phase – margin of the system is

**[EE ESE - 2008]**

(a) Infinite
(b) Greater than zero but not finite
(c) Zero
(d) Less than zero

**77.** What is the value of the damping ratio of a second order system when the value of the resonant peak is unity?

**[EC ESE - 2008]**

(a) $\sqrt{2}$ (b) Unity
(c) $1/\sqrt{2}$ (d) Zero

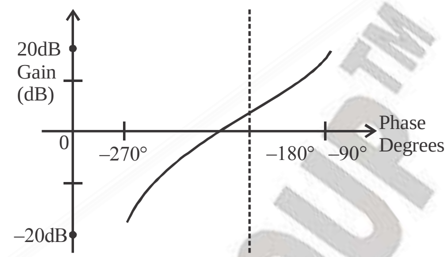
**78.** What is the gain margin of a system when the magnitude of the polar at phase cross over is 'a'?

**[EC ESE - 2008]**

(a) 1/a (b) – a
(c) Zero (d) a

**79.** The gain-phase plot of a linear control system is shown in the below figure. What are the gain-margin (GM) and the phase-margin (PM) of the system?

**[EC ESE - 2008]**




(a) GM > 0 dB and PM > 0 degree
(b) GM > 0 dB and PM < 0 degree
(c) GM < 0 dB and PM > 0 degree
(d) GM < 0 dB and PM < 0 degree

**80.** Match List-I (Shape of Nyquist Plot) with List-II (Gain Margin) and select the correct answer using the codes given below the lists:

**List-I**

A. The plot does not intersect negative real axis.
B. The plot intersects negative real axis between 0 and (–1, j0)
C. The plot passes through the point (–1, j0)
D. The plot encloses the point (–1, j0)

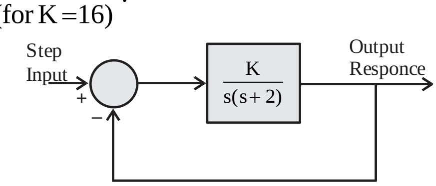
**List-II**

(i) < 0 dB
(ii) 0 dB
(iii) > 0 dB
(iv) ∞ dB

**[EC ESE - 2008]**

**Codes**:
(a) A-iii, B-iv, C-i, D-ii
(b) A-iv, B-iii, C-ii, D-i
(c) A-iii, B-iv, C-ii, D-i
(d) A-iv, B-iii, C-i, D-ii

**81.** A closed loop system is shown in the below figure. What is the ratio of output frequencies $\frac{\omega(\text{for } K=32)}{\omega(\text{for } K=16)}$?




**[EC ESE - 2008]**

(a) 1.40 (b) 1.42
(c) 1.44 (d) 1.46

**82.** A system with gain margin close to unity or a phase margin close to zero is

**[EC ESE - 2008]**

(a) Relatively stable (b) Oscillatory
(c) Stable (d) High stable

**83.** In case of d.c. servo-motor the back-emf is equivalent to an "electric friction" which tends to

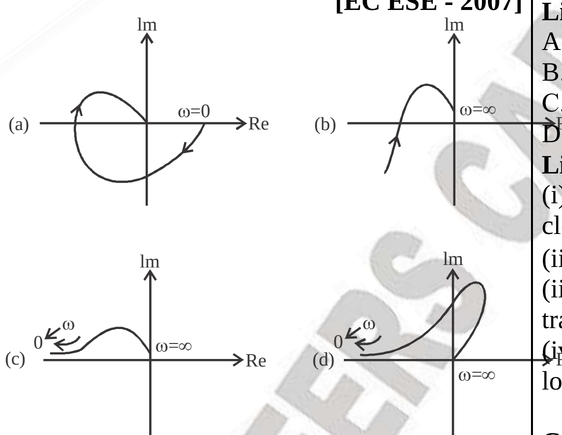
**[EC ESE - 2008]**

(a) Improve stability of the motor
(b) Slowly decrease stability of the motor
(c) Very rapidly decrease stability of the motor
(d) Have no effect on stability

**84.** Which one of the following polar plots corresponds to

$$G(j\omega) = \frac{1}{(j\omega)^2 (1 + j\omega T)}?$$

**[EC ESE - 2007]**




**85.** Consider the following statements with reference to relative stability of a system:
1.Phase margin is related to effective damping of the system.
2.Gain margin gives better estimate of damping ratio than phase margin.
3.When expressed in dB, gain margin is negative for a stable system.
Which of the statements given above are correct?

**[EC ESE - 2007]**

(a) 1 and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3

**86.** Which one of the following statements is correct for gain margin and phase margin of two closed-loop systems having loop functions G(s) H(s) and exp (–s) g(s) H(s)?

**[EC ESE - 2007]**

(a)Both gain and phase margins of the two systems will be identical.
(b)Both gain and phase margins of G(s) H(s) will be more
(c)Gain margins of the two systems are the same but phase margin of G(s) H(s) will be more
(d)Phase margins of the two systems are the same but gain margin of G(s) H(s) will be less

**87.** Match List-I (Plot/Diagram/Chart) with List-II (Characteristic) and select the correct answer using the code given below the lists:

**List-I**
A. Constant M loci
B. Constant N loci
C. Nichol's chart
D. Nyquist plot

**List-II**
(i) Constant gain and phase shift loci of the closed-loop system
(ii) Plot of loop gain with variation of ω
(iii) Circles of constant gain for closed-loop transfer function.
(iv) Circles of constant phase shift of closed-loop transfer function.

**[EC ESE - 2007]**

**Codes**:
(a) A-iii, B-iv, C-ii, D-i
(b) A-iii, B-iv, C-i, D-ii
(c) A-iv, B-iii, C-ii, D-i
(d) A-iv, B-iii, C-i, D-ii

**88.** A controller transfer function is given by C(s) = (2s + 1)/(0.25s + 1). What is the nature and parameter?

**[EC ESE - 2007]**

(a) Lag controller, $\alpha = 10$
(b) Lag controller, $\alpha = 2$
(c) Lead controller, $\beta = 0.1$

(d) Lead controller, $\beta = 0.2$

**89.** Which one of the following is the correct statement?
For the minimum phase system to be stable,

**[EE ESE - 2007]**

(a) Phase margin should be negative and gain margin positive
(b) Phase margin should be positive and gain margin negative
(c) Both gain margin and phase margin should be positive
(d) Both gain margin and phase margin should be negative

**90.** Consider the following statements in connection with frequency domain specifications of a control system:
(i) Resonant peak and peak overshoot are both functions of the damping ratio $\xi$ only.
(ii) The resonant frequency $\omega_r = \omega_n$ for $\xi > 0.707$.
(iii) Higher the resonant peak, higher is the maximum overshoot of the step response
Which of the statements given above are correct?

**[EE ESE - 2007]**

(a) i and ii (b) ii and iii
(c) i and iii (d) i, ii and iii

**91.** At which frequency does the Bode magnitude plot for the function $K/s^2$ have gain crossover frequency ?

**[EE ESE - 2007]**

(a) $\omega = 0$ r/s (b) $\omega = \sqrt{K}$ r/s
(c) $\omega = K$ r/s (d) $\omega = K^2$ r/s

**92.** Which one of the following is correct ?
A unity feedback system with forward patch transfer function

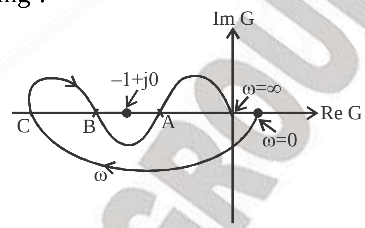
$$G(s) = \frac{K}{s(1+sT_1)(1+sT_2)}$$

is stable provided the value of K is given by

**[EE ESE - 2007]**

(a) $K < \frac{T_1+T_2}{T_1T_2}$ (b) $K < \frac{T_1T_2}{T_1+T_2}$
(c) $K > \frac{T_1+T_2}{T_1T_2}$ (d) $K > \frac{T_1T_2}{T_1+T_2}$

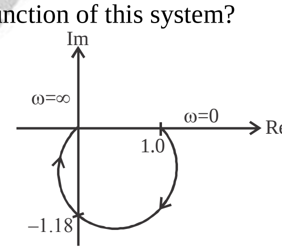
**93.** Nyquist plot of a system is shown in the below figure. The system is which one of the following ?




**[EE ESE - 2007]**

(a) Marginally stable
(b) Conditionally stable
(c) Stable
(d) Unstable

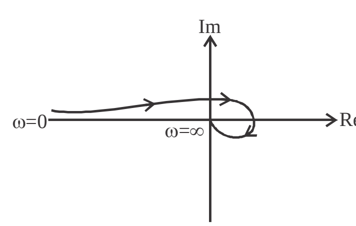
**94.** The polar plot of frequency response of a linear under damped second order system is shown in the figure given below. What is the transfer function of this system?




**[EE ESE - 2007]**

(a) $\frac{8}{s^2+10s+1}$ (b) $\frac{8}{s^2+8.48s+10}$
(c) $\frac{100}{s^2+8.48s+100}$ (d) $\frac{1000}{s^2+10s+8.48}$

**95.** The Nyquist plot of a system is sketched below:



Corresponding to this plot, what is the open – loop transfer function ?

**[EE ESE - 2006]**

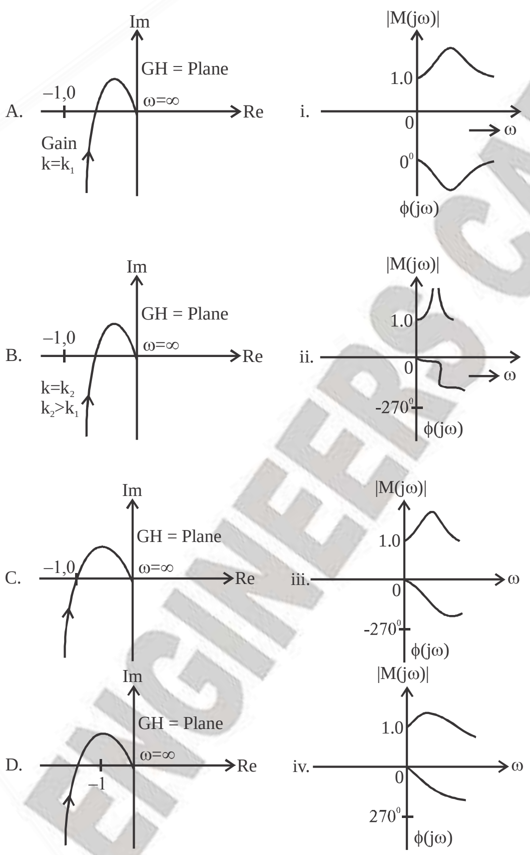
(a) $\dfrac{k}{(1+sT_1)(1+sT_2)(1+sT_3)}$

(b) $\dfrac{1}{s(1+sT_1)(1+sT_2)(1+sT_3)}$

(c) $\dfrac{k}{s^2(1+sT_1)(1+sT_2)}$

(d) $\dfrac{k}{s^2(1+sT_1)(1+sT_2)(1+sT_3)}$

**96.** Match List-I (Nyquist Plot) with List-II (Frequency Response) and select the correct answer using the code given below the lists.




**[EE ESE - 2006]**

**Codes:**

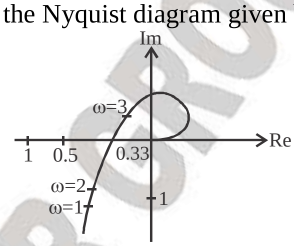
(a) A-iv, B-iii, C-ii, D-i

(b) A-iv, B-ii, C-i, D-iii

(c) A-ii, B-i, C-iii, D-iv

(d) A-ii, B-iv, C-iii, D-i

**97.** What is the appropriate value of the gain margin in the Nyquist diagram given below?



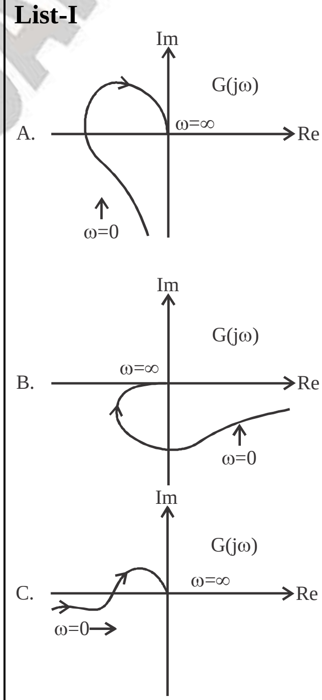

**[EE ESE - 2006]**

(a) 0.67 (b) 3.0

(c) 1.0 (d) 1/3

**98.** Match List-I (Polar Plot of system) with List-II (System Type) and select the correct answer using the code given below the lists:

**List-I** **List-II**




(i) Type 0

(ii) Type 1

(iii) Type 2

**[EE ESE - 2006]**

**Codes:**
(a) A-i, B-iii, C-ii
(b) A-ii, B-i, C-iii
(c) A-iii, B-ii, C-i
(d) A-i, B-ii, C-iii

**99.** Consider the following statements:
The gain cross – over point is the point where
(i) The magnitude $|G(j\omega)| = 1$ in polar plot.
(ii) The magnitude curve of $G(j\omega)$ crosses zero dB line in Bode plot
(iii) Magnitude vs phase plot touches the zero dB loci in Nichol's chart
Which of the statements given above are correct?

**[EE ESE - 2006]**

(a) Only i and ii
(b) Only i and iii
(c) Only ii and iii
(d) i, ii and iii

**100.** A system function has a pole at s = 0 and a zero at s = – 1. The constant multiplier is unity. For an excitation cost, what is the steady-state response ?

**[EE ESE - 2006]**

(a) $\sqrt{2}\sin(t+45^0)$
(b) $\sqrt{2}\sin(t-45^0)$
(c) $\sin(t-45^0)$
(d) sin t

**101. Assertion (A):** All the systems which exhibit overshoot in transient response will also exhibit resonance peak in frequency response.
**Reason (R):** Large resonance peak in frequency response corresponds to a large overshoot in transient response.

**[EE ESE - 2006]**

(a) Both A and r are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**102.** For a stable system, what are the restrictions on the gain margin and phase margin?

**[EC ESE - 2006]**

(a) Both gain margin and phase margin are negative
(b) Gain margin is negative and phase margin is positive
(c) Gain margin is positive and phase margin is negative
(d) Both gain margin and phase margin are positive

**103.** Consider the following statements:
For the first order transient systems, the time constant is
1. A specification of transient response
2. Reciprocal of real-axis pole location
3. An indication of accuracy of response
4. An indication of speed of the
Which of the statements given above are correct?

**[EC ESE - 2006]**

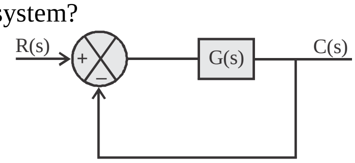
(a) Only 1 and 2
(b) Only 1, 2 and 4
(c) Only 3 and 4
(d) 1, 2, 3 and 4

**104.** Which one of the following statements is correct?
Nichol's chart is useful detailed study and analysis of

**[EC ESE - 2006]**

(a) Closed loop frequency response
(b) Open loop frequency response
(c) Close loop and open loop frequency responses
(d) None of the above

**105.** Consider the unity feedback system with $G(s) = \dfrac{2}{s(s+1)(2s+1)}$. What is the gain margin of the system?




**[EC ESE - 2006]**

(a) 3/4
(b) 4/3
(c) 1/2
(d) 3/5

**106.** Consider the following statements regarding the asymptotic Bode plots used for frequency response analysis:

1.The deviation of the actual magnitude response for a zero on real axis is 3 dB at the corner frequency.
2.The phase angle for a pair of complex conjugate poles at undamped frequency depends upon the value of damping ratio.
What of the statements given above is/are correct?

**[EC ESE - 2006]**

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 and 2

**107.** Consider the following statements for a minimum phase system:
1.All the poles of the transfer function should lie in the left of s-plane.
2.The zeros of the transfer function can lie anywhere in the s-plane.
3.Given the magnitude characteristic over the entire frequency range, the phase angle characteristic can be uniquely determined.
Which of the statements given above are correct?

**[EC ESE - 2006]**

(a) 1, 2 and 3 (b) Only 1 and 2
(c) Only 2 and 3 (d) Only 1 and 3

**108.** For a unity feedback control system the damping ratio is 0.421. What is the resonance magnitude?

**[EC ESE - 2006]**

(a) $M_r = 1$ (b) $M_r = 0.707$
(c) $M_r = 1.30$ (d) $M_r = 1.95$

**109.** Match **List-I** (Frequency Response) with **List-II** (Time Response) and select the correct answer using the code given below the lists:

**List-I**
A. Bandwidth
B. Phase margin
C. Response peak
D. Gain margin

**List-II**
(i) Overshoot
(ii) Stability
(iii) Speed of time response
(iv) Damping ratio

**[EC ESE - 2005]**

**Codes**:
(a) A-iii, B-ii, C-i, D-iv
(b) A-i, B-ii, C-i, D-ii
(c) A-iii, B-iv, C-i, D-ii
(d) A-i, B-ii, C-iii, D-iv

**110.** If the gain of the open loop system is doubled, the gain margin of the system is

**[EC ESE - 2005]**

(a) Not affected
(b) Doubled
(c) Halved
(d) One fourth of original value

**111.** Which one of the following methods can determine the closed loop system resonance frequency of operation?

**[EC ESE - 2005]**

(a) Root locus method
(b) Nyquist method
(c) Bode plot
(d) M and N circle method

**112.** For a stable closed loop system, the gain at phase cross-over frequency should always be:

**[EC ESE - 2005]**

(a) > 20 dB (b) > 6 dB
(c) < 6 dB (d) < 0 dB

**113.** For the minimum phase system to be stable:

**[EC ESE - 2005]**

(a) Phase margin should be negative and gain margin should be positive
(b) Phase margin should be positive and gain margin should be negative
(c) Both phase margin and gain margin should be positive
(d) Both phase margin and gain margin should be negative.

**114.** Consider the following statements:
The frequency response of a control system has very sharp cut off characteristics. This implies that:
1. It has large peak resonance

2. It has large bandwidth
3. It is a less stable system.
Which of the statements given above is/are correct?

**[EC ESE - 2005]**

(a) 1 only (b) 2 and 3
(c) 1 and 3 (d) 1, 2 and 3

**115. Assertion (A):** The variation in gain of the system does not alter the phase angle plot in the Bode diagram.
**Reason (R):** The phase margin of the system is not affected by the variation in gain of the system.

**[EE ESE - 2005]**

(a) Both A and R are true but R is the correct explanation of A.
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**116.** Match List-I with List-II and select the correct answer using the codes given below the lists :
**List-I**
A. Breakaway point
B. Phase margin
C. Gain Margin
D. Second order system
**List-II**
(i) Stable
(ii) Phase cross - over frequency
(iii) Gain cross - over frequency
(iv) Root locus

**[EE ESE - 2005]**

**117.** Encirclement of origin of 1 + G(s) plane corresponds to encirclement of a point in the –1 + G(s) plane, given by

**[EE ESE - 2005]**

(a) 1 + j0 (b) 0 + j0
(c) – 2 + j0 (d) – 1 + j0

**118.** The constant M – circles corresponding to the magnitude (M) of the closed loop transfer function of a linear system for values of M greater than one lie in the G – plane and to the

**[EE ESE - 2005]**

(a) Right of the M = 1 line
(b) Left of the M = 1 line
(c) Upper side of the M = ± j1 line
(d) Lower side of the M = – j1 line

**119.** Match List-I (Nyquist Plot of Loop Transfer Function of a Control System) with List-II (Gain Margin in dB) and select the correct answer using the code given below the lists :
**List-I**
A. Does not intersect negative real axis
B.Intersects the negative real axis between 0 and (– 1, j0)
C.Passes through (–1, j0)
D.Encloses (–1, j0)
**List-II**
(i) > 0
(ii) ∞
(iii) < 0
(iv) 0

**[EE ESE - 2005]**

**Codes:**
(a) A-ii, B-iv, C-i, D-iii
(b) A-iii, B-i, C-iv, D-ii
(c) A-ii, B-i, C-iv, D-iii
(d) A-iii, B-iv, C-i, D-ii

**120.** Match List-I (Plot/Mode) with List-II (Related parameter) and select the correct answer using the codes given below:
**List-I**
A. Root locus plot
B. Bode plot
C. Nyquist plot
D. Signal flow chart
**List-II**
(i) Corner frequency
(ii) Breakway point
(iii) Critical point
(iv) Transmittance

**[EE ESE - 2004]**

**Codes:**
(a) A-iv, B-iii, C-i, D-ii
(b) A-iv, B-i, C-iii, D-ii
(c) A-ii, B-iii, C-i, D-iv
(d) A-ii, B-i, C-iii, D-iv

**121.** For a unity feedback system, the origin of the s-plane is mapped in the z-plane by transformation $z=e^{sT}$ to which one of the following

**[EE ESE - 2004]**

(a) Origin (b) 1 + j0
(c) – 1 + j0 (d) 1 + j1

**122.** Consider the following statements for a counterclockwise Nyquist path

(i) For a stable loop system, the Nyquist plot of G(s) H(s) should encircle (– 1, j0) point as many times as there are poles of G(s) H(s) in the right half of the s-plane, the encirclements, if there are any must be made in the counter-clockwise direction.

(ii) If the loop gain function G(s) H(s) is a stable function, the closed loop system is always stable.

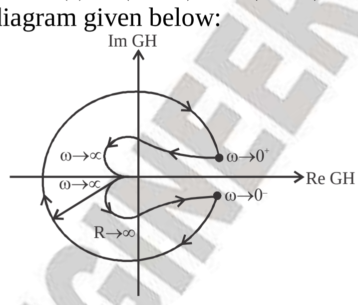
(iii) If the loop gain function G(s) H(s) is stable function, for a stable closed- loop system, the Nyquist plot of G(s) H(s) must not enclose the critical point (– 1 , j0)

Which of these statements is/are correct?

**[EE ESE - 2004]**

(a) Only i (b) i and ii
(c) i and iii (d) Only iii

**123.** Consider the following Nyquist plot of a feedback system having open loop transfer function $GH(s) = (s + 1)/ [s^2 (s - 2)]$ as shown in the diagram given below:




What is the number of closed loop poles in the right half of the s – plane?

**[EE ESE - 2004]**

(a) 0 (b) 1
(c) 2 (d) 3

**124.** A minimum phase unity feedback system has a Bode plot with a constant slope of –20 dB/decade for all frequencies. What is the value of maximum phase margin for the system ?

**[EE ESE - 2004]**

(a) 0° (b) 90°
(c) – 90° (d) 180°

**125.** The Nyquist plot for the closed-loop control system with the loop transfer function $G(s)H(s) = \frac{100}{s(s+10)}$ is plotted. Then, the critical point ( – 1, j0) is

**[EE ESE - 2004]**

(a) Never enclosed
(b) Enclosed under certain conditions
(c) Just touched
(d) Enclosed

**126.** A unity feedback control system has a forward loop transfer function as $\frac{e^{-Ts}}{[s(s+1)]}$. Its phase value will be zero at frequency $\omega_1$. Which one of the following equations should be satisfied by $\omega_1$

**[EE ESE - 2004]**

(a) $\omega_1 = \cot(T\omega_1)$ (b) $\omega_1 = \tan(T\omega_1)$
(c) $T\omega_1 = \cot(\omega_1)$ (d) $T\omega_1 = \tan(\omega_1)$

**127.** Match List-I (Type of plots) with List-II (Functions) and select the correct answer using the codes given below:

**List-I**
A. Bode plots
B. Polar plots
C. Nyquist plots
D. Nichols chart

**List-II**
(i)Open loop response due to damped sinusoidal inputs as a function of complex frequency
(ii)Open loop response due to undamped sinusoidal as a function of real frequency
(iii)Closed loop response due to sinusoidal inputs as a function of real frequency
(iv)Open loop magnitude and phase angle responses for undamped sinusoidal inputs

plotted separately as a function of real frequency.

**[EE ESE - 2004]**

**Codes:**
(a) A-ii, B-iv, C-iii, D-i
(b) A-ii, B-iv, C-i, D-iii
(c) A-iv, B-ii, C-iii, D-i
(d) A-iv, B-ii, C-i, D-iii

**128.** Which one of the following statements is correct in the respect of the theory of stability?

**[EC ESE - 2004]**

(a)Phase margin is the phase angle lagging, in short of 180°, at the frequency corresponding to a gain of 10.
(b)Gain margin is the value by which the gain falls short of unity, at a frequency corresponding to 90° phase lag.
(c)Routh-Hurwitz criterion can determine the degree of stability.
(d)Gain margin and phase margin are the measure of the degree of stability.

**129.** A tachometer feedback is used as an inner loop in a position control servo-system. What is the effect of feedback on the gain, of the sub-loop incorporating tachometer and on the effective time constant of the system?

**[EC ESE - 2004]**

(a) Both are reduced
(b) Gain is reduced but the time constant is increased.
(c) Gain is increased but the time constant is reduced.
(d) Both are increased.

**130. Assertion (A)**: The bandwidth of a control system indicates the noise filtering characteristic of the system.
**Reason (R)**: The bandwidth is a measure of ability of a control system to reproduce the input signal.

**[EC ESE - 2004]**

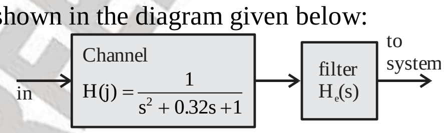
(a) Both A and R are true and r is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

**131. Assertion(A)**: The error detector in a position control system using synchro pairs employs synchro transmitter for reference signal and synchro control transformer for the feed back signal.
**Reason (R)**: Synchro control transformer rotor has a uniform magnetic reluctance.

**[EC ESE - 2004]**

(a) Both A and R are true and r is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

**132.** A filter at the input to a processing system is shown in the diagram given below:




The channel works for toll quality telephone use. If the filter $H_e$ (s) is to be designed so that linear distortion is minimized, then $H_e(s)$ should have which one of the following?

**[EC ESE - 2004]**

(a)Constant delay
(b)Constant phase
(c)Inverse relationship with H(s)
(d)Inverse relationship with H(s) and constant delay.

**133.** Which one of the following statements is correct?
The effects of phase lead compensator on gain cross-over frequency ($\omega_{cg}$) and the bandwidth (BW) are

**[EC ESE - 2004]**

(a) That both are decreased
(b) That $\omega_{cg}$ is decreased but BW is increased
(c) That $\omega_{cg}$ is increased but BW is decreased
(d) That both are increased

**134.** Which one of the following statements is correct?

**[EC ESE - 2004]**

(a) Phase margin remains the same
(b) Phase margin increases
(c) Phase margin decreases
(d) Gain margin increases

**135.** What is the value of M for the constant M circle represented by the equation.
$8x^2 + 18x + 8y^2 + 9 = 0$, where x = Re |G(jω)| and y = lm |G(jω)|?

**[EC ESE - 2004]**

(a) 0.5 (b) 2
(c) 3 (d) 8

**136.** All the constant N-circles in G-planes cross the real axis at the fixed points. Which are these points?

**[EC ESE - 2004]**

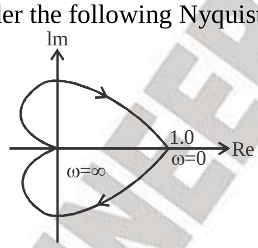
(a) –1 and origin (b) Origin and +1
(c) –0.5and + 0.5 (d) –1 and + 1

**137.** The forward path transfer function of a unity feedback system is given by $G(s) = \frac{1}{(1+s)^2}$. What is the phase margin for this system?

**[EC ESE - 2004]**

(a) $-\pi$ rad (b) 0 rad
(c) $\pi/2$ rad (d) $\pi$ rad

**138.** Consider the following Nyquist plot:




With which one of the following transfer function, does the above Nyquist plot match?

**[EC ESE - 2004]**

(a) $\frac{1}{(s+1)^3}$ (b) $\frac{1}{(s+1)^2}$
(c) $\frac{1}{(s^2+2s+2)}$ (d) $\frac{1}{(s+1)}$

**139.** The forward path transfer function of a unity feedback system is given by

$$G(s) = \frac{100}{s^2 + 10s + 100}$$

The frequency response of this system will exhibit the resonance peak at

**[EC ESE - 2004]**

(a) 10 rad/s (b) 8.66 rad/s
(c) 7.07 rad/s (d) 5 rad/s

**140.** Constant M circles have their center and radius as

**[EC ESE - 2003]**

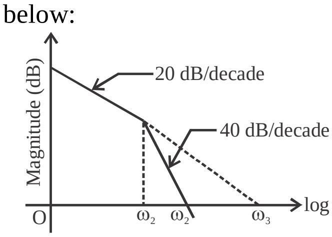
(a) $\left(\frac{-M^2}{M^2-1}, 0\right)$ and $\left(\frac{M^2}{M^2-1}\right)$

(b) $\left(\frac{-M^2}{M^2-1}, 0\right)$ and $\left(\frac{M}{M^2-1}\right)$

(c) $\left(0, \frac{-M^2}{M^2-1}\right)$ and $\left(\frac{M^2}{M^2-1}\right)$

(d) $\left(0, \frac{-M^2}{M^2-1}\right)$ and $\left(\frac{M}{M^2-1}\right)$

**141.** Consider the following statements regarding the frequency response of a system as shown below:



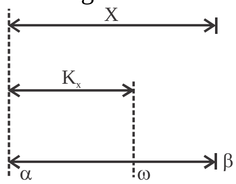

1. The type of the system is one.
2. $\omega_3$ = static error coefficient (numerically)
3. $\omega_2 = \frac{\omega_1 + \omega_3}{2}$

Select the correct answer using the codes given below:

**[EC ESE - 2003]**

(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) 1 and 3

**142.** The frequency ω in terms of frequencies α and β in the above figure is



Frequency in log scale

**[EC ESE - 2003]**

(a) K(logα – logβ)
(b) $(\beta/\alpha)^K$
(c) $\alpha^K.\ \beta^{K-1}$
(d) $\alpha^{1-K}\ .\ \beta^K$

**143.** The phase margin (PM) and the damping ratio (ξ) are related by

**[EC ESE - 2003]**

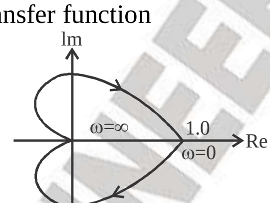
(a) $PM = 90° - \tan^{-1}\left\{\frac{\sqrt{-2\xi^2+\sqrt{1+4\xi^4}}}{2}\right\}$

(b) $PM = \tan^{-1}\left\{\frac{2\xi}{-2\xi^2+\sqrt{1+4\xi^4}}\right\}$

(c) $PM = 90° + \tan^{-1}\left\{\frac{\sqrt{2\xi^2+\sqrt{1+4\xi^4}}}{2\xi}\right\}$

(d) $PM = 180° - \tan^{-1}\left[\frac{\sqrt{2\xi^2-\sqrt{1-4\xi^4}}}{2}\right]$

**144.** The Nyquist plot shown below, matches with the transfer function




**[EC ESE - 2003]**

(a) $\frac{1}{(s+1)^3}$
(b) $\frac{1}{(s+1)^2}$
(c) $\frac{1}{(s^2+2s+2)}$
(d) $\frac{1}{(s+1)}$

**145.** Consider the following open loop frequency response of a unity feedback system.

| **ω.rad/s→:** | 2 | 3 | 4 | 5 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|---|
| **\|G(jω)\|→:** | 7.5 | 4.8 | 3.15 | 2.25 | 1.70 | 1.00 | 0.64 |
| **∠G(jω)→:** | -118° | -130° | -140° | -150° | -157° | -170° | -180° |

The gain and phase margin of the system are respectively.

**[EC ESE - 2003]**

(a) 0.00 dB, –180°
(b) 3.86 dB, –180°
(c) 0.00 dB, –10°
(d) 3.86 dB, 10°

**146.** Consider the following techniques:
1. Bode plot
2. Nyquist plot
3. Nichol's chart
4. Routh-Hurwitz criterion

Which of these techniques are used to determine relative stability of a closed loop linear system?

**[EC ESE - 2003]**

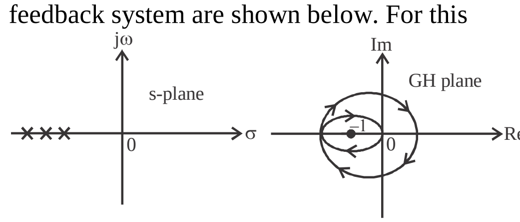
(a) 1 and 2
(b) 1 and 4
(c) 1, 2 and 3
(d) 2, 3 and 4

**147.** Which one of the following techniques is utilized to determine the actual point at which the root locus crosses the imaginary axis?

**[EC ESE - 2003]**

(a) Nyquist technique
(b) Routh-Hurwitz criterion
(c) Nichol's criterion
(d) Bode technique

**148.** The pole – zero map and the Nyquist plot of the loop transfer function GH(s) of a feedback system are shown below. For this

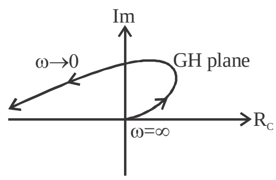



**[EE ESE - 2003]**

(a)Both open loop and closed loop system are stable.
(b)Open loop system is stable but closed loop system is unstable.
(c)Open loop system is unstable but closed loop system is stale.

(d)Both open loop and closed loop systems are unstable

**149.** The Nyquist plot of a control system is shown below. For this system, G(s) H(s) is equal to




**[EE ESE - 2003]**

(a) $\frac{K}{s(1+sT_1)}$

(b) $\frac{K}{s^2(1+sT_1)}$

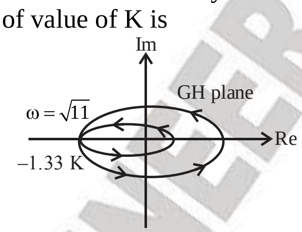
(c) $\frac{K}{s^3(1+sT_1)}$

(d) $\frac{K}{s^2(1+sT_1)\times(1+sT_1)}$

**150.** The Nyquist plot of a unity feedback system having open loop transfer function

$$G(s)=\frac{K(s+3)(s+5)}{(s-2)(s-4)}$$

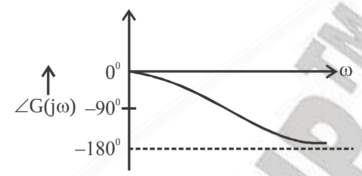
is as shown below. For the system to be stable, the range of value of K is




**[EE ESE - 2003]**

(a) 0 < K < 1.33
(b) 0 < K < 1/1.33
(c) K > 1.33
(d) K > 1/1.33

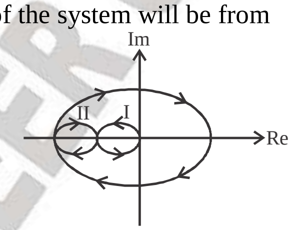
**151.** The Bode phase angle plot of a system is shown below. The type of the system is




**[EE ESE - 2003]**

(a) 0
(b) 1
(c) 2
(d) 3

**152.** Consider the Nyquist diagram for given KG(s)H(s) . The transfer function KG(s)H(s) has no poles and zeros in the right half of s – plane. If the (– 1, j0) point is located first in region I and then in region II, the change in stability of the system will be from




**[EE ESE - 2002]**

(a) Unstable to stable
(b) Stable to stable
(c) Unstable to unstable
(d) Stable to unstable

**153.** List-I and List-II show the transfer function and polar plots respectively. Match List-I with List-II and select the correct answer:

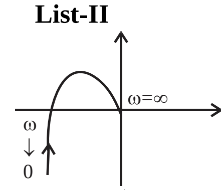
**List-I**

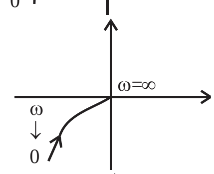
A. $\frac{1}{s(1+sT)}$

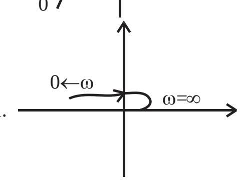
B. $\frac{1}{(1+sT_1)(1+sT_2)}$

C. $\frac{1}{s(1+sT_1)(1+sT_2)}$

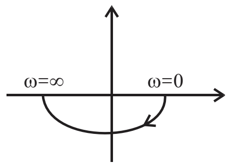
**List-II**

i. 


ii. 


iii. 

D. $\frac{1}{s^2(1+sT_1)(1+sT_2)}$ iv.




**[EE ESE - 2002]**

**Codes:**

(a) A-ii, B-i, C-iv, D-iii
(b) A-iii, B-iv, C-i, D-ii
(c) A-ii, B-iv, C-i, D-iii
(d) A-iii, B-i, C-iv, D-ii

**154. Assertion (A)**: $G(s)=\frac{10(s-25)}{s(s+1)(s+5)}$ represents a non-minimum phase transfer function.

**Reason (R)**: A minimum phase transfer function has the property that its magnitude and phase are uniquely related.

**[EC ESE - 2002]**

(a) Both A and R are true and r is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

**155.** The constant N loci represented by the equation $x^2 + x + y^2 = 0$ where $x = \text{Re}\,|G(j\omega)|$ and $y = \text{lm}\,|G(j\omega)|$ is for the value of phase angle equal to

**[EC ESE - 2002]**

(a) –45° (b) 0°
(c) +45° (d) +90°

**156.** Consider the following performance characteristics:
1. Reduced velocity constant for a given relative stability
2. Reduced gain cross-over frequency
3. Reduced bandwidth
4. Reduced resonance peak of the system
Which of these performance characteristic are achieved with the phase-lag compensation?

**[EC ESE - 2002]**

(a) 1 and 2 (b) 1 and 3
(c) 2, 3 and 4 (d) 1, 2, 3 and 4

**157.** Consider the following statements associated will phase and gain margins:
1.They are a measure of closeness of the polar plot to the –1 + j0 point.
2.For a non–minimum phase to be stable it must have positive phase and gain margins.
3.For a minimum phase system to be stable, both the margins must be positive.
Which of the above statements is/are correct?

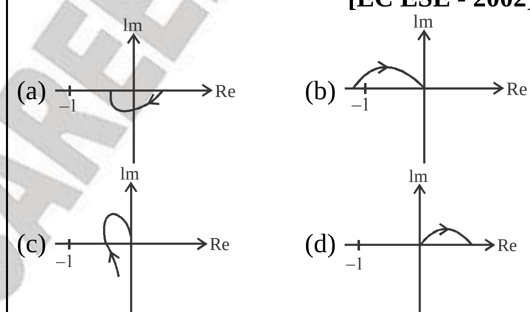
**[EC ESE - 2002]**

(a) 2 and 3 (b) 1 and 3
(c) 1 and 2 (d) 1 alone

**158.** Which of the following is the Nyquist diagram for the open loop function.

$$G(s).H(s)=\frac{5}{s(1+0.1s)(1+0.01s)}?$$

**[EC ESE - 2002]**




**159.** Consider the following statements:
Nichol's chart gives information about
1.Closed loop frequency response
2.The value of the peak magnitude of the closed loop frequency response $M_p$
3.The frequency at which $M_p$ occurs
Which of the following above statements are correct?

**[EC ESE - 2002]**

(a) 2 and 3 (b) 1 and 2
(c) 1 and 3 (d) 1, 2 and 3

**160.** The Nyquist plot of

$$G(s).H(s)=\frac{10}{s^2(1+0.5s).(1+s)}$$

**[EC ESE - 2002]**

(a) Will start (ω = ∞) in the first quadrant and will terminate (ω = 0) in the second quadrant

(b) Will start (ω = ∞) in the fourth quadrant and will terminate (ω = 0) in the second quadrant.
(c) Will start (ω = ∞) in the second quadrant and will terminate (ω = 0) in the third quadrant
(d) Will start (ω = ∞) in the first quadrant and will terminate (ω = 0) in the fourth quadrant.

**161. Assertion (A)**: The stator winding of a control transformer has higher impedance per phase.
**Reason (R)**: The rotor of control transformer is cylindrical in shape.

**[EC ESE - 2001]**

(a) Both A and R are true and r is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

**162.** The constant M-circle is represented by the equation $x^2 + 2.25x + y^2 = 1.125$ where x = Re [G(jω)] and y = lm [G(jω)] has the value of M equal to

**[EC ESE - 2001]**

(a) 1 (b) 2
(c) 3 (d) 4

**163.** A constant N-circle having center at (–1/2 + j0) in the G-plane, represents the phase angle equal to.

**[EC ESE - 2001]**

(a) 180° (b) 90°
(c) 45° (d) 0°

**164.** An open loop transfer function of a unity feedback control system has two finite zeros, two poles at origin and two pairs of complex conjugate poles. The slope high frequency asymptote in Bode magnitude plot will be

**[EC ESE - 2001]**

(a) +40 dB/decade (b) 0 dB/decade
(c) –40 dB/decade (d) –80 dB/decade

**165.** The open-loop transfer function of a unity feedback control system is given as

$$G(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$$

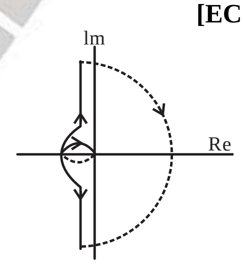
The phase crossover frequency and the gain margin are, respectively (Correct the options)

**[EC ESE - 2001]**

(a) $\frac{1}{\sqrt{T_1T_2}}$ and $\frac{T_1+T_2}{T_1T_2}$
(b) $\sqrt{T_1T_2}$ and $\frac{T_1+T_2}{T_1T_2}$
(c) $\frac{1}{\sqrt{T_1T_2}}$ and $\frac{T_1T_2}{T_1+T_2}$
(d) $\sqrt{T_1T_2}$ and $\frac{T_1T_2}{T_1+T_2}$

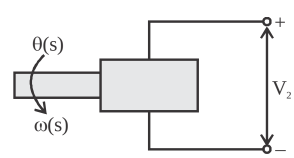
**166.** Nyquist plot shown in the given figure is for a type

**[EC ESE - 2001]**




(a) Zero system (b) One system
(c) Two system (d) Three system

**167.** Which one of the following relations holds good for the tachometer shown in the given figure?




**[EC ESE - 2001]**

(a) $V_2(s) = sk_1\omega(s)$ (b) $V_2(s) = k_ts^2\theta(s)$
(c) $V_2(s) = k_ts^2\omega(s)$ (d) $V_2(s) = k_ts\theta(s)$

**168.** The open loop transfer function of a system is

$$G(s)H(s) = \frac{K}{(1+s)(1+2s)(1+3s)}$$

The phase crossover frequency $\omega_{pc}$ is

**[EC ESE - 2001]**

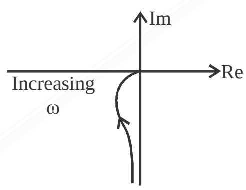
(a) $\sqrt{2}$ (b) 1
(c) Zero (d) $\sqrt{3}$

**169.** In the Nyquist lot cuts the negative real axis at a distance of 0.4, then the gain margin of the system is

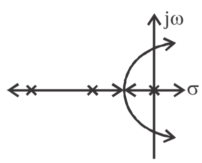
**[EE ESE - 2001]**

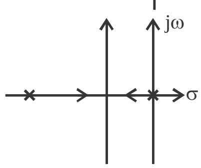
(a) 0.4 (b) – 0.4
(c) 4 (d) 2.5

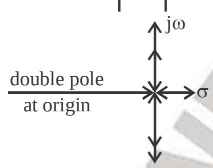
**170.** The Nyquist plot of a servo system is shown in the figure. The root loci for the system would be




**[EE ESE - 2001]**

(a)


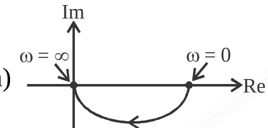

(b)


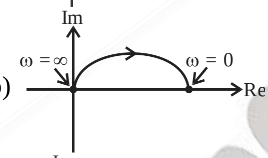

(c)


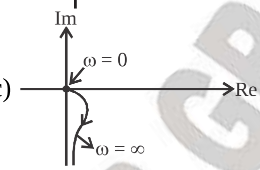

(d) None of the drawn plot of (a), (b), (c) of the question.

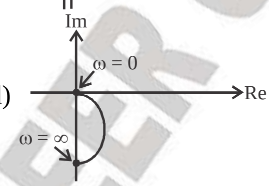
**171.** The transfer function of a certain system is given by $G(s)=\frac{s}{(1+s)}$. The Nyquist plot of the system is

**[EE ESE - 2001]**

(a)



(b)



(c)



(d)



**172.** The phase angle of the system $G(s)=\frac{s+5}{s^2+4s+9}$, varies between

**[EE ESE - 2001]**

(a) 0° and 90° (b) 0° and –90°
(c) 0° and – 180° (d) –90° and 1– 180°

**173.** Which one of the following features is NOT associated with Nichols chart?

**[EC ESE - 2000]**

(a) (0 dB, –180°) point on Nichols charts represents the critical point (–1 + j0)
(b) It is symmetric about –180°
(c) The M loci are centered about (0, dB, –180°) point
(d) The frequency at the intersection of the G(jω) locus and M = +3 dB locus gives bandwidth of the closed-loop system.

**174.** Which one of the following equations represents the constant magnitude locus in G-plane for M = 1?
[x-axis is Re G(jω) and y-axis is lm G(jω)]

**[EC ESE - 2000]**

(a) x = –0.5 (b) x = 0
(c) $x^2 + y^2 = 1$ (d) $(x + 1)^2 + y^2 = 0$

**175.** The polar plot (for positive frequencies) for the open-loop transfer function of a unity feedback control system is shown in the given figure.

The phase margin and the gain margin of the system are respectively:

**[EC ESE - 2000]**

(a) 150° and 4 (b) 150° and 3/4
(c) 30° and 4 (d) 30° and 3/4

**176.** The open-loop transfer function G(s) of a unity feedback control system is $\frac{1}{s(s+1)}$. The system is subjected to an input r(t) = sin t. The steady state error will be

**[EC ESE - 2000]**

(a) Zero (b) 1
(c) $\sqrt{2}\sin\left(1-\frac{\pi}{4}\right)$ (d) $\sqrt{2}\sin\left(1+\frac{\pi}{4}\right)$

**177.** Match List-I (Scientist) with List-II (Contribution in the area of) and select the correct:

**List-I**
A. Bode
B. Evans
C. Nyquist

**List-II**
(i) Asymptotic plots
(ii) Polar plots
(iii) Root-locus technique
(iv) Constant M and N plots

**[EC ESE - 2000]**

**Codes**:
(a) A-i, B-iv, C-ii
(b) A-ii, B-iii, C-iv
(c) A-iii, B-i, C-iv
(d) A-i, B-iii, C-ii

**178.** Consider the following servomotors:
1. AC two-phase servomotor
2. DC servomotor
3. Hydraulic servomotor
4. Pneumatic servomotor

The correct sequence of these servomotors in increasing order of power handling capacity is

**[EC ESE - 2000]**

(a) 2, 4, 3, 1 (b) 4, 2, 3, 1
(c) 2, 4, 1, 3 (d) 4, 2, 1, 3

**179.** Match List-I (Functional components) with List-II (Devices) and select the correct answer:

**List-I**
A. Error detector
B. Servometer
C. Amplifier
D. Feedback

**List-II**
(i) Three-phase FHP induction motor
(ii) A pair of synchronous transmitter and control transformer
(iii) Tachogenerator
(iv) Armature controlled FHP DC motor
(v) Option not found

**[EC ESE - 2000]**

**Codes**:
(a) A-ii, B-iv, C-i, D-v
(b) A-iv, B-ii, C-v, D-iii
(c) A-ii, B-iv, C-v, D-iii
(d) A-i, B-ii, C-iii, D-v

**180. Assertion (A)**: The largest undershoot corresponding to a unit step input to an underdamped second order system with damping ratio ξ and undamped natural frequency of oscillation $\omega_n$ is $e^{-2\xi\pi\sqrt{1-\xi^2}}$.

**Reason (R)**: The overshoots and undershoots of a second order underdamped system is $e^{-\xi n\pi/\sqrt{1-\xi^2}}$, n = 1, 2, …..

**[EC ESE-2000]**

(a) Both A and R are true and r is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

**181. Assertion (A)**: An on-off controller gives rise to imaginary axis gives rise to self-sustained oscillation in the output.

**Reason (R)**: Location of a pair of poles on the imaginary axis gives rise to self-sustained oscillation in the output.

**[EC ESE-2000]**

(a) Both A and R are true and r is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

**182. Assertion (A)**: Synchronous counter has high speed of operation than ripple counter.
**Reason (R)**: Synchronous counter uses high speed flip-flops.

**[EC ESE - 2000]**

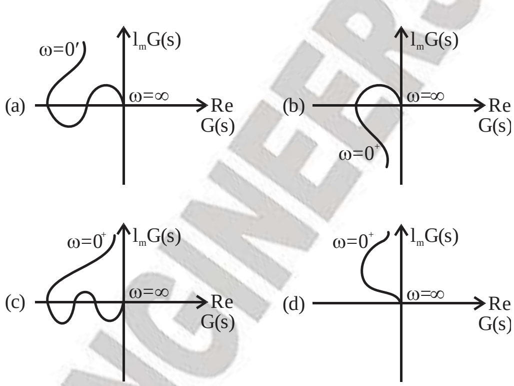
(a)Both A and R are true and r is the correct explanation of A
(b)Both A and R are true but R is NOT the correct explanation of A
(c)A is true but R is false
(d)A is false but R is true.

**183.** The open-loop transfer function of a unity negative feedback system is

$$G(s) = \frac{K(s+10)(s+20)}{s^3(s+100)(s+200)}$$

The polar plot of the system will be

**[EC ESE - 1999]**

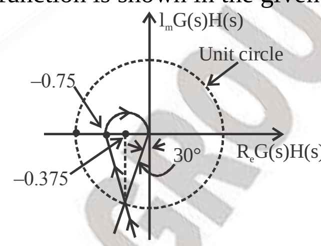



**184.** The polar plot of a transfer function passes through the critical point (–1, 0). The gain margin is

**[EC ESE - 1999]**

(a) Zero
(b) –1 dB
(c) 1 dB
(d) Infinity

**185.** A portion of the polar of an open-loop transfer function is shown in the given figure.




The phase margin and gain margin will be respectively.

**[EC ESE - 1999]**

(a) 30° and 0.75
(b) 60° and 0.375
(c) 60° and 0.75
(d) 60° and 1/0.75

**186.** The constant –M circle for M = 1 is the

**[EC ESE - 1999]**

(a) Straight line; x = –1/2
(b) Critical point (–1, j0)
(c) Circle with r = 0.33
(d) Circle with r = 0.67

**187.** The radius of constant-N circle for N = 1 is

**[EC ESE - 1999]**

(a) 2
(b) $\sqrt{2}$
(c) 1
(d) $1/\sqrt{2}$

**188. Assertion (A)**: The phase angle plot in Bode diagram is not affected by the variation in the gain of the system.
**Reason (R)**: The variation in the gain of the system has no effect on the phase margin of the system.

**[EC ESE - 1999]**

(a) Both A and R are true and r is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

# SOLUTIONS

**Sol.1. (a)**

**Sol.2. (b)**
For open loop system no of poles in right half of s plane (P) = 1
$n = p^+ - z^+$
For stability $Z^+ = 0$
N = P = 1

**Sol.3. (b)**
The T.F. of given Bode plot.

$$\text{T.F.} = \frac{k_1\left(\frac{S}{20}+1\right)}{S\left(\frac{S}{2}+1\right)} = \frac{k(s+20)}{s(s+2)}$$

**Sol.4. (c)**

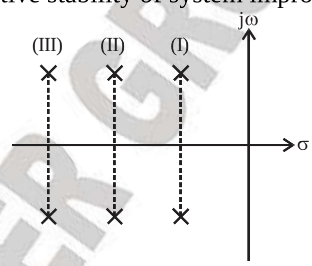
$$\text{T.F.} = \frac{ks^2}{\left(\frac{S}{10}+1\right)^5}$$

**Sol.5. (c)**
Low – frequency asymptote slope depends upon the poles or zeros at origin.
= (–20) × 2
= –40 dB/decade

**Sol.6. (d)**
From bode plot we can determine the open loop transfer function but to determine function but to determine the roots of closed – loop control system we have to know G(s) or H(s) separately. So, statement – I is wrong.

**Sol.7. (b)**
The slop of highest frequency asymptote
= (Z – P) × 20 dB/dec
= (2 – 14) × 20
= –240 dB/dec

**Sol.8. (c)**
Gain Margin and Phase margin of the system gives relative stability.
Relative stability is analysis of how fast transient has died out in the system. If we moves away from jω axis in left half of s plane then relative stability of system improves.




(iii) is relatively more stable to (ii)
(ii) is relatively more stable to (i).

**Sol.9. (d)**

**Sol.10. (a)**

$$G(s)\,H(s) = \frac{2K}{s(s+1)(s+5)}$$

For marginal stability we need to find frequency of sustained oscillation.
If G(s) H(s) ⇒ s(s + 1) (s + 5) + 2k = 0
⇒ $s^3 + 6s^2 + 5s + 2k = 0$
Now from Rough Huswitz criteria

| | | |
|---|---|---|
| $S^3$ | 1 | 5 |
| $S^2$ | 6 | 2K |
| $S^1$ | $\frac{30-2k}{6}$ | |
| $S^0$ | 2k | |

So k = 15
Now we get that k = 15
So $6s^2 + 30 = 0$

$$\omega_{oscillation} = \sqrt{5}\ \text{rad/sec}$$

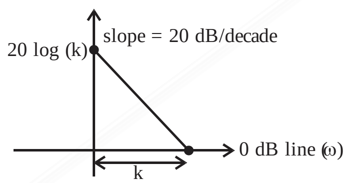
**Sol.11. (b)**

Gain margin is the factor by which the gain of system should be increased to drive it to marginally stable condition on drive it to oscillations.

**Sol.12. (b)**

**Sol.13. (b)**
For type-I system, the intersection of initial slope of bode plot with 0 dB axis give error constant

For example $\frac{k}{s(s+p)}$




**Sol.14. (b)**

$$GH(s) = \frac{25}{s(s+6)}$$

q(s) = 1 + GH(s) = $s^2 + 6s + 25 = 0$

$\omega_n = 5;\ \xi = 0.6$

$$\therefore W_r = \omega_n\sqrt{1-2\xi^2} = 2.67$$

$$\therefore M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = 1.04$$

**Sol.15. (d)**
All the mentioned plots are popular and commonly used in control analysis.

**Sol.16. (c)**

$$GH(s) = \frac{5(s^2+10S+100)}{S^2(S+15S+1)}$$

Corner frequency in Bode plot is defined for finite poles and zeros, which are complex in given system.
For complex pole, zeros corner frequency.
∴ Hence 10, 1.

**Sol.17. (a)**

$$GH(s) = \frac{K}{S(s+5)}$$

q(s) = 1 + GH(s) = $S^2 + 5S + K = 0$

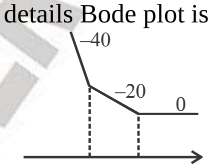
$$\omega_n = \sqrt{K};\ \xi = \frac{5}{2\sqrt{K}}$$

$$\therefore PM = 100\,\xi = 100 \times \frac{5}{2\sqrt{K}} = 45°$$

$\therefore K \approx 35$

**Sol.18. (a)**
GM and PM of unstable system are always negative.

**Sol.19. (b)**
As per given details Bode plot is



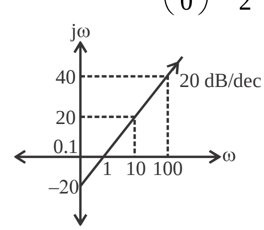

Initial slope indicates 2 poles at origin
Final slope indicates
P = Z
∴ P = Z = 2

**Sol.20. (a)**
Nichols chart provides complete frequency response of a system.

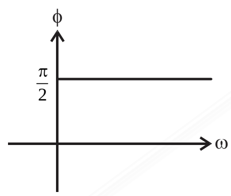
**Sol.21. (c)**

**Sol.22. (d)**
G(s) = s

$$Mag = |j\omega| = \omega;\ \phi = \tan^{-1}\left(\frac{\omega}{0}\right) = \frac{\pi}{2}$$



| w | $M_{dB}$ | $\phi$ |
|---|---|---|
| 0.1 | –20 | $\frac{\pi}{2}$ |
| 1 | 0 | $\frac{\pi}{2}$ |
| 10 | 20 | $\frac{\pi}{2}$ |
| 100 | 40 | $\frac{\pi}{2}$ |




**Sol.23. (a)**

**Sol.24. (b)**
For ramp input applied second order system, the steady state error

$$= \frac{2\xi}{\omega_n}$$

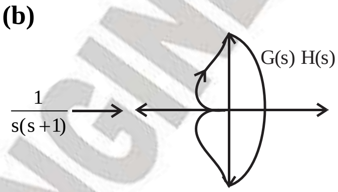
And also slope at one of corner frequency is = ± 40 dB/decade.

**Sol.25. (d)**

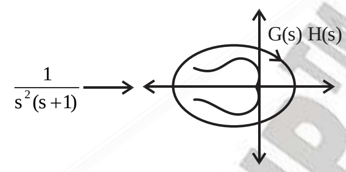
**Sol.26. (a)**

**Sol.27. (c)**
P. M. = $180^0 + \phi$
= $180^0 + (-125^0) = 55^0$

**Sol.28. (b)**

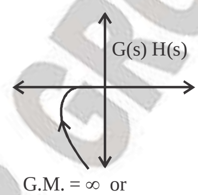


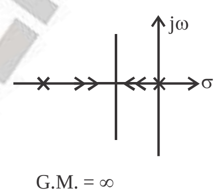

After adding pole at origin




So, nyquist plot of system will rotate by $90^0$ I clockwise direction.

**Sol.29. (d)**




i.e. we can increase gain K from 0 to ∞.

**Sol.30. (b)**

$$G(s)\,H(s) = \frac{K}{s^3 + 6s^2 + 11s + 6}$$

$$G(\omega)\,H(\omega) = \frac{L}{-6\omega^{-1} + 6 + 1(11\omega - \omega^2)}$$

Phase crossover frequency is obtained by equation |G(ω) H(ω) = –180°

$$-180° = -\tan^{-1}\left[\frac{11\omega - \omega^3}{6 - 6\omega^2}\right]$$

$$\frac{11\omega - \omega^3}{6 - 6\omega} = 0$$

$11\omega - \omega^3 = 0$

$\omega = \pm\sqrt{11}$

But frequency can't be negative so $\omega = \sqrt{11}$

**Sol.31. (*)**

**Sol.32. (*)**

**Sol.33. (*)**

**Sol.34. (*)**

**Sol.35. (*)**

**Sol.36. (*)**

**Sol.37. (a)**

Number of encirclements of origin in the clockwise direction = Z – P = 3 – 2 = 1.

**Sol.38. (b)**

$$G_c(s) = \frac{5(1+0.3s)}{1+0.1s}$$

The two corner frequencies are

$\omega = \frac{1}{0.3}$ lower corner frequency

$\omega = \frac{1}{0.1}$ upper corner frequency

The maximum phase lead $\phi_m$ occurs at mid frequency $\omega_m$.

$$\omega_m = \sqrt{\omega_1\omega_2} = \sqrt{\frac{1}{0.3} \times \frac{1}{0.1}}$$

$$\Rightarrow \omega_m = \frac{10}{\sqrt{3}}$$

$$\therefore \phi_m = \tan^{-1}(0.3\omega_m) - \tan^{-1}(0.1\omega_m)$$

$$= \tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}} = 30^0$$

**Sol.39. (c)**

**Sol.40. (b)**

$$G(s)H(s) = \frac{K}{s(1+0.2s)(1+0.5s)}$$

$$G(j\omega)H(j\omega) = \frac{K \times 5 \times 20}{j\omega(5+j\omega)(20+j\omega)}$$

$$= \frac{100K}{j\omega(-\omega^2 + 25j\omega + 100)}$$

$$= \frac{100K}{-j\omega^3 - 25\omega^2 + 100j\omega}$$

at $\omega = \omega_{pc}$

$\Rightarrow 100\omega_{pc} - \omega^3_{pc} = 0$

$\Rightarrow \omega^2_{pc} = 100$

$\Rightarrow \omega_{pc} = 10$ r/s

**Sol.41. (b)**

$$G(s) = \frac{K}{3+10K+K^2}$$

The range of K is calculated through Routh array using the characteristic equation 1 + G(s) H(s) = 0.

$$1 + \frac{K}{s(s+1)(s+2)} = 0$$

$s^3 + 3s^2 + 2s + K = 0$

| | | |
|---|---|---|
| $s^3$ | 1 | 2 |
| $s^2$ | 3 | K |
| $s^1$ | $\frac{3\times 2 - K}{3}$ | |
| $s^0$ | $\frac{\left(\frac{6-K}{3}\right)\times K}{\left(\frac{6-K}{3}\right)}$ | |

For the stable system first column values of the Routh array should always be greater than zero, thus,

6 – K > 0 ⇒ K < 6

also K >

Now the range is 0 < K < 6

**Sol.42. (c)**

$$T = \frac{(1+2K)}{(3+4K)}$$

Sensitivity,

$$S_K^T = \frac{\partial T/T}{\partial K/K} = \frac{\partial T}{\partial K}.\frac{K}{T}$$

$$\frac{\partial T}{\partial K} = \frac{2(3+4K) - 1(1+2K).4}{(3+4K)^2} = \frac{2}{(3+4K)}$$

$$S_K^T = \frac{2}{(3+4K)^2}.\frac{K}{(1+2K)}.(3+4K)$$

$$= \frac{2K}{3+10K+K^2}$$

**Sol.43. (a)**

$$G(s) = \frac{(2s+5)}{(s+5)(s+4)}$$

The dc gain is always calculated in ($1 \pm T_s$) form i.e. time constant form.

$$\text{So } G(s) = \frac{5\left(1+\frac{2}{5}s\right)}{455(1+0.2s)(1+0.25s)}$$

$$= 0.25\frac{(1+0.4s)}{(1+0.2s)(1+25s)}$$

So dc gain is calculated at s = 0

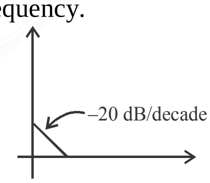
G(s) = 0.25

**Sol.44. (d)**

The type-1 system is given as

$$G(s)H(s) = \frac{K}{s(s+\omega_1)}$$

Since this represents a slope of –20 dB/decade at the low frequency.




**Sol.45. (a)**

$$G(s)H(s) = \frac{1}{s(s+1)}$$

Condition to calculate gain cross-over frequency is

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 0$$

$$\left|\frac{1}{\omega\sqrt{\omega^2+1}}\right| = 1$$

$\therefore$ ω = 0.768 ≈ 1 rad/sec

$\therefore$ angle $\phi = -90° - \tan^{-1}\omega = 135°$

Phase margin is calculated as 180 + ϕ = 45°

**Sol.46. (b)**

To calculate the gain, we need to first calculate the phase cross-over frequency.

$$G(s)H(s) = \frac{10}{(s+5)^3} = \frac{10}{(j\omega+5)^3}$$

$$\text{So } |\angle G(s)H(s)|_{\omega=\omega_{pc}} = -180°$$

$$\Rightarrow -3\tan^{-1}\frac{\omega}{5} = -180°$$

$$\therefore \quad \omega_{pc} = 5\sqrt{3}\text{ rad/sec}$$

$$X = |G(j\omega)H(j\omega)|_{\omega=5\sqrt{3}}$$

$$= \frac{1}{\left(\sqrt{\omega^2+5^2}\right)^3} = 0.01$$

$$\therefore \text{ Gain margin} = 20\log\frac{1}{X} = 40\text{ dB}.$$

**Sol.47. (a)**

The N circles are always drawn between –1 and origin for different values of N.

**Sol.48. (a)**

When M = 1 is put in the magnitude equation i.e.

$x^2(M^2-1) + 2x\,M^2 + Y^2(M^2-1) + M^2 = 0$

2x + 1 = 0

Thus, it is a straight line x = 0.5.

**Sol.49. (c)**

Let the open-loop transfer function is

$$G(s)H(s) = \frac{K}{s(s+\omega_c)}$$

$$\angle G(s)H(s) = -90° - \tan^{-1}\frac{\omega}{\omega_c} \qquad \ldots\text{(i)}$$

Thus it can be seen from equation (i) that phase angle does not depend on the gain of the system.

**Statement 2:** Phase margin depends on the gain cross-over frequency $\omega_{gc}$ and $\omega_{gc}$ can be calculated as

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 1$$

$$\frac{K}{\omega\sqrt{\omega^2+\omega_c^2}} = 1$$

and PM = 180° + ϕ

$$\phi = -90° - \tan^{-1}\frac{\omega_{gc}}{\omega_c}$$

Thus ϕ depends on the $\omega_{gc}$ and $\omega_{gc}$ depends on the gain K. So variation in gain affects the phase margin.

**Sol.50. (a)**

As we know from the formulae

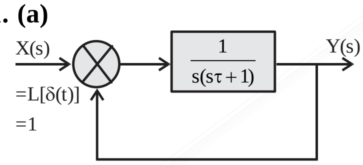
Rise time, $t_r = \frac{0.35}{\text{Bandwidth}}$

Thus it can be seen that rise time is inversely proportional to bandwidth.

Also $\omega_n \sqrt{1-\xi^2}$

Increasing $\omega_n$ causes increase in $\omega_d$ and thus bandwidth increase and rise time reduces.

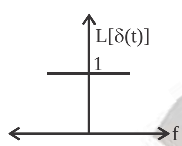
**Sol.51. (a)**




$$\frac{Y(s)}{X(s)} = H(s) = \frac{1}{s^2\tau + s + 1}$$

$$Y(s) = \frac{1}{\tau s^2 + s + 1}$$

Thus it can be seen from the equation that the system oscillates at natural frequency. Since the impulse response contains all the frequency components having frequency response as:




**Sol.52. (b)**

Consider an open loop transfer function G(s) H(s) as

$$G(s)H(s) = \frac{K(s+z_1)(s+z_2).....(s+z_m)}{(s+p_1)(s+p_2).....(s+p_n)}, m < n$$

The characteristic equation of the transfer function is given as:

1 + G(s) H(s) = 0 = q(s)

$$q(s) = 1 + \frac{k(s+z_1)(s+z_2)....(s+z_m)}{(s+p_1)(s+p_2)....(s+p_n)}$$

$$= \frac{(s+z_1')(s+z_2')....(s+z_n')}{(s+p_1)(s+p_2)....(s+p_n)}$$

Numerator of above equation determines closed loop poles because characteristic equation determine the closed loop poles. Denominator of above equation determines the open loop poles.

Observing the encirclement about origin for 1 + G(s) H(s) = 0 is same as the observing encirclement about –1 for G(s) H(s) . (i.e. open loop transfer function) to determine stability of a closed loop system.

**Sol.53. (d)**

Equation for N-circles is

$$\left[x+\frac{1}{2}\right]^2 + \left[y-\frac{1}{2N}\right]^2 = \frac{1}{4} + \left[\frac{1}{2N}\right]^2$$

Hence, Center = $\left(-\frac{1}{2}, \frac{1}{2N}\right)$

and, radius = $\sqrt{\frac{1}{4} + \left(\frac{1}{2N}\right)^2}$

**Sol.54. (b)**

System will be marginally stable if gain cross over frequency is equal to phase cross over frequency and gain margin is equal to phase margin also.

**Sol.55. (c)**

System will be stable, only when gain margin in dB and phase margin in degrees both are positive.

**Sol.56. (d)**

$$G(s) = \frac{10}{0.66s^2 + 2.33s + 1}$$

$$= \frac{10}{(s+0.5)(s+3.03)}$$

Hence corner frequencies are 0.5 and 3.03 rad/sec.

**Sol.57. (c)**

Transfer function = $\frac{10(s+1)}{(s+2)}$

$$\Rightarrow T(s) = \frac{10(s+1)}{(s+2)}$$

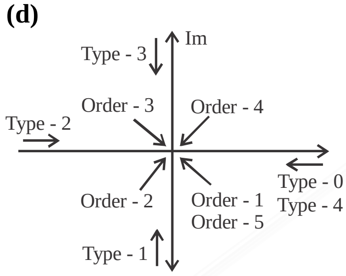
$$\Rightarrow T(j\omega) = \frac{10(1+j\omega)}{(2+j\omega)}$$

Phase of the T(jω) is given by

$\angle T(j\omega)|_{\omega=1} = \tan^{-1} - \tan^{-1}\dfrac{1}{2} > 0$

Hence voltage response of the system leads the current.

**Sol.58. (d)**




General shape

**Sol.59. (c)**
G(s) can be written as for frequency ω = 0.1 rad/sec. $G(s) = \dfrac{100}{8}$ as other corner frequencies are greater tha 0.1 rad/sec.

$|G(s)|_{s=j.01} = \dfrac{100}{j0.1}$

|G(j0.1) = 1000
Gain = 20 $\log_{10}$ 1000 = 60 dB
Here, option (c) is correct.

**Sol.60. (c)**

**Sol.61. (b)**
Characteristic equation
$s^2 + 2s + 100 = 0$
Comparing with
$s^2 + 2\xi\omega_n + \omega^2_n = 0$
Resonant frequency = $\sqrt{100} = 10$ rad / sec
BW = $2\xi\omega_n$ = 2 rad /sec

**Sol.62. (c)**
Damped frequency
$\omega_d = \sqrt{8}$ rad / s
BW = 2 rad/s
$\omega_d = \sqrt{\omega_n^2 - \alpha^2}$

Where, $\alpha = \dfrac{BW}{2}$

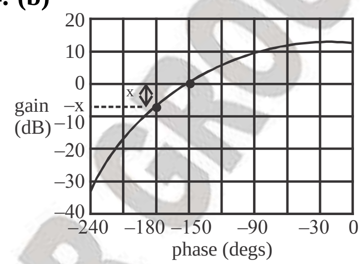
α = 1

$\sqrt{8} = \sqrt{\omega_n^2 - 1}$

$\omega_n$ = 3 rad/sec

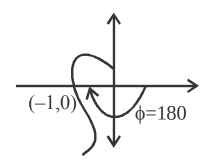
**Sol.63. (b)**

**Sol.64. (b)**




From the graph
GM = 0 – (–X) = X(+ve)
PM = 180 – 150 = 30 (+ve)

**Sol.65. (a)**



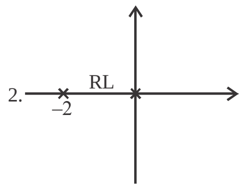

PM = 180 + ϕ
= 180 – 180 = 0

**Sol.66. (d)**
Value of phase angle at gain cross-over frequency = ϕ = –90°
∴ phase margin = 180 + ϕ
⇒ PM = 180° – 90°
⇒ PM = +90°

**Sol.67. (a)**
Consider

$G(s)H(s) = \dfrac{K}{s(s+2)}$

1. P =2, Z = 0, P – Z = 2




2. $\theta = \frac{(2q+1)180}{P-Z}$

3. Centroid = $\frac{0+(-2)}{2} = -1$

4. B.A. point

$s^2 + 2s + K = 0$

$\Rightarrow \frac{dK}{ds} = -4s - 2 = 0$

$\Rightarrow s = -0.5$

Hence, Root locus diagram

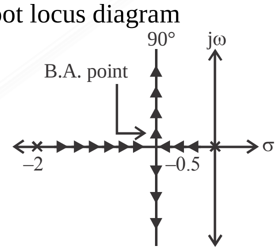
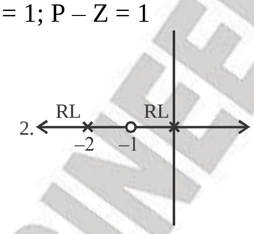



Fig.1

$\Rightarrow$ Now adding a open zero in the system i.e.

$G(s)H(s) = \frac{K(s+1)}{s(s+2)}$

1. P = 2; Z = 1; P – Z = 1




3. $\theta = \frac{(2q+1)180}{P-Z}$

$\Rightarrow \theta = 180$

Now drawing the root locus

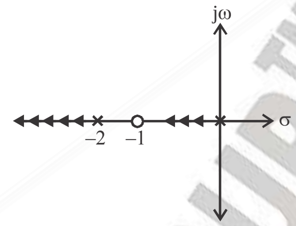



Fig.2

From figure (1) and (2) it is clear that by adding a zero root locus is shifting towards left making system more stable.

**Sol.68. (c)**

N = P – Z

N is Number of encirclement

P is Number of open loop poles lying in RHS of s – plane

Z is Number of closed loop poles lying in RHS of s – plane

Z = P – N

Z = 1 – N

For Z = 0, N = +1

**Sol.69. (c)**

**Sol.70. (c)**

**Sol.71. (c)**

**Sol.72. (d)**

**Sol.73. (d)**

Initial slope gives number of poles at origin or type of the system.

**Sol.74. (b)**

N = Z – P for clockwise encirclement

where, N is No of encirclement

P is No of open loop pole on right

Z is No of closed pole on right

**Sol.75. (c)**

$\frac{6dB}{octave} = 20dB/decade$

**Sol.76. (c)**

Angle at phase crossover frequency

$= -180^0$

So, PM = $180^0 + \phi = 0$

**Sol.77. (d)**

$$M_P e^{-\left(\xi\pi/\sqrt{1-\xi^2}\right)=1}$$

$$\frac{-\xi\pi}{\sqrt{1-\xi^2}} = 0$$

$\Rightarrow \xi = 0$

**Sol.78. (a)**

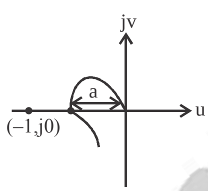
$$GM = \frac{1}{a}$$

Where a = gain at phase crossover

**Sol.79. (d)**

Since gain (dB) > 0 at $\omega_{pc}$,
therefore, GM < 0
PM = 180 + $\angle GH|\omega_{gc}$
Since, $\angle GH|_{\omega gc} < -180°$
therefore, PM < degree

**Sol.80. (b)**




GM = 20 log (1/a)
for a < 1, GM > 0 dB
for a = 1, GM = 0 dB
for a > 1, GM < 0 dB

**Sol.81. (c)**

**Sol.82. (b)**

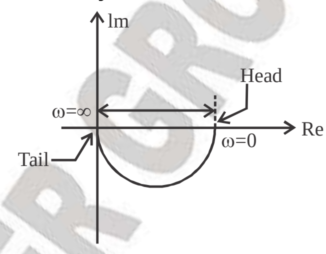
System with G.M. ≈ 1
P.M. ≈ 0
is oscillatory.

**Sol.83. (a)**

**Sol.84. (c)**

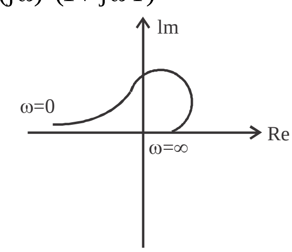
(i) When a pole is added at origin the tail and head of the plot shift by 90° in clockwise direction.
(ii) When a pole is added at negative real axis, the tail of the pole remains at same position whereas head of plot is shifted by 90° in clockwise direction.

Polar plot of $\frac{1}{1+j\omega T}$ is shown below.




When two poles are added at origin, head and tail both will shift by 90° × 2 = 180° in the clockwise direction.
Therefore, polar plot of

$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega T)}$ is shown below:

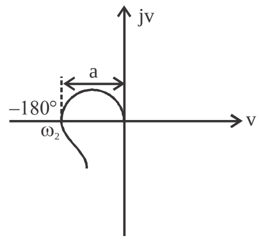



**Sol.85. (a)**

(i) Phase Margin,

$$PM = 180° - \tan^{-1}\frac{\sqrt{1-2\xi^2}}{\xi}$$

Thus, the phase margin is related to effective damping of the system.
(ii) Gain Margin: It is factor by which the system gain can be increased to drive to the verge of instability.
GM = 1/a
Where a = $|G(j\omega)\, H(j\omega)|_{\omega = \omega 2}$
$GM|_{dB} = -20 \log_{10} a$




For a stable system, a < 1. Therefore, for a stable system, GM(dB) should always be positive.

**Sol.86. (c)**
The factor exp (–st) is the cause of the term transportation lag (time delay). The effect of $e^{-st}$ term is simply to rotate each point of the G(s) H(s) plot by an angle of ωT rad in the clockwise direction. So the phase margin of the system reduces as T increases. But since $|e^{-s}| = 1$, therefore, the gain margins of both the system are the same.

**Sol.87. (b)**

**Sol.88. (c)**
C(s) = (2s + 1)/(0.2s + 1)
Comparing with the sinusoidal transfer function of the lead controller.

$$G_C(s) = \frac{1+s\tau}{1+\beta s\tau}; \beta < 1$$

τ = 2
βτ = 0.2

$$\Rightarrow \beta = \frac{0.2}{2} = 0.1$$

**Sol.89. (c)**
From polar plot gain should be less than 1 so GM should be (+ve) as GM = $-20 \log_{10} a$ where a is gain at phase cross – over PM = 180 + ϕ at gain crossover.

**Sol.90. (c)**

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

So, $\omega_r = \omega_n$ for ξ = 0. Hence point 2 is not correct.

**Sol.91. (b)**
Gain crossover is the frequency at which gain of T.F. is unity $|G(j\omega)| = \frac{K}{\omega_{gc}^2} = 1$

$$\omega_{gc} = \sqrt{K}$$

**Sol.92. (a)**
Change equation 1 + G(s) = 0
⇒ $s(1 + sT_1)(1 + sT_2) + K = 0$
⇒ $s^3T_1T_2 + s_2(T_1 + T_2) + s + K = 0$
Routh Array

| | | |
|---|---|---|
| $S^3$ | $T_1T_2$ | 1 |
| $S^2$ | $(T_1+T_2)$ | K |
| S | $\frac{(T_1+T_2)-(T_1T_2)K}{T_1+T_2}$ | |

and $\frac{(T_1+T_2)-(T_1T_2)K}{T_1+T_2} > 0$

So, $K < \frac{T_1+T_2}{T_1T_2}$

**Sol.93. (b)**
In between B and A of plot system is stable.
It is not enclosing (– 1 + j0) point.

**Sol.94. (c)**
At ω = 0, only in option (c) magnitude is equal to 1. On calculating ξ only this option gives ξ < 1.

**Sol.95. (d)**
It is a type – 2 and order – 5 plot.

**Sol.96. (a)**

**Sol.97. (b)**

$$GM = \frac{1}{0.33} \simeq 3$$

**Sol.98. (b)**

**Sol.99. (a)**

**Sol.100. (a)**

$$C(s) = \frac{(s+1)}{s} \cdot \frac{s}{s^2+1}$$

$$\therefore\ C(s) = \frac{s}{s^2+1} + \frac{1}{s^2+1}$$

$\therefore$ c(t) = cos t + sin t

$$= \sqrt{2}\sin(t+45^0)$$

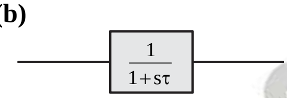
**Sol.101. (a)**

For $\xi = \frac{1}{\sqrt{2}}$ frequency response para – meters eg. $M_r$ resonant peak and time response parameters eg. $M_p$ peak over – shoot are well correlated. For $\xi > \frac{1}{\sqrt{2}}$ the resonant peak $M_r$ does not exist and the correlation breaks down. This is not a serious problem as for this range of $\xi$, the step response oscillations are well – damped and $M_p$ is hardly perceptible.

**Sol.102. (d)**

For a stable system, both GM and PM should be positive.

**Sol.103. (b)**




(i) Time constant is a specification of transient response.
(ii) s = –1/τ
(iii) Time constant is an indication of speed of the response.

**Sol.104. (a)**

**Sol.105. (a)**

$\angle G(j\omega) = -90° - \tan^{-1}\omega - \tan^{-1} 2\omega = -180°$

$$\Rightarrow\ \tan^{-1}\left(\frac{\omega+2\omega}{1-2\omega}\right) = 90°$$

$\Rightarrow 1 - 2\omega^2 = 0$

$$\Rightarrow\ \omega = \frac{1}{\sqrt{2}}\text{ rad/s}$$

$$|G(j\omega)|_{\omega=\frac{1}{\sqrt{2}}} = \frac{2}{\frac{1}{\sqrt{2}}\cdot\sqrt{\frac{1}{2}+1}\cdot\sqrt{\frac{4}{2}+1}}$$

$$= \frac{2\sqrt{2}}{\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{6}}{\sqrt{2}}} = \frac{4}{3}$$

$$\text{Gain margin} = \left.\frac{1}{|G(j\omega)|}\right|_{\omega=\omega_{pc}} = \frac{3}{4}$$

**Sol.106. (a)**

(i) Error in log-magnitude for $0 < \omega \le (1/T)$ is given by $-10\log(1+\omega^2T^2) + 10\log 1$. Therefore, error at corner frequency w = 1/T is –10 log (1 + 1) + 10 log = –3db. So, the deviation of the actual magnitude response for a zero on real axis is 3 dB at the corner frequency.

(ii) Quadratic factor for a complex conjugate poles is

$$\frac{1}{1+j2\xi u - u^2}\text{ where } u = \frac{\omega}{\omega_n}.$$

Phase angle of quadratic factor at undamped, frequency, i.e.

$$\omega = \omega_n\left(\Rightarrow u = \frac{\omega}{\omega_n} = 1\right)$$

$$\phi = -\tan^{-1}\left(\frac{2\xi}{1-1}\right) = -\tan^{-1}\infty = -90°.$$

So phase angle is independent of $\xi$.

**Sol.107. (b)**

When transfer function has no pole and zero in RHS of s-plane, it is called minimum phase transfer function.

**Sol.108. (c)**

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$M_r = \frac{1}{2\times 0.421\sqrt{-(0.421)^2}}$$

$M_r$ = 1.30

**Sol.109. (c)**

(i) Rise time $\propto \dfrac{1}{BW}$

Speed of time response $\propto \dfrac{1}{\text{Band width}}$

(ii) Phase margin = 180° + $\phi$,

$= 180° - \tan^{-1}\dfrac{\sqrt{1-2\xi^2}}{\xi}$

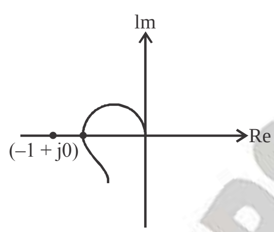
(iii) Response peak is called overshoot.
(iv) Gain margin tells about the stability of the system.

**Sol.110. (c)**

Gain margin = $\dfrac{1}{\text{Gain}}$

**Sol.111. (d)**

**Sol.112. (d)**
For a stable closed loop system, the gain at phase crossover frequency should be less than 1.




Gain < 20 log 1 dB
$\Rightarrow$ Gain < 0 dB

**Sol.113. (c)**
For a minimum phase system to be stable, both phase margin and gain margin should be positive.

**Sol.114. (c)**
The sharper the cutoff characteristic, the larger the peak resonance and the lesser stable the system.

**Sol.115. (c)**

**Sol.116. (a)**

**Sol.117. (c)**

**Sol.118. (b)**

**Sol.119. (c)**
Refer Nyquist stability criteria i.e. concept of gain Margin.

**Sol.120. (d)**

**Sol.121. (b)**
For origin pu s = 0 in $e^{sT} = Z$, Z = 1
hence imaginary part is 0.

**Sol.122. (c)**

**Sol.123. (c)**
N =– 1, $P_+$ = 1
$\Rightarrow Z_+ = P_+ - N = 1 - (-1) = 2$

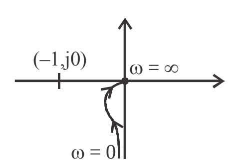
**Sol.124. (b)**

From given Bode plot G(s)H(s) = $\dfrac{k}{j\omega}$ as

H(s) = 1
$\angle G(j\omega)\,H(j\omega) = -90^0$
$\therefore$ PM (maximum) = $-90^0 + 180^0 = 90^0$

**Sol.125. (a)**

It is a type – I and order II transfer function




So never enclosed.

**Sol.126. (a)**

$$-\omega_1 T - \tan^{-1}\left(-\frac{1}{\omega_1}\right) = 0$$

$$\Rightarrow \tan(-\omega_1 T) = \frac{1}{\omega_1}$$

$$\Rightarrow \omega_1 = \cot(\omega_1 T)$$

**Sol.127. (d)**

**Sol.128. (d)**

**Sol.129. (a)**
Techometer feedback reduces both gain and effective time constant.

**Sol.130. (d)**

**Sol.131. (d)**

**Sol.132. (d)**
To minimize the distortion $H_e(s)$ should have the inverse relationship with H(s) and constant delay.

**Sol.133. (d)**
Phase lead compensator acts like a high pass filter. So gain crossover frequency and bandwidth both increase.

**Sol.134. (c)**
The introduction of a time delay element decreases both phase margin and gain margin.

**Sol.135. (c)**
$8x^2 + 18x + 8y^2 + 9 = 0$

$$\Rightarrow\ x^2+\frac{9}{4}x+y^2+\frac{9}{8}=0$$

$$\Rightarrow\ \left(x+\frac{9}{8}\right)^2+y^2=\frac{81}{64}-\frac{9}{8}$$

Center of constant – M circle is $\left(\frac{-M^2}{M^2-1},0\right)$

So $\frac{M^2}{M^2-1}=\frac{9}{8}$

$\Rightarrow 8M^2 = 9M^2 - 9$
$\Rightarrow M^2 = 9$
$\Rightarrow M = 3$

**Sol.136. (a)**
Constant – N circle equation is

$$\left(x+\frac{1}{2}\right)^2+\left(y-\frac{1}{2N}\right)^2=\frac{N^2+1}{4N^2}$$

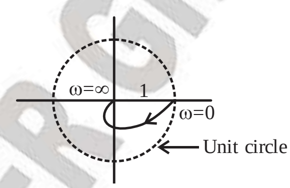
Where $N = \tan\alpha$

Center of circles is at $\left(-\frac{1}{2},\frac{1}{2N}\right)$

Radius is $\frac{\sqrt{N^2+1}}{2N}$

Constant – N circles always pass through (–1, 0) and (0, 0).

**Sol.137. (d)**
Polar plot of the given transfer function is shown below:




$\angle G(j\omega)|_{\omega=\omega gc} = 0°$
$PM = 180° + \angle G(j\omega)|_{\omega=\omega gc}$
$PM = 180°$

**Sol.138. (b)**
When a pole is added at negative real axis, the tail of the plot remains at same position whereas the head of plot shifts by 90° in clockwise direction.

**Sol.139. (c)**

$$G(s)=\frac{100}{s^2+10s+100}$$

$\omega_n=\sqrt{100}$ $\omega_n$ = 10 rad/s

$$\xi=\frac{10}{2\omega_n}\Rightarrow\xi=\frac{10}{2\times10}=\xi=0.5$$

$$\omega_r=\omega_n\sqrt{1-2\xi^2}=10\sqrt{1-2(0.5)^2}$$

$\omega_r$ = 7.07 rad/s

**Sol.140. (b)**
Constant – M circle equation is

$$\left(x+\frac{M^2}{M^2-1}\right)^2+y^2=\left(\frac{M}{M^2-1}\right)^2$$

So, the center is $\left(\frac{-M^2}{M^2-1},0\right)^2$

and radius is $\frac{M}{M^2-1}$

**Sol.141. (b)**

(i) The type of the system can be found out by the initial slope. The type of the system is n where the initial slope is –20n dB/decade.

(ii) $\omega_2 \neq \frac{\omega_1+\omega_3}{2}$

**Sol.142. (d)**

$x = C\log\frac{\beta}{\alpha}$ where C is a constant.

$Kx = C\log\frac{\omega}{\alpha} \Rightarrow K = \frac{\log(\omega/\alpha)}{\log(\beta/\alpha)}$

$\Rightarrow \log\frac{\omega}{\alpha} = K\log\frac{\omega}{\alpha} = K\log\frac{\beta}{\alpha}$

$\Rightarrow \log\frac{\omega}{\alpha} = \log\left(\frac{\beta}{\alpha}\right)^K$

$\Rightarrow \frac{\omega}{\alpha} = \left(\frac{\beta}{\alpha}\right)^K$

$\Rightarrow \omega = \alpha^{1-K}.\beta^K$

**Sol.143. (b)**

The open-loop transfer function for a unity feedback second-order system is

$$G(s)H(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

$$G(j\omega)H(j\omega) = \frac{\omega_n^2}{j\omega(j\omega+2\xi\omega_n)}$$

At gain crossover frequency $\omega = \omega_{gc}$, $|G(j\omega)H(j\omega)| = 1$

$$\Rightarrow \frac{\omega_n^2}{\omega_{gc}\sqrt{\omega_{gc}^2+4\xi^2\omega_n^2}} = 1$$

$\Rightarrow \omega_n^4 = \omega_{gc}^2(\omega_{gc}^2 + 4\xi^2\omega_n^2)$

$\Rightarrow \omega_{gc}^4 + 4\xi{=}2\,{}^{\omega}{}_n^{2}\,\omega_{gc}^2 - \omega_n^4 = 0$

Dividing by $\omega_n^4$;

$$\left(\frac{\omega_{gc}}{\omega_n}\right)^4 + 4\xi^2\left(\frac{\omega_{gc}}{\omega_n}\right)^2 - 1 = 0$$

$$\Rightarrow \left(\frac{\omega_{gc}}{\omega_n}\right)^2 = \frac{-4\xi^2+\sqrt{16\xi^4+4}}{2}$$

$$\Rightarrow \left(\frac{\omega_{gc}}{\omega_n}\right)^2 = -2\xi^2+\sqrt{4\xi^4+1}$$

Negative sign has been discarded as square cannot be negative.

PM = 180° + $\angle G(j\omega)H(j\omega)|_{\omega=\omega gc}$

$= 180° - 90° - \tan^{-1}\left(\frac{\omega}{2\xi\omega_n}\right)$

$= 90° - \tan^{-1}\left\{\frac{\sqrt{4\xi^4+1}-2\xi^2}{2\xi}\right\}$

$= \cot^{-1}\left\{\frac{-2\xi^2+\sqrt{1+4\xi^4}}{2\xi}\right\}$

$\left[\because \tan^{-1}5+\cot^{-1}x = \frac{\pi}{2}\right]$

$PM = \tan^{-1}\left\{\frac{2\xi}{-2\xi^2+\sqrt{1+4\xi^4}}\right\}$

$\left[\because \tan^{-1}x\cot^{-1}\left(\frac{1}{x}\right)\right]$

**Sol.144. (b)**

When a pole is added at negative real axis, the tail of the plot remains at same position whereas the head of plot shifts by 90° in clockwise direction.

**Sol.145. (d)**

$$GM = \frac{1}{|G(j\omega)|}\bigg|_{\omega=\omega_{pc}} = \frac{1}{0.64}$$

$$GM = 20\log\left(\frac{1}{0.64}\right) = 3.86\,dB$$

PM = 180° + $\angle G(j\omega)|_{\omega=\omega gc}$

= 180° – 170°

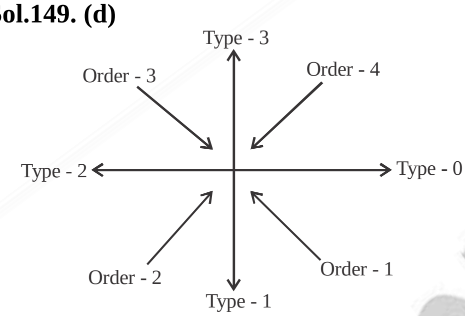
⇒ PM = 10°

**Sol.146. (c)**

**Sol.147. (b)**
The corresponding value of ω is found out from auxiliary equation in Routh array.

**Sol.148. (b)**
Refer Nyquist plot and condition for closed loop stability. Open loop system is stable as all the open loop ples are in left hand of s – plane. While closed loop system is unstable because Nyquist plot encloses twice the point (– 1 +j0) but for stability it should not enclose it as N = P = 0.

**Sol.149. (d)**




**Sol.150. (d)**
– 1.33 K =– 1

$K_{marg} = \dfrac{1}{1.33}$

$G(s)=\dfrac{K(s+3)(s+5)}{(s-2)(s-4)}$

using N = P – Z

| **Case 1:** | **Case 2:** |
|---|---|
| K<< 0.75 | K > 0.75 |
| K<< 0.75 | 2 – 2 – Z |
| 0 = 2 – Z | Z = 0; hence stable |
| Z = 2; hence unstable | |

$K > \dfrac{1}{1.33}$

Apply the concept of gain margin. For K = 1, real part of G(jω) H(jω) is 1.33. For the system to be stable K.133 ≤ to avoid the encirclement of point (–1 + j0).

**Sol.151. (a)**
The system is an all – pass system having transfer function

$G(j\omega)=\dfrac{1-j\omega T}{1+j\omega T}$

**Sol.152. (d)**
For region – I: N(number of encirclement of point [– 1 + j0]) = 1 – 1 = 0 so system is stable.
For region – II: N = – 1 – 1 = – 2 (clockwise) ≠ 0 so system is unstable.

**Sol.153. (c)**

**Sol.154. (b)**

**Sol.155. (c)**

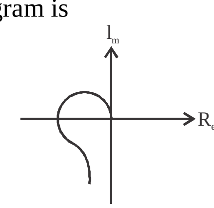
**Sol.156. (c)**
Phase lag compensation has the following features:
(i) Poles in nearer to origin
(ii) Bandwidth reduces
(iii) Gain crossover frequency reduces
(iv) Phase crossover frequency reduces
(v) Resonance peak reduces

**Sol.157. (b)**

**Sol.158. (c)**

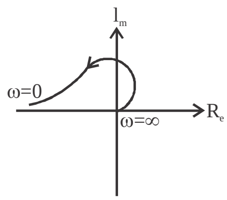
$G(s)H(s)=\dfrac{5}{s(1+0.1s)(1+0.01s)}$

Nyquist diagram is




**Sol.159. (d)**

**Sol.160. (a)**
Nyquist plot approximated is shown below:




**Sol.161. (b)**

**Sol.162. (c)**

$x^2 + 2.25x + y^2 = -1.125$

$\Rightarrow (x + 1.125)^2 + y^2 = 0.140625$

M-circle equation is

$$\left(x+\frac{M^2}{M^2-1}\right)^2 + y^2 = \left(\frac{M}{M^2-1}\right)^2$$

Comparing,

$$\frac{M^2}{M^2-1} = 1.125$$

$$\Rightarrow M^2 = \frac{1.125}{0.125} = 9$$

$\Rightarrow M = 3$

**Sol.163. (b)**

Center of N-circle is $\left(\frac{-1}{2};\frac{+1}{2N}\right)$

so $\frac{+1}{2N} = 0$

$N = \infty$

$\tan\alpha = \infty$

$\alpha = 90°$

**Sol.164. (d)**

No. of poles, n = 6

No. of zeros, m = 2

Slope of high frequency asymptote

= –20 (n-m) = –20 (6-2)

= –80 dB/decade

**Sol.165. (a)**

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T_1)(1+j\omega T_2)}$$

At $\omega_{pc}$, $\angle G(j\omega) = -180°$

$-90° - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 = -180°$

$$\Rightarrow \tan^{-1}\frac{\omega T_1 + \omega T_2}{1-\omega^2 T_1 T_2} = 90°$$

$$\Rightarrow \frac{\omega T_1 + \omega T_2}{1-\omega^2 T_1 T_2} = \tan 90° = \infty$$

$\Rightarrow 1 - \omega^2 T_1 T_2 = 0$

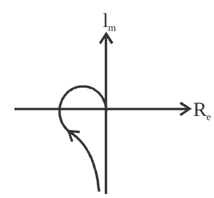
$$\Rightarrow \omega_{pc} = \frac{1}{\sqrt{T_1 T_2}}$$

$$|G(j\omega)|_{\omega=\omega_{pc}} = \frac{1}{\omega_{pc}\sqrt{1+\omega_{pc}^2 T_1^2}\sqrt{1+\omega_{pc}^2 T_2^2}}$$

$$= \frac{1}{\frac{1}{\sqrt{T_1 T_2}}.\sqrt{1+\frac{T_1}{T_2}}.\sqrt{1+\frac{T_2}{T_1}}} = \frac{T_1 T_2}{T_1 + T_2}$$

$$GM = \frac{1}{|G(j\omega)|}\Bigg|_{\omega=\omega_{pc}} = \frac{T_1 + T_2}{T_1 T_2}$$

**Sol.166. (b)**




$$G(s)H(s) = \frac{1}{sT_1(1+sT_2)(1+sT_3)}$$

**1.** When a pole is added at negative real axis, the tail of the plot remains at the same position whereas head of plot shifts by 90° in clockwise direction.

**2.** When a pole is added at origin, both the tail and head of the plot shift by 90° in clockwise direction.

**Sol.167. (d)**

$$V_2(t) = K_t\frac{d\theta}{dt}$$

$\Rightarrow V_2(s) = K_t s\theta(s)$

**Sol.168. (b)**

At $\omega_{pc}$, $\angle G(j\omega) = -180°$

$\Rightarrow -\tan^{-1}\omega_c - \tan^{-1} 2\omega_{pc} - \tan^{-1} 3\omega_{pc} = -180°$

$\Rightarrow \tan^{-1}\omega_{pc} + \tan^{-1} 2\omega_{pc} = 180 - \tan^{-1} 3\omega_{pc}$

$$\Rightarrow \tan^{-1}\left(\frac{3\omega_{pc}}{1-2\omega_{pc}^2}\right) = 180 - \tan^{-1} 3\omega_{pc}$$

$$\Rightarrow \frac{3\omega_{pc}}{1-2\omega_{pc}^2} = -3\omega_{pc}$$

$\Rightarrow 1 - 2\omega_{pc}^2 = -1$

$\Rightarrow 2\omega_{pc}^2 = 2$

$\Rightarrow \omega_{pc}^2 = 1$

$\Rightarrow \omega_{pc} = 1$ rad/s

**Sol.169. (d)**

$$GM = \frac{1}{0.4} = 2.5$$

**Sol.170. (b)**

Open – loop transfer function is given by $\frac{1}{s(1+sT)}$ as per Nyquist plot.

**Sol.171. (b)**

**Sol.172. (b)**

Put $s = j\omega$ and solve for

$$\phi = \tan^{-1}\left(\frac{-11\omega-\omega^3}{45-\omega^2}\right)$$

**Sol.173. (d)**

**Sol.174. (a)**

Closed loop frequency response,

$$T(j\omega) = \frac{G(j\omega)}{1+G(j\omega)} = \frac{x+jy}{1+x+jy}$$

$$\text{Magnitude, } M = \left[\frac{x^2+y^2}{(1+x)^2+y^2}\right]^{-1/2}$$

$$\Rightarrow M^2 = \frac{x^2+y^2}{x^2+2x+1+y^2}$$

Putting M = 1,

$$1 = \frac{x^2+y^2}{x^2+2x+1+y^2}$$

$\Rightarrow x^2 + 2x + 1 + y^2 = x^2 + y^2 \Rightarrow 2x = -1$

$\Rightarrow x = -0.5$

**Sol.175. (a)**

Phase margin = 180° – 30° = 150°

$$\text{Gain margin} = = \frac{1}{0.25} = 4$$

**Sol.176. (a)**

$$\text{Steady state error } e_{ss} = \lim_{s\to 0}\frac{sR(s)}{1+G(s)}$$

$$R(s) = \frac{1}{s^2+1}$$

$$e_{ss} = \lim_{s\to 0}\frac{s.\frac{1}{s^2+1}}{1+\frac{1}{s(s+1)}}$$

$$e_{ss} = \lim_{s\to 0}\frac{s^2(s+1)}{(s^2+1)\{s(s+1)+1\}} \quad ,e_{ss} = 0$$

**Sol.177. (d)**

Bode : Asymptotic plots

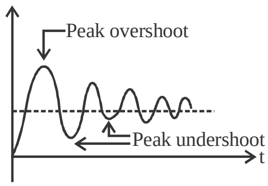
Evans : Root – locus technique

Nyquist : Polar plots

**Sol.178. (c)**

**Sol.179. (c)**

**Sol.180. (a)**




$M_p = e^{-\xi n\pi/\sqrt{1-\xi^2}}$ for n = 1, 2, 3, …..

**Sol.181. (a)**

**Sol.182. (c)**

**Sol.183. (a)**
$\angle G(j\omega)|_{\omega = 0} = -270°$
$\angle g(j\omega)|_{\omega = \infty} = -270°$
The polar plot intersects with the negative real axis as in making imaginary term of G(jω) to be zero, the solution exists.

**Sol.184. (a)**
Gain margin = 1/1 = 1
= 20 log log 1 dB = 0 dB

**Sol.185. (d)**
Phase margin = 90° – 30° = 60°

**Sol.186. (a)**
Equation for constant–M circle is

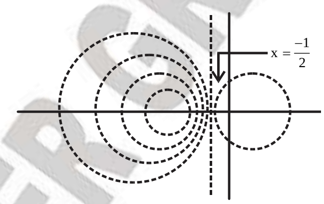
Gain margin = 1/0.75

$$\left(X+\frac{M^2}{M^2-1}\right)^2 + y^2 = \frac{M^2}{(M^2-1)^2}$$

Whose center is $\left(-\frac{M^2}{M^2-1}, 0\right)$

and radius is $\frac{M}{M^2-1}$

The constant –M circle is the straight line at $x = -\frac{1}{2}$.



Locus of constant-M circles

**Sol.187. (d)**
Equation for constant – N circle is

$$\left(x+\frac{1}{2}\right)^2 + \left(y-\frac{1}{2N}\right)^2 = \frac{N^2+1}{4N^2}$$

Whose center is $\left(\frac{-1}{2}, \frac{1}{2N}\right)$

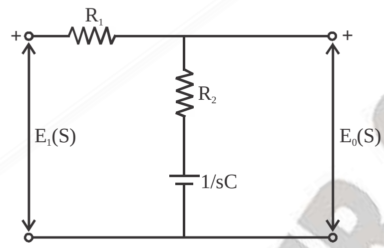
and radius is $\frac{\sqrt{N^2+1}}{2N}$

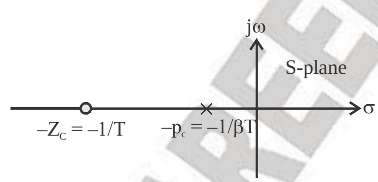
**Sol.188. (c)**

# CHAPTER - 9
# *COMPENSATORS*

## 9.1 LAG COMPENSATOR

A compensator having the characteristic of a lag network is called a lag compensator. Lag compensation results in a large improvement in steady state performance but results in a slower response due to reduced band width. Lag compensator is essentially a low pass filter and so high frequency noise signals are attenuated.




Transfer function of lag compensator, $G_c(s) = \dfrac{s+z_c}{s+p_c} = \left(\dfrac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}\right)$

### 9.1.1 Frequency Response of a Lag Compensator

Consider the general form of lag compensator

$$G_c(s) = \frac{s+(1/T)}{s+(1+\beta T)} = \beta\frac{(1+sT)}{(1+s\beta T)}$$

The sinusoidal transfer function of lag compensator is obtained by letting s = jω

$$\therefore G_c\,(j\omega) = \beta\,\frac{(1+j\omega t)}{(1+j\omega\beta T)}$$

When ω = 0, $G_c(j\omega)=\beta$

$$G_c(j\omega) = \frac{1+j\omega T}{1+j\omega\beta T} = \frac{\sqrt{1+(\omega T)^2}\angle\tan^{-1}\omega T}{\sqrt{1+(\omega\beta T)^2}\angle\tan^{-1}\omega\beta T}\text{...(i)}$$

The sinusoidal transfer function has two corner frequencies and they are denoted as $\omega_{c1}$ and $\omega_{c2}$

Here, $\omega_{cl} = 1/\beta T$ and $\omega_{c2} = 1/T$

Since, $\beta T > T$, $\omega_{c1} < \omega_{c2}$

The approximate magnitude plot of lag compensator is shown in figure. The magnitude plot of $G_c(j\omega)$ is a straight line through 0 dB upto $\omega_{c1}$, Then it has as slope of –20 dB /dc upto $\omega_{c2}$ it is a straight line with a constant gain of 20 log (1/β)

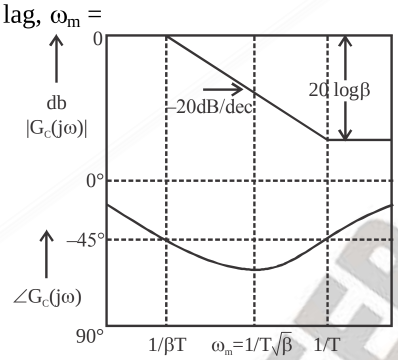
Let $\phi = \angle G_c(j\omega)$, therefore $\phi = \tan^{-1}\omega T - \tan^{-1}\omega\beta T$

As $\omega \to 0, \phi \to 0$;

As $\omega \to \infty, \phi \to 0$

As 'ω' is varied from 0 to ∞, the phase angle decreases from 0 to a maximum value of $\phi_m$ At $\omega = \omega_m$, then increases from maximum value to 0.

Frequency of maximum phase lag, $\omega_m$ =

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### 9.1.2 Determination of $\omega_n$ and $\phi_m$

The frequency $\omega_m$ can be determined by differentiating ϕ with respect to ω and equating to dϕ/dω to zero as shown below.

From Eq. (i) we get

Phase of $G_c(j\omega)$, $\phi = \angle G_c(j\omega) = \tan^{-1} - \tan^{-1}\omega\beta T$

On differentiating the above equation, we get

$$\omega_m = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{(1/\beta T).(1/T)} = \frac{1}{T\sqrt{\beta}} \qquad \left(\text{Note: } d/dt(\tan\theta = \frac{1}{1+\theta^2}\right)$$

When $\omega = \omega_m$, $d\phi/d\omega = 0$

Hence, replace by $\omega_m$ in above equation and equate to zero.

$$\frac{1}{1+(\omega_m T)^2} - \frac{1}{1+(\omega_m\beta T)^2}\beta T = 0$$

On cross multiplication we get,

$1 + (\omega_m\beta T)^2 = \beta [1 + (\omega_m T)^2]$

$(\omega_m\beta T)^2 - \beta(\omega_m T)^2 = \beta - 1$

$\beta(\omega_m T)^2(\beta - 1) = (\beta - 1)$

$\omega_m^2 = 1/T^2\beta \qquad \therefore \ \omega_m = 1/T\sqrt{\beta}$

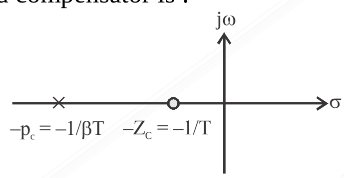
Frequency corresponding to maximum phase lag, $\omega_m = 1/T\sqrt{\beta}$ … (i)

$\therefore$ Maximum lag angle , $\phi_m = \ \phi_m = \tan^{-1}\left(\frac{1-\beta}{2\sqrt{\beta}}\right)$

## 9.2 LEAD COMPENSATOR

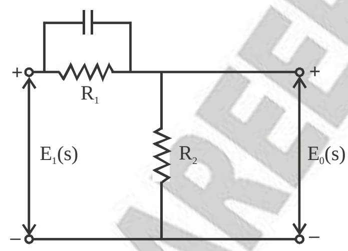
1. A compensator having the characteristics of a lead network is called a lead compensator.
2. The lead compensation increases the band width.
3. It improves the speed of the response and also reduces the amount of overshot. 4. It appreciably improves the transient response.
5. A lead compensator is basically a high filter and so it amplifies high frequency noise signals.

The s-plane representation of lead compensator is :




Transfer function of a lead compensator, $G_c(s) = \dfrac{s + Z_c}{s + (1/\alpha T)}$

**Electrical lead net work**




### 9.2.1 Frequency Response of a Lead Compensator

Consider the general form of lead compensator,

$$G_c(s) = \frac{s + (1/T)}{s + (1 + \alpha T)} = \alpha \frac{(1 + sT)}{(1 + \alpha s T)}$$

The sinusoidal transfer function of a lead compensator is obtained letting s = jω

$$\therefore\ G_c(j\omega) = \alpha \frac{(1 + j\omega T)}{(1 + j\omega\alpha T)};\ \ ;\ \text{when } \omega = 0,\ G_c(j\omega) = \alpha$$

Let us assume that the attenuation α is eliminated by a suitable amplifier network. Now, $G_c(j\omega)$ is given by

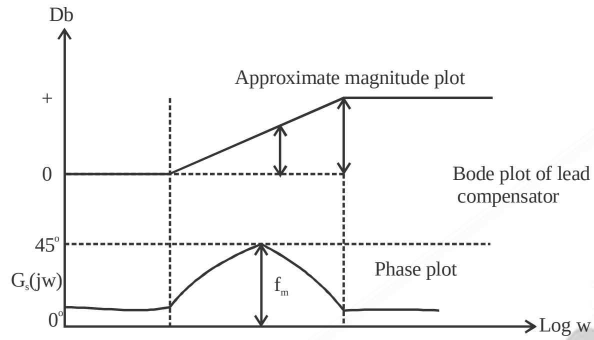
$$G_c(j\omega)\ \frac{1 + j\omega T}{1 + j\omega\alpha T} = \frac{\sqrt{1 + (\omega T)^2}\angle \tan^{-1}\omega T}{\sqrt{1 + (\alpha\omega T)^2}\angle \tan^{-1}\omega\alpha T}\ ;\ \text{when } \omega = 0,\ G_c(j\omega) = \alpha$$

The sinusoidal transfer function has two corner frequencies $\omega_{c1}$ and $\omega_{c2}$

Here, $\omega_{c1} = 1/T$ and $\omega_{c2} = 1/\alpha T$

Since, $T > \alpha T$, $\omega_{c1} < \omega_2$

The approximate magnitude plot of lead compensator is shown below.




Let $\phi - \tan^{-1} \omega T - \tan^{-1}{}_{\omega\alpha T}$
As $\omega \to 0, \phi \to 0$
As $\omega \to \infty, \phi \to 0$
Frequency of maximum phase lead,

$$\omega_m = \sqrt{\omega_{c1}\omega_{c2}} = \sqrt{(1/\alpha T).(1/T)} = \frac{1}{T\sqrt{\alpha}}$$

**9.2.2 Determination of $\omega_m$, $\phi_m$ and $\alpha$**

The frequency $\omega_m$ can be determined by differentiating $\phi$ with respect to $\omega$ and equating $d\phi/d\omega$ to zero.
Phase of $G_c(j\omega)$, $\phi = \tan^{-1} \omega T - \tan^{-1} \alpha\omega T$
On differentiating the above equation with respect to $\omega$ and equating $d\phi/d\omega$ to zero, we get the frequency corresponding to maximum phase lead as,
$\omega_m = 1/T \sqrt{\alpha}$
Also can we can express $\phi_m$ in terms of $\alpha$ and in terms of $\phi_m$ as shown below.

$$;\phi_m = \tan^{-1}\left(\frac{1-\alpha}{2\sqrt{\alpha}}\right); \alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}$$

**9.3 LAG-LEAD COMPENSATOR**

A compensator having the characteristics of lag-lead network is called a lag lead compensator. A lag lead compensator improves both transient and steady state response.
The transfer function of lag lead compensator

$$G_c(s) = \frac{(s+1/T_1)}{(s+1/\beta T_1)} \quad \frac{(s+1/T_2)}{(s+1/\alpha/T_2)}$$

Where $\beta > 1$ and $0 < \alpha < 1$

**9.3.1 Frequency Response of Lag - Lead compensator**

Consider the transfer function of Lag- Lead compensator

$$G_c(s) = \frac{(s+1/T_1)(s+1/T_2)}{(s+1/\beta T_1)(s+1/\alpha T_2)} = \alpha\beta\frac{(1+sT_1)(1+sT_2)}{(1+s\beta T_1)(1+s\alpha T_2)}$$

This sinusoidal transfer function of a lag – lead compensator is obtained by letting $= j\omega$

$$\therefore\ G_c(j\omega)\ \ \alpha\beta\frac{(1+j\omega T_1)(1+j\omega T_2)}{(1+j\omega\beta T_1)(1+j\omega\alpha T_2)}$$

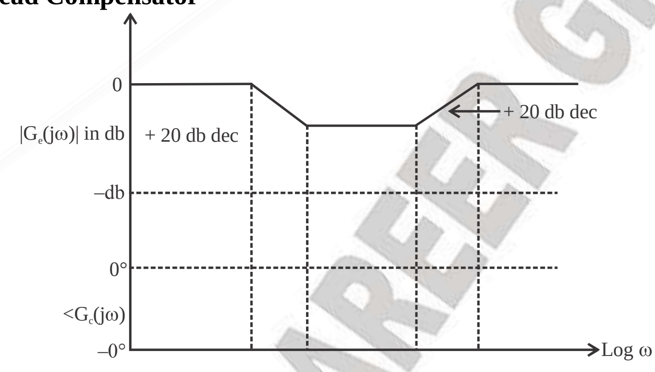
For a single lag – lead compensator, $\alpha\beta = 1$. Hence from above equation, we can say that the lag – lead compensator provides a de gain of unity.

$$\therefore\ G_c(j\omega) = \frac{(1+j\omega T_1)(1+j\omega T_2)}{(1+j\omega\beta T_1)(1+j\omega\alpha T_2)}$$

The sinusoidal transfer function shown in above equating has four corner frequency and they are $\omega_{c1}$, $\omega_{c2}$, $\omega_{c3}$ and $\omega_{c4}$ , where $\omega_{c1} < \omega_{c2} < \omega_{c3} < \omega_{c4}$/

Hence $\omega_{c1} = 1/\beta T_1$ : $\omega_{c2} = 1/T_1$; $\omega_{c3} = 1/T_2$ and $\omega_{c4} = 1/\alpha T_2$

### 9.4.1 Effects of Lead Compensator




1. Improves the transient response
2. It improves stability
3. It increase the bandwidth
4. Signal to noise ratio at the o/p is less than the input i.e. It increase the effect of noise
5. It helps to increase error constant upto some extent.

### 9.4.2 Effects of Lag Compensator

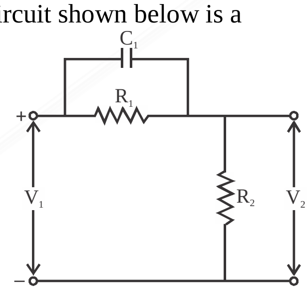
1. Improves the steady state response. It increase the error constant to a great extent and hence steady state error decrease.
2. Decreases the bandwidth.
3. Reduces the effect of noise
5. Reduces the stability margin i.e. the system becomes lesser stable.
6. Does not affect the transient response.

# ASSIGNMENT

**1.** A negative feedback control system has a transfer function $G(s) = \frac{k}{s+2}$. We select a compensator $G_c(s) = \frac{s+a}{s}$ in order to achieve zero steady state error for a step input. Select 'a' and 'k' so that the overshoot to a step is approximately 5% and the settling time (with a 2% criterion) is approximately 1 second.

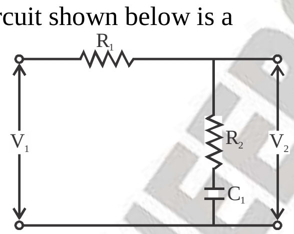
(a) k=8, a = 5.6 (b) k=8, a = 6.6
(c) k=6, a = 5.6 (d) k=6, a =6.6

**2.** The circuit shown below is a




(a) Lag network (b) Lead network
(c) Lead-lag network (d) None

**3.** The circuit shown below is a




(a) Lag network (b) Lead network
(c) Lead-lag network (d) None

**4.** The transfer function of a simple RC network functioning as a controller is $G_c(s) = \frac{(s+z_1)}{(s+p_1)}$. The required condition for the RC network to act as a phase lead controller is

(a) $p_1 < z_1$ (b) $p_1 > z_1$
(c) $p_1 = z_1$ (d) None of these

**5.** The damping of the system can be increased by using a compensator having a pair of complex roots as

(a) Phase lead (b) Phase lag lead
(c) Phase lag (d) None of these

**6.** If poles are added in a transfer function it will cause

(a) Lag compensation
(b) Lead compensation
(c) Lead-lag compensation
(d) None of these

**7.** If zero are added in a transfer function, it will cause

(a) Lag compensation
(b) Lead compensation
(c) Lead-lag compensation
(d) None of these

**8.** The transfer function of a lead compensator is $G_c(s) = \frac{1+0.12s}{1+0.04s}$. The maximum phase shift that can be obtained from this compensator is

(a) $60^o$ (b) $45^o$
(c) $30^o$ (d) $15^o$

**9.** Consider the following statements in case of phase lead compensation:

(a) Improvement of gain and phase margins
(b) Less rise time and more settling time
(c) Bandwidth is increased
(d) Affect the steady-state error

Which of these statements are correct?

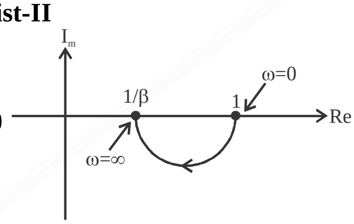
(a) 1, 2 and 3 (b) 1 and 3
(c) 2 and 3 (d) 2 and 4

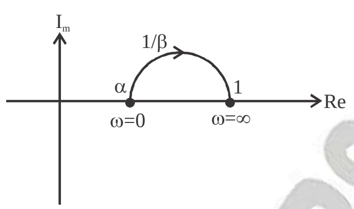
**10.** Consider the following statement in case of phase lag compensation:

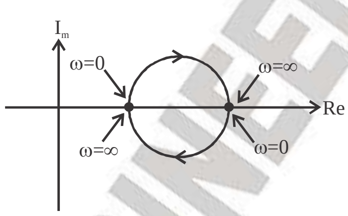
(a) It is a low pass filter.

(b) It approximately act as a proportional plus integral controller.
(c)The bandwidth of the system is reduced.
(d) Rise time and setting time are large.
Which of these statements are correct?
(a) 1, 2, 3 and 4 (b) 1, 2 and 3
(c) 2, 3 and 4 (d) 1, 3 and 4

**11.** Match List-I (Type of compensator) with List-II (Polar plot) and select the correct answer using the code given below the lists:
**List-I**
A. Phase lead
B. Phase lag
C. Lead-lag
**List-II**

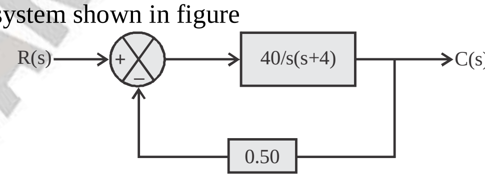
(i)



(ii)


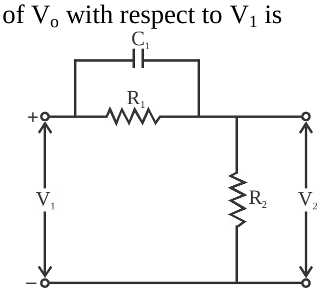

(iii)



**Codes:**
(a) A-i, B-ii, C-iii
(b) A-i, B-iii, C-ii
(c) A-ii, B-i, C-iii
(d) A-ii, B-iii, C-i

**12.** Match List-I with List-II and select the correct answer using the code given below:
**List-I**
A. Ac servometer
B. dc amplifier
C. Lead network
D. Lag network
**List-II**
(i) $\frac{s+2}{s+p}(2<p)$
(ii) $\frac{1+T_1s}{1+T_2s}(T_1<T_2)$
(iii) $\frac{k}{1+Ts}$
(iv) $\frac{k}{s(1+Ts)}$
**Codes:**
(a) A-iv, B-iii, C-i, D-ii
(b) A-iii, B-iv, C-i, D-ii
(c) A-iv, B-i, C-iii, D-ii
(d) A-iii, B-ii, C-iv, D-i

**13.** Calculate the sensitivity of the closed-loop system shown in figure


below with respect to the forward path transfer function at $\omega = 1.3 \text{rad/sec}$.
(a) 1.05 (b) −1.05
(c) 0.287 (d) 2.87

**14.** For the given network, the maximum phase lead $\phi_m$ of $V_o$ with respect to $V_1$ is


(a) $\sin^{-1}\left(\frac{R_1}{2R_2}\right)$ (b) $\sin^{-1}\left(\frac{R_1}{R_1+2R_2}\right)$

(c) $\sin^{-1}\left(\frac{R_1}{R_1+3R_2}\right)$ (d) $\sin^{-1}\left(\frac{R_1}{2R_2C_1}\right)$

**15.** If the transfer function of a phase lead compensator is (s + a)/(s + b) and that of a lag compensator is (s + p)/(s + q), then which one of the following must be satisfied?
(a) A > b and p > q
(b) A > b and p < q
(c) A < b and p < q
(d) A < b and p > q

**16.** The transfer function of a phase lead network can be written as

(a) $\frac{1+sT}{1+s\beta T};\beta>1$ (b) $\frac{\alpha(1+sT)}{1+s\alpha T};\alpha<1$

(c) $\frac{\beta(1+sT)}{1+s\beta T+T};\beta<1$ (d) $\frac{(1+sT)}{\alpha(1+sT)};\alpha>1$

**17.** For a stable system, what are the restrictions on the gain margin and phase margin?
(a) Both gain margin and phase margin
(b) Gain margin is negative and phase margin is positive
(c) Gain margin is positive and phase margin is negative
(d) Both gain margin and phase margin are positive

**18.** A property of phase lead compensation is that the
(a) Overshoot is increased
(b) Bandwidth of closed loop system is reduced.
(c) Rise time of close loop system is reduced
(d) Gain margin is reduced.

**19.** The transfer function is $\frac{1+0.5s}{1+s}$. It is represents a
(a) Lead network
(b) Lag network
(c) Lag - lead network
(d) Proportional network

# SOLUTIONS

**Sol. 1. (c)**
The characteristics equation is $1+G_c(s)G(s)H(s)=0$

$$1+\frac{k(s+a)}{s(s+2)}=0$$

$s^2+(2+k)s+ak=0$ …(i)

Compare equation (i) to standard equation $s^2+2\xi\omega_n s+\omega_n^2=0$ we get,

$\omega_n = \sqrt{ak}$ and $2\xi\omega_n = 2+k$

Now, Peak overshoot, $M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$

$$0.05 = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

In (0.05), $\frac{-\pi\xi}{\sqrt{1-\xi^2}}, \xi = 0.689$

And also, settling time (for 2%) = $\frac{4}{\xi\omega_n}$

$$1=\frac{4}{\xi\omega_n}, \quad \omega_n = \frac{4}{\xi} = \frac{4}{0.689} = 5.79$$

$2\xi = 2+k$

Or k = 6

$\omega_n = \sqrt{ak} = \sqrt{6a}$

a = 5.6

$\therefore$ k = 6 and a = 5.6

**Sol. 2. (b)**

**Sol. 3. (a)**

**Sol. 4.**
To act as a phase lead controller $p_1 > z_1$

**Sol. 5. (a)**

**Sol. 6. (a)**

**Sol. 7. (b)**

**Sol. 8.**
The standard transfer function of lead compensator is

$$G_c(j\omega) = \frac{\alpha(1+j\omega T)}{(1+j\omega\alpha T)} \quad \text{…(i)}$$

and given transfer function $G_c(s) = \frac{1+0.12s}{1+0.04s}$

Put $s = j\omega$ then,

$$G_c(j\omega) = \frac{1+j0.12\omega}{1+j0.04\omega} \quad \text{…(ii)}$$

Compare equation (i) and (ii) we get,

T = 0.12 and $\alpha T = 0.04$

$$\alpha = \frac{0.04}{T} = \frac{0.04}{0.12} = \frac{1}{3}$$

Maximum phase shift = $\sin\phi_m = \frac{1-\alpha}{1+\alpha}$

$$\sin\phi_m = \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{1}{2}$$

$\phi_m = \sin^{-1}(0.5) = 30°$

**Sol. 9.**
Effect of phase lead compensation

Less overshoot, less rise time and less settling time are obtained due to increase of damping and hence there is improvement in the transient response.

Bandwidth of the closed-loop system is increased and hence the response is faster.

**Sol. 13.**

$$G(s) = \frac{40}{s(s+4)}$$

H(s) = 0.50

$$S_G^M = \frac{1}{1+G(s)H(s)} = \frac{s^2+4s}{s^2+4s+20}$$

$$S_G^M\Big|_{s=j\omega} = \frac{1}{1+G(s)H(s)} = \frac{s^2+4s}{s^2+4s+20}$$

$$\frac{-169+j55}{-169+j52+20}=\frac{-1.69+j52}{18.31+j52}$$

(put ω = 1.3 rad/sec)

$$\left|S_G^M\right|=0.287$$

**Sol. 14. (b)**

**Sol. 15. (d)**
In phase lead compensator, zero is nearer to origin. In phase lag compensator, pole is nearer to origin.

**Sol. 16. (b)**
Phase lead compensation improves transient response. Phase lag compensation improves steady state response

**Sol. 17. (d)**
For a stable system, both GM and PM should be positive.

**Sol. 18. (c)**

**Sol. 19. (b)**

$$G(s)=\frac{1+0.5s}{1+s}$$

Comparing it with $\alpha\left(\frac{1+s\tau}{1+s\tau}\right)$

$\tau = 0.5$

$\alpha\tau = 1 \Rightarrow = \frac{1}{0.5}$

$\alpha = 2$

Since $\alpha > 1$, It is a lag compensator

# GATE QUESTIONS

**1.** The transfer function C(s) of a compensator is given below:

$$C(s) = \frac{\left(1+\frac{s}{0.1}\right)\left(1+\frac{s}{100}\right)}{(1+s)\left(1+\frac{s}{10}\right)}$$

The frequency range in which the phase (lead) introduce by the compensator reaches the maximum is

**[GATE - 2017]**

(a) $0.1 < \omega < 1$ (b) $1 < \omega < 10$
(c) $10 < \omega < 100$ (d) $\omega > 100$

**2.** Which of the following statement is incorrect?

**[GATE - 2017]**

(a)Lead compensator is used to reduce the settling time.
(b)Lag compensator is used to reduce the steady state error.
(c)Lead compensator may increase the order of a system
(d)Lag compensator always stabilizes an unstable system.

**Common data for Q. 3 and Q. 4**
The transfer function of a compensator is given as $G_c(s) = \frac{s+a}{s+b}$

**3.** $G_c(s)$ is a lead compensator if

**[GATE - 2012]**

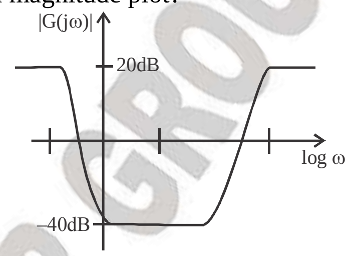
(a) a = 1, b = 2 (b) a = 3, b = 2
(c) a = –3, b = –1 (d) a = 3, b = 1

**4.** The phase of the above lead compensator is maximum at

**[GATE - 2012]**

(a) $\sqrt{2}\text{rad/s}$ (b) $\sqrt{3}\text{rad/s}$
(c) $\sqrt{6}\ \text{rad/s}$ (d) $1/\sqrt{3}\text{rad/s}$

**5.** The magnitude plot of a rational transfer function G(s) with real coefficients is shown below. Which of the following compensators has such a magnitude plot?




**[GATE - 2009]**

(a) Lead compensator
(b) Lag compensator
(c) PID compensator
(d) Lead - lag compensator

**6.** The transfer functions of two compensators are given below :

$$C_1 = \frac{10(s+1)}{(s+10)}, C_2 = \frac{s+10}{10(s+1)}$$

Which one of the following statements is correct?

**[GATE - 2008]**

(a) $C_1$ is lead compensator and $C_2$ is a lag compensator
(b) $C_1$ is a lag compensator and $C_2$ is a lead compensator
(c) Both $C_1$ and $C_2$ are lead compensator
(d) Both $C_1$ and $C_2$ are lag compensator

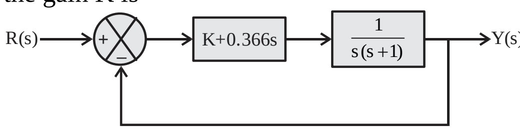
**7.** The open loop transfer function of a plant is given as $G(s) = \frac{1}{s^2-1}$. If the plant is operated in a unity feedback configuration, then the lead compensator that an stabilize this control system is

**[GATE - 2007]**

(a) $\frac{10(s-1)}{s+2}$ (b) $\frac{10(s+4)}{s+2}$

(c) $\frac{10(s+2)}{s+10}$ (d) $\frac{2(s+2)}{s+10}$

**8.** If the compensated system shown in the figure has a phase margin of $60^o$ at the crossover frequency of 1 rad/sec, then value of the gain K is




**[GATE - 2005]**

(a) 0.366 (b) 0.732
(c) 1.366 (d) 2.738

**9.** A lead compensator used for a closed loop controller has the following transfer function $\frac{K\left(1+\frac{s}{a}\right)}{\left(1+\frac{s}{b}\right)}$ For such a lead compensator

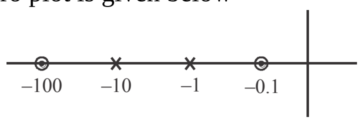
**[GATE - 2003]**

(a) a < b (b) b < a
(c) a > Kb (d) a<Kb

# SOLUTIONS

**Sol.1. (a)**

Pole zero plot is given below




$$\text{Lead}\, G(s) = \frac{s+0.1}{s+1}$$

$$\angle G(s) = \angle \tan^{-1}\frac{\omega}{0.1} - \tan^{-1}\frac{\omega}{1}$$

Phase lead occur from $\omega = 0.1$ to $\omega = 1$

Range $0.1 < \omega < 1$

**Sol.2. (d)**

Lag compensator reduces the steady state error but it cannot stabilizes an unstable system.

**Sol.3. (a)**

$$G_c(s) = \frac{s+a}{s+b} = \frac{j\omega+a}{j\omega+b}$$

Phase lead angle, $\phi = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{b}\right)$

$$\tan^{-1}\left(\frac{\frac{\omega}{a}-\frac{\omega}{b}}{1+\frac{\omega^2}{ab}}\right) = \tan^{-1}\left(\frac{\omega(b-a)}{ab+\omega^2}\right)$$

For phase lead compensation $\phi > 0$

$b - a > 0$

$b > a$

> For phase lead compensator zero is nearer to the origin as compared to pole, so option (C) cannot be true.

**Sol.4. (a)**

$$\phi = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan{-1}\left(\frac{\omega}{b}\right)$$

$$\frac{d\phi}{d\omega} = \frac{1/a}{1+\left(\frac{\omega}{a}\right)^2} - \frac{1/b}{1+\left(\frac{\omega}{b}\right)^2} = 0$$

$$\frac{1}{a} - \frac{1}{b} = \frac{\omega^2}{ab}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$\omega = \sqrt{ab} = \sqrt{1\times 2} = \sqrt{2}\,\text{rad/sec}$$

**Sol.5. (c)**

This compensator is roughly equivalent to combining lead and lad compensators in the same design and it is referred also as PID compensator.

**Sol.6. (a)**

For $C_1$ Phase is given by

$$Q_{C_1} = \tan^{-1}(\omega) - \tan^{-1}\left(\frac{\omega}{10}\right)$$

$$= \tan^{-1}\left(\frac{\omega - \frac{\omega}{10}}{1+\frac{\omega^2}{10}}\right)$$

$$= \tan^{-1}\left(\frac{9\omega}{10+\omega^2}\right) > 0 \quad \text{(Phase lead)}$$

Similarly for $C_2$, phase is

$$\theta_{C_2} = \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}(\omega)$$

$$= \tan^{-1}\left(\frac{\frac{\omega}{10} - \omega}{1+\frac{\omega^2}{10}}\right)$$

$$= \tan^{-1}\left(\frac{-9\omega}{10+\omega^2}\right) \quad \text{(Phase lag)}$$

**Sol.7. (a)**

$$G(s) = \frac{1}{s^2-1} = \frac{1}{(s+1)(s-1)}$$

The lead compensator C(s) should first stabilize the plant i.e. remove $\frac{1}{(s-1)}$ term. From only options (A), C(s) can remove this term .Thus

$$G(s)C(s) = \frac{1}{(s+1)(s-1)} \times \frac{10(s-1)}{(s+2)}$$

$= \frac{10}{(s+1)(s+2)}$ Only option (a) is satisfies.

**Sol.8. (c)**
Open loop transfer function of the system is given by

$$G(s)H(s) = (K+0.366s)\left[\frac{1}{s(s+1)}\right]$$

$$G(j\omega)H(j\omega) = \frac{K + j0.366\omega}{j\omega(j\omega+1)}$$

Phase margin of the system is given as

$\phi_{PM} = 60^o = 180^o + \angle G(j\omega_g)H(j\omega_g)$

Where $\omega_g \to$ gain cross over frequency = 1 rad/sec

So, $60° = 180° + \angle G(j\omega_g)H(j\omega_g)$

$= 90° + \tan^{-1}\left(\frac{0.366}{K}\right) - \tan^{-1}(1)$

$= 90° - 45° + \tan^{-1}\left(\frac{0.366}{K}\right)$

$15° = \tan^{-1}\left(\frac{0.366}{K}\right)$

$\frac{0.366}{K} = \tan 15°$

$K = \frac{0.366}{0.267} = 1.366$

**Sol.9. (a)**
Transfer function of lead compensator is given by

$$H(s) = \frac{K\left(1+\frac{s}{a}\right)}{\left(1+\frac{s}{b}\right)}$$

$$H(j\omega) = K\left[\frac{1+j\left(\frac{\omega}{a}\right)}{1+j\left(\frac{\omega}{b}\right)}\right]$$

So, phase response of the compensator is

$\theta_h(\omega) = \tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{b}\right)$

$= \tan^{-1}\left(\frac{\frac{\omega}{a}-\frac{\omega}{b}}{1+\frac{\omega^2}{ab}}\right) = \tan^{-1}\left[\frac{\omega(b-a)}{ab+\omega^2}\right]$

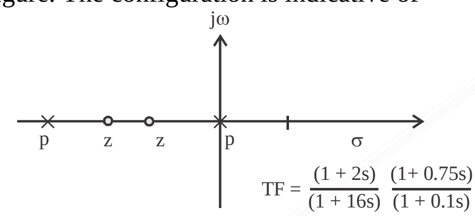
$\theta_h$ should be positive for phase lead compensation

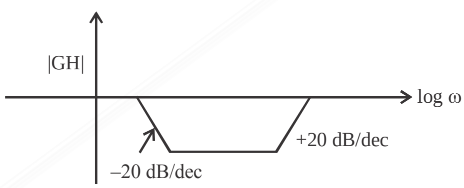
So, $\theta_h(\omega) = \tan^{-1}\left[\frac{\omega(b-a)}{ab-\omega^2}\right] > 0$

$b > a$

# ESE OBJ QUESTIONS

**1.** The pole – zero configuration of the transfer function of a compensator and the corresponding Bode plot are shown in the figure. The configuration is indicative of




**[EE ESE - 2018]**

(a) Lag compensator
(b) Lag-lead compensator
(c) P–D compensator
(d) Lead compensator

**2. Statement (I):** Phase lag network is used to increase stability as well as bandwidth of the system.
**Statement (II):** Phase lead network increases bandwidth of the system.

**[EE ESE - 2017]**

(a)Both Statement (I) and Statement (II) are individually true and statement (II) is the correct explanation of Statement (I).
(b)Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
(c)Statement (I) is true but Statement (II) is false
(d)Statement (I) is false but Statement (II) is true.

**3.** A phase – lead network has its transfer function $G_C(s) = \dfrac{(1+0.04s)}{(1+0.01s)}$. What is the frequency at which the maximum phase – lead occurs?

**[EE ESE - 2017]**

(a) 25 rad/sec (b) 50 rad/sec
(c) 75 rad/sec (d) 100 rad/sec

**4.** A linear time – invariant control system with unsatisfactory steady state error is to be compensated. Which is/are the correct type of cascade compensation to be provided ?
1. Lead
2. Lag
3. Lag-lead
Select the correct answer using the codes given below:

**[EE ESE - 2017]**

(a) 1 only (b) 2 only
(c) 3 only (d) 1, 2 and 3 only

**5.** For a lead compensator, whose transfer function is given by $K = \dfrac{s+a}{s+b}$; a, b ≥ 0

**[EC ESE - 2017]**

(a) a < b (b) a > b
(c) a ≥ Kb (d) a = 0

**6. Statement (I):** Elements with non – minimum phase transfer functions introduce large phase lags with increasing frequency resulting in complex compensation problems.
**Statement (II):** Transportation lag commonly encountered in process control systems is a non – minimum phase elements.

**[EC ESE - 2017]**

(a) Both Statement (I) and Statement (II) are individually true and statement (II) is the correct explanation of Statement (I).

(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
(c) Statement (I) is true but Statement (II) is false
(d) Statement (I) is false but Statement (II) is true.

**7.** An R-C network has the transfer functions

$$G_C(s) = \frac{s^2 + 10s + 24}{s^2 + 10s + 16}$$

The network could be used as
1. Lead compensator
2. Lag compensator
3. Lag-lead compensator
Which of the above is/are, correct?

**[EE ESE - 2016]**

(a) 1 only (b) 2 only
(c) 3 only (d) 1, 2 and 3

**8. Statement (I)**: For type-II or higher system lead compensator may be used.
**Statement (II)**: Lead compensator increases the margin of stability.

**[EE ESE - 2016]**

(a) Both Statement (I) and Statement (II) are individually true and statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I)
(c) Statement (I) is true but Statement (II) is false
(d) Statement (I) is false but Statement (II) is true.

**9.** A phase lead compensator has its transfer function, $G_C(s) = \frac{1+0.5s}{1+0.05s}$. The maximum phase lead and the corresponding frequency, respectively are nearly.

**[EC ESE - 2015]**

(a) $\sin^{-1}$ (0.9) and 6 r/s
(b) $\sin^{-1}$ (0.82) and 6 r/s
(c) $\sin^{-1}$ (0.9) and 4 r/s
(d) $\sin^{-1}$ (0.82) and 6 r/s

**10.** Consider the following statements:
1. Lead compensation decreases the bandwidth of the system:
2. Lag compensation increases the bandwidth of the system?
Which of the above statements is/are correct?

**[EC ESE - 2016]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

**11.** The transfer function of a controller is $G_C(s) = \frac{1+3s}{1+s}$. The maximum phase control provided by this controller is

**[EC ESE - 2015]**

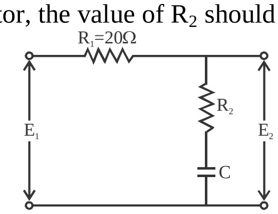
(a) 30° lead (b) 30° lag
(c) 45° lead (d) 45° lag

**12.** Consider the following statements:
The effect of phase lead network is given as
1. Increased velocity constant
2. Increased phase margin
3. Increased bandwidth
4. Slower response
Which of the above statements are correct?

**[EC ESE - 2015]**

(a) 1, 2 and 3 only (b) 1, 2 and 4
(c) 2, 3 and 4 only (d) 1, 2, 3 and 4

**13.** For the following network to work as lag compensator, the value of $R_2$ should be




**[EE ESE - 2015]**

(a) $R_2 \geq 20\ \Omega$ (b) $R_2 \leq 10\ \Omega$
(c) $R_2C \leq \frac{R_1^2C}{2}$ (d) Any value of $R_2$

**14.** Time response of an indicating instrument is decided by which of the following systems ?

**[EE ESE - 2015]**

(a) Mechanical system provided by pivot and jewel bearing
(b) Controlling system

(c) Deflecting system
(d) Damping system

**15. Statement (I):** The state feedback design is more realistic than conventional fixed configuration controller design.
**Statement (II):** The disadvantage with the state feedback is that all the states must be sensed and fed back for control.

**[EE ESE - 2014]**

(a) Both Statement (I) and Statement (II) are individually true and statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

**16.** The effect of integral controller on the steady-state error $e_{ss}$ and that on the relative stability $R_s$ of the system is

**[EE ESE - 2014]**

(a) Both are increased
(b) $e_{ss}$ is increased but $R_s$ is reduced
(c) $e_{ss}$ is reduced but $R_s$ is increased
(d) Both are reduced

**17.** The correct sequence of steps needed to improve system stability is

**[EE ESE - 2014]**

(a) Insert derivation action, use negative feedback and reduce gain.
(b) Reduce gain, use negative feedback and insert derivation action.
(c) Reduce gain, insert derivation action and use negative feedback
(d) Use negative feedback, reduce gain and insert derivation action.

**18.** Two compensator have transfer functions $G_1(s) = \frac{5(s+10)}{(s+50)}$ and $G_2(s) = \frac{(s+50)}{5(s+10)}$ respectively.

**[EC ESE - 2014]**

(a) Both are lag
(b) Both are lead
(c) $G_1$ is lead and $G_2$ is lag
(d) $G_1$ is lag and $G_2$ is lead

**19.** By adding zero to the system transfer function, the improvement to transient response is called is:

**[EC ESE - 2014]**

(a) Phase lead compensation
(b) Phase lag compensation
(c) Phase lag and phase lead compensation
(d) Phase lead and phase lag compensation

**20.** The network having transfer $G(s) = \dfrac{1+\frac{s}{4}}{1+\frac{s}{25}}$ will provide maximum phase lead at a frequency of:

**[EC ESE - 2014]**

(a) 4 rad/sec (b) 25 rad/sec
(c) 10 rad/sec (d) 100 rad/se

**21.** In a closed loop system for which the output is the speed of a motor, the output rate control can be used to

**[EE ESE - 2013]**

(a) Reduce the damping of the system
(b) Limit the torque output of the motor
(c) Limit the speed of the motor
(d) Limit the acceleration of the motor

**22.** An effect of phase – lag compensation on servo – system performance is that

**[EE ESE - 2013]**

(a) For a given relative stability the velocity constant is increased
(b) For a given relative stability, the velocity constant is decreased
(c) The bandwidth of the system is increased
(d) The time response of the system is made faster

**23. Statement (I):** The rotor of a servomotor is built with resistance so that is X/R ratio becomes small.
**Statement (II):** The servomotor has good accelerating characteristics.
**[EE ESE - 2012]**
**Codes:**
(a)Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b)Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c)Statement (I) is true but Statement (II) is false.
(d)Statement (I) is false but Statement (II) is true.

**24. Statement (I):** Control system components for aviation systems are designed for 400 Hz.
**Statement (II):** The weight of the components reduces when designed for higher frequencies.
**[EE ESE - 2012**
**Codes:**
(a)Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b)Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c)Statement (I) is true but Statement (II) is false.
(d)Statement (I) is false but Statement (II) is true.

**25.** In position control system, the device used for providing rate feedback is called.
**[EE ESE - 2012]**
(a) Potentiometer (b) Synchro
(c) Tachogenerator (d) Servomotor

**26.** The following transfer function represents a phase – lead compensator
**[EE ESE - 2012]**
(a) $\frac{s+4}{s+6}$ (b) $\frac{4s+2}{6s+1}$
(c) $\frac{s+4}{3s+6}$ (d) $\frac{1}{s}$

**27.** Given a badly underdamped control system, the type of cascade compensator to be used to improve its damping is
**[EE ESE - 2012]**
(a) Phase – lead (b) Phase – lag
(c) Phase – lag – lead (d) Notch filter

**28.** The phase – lead compensation is used to
**[EE ESE - 2012]**
(a) Increase rise time and decrease overshoot.
(b) Decrease both rise time and overshoot.
(c) Increase both rise time and overshoot.
(d) Decrease rise time and increase overshoot.

**29.** What is the effect of lag compensator on the system bandwidth and the signal – to – noise ratio?
**[EE ESE - 2012]**
(a) Bandwidth is increased and signal – to – noise ratio is improved.
(b) Bandwidth is increased and signal – to – noise ratio is deteriorated.
(c) Bandwidth is reduced and signal – to noise ratio is deteriorated.
(d) Bandwidth is reduced and signal – to – noise ratio is improved.

**30.** A phase lead compensating network has its transfer function $G_C(s) = \frac{10(1+0.04s)}{(1+0.01s)}$. The maximum phase lead occurs at a frequency of
**[EC ESE - 2012]**
(a) 50 rad/s (b) 25 rad/s
(c) 10 rad/s (d) 4 rad/s

**31.** Considering the filtering property, the lead compensators and lag compensators are categorized respectively as
**[EC ESE - 2012]**
(a) Low pass and high pass filters
(b) High pass and low pass filters
(c) High pass and high pass filters
(d) Low pass and low pass filters

**32.** The necessary conditions for poles and zeros of the transfer function of a bridge-T network containing only resistors and capacitors and used as a compensator are

**[EC ESE - 2012]**

(a) All the poles and zeros must be imaginary
(b) Poles and zeros both can be complex
(c) Poles can be complex but zeros must be real
(d) Zeros can be complex but poles must be real

**33.** What is the transfer function of a phase lead compensator? The values of $\beta$ and $\tau$ are given as $\beta < 1$ and $\tau > 0$.

**[EC ESE - 2011]**

(a) $\frac{\beta(\tau s+1)}{(\beta\tau s+1)}$ (b) $\frac{\beta(\beta\tau s+1)}{(\tau s+1)}$

(c) $\frac{\beta(\beta\tau s-1)}{(\tau s+1)}$ (d) $\frac{\beta(\beta\tau s-1)}{(\tau s-1)}$

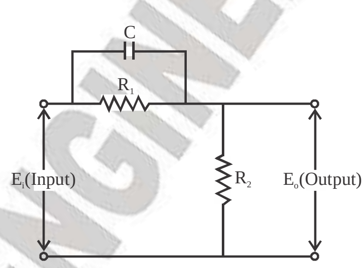
**34.** What is the transfer function of a phase lag compensator? The values of $\alpha$ and $\tau$ are given as $\alpha > 1$ and $\tau > 0$.

**[EC ESE - 2011]**

(a) $\frac{1}{\alpha}\frac{\left(S+\frac{1}{\tau}\right)}{\left(S+\frac{1}{\alpha\tau}\right)}$ (b) $\frac{1}{\alpha}\frac{\left(S-\frac{1}{\tau}\right)}{\left(S-\frac{1}{\alpha\tau}\right)}$

(c) $\frac{1}{\alpha}\frac{\left(S+\frac{1}{\tau}\right)}{\left(S-\frac{1}{\alpha\tau}\right)}$ (d) $\frac{1}{\alpha}\frac{\left(S-\frac{1}{\tau}\right)}{\left(S+\frac{1}{\alpha\tau}\right)}$

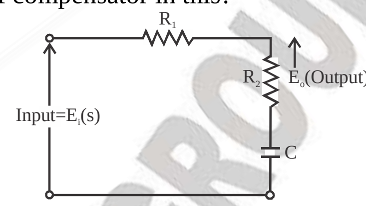
**35.** The circuit diagram of an electrical network is given in figure. What type of compensator is this?




**[EC ESE - 2011]**

(a)Phase lag compensator
(b)Phase lead compensator
(c)Lag-lead compensator
(d)Neither phase lag nor phase lead compensator

**36.** An electrical network shown in figure. What type of compensator in this?




**[EC ESE - 2011]**

(a) Phase lead compensator
(b) Phase lag compensator
(c) Lag-lead compensator
(d)Neither phase lead nor phase lag compensator

**37.** What are the gain and phase angle of a system having the transfer function G(s) = (s + 1) at a frequency of 1 rad/sec?

**[EC ESE - 2011]**

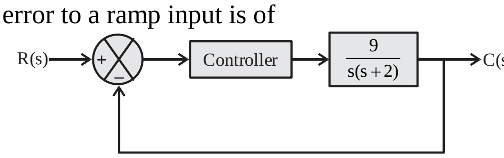
(a) 0.41 and 0° (b) 1.41 and 45°
(c) 1.41 and –45° (d) 2.41 and 90°

**38.** For a tachometer, if $\theta(t)$ is the rotor displacement, e(t) is the output voltage an K is the tachometer constant, then the transfer function is defined as

**[EE ESE - 2011]**

(a) $Ks^2$ (b) K/s
(c) Ks (d) K

**39.** In the control system shown below the controller which can give zero steady – state error to a ramp input is of




**[EE ESE - 2011]**

(a) Proportional type
(b) Integral type
(c) Derivative type

(d) Proportional plus derivative type

**40.** The transfer function of a phase-lead compensator is given by $G(s) = \frac{1+3Ts}{1+Ts}, T > 0.$ The maximum phase shift provided by such a compensator is

**[EE ESE - 2010]**

(a) 90° (b) 60°
(c) 45° (d) 30°

**41.** How can the bandwidth of a control system be increased?

**[EC ESE - 2009]**

(a) By the use of phase lead network
(b) By the use of phase lag network
(c) By the use of both phase-lag and phase-lead network
(d) By the use of cascaded amplifiers in the system

**42.** Consider the following statements in connection with two - position controller.
(i) If the controller has a 4% neutral zone, its position error band will be 2% and negative error band will be 8%.
(ii) The neutral zone is also known as dead
(iii) The controller action of a two - position controller is very similar to that of a pure on - off controller.
(iv) Air - conditioning system works essentially on a two - position control basis.
Which of the above statements are correct ?

**[EE ESE - 2009]**

(a) i, ii and iii only (b) ii, iii and iv only
(c) ii and iv only (d) i, ii, iii and iv

**43. Assertion (A):** The stator windings of a control transformer has higher impedance per phase.
**Reason (R):** The rotor of a control transformer is a cylindrical in shape.

**[EE ESE - 2009]**

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**44.** Consider the following statements with reference to hydraulic systems:
(i) A small size actuator can develop a very large force of torque.
(ii) A source with supply and return line is required.
(iii) It is insensitive to temperature changes.
Which of the above statements is/are correct?

**[EE ESE - 2009]**

(a) i only (b) ii only
(c) i and ii (d) ii and iii

**45.** The poles and zeroes of an all-pass network are located in which part of the s-plane ?

**[EE ESE - 2009]**

(a) Poles and zeroes are in the right half of s-plane.
(b) Poles and zeroes are in the left half of s-plane.
(c) Poles in the right half and zeroes in the left half of s-plane.
(d) Poles in the left half and zeroes in the right half of s-plane.

**46.** If D is the rotor diameter and L, the axial length, then a high performance a.c. servomotor is characterized by which one of the following ?

**[EE ESE - 2009]**

(a) Large D and Large L
(b) Large D and Small L
(c) Small D and Small L
(d) Small d and Large L

**47.** Consider the following statements:
(i) A phase lead network provides a positive phase angle over the frequency range of interest.
(ii) Armature controlled d.c. servo motor is inherently a closed - loop system.
(iii) Phase lag network provides significant amplification over the frequency range of interest.
(iv) Transfer functions with zeros in the right half of a s - plane is an non - minimum phase system.
Which of these statements is/are correct?

**[EE ESE - 2009]**

(a) iii only (b) i and ii only

(c) i, ii and iv (d) ii, iii and iv

**48.** The transfer function of a phase - lead compensator is given by:

$$G(s) = \frac{1+3Ts}{1+Ts} \text{ where } T > 0$$

What is the maximum shift provided by such a compensator?

**[EE ESE - 2009]**

(a) $90^0$ (b) $60^0$
(c) $45^0$ (d) $30^0$

**49.** Consider the following statements:
(i) Bandwidth is increased.
(ii) Peak overshoot in the step response is increased
Which of these are the effects of using lead compensation in a feedback system?

**[EE ESE - 2009]**

(a) i only (b) ii only
(c) Both i and ii (d) Neither i nor ii

**50.** Synchro machines are used for which one of the following?

**[EE ESE - 2008]**

(a) Converting single-phase supply to 3-ϕ supply
(b) Stepping up low frequency signal to high frequency
(c) Detection of positional error in a.c. servo system
(d) Detection of positional error in d.c. servo system

**51.** A tachometer is added to a servo – mechanism because

**[EE ESE - 2008]**

(a)It is easily adjustable
(b)It can adjust damping
(c)It converts velocity of the shaft to a proportional d.c. voltage

**52.** For a tachometer if θ(t) is the rotor displacement, e(t) is the output voltage and $K_t$ is the tachometer constant, then the transfer function is defined as

**[EE ESE - 2008]**

(a) $K_t.s^2$ (b) $k_t.s$
(c) $K_t/s$ (d) $K_t$

**53.** The transfer function of a P-I controller is

**[EE ESE - 2008]**

(a) $K_p + K_{i.s}$ (b) $K_p + (K_i/s)$
(c) $(K_p/s) + K_i.s$ (d) $K_p.s + (K_i/s)$

**54.** To detect the position error in a position control system, which of the following may be used ?
(i) Potentiometers
(ii) Synchros
(iii) LVDT
Select the correct answer using the code given below:

**[EE ESE - 2008]**

(a) i and ii (b) i and iii
(c) ii and iii (d) i, ii and iii

**55.** Consider the following statements for a PI compensator for a control system:
1. It is equivalent to adding a zero at origin.
2. It reduces overshoot.
3. It improves order of the system by 1.
4. It improves steady-state error of the system.
Which of the statements given above are correct?

**[EC ESE - 2007]**

(a) 1, 2, 3 and 4 (b) 1, 2 and 3 only
(c) 2, 3 and 4 only (d) 1 and 4 only

**56.** For a stepper motor, what is the correct relationship between the maximum slew rate (MSR) and the load ?

**[EE ESE - 2007]**

(a) MSR decreases as load is reduced
(b) MSR increases considerably as load is increased
(c) MSR increases as load is reduced
(d) MSR remains the same even if the load is changed

**57.** Which one of the following is the correct statement ?
The rotor resistance to reactance ratio and the moment of inertia of an ac servomotor in

comparison to an ordinary 2 - ϕ induction motor of similar rating area respectively,

**[EE ESE - 2007]**

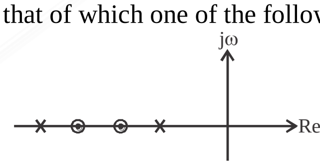
(a) Lower and lower (b) Lower and higher
(c) Higher and higher (d) Higher and lower

**58.** Which of the following are the characteristics of a phase – lead controller ?
(i) When used properly it can increase the damping of the system.
(ii) It improves rise time.
(iii) It improves setting time.
(iv) It affects the steady state error.
Select the correct answer using the code given below:

**[EE ESE - 2007]**

(a) i, ii and iv (b) i, iii and iv
(c) ii, iii and iv (d) i, ii and iii

**59.** The pole – zero plot shown below in the figure is that of which one of the following ?




**[EE ESE - 2007]**

(a) Integrator
(b) PD controller
(c) PID controller
(d) Lag-lead compensator

**60.** The phase lead compensation is used for which one of the following ?

**[EE ESE - 2007]**

(a) To increase rise time and decrease overshoot
(b) To decrease both rise time and overshoot
(c) To increase both rise time and overshoot
(d) To decrease rise time and increase overshoot

**61.** What is the effect of lag compensator on the system bandwidth (BW) and the signal to noise ratio (SNR) ?

**[EE ESE - 2007]**

(a) BW is reduced and SNR is improved
(b) BW is reduced and SNR is dete – riorated
(c) BW is increased and SNR is improved.
(d) BW is increased and SNr is dete - riorated

**62.** What is the effect of providing distance – velocity lag/transportation lag ?

**[EE ESE - 2007]**

(a) To increase the phase margin
(b) To reduce the phase margin
(c) To alter the gain at a given ω
(d) To improve the transient response of the system.

**63.** Microsyn is based on the principle of

**[EE ESE - 2007]**

(a) DC motor
(b) Resolver
(c) Saturable reactor
(d) Rotating differential transformer

**64.** Which one of the following is required for stability of an arc servomotor ?

**[EE ESE - 2007]**

(a) A negative slope on the torque – speed curve
(b) A linearized torque – speed curve
(c) The ratio of the rotor reactance to rotor resistance should be high
(d) The rotor diameter should be less and axial length large

**65.** Match **List-I** with **List-II** and select the correct answer using the code given the below the lists :

**List-I**
A. Synchros
B. Operational amplifier
C. Stepper motor
D. Tacho-generator

**List-II**
(i) Controller
(ii) Error detector
(iii) Actuator
(iv) Feedback element

**[EE ESE - 2006]**

**Codes:**
(a) A-iii, B-i, C-ii, D-iv
(b) A-ii, B-iv, C-iii, D-i
(c) A-iii, B-iv, C-ii, D-i
(d) A-ii, B-i, C-iii, D-iv

**66.** Match **List-I** (Application) with **List-II** (Control System Component) and select the correct answer using the code given the below the lists:

**List-I**

A. Measuring inclination of frames in inertial navigation system

B. Used as an actuator element in computer printer

C. For low power applications

**List-II**

(i) Gyroscope

(ii) Servomotor

(iii) Stepper Motor

(iv) Schrage Motor

**[EE ESE - 2006]**

**Codes:**

(a) A-ii, B-iii, C-iv

(b) A-i, B-iv, C-ii

(c) A-i, B-iii, C-ii

(d) A-ii, B-i, C-iv

**67.** The effect of tachometer feedback in a control system is to reduce

**[EE ESE -2006]**

(a) Only time constant

(b) Only gain

(c) Damping

(d) Both gain and time constant

**68.** The transfer function of a phase lead compensator is found to be of the form $\frac{s+z_1}{s+p_1}$ and that of a lag compensator to be of the form $\frac{s+z_3}{s+p_2}$.

Then which of the following conditions must be satisfied?

**[EE ESE - 2006]**

(a) $z_1 > p_1$ and $z_2$ and $p_2$

(b) $z_1 > p_1$ and $z_2 < p_2$

(c) $z_1 < p_1$ and $z_2 < p_2$

(d) $z_1 < p_1$ and $z_2 > p_2$

**69.** What is the effect of phase lead compensator on gain cross – over frequency ($\omega_{gc}$) and on the bandwidth ($\omega_b$) ?

**[EE ESE - 2006]**

(a) Both are increased

(b) $\omega_{gc}$ is increased but $\omega_b$ is decreased

(c) $\omega_{gc}$ is decreased but $\omega_b$ is increased

(d) Both are decreased

**70. Assertion (A):** With lag-lead compensation, the bandwidth of the system is not affected much.

**Reason (R):** The effect of lag and lead compensations at high frequencies cancel one another.

**[EE ESE - 2006]**

(a)Both A and R are true and R is the correct explanation of A.

(b)Both A and R are true but R is NOT the correct explanation of A.

(c)A is true but R is false.

(d)A is false R is true.

**71. Assertion (A):** DC servomotors are more commonly used in armature controlled mode instead of in field controlled mode.

**Reason (R):** Armature controlled DC motors have higher starting torque than field controlled motors.

**[EE ESE - 2006]**

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is NOT the correct explanation of A.

(c) A is true but R is false.

(d) A is false R is true.

**72. Assertion (A):** For a control system having synchro pair as error detector dc amplifier as control amplifier, a phase sensitive detector is required to demodulate in place of ordinary diode detector.

**Reason (R):** Synchro output is a suppressed carrier amplitude modulated signal which cannot be demodulated by ordinary diode detector.

**[EE ESE – 2006]**

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is NOT the correct explanation of A.
(c) A is true but R is false.
(d) A is false R is true.

**73.** A phase lead compensating network consists of only capacitor and resistors. The locations of its pole and zero in s-plane are at $p_c$ and $z_c$ respectively. Which of the following conditions must be satisfied?

**[EC ESE-2006]**

(a) Both $p_c$ and $Z_c$ in LHS and $p_c < Z_c$
(b) Both $P_c$ and $Z_c$ in LHS and $p_c > Z_c$
(c) $p_c$ is in LHS and $Z_c$ can be in RHS
(d) $Z_c$ is in LHS and $p_c$ can be in RHS

**74.** What is the effect of phase-lag compensation on the performance of a servo system?

**[EC ESE-2005]**

(a) For a given relative stability, the velocity constant is increased.
(b) For a given relative stability, the velocity constant is decreased.
(c) The bandwidth of the system is increased.
(d) The time response is made faster.

**75.** Match **List-I** (Compensation) with **List-II** (Characteristic) and select the correct answer using the code given below lists:

**List-I**
A. Lag
B. Lead
C. Lag-Lead
D. Rate

**List-II**
(i) Increases bandwidth
(ii) Attenuation
(iii) Increases damping factor
(iv) Second order

**[EC ESE - 2005]**

**Codes**:
(a) A-iii, B-i, C-iv, D-ii
(b) A-ii, B-iv, C-i, D-iii
(c) A-iii, B-iv, C-i, D-ii
(d) A-ii, B-i, C-iv, D-iii

**76.** The transfer function of phase-lead compensator is given by $G(s)=\frac{1+aTs}{1+Ts}$ where T > 0, a > 1.What is the maximum phase shift provided by this compensator?

**[EC ESE - 2005]**

(a) $\tan^{-1}\left(\frac{a+1}{a-1}\right)$
(b) $\tan^{-1}\left(\frac{a-1}{a+1}\right)$
(c) $\cos^{-1}\left(\frac{a-1}{a+1}\right)$
(d) $\sin^{-1}\left(\frac{a-1}{a+1}\right)$

**77.** Match List-I (System) with List-II (Transfer function) and select the correct answer using the code given below:

**List-I**
A. Lag Network
B. AC Servomotor
C. Field Controlled dc servomotor
D. Tacho-generator

**List-II**
(i) $K\left(\frac{1+aTs}{1+Ts}\right)$
(ii) $K_1 s$
(iii) $\frac{K}{s(1+s\tau_m)(1+s\tau_f)}$
(iv) $\frac{K_m}{s(1+s\tau_m)}$

**[EE ESE - 2005]**

**Codes:**
(a) A-iii, B-ii, C-i, D-iv
(b) A-i, B-iv, C-iii, D-ii
(c) A-iii, B-iv, C-i, D-ii
(d) A-i, B-ii, C-iii, D-iv

**78.** In a speed control system, output rate feedback is used to

**[EE ESE - 2005]**

(a) Limit the speed of motor
(b) Limit the acceleration of the motor
(c) Reduce the damping of the system
(d) Increase the gain margin

**79.** Match List-I (Name of the Control System Component) with List-II (Use of the

Component in Control System) and select the correct answer using the code given below:

**List-I**

A . Amplidyne
B . Potentiometer
C. Stepper motor
D. AC tacho - generator

**List-II**

(i) Feed back element
(ii) Actutator
(iii) Control Amplifier
(iv) Error detector

**[EE ESE - 2005]**

**Codes:**

(a) A-iii, B-i, C-ii, D-iv
(b) A-ii, B-iv, C-iii, D-i
(c) A-iii, B-iv, C-ii, D-i
(d) A-ii, B-i, C-iii, D-iv

**80.** Consider the following statements regarding compensators used in control systems:

(i) For type-2 or higher systems, lag compensator is universally used to overcome the undesirable oscillatory transient response.
(ii) In case of lag- lead compensator, a lag and a lead compensator are basically connected in parallel.
(iii) The S-plane representation of the lead compensator has a zero closer to the origin than the pole.
(iv) A lag compensator improves the steady state behavior of a system while nearly maintaining its transient response.

Which of the statements given above are correct?

**[EE ESE - 2005]**

(a) i,ii and iii (b) ii, iii and iv
(c) i and ii (d) iii and iv

**81.** If the rotor axis of synchro transmitter is along the axis of $S_2$ stator winding, when will be the electrical zeroing ?

**[EE ESE - 2005]**

(a) $V_{s1}$, $V_{s2}$ is maximum
(b) $V_{s2}$, $V_{s3}$ is maximum
(c) $V_{s2}$, $V_{s3}$ is minimum
(d) $V_{s3}$, $V_{s1}$ is minimum

**82.** Consider the following statements:

A first order system with a proportional controller exhibits an offset to a step input. In order to reduce the offset, it is necessary to

(i) increase the gain of proportional controller
(ii) add a derivative mode
(iii) add an integral mode

Select the correct answer using the code given below:

**[EE ESE - 2005]**

(a) i, ii and iii (b) i and ii
(c) ii and iii (d) i and iii

**83.** In the block diagram of a separately excited dc motor, how does the armature induced emf appear as ?

**[EE ESE - 2005]**

(a) Positive feedback
(b) Negative feedback
(c) Disturbance input
(d) Output

**84.** A linear ac servomotor must have:

**[EE ESE - 2005]**

(a) High rotor resistance
(b) High rotor reactance
(c) A large air gap
(d) Both high rotor resistance and reactance

**85.** Consider the following statements:

(i) Servomotors have lighter rotor as compared to ordinary motors and hence lower inertia
(ii) Back e.m.f. in field controlled d.c. motors acts as minor loop feedback and results in increased damping and improved transient response
(iii) Permanent magnet d.c. servomotors can be used in either armature-controlled or field-controlled nodes.

Which of the above statements are not correct?

**[EE ESE - 2004]**

(a) i and ii (b) ii and iii
(c) i and iii (d) i, ii and iii

**86.** Match List-I (Name of the Component) with List-II (Type of the Component) and select the correct answer using the codes given below:

**List-I**
A. Amplidyne
B. Potentiometer
C. Stepper motor
D. AC Tachogenerator

**List-II**
(i) Rate feedback
(ii) Actuator
(iii) Servo amplifier
(iv) Error detector

**[EE ESE - 2004]**

**Codes:**
(a) A-iii, B-ii, C-i, D-iv
(b) A-i, B-ii, C-iii, D-iv
(c) A-iii, B-iv, C-ii, D-i
(d) A-i, B-iv, C-iii, D-ii

**87.** Which one of the following is not a correct reason to select feedback compensation over cascaded one ?

**[EE ESE - 2004]**

(a) No amplification is required as the energy transfer is from higher to lower level.
(b) Suitable devices are not available for compensation (series)
(c) It is economical
(d) Provides greater stiffness against load disturbances

**88.** Which one of the following is the correct expression for the transfer function of an electrical RC phase-lag compensating network?

**[EE ESE - 2004]**

(a) $\frac{RCS}{(1+RCS)}$ (b) $\frac{RC}{(1+RCS)}$
(c) $\frac{1}{(1+RCS)}$ (d) $\frac{1}{(1+RCS)}$

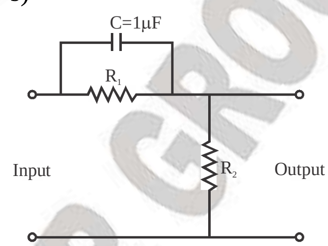
**89.** Consider the following statements for phase-lead compensation:
1.Phase-lead compensation shifts the grain cross-over frequency to the right.
2.The maximum phase-lead angle occurs at the arithmetic mean of the corner.
3.Phase-lead compensation is effective when the slope of the uncompensated system near the gain cross-over is low.

What of the statements given above are correct?

**[EC ESE - 2004]**

(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) 1 and 3

**90.** The transfer function of a phase lead network, as shown in the figure below is $\frac{K(1+0.3s)}{(1+0.17s)}$.

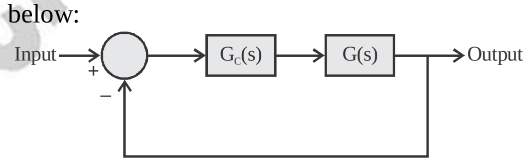



The values of $R_1$ and $R_2$ are respectively.

**[EC ESE - 2003]**

(a) 300 kΩ and 300 kΩ
(b) 300 kΩ and 400 kΩ
(c) 400 kΩ and 300 kΩ
(d) 400 kΩ and 400 kΩ

**91.** A closed loop system, employing lag-lead compensator $G_c(s)$ is shown in the figure given below:




$$G(s)=\left[\frac{1+\tau_1 s}{1+\frac{\tau_1 s}{\beta}}\right]\left[\frac{1+\tau_2 s}{1+\beta\tau_2 s}\right],\beta<1$$

If g(s) has 3 poles in the left half of a s-plane, then the shape of the Bode plot for $|G(s)\,G_c(s)|$ in the highest frequency range will be

**[EC ESE - 2003]**

(a) –20 dB/decade (b) –40 dB/decade
(c) –60 dB/decade (d) –80 dB/decade

**92. Assertion (A):** Servomotors normally have heavier rotors and lower R/X ratio as compared to ordinary motors of similar ratings.

**Reasons (R):** Servomotors should have smaller electrical and mechanical time constants for faster response.

**[EE ESE - 2003]**

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**93. Assertion (A):** Tachogenerator feedback is used as minor loop feedback in position control systems to improve stability.
**Reason (R):** Tachogenerator provides velocity feedback which decreases the damping in the system.

**[EE ESE - 2003]**

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**94. Assertion (A):** Use of lead compensation results in increased system bandwidth.
**Reason (R):** The angular contribution of the compensator pole is more than that of the compensator zero.

**[EE ESE - 2003]**

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**95.** Match List-I (Component) with List-II (Purpose) and select the correct answer:
**List-I**
A. Input potentiometer in D.C. system
B. Synchro pair in a.c. system
C. Motor
D. Feedback tachogenerator
**List-II**
(i) Actuator
(ii) Error detector
(iii) Transducer

**[EE ESE - 2003]**

**Codes:**
(a) A-iii, B-ii, C-iii, D-i
(b) A-ii, B-ii, C-i, D-iii
(c) A-ii, B-iii, C-iii, D-i
(d) A-iii, B- ii, C- i, D-iii

**96.** Which one of the following statements is NOT correct ?

**[EE ESE - 2003]**

(a) The transfer function of a lag–lead compensation network is
$$\frac{(1+sT_a a)(1+sT_2 b)}{(1+sT_1)(1+sT_2)}(a>1, b<1)$$
(b) Bridged T – network is used for cancellation compensation
(c) Phase- lag compensation improves steady state response and often results in reduced rise time
(d) Compensating network can be introduced in the feedback path of a control system

**97.** A property of phase - lead compensation is that the

**[EE ESE - 2003]**

(a) Overshoot is increased
(b) Bandwidth of closed loop system is reduced
(c) Rise-time of closed loop system is reduced
(d) Gain margin is reduced

**98.** Backlash in a stable control system may cause

**[EE ESE - 2002]**

(a) Underdamping
(b) Overdamping
(c) High level oscillations
(d) Low level oscillations

**99.** Consider the following statements regarding A.C. servomotor:

**[EE ESE - 2002]**

(i) The torque – speed curve has negative slope.
(ii) It is sensitive to noise

(iii) The rotor has high resistance and low inertia
(iv) It has slow acceleration

**100.** Indicate which one of the following transfer functions represents phase lead compensator ?

**[EE ESE - 2002]**

(a) $\frac{s+1}{s+2}$ (b) $\frac{6s+3}{6s+2}$
(c) $\frac{s+5}{3s+2}$ (d) $\frac{s+8}{s+5s+6}$

**101.** Match List-I with List-II and select the correct answer:

**List-I**
A. Phase lag controller
B. Addition of zero at origin
C. Derivative output compensation
D. Derivative error compensation

**List-II**
(i) Improvement in transient response
(ii) Reduction in steady – state error
(iii) Reduction in settling time
(iv) Increase in damping constant

**[EE ESE - 2002]**

**Codes:**
(a) A-iv, B-iii, C-i, D-ii
(b) A-ii, B-i, C-iii, D-iv
(c) A-iv, B-i, C-iii, D-ii
(d) A-ii, B-iii, C-i, D-iv

**102.** Consider the following statements regarding a phase-lead compensator:
1. It increases the bandwidth of the system.
2. It helps in reducing the steady state error due to ramp input.
3. It reduces the overshoot due to step input.
Which of the above statements is/are correct?

**[EC ESE - 2002]**

(a) 1 and 2 (b) 1 and 3
(c) 2 and 3 (d) 1 alone

**103.** Which one of the following compensations is adopted for improving transient response of a negative unity feedback system?

**[EC ESE - 2001]**

(a) Phase lead compensation
(b) Phase lag compensation
(c) Gain compensation
(d) Both phase lag compensation and gain compensation

**104.** The transfer function of a phase lead network can be written as.

**[EC ESE - 2001]**

(a) $\frac{1+sT}{1+s\beta T};\beta>1$ (b) $\frac{\alpha(1+sT)}{1+s\alpha T};\alpha 1$
(c) $\frac{\beta(1+sT)}{1+s\beta T+T};\beta<1$ (d) $\frac{(1+sT)}{\alpha(1+sT)};\alpha>1$

**105.** The transfer function of phase–lead compensator is given by

$G(s) = \frac{1+aTs}{a+Ts}$, where T > 0, a > 1.

What is the maximum phase shift provided by this compensator?

**[EC ESE - 2001]**

(a) $\tan^{-1}\left(\frac{a+1}{a-1}\right)$ (b) $\tan^{-1}\left(\frac{a-1}{a+1}\right)$
(c) $\cos^{-1}\left(\frac{a-1}{a+1}\right)$ (d) $\sin^{-1}\left(\frac{a-1}{a+1}\right)$

**106.** Match List-I with List-II and select the correct answer:

**List-I**
A. $e^{-as}$
B. $\frac{1-s}{1+s}$
C. $\frac{1+as}{1+bs}, a<b$
D. $\frac{K}{s(1+as)}$

**List-II**
(i) All-pass filter
(ii) Transport delay
(iii) Lag network
(iv) Servomotor

**[EC ESE - 2001]**

**Codes**:
(a) A-iv, B-iii, C-i, D-ii
(b) A-ii, B-i, C-iii, D-iv

(c) A-ii, B-iii, C-i, D-iv
(d) A-iv, B-i, C-iii, D-ii

**107.** The torque–speed characteristic of two–phase induction motor is largely affected by

**[EE ESE - 2001]**

(a) Voltage
(b) R/X and speed
(c) X/R
(d) Supply voltage and frequency

**108.** A synchro transmitter consists of a

**[EE ESE - 2001]**

(a) Salient pole rotor winding excited by an ac supply and a three – phase balanced stator winding
(b) Three–phase balance stator winding excited by a three phase balanced ac signal and rotor connected to a dc voltage source
(c) Salient pole rotor winding excited by a dc signal
(d) Cylindrical rotor winding and a stepped stator excited by pulses.

**109.** The transfer function of a phase lag compensator is given by $\frac{1+Ts}{1+aTs}$, where a > 1 and T > 0. The maximum phase shift provided by such a compensator is

**[EE ESE - 2001]**

(a) $\tan^{-1}\left(\frac{a+1}{a-1}\right)$ (b) $\tan^{-1}\left(\frac{a-1}{a+1}\right)$
(c) $\sin^{-1}\left(\frac{a+1}{a-1}\right)$ (d) $\sin^{-1}\left(\frac{1-a}{a+1}\right)$

**110.** The Laplace transform of a transportation lag of 5 seconds is

**[EE ESE - 2001]**

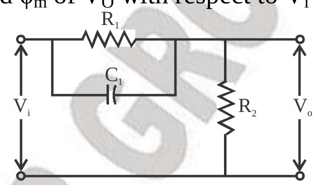
(a) exp (– 5s) (b) exp (5s)
(c) $\frac{1}{s+5}$ (d) $\exp\left(\frac{-s}{5}\right)$

**111.** The transfer function $\frac{V_2(s)}{V_1(s)} = \frac{10s}{s^2+101s+100}$ is for an active

**[EE ESE - 2001]**

(a) Low pass filter (b) Band pass filter
(c) High pass filter (d) All pass filter

**112.** For the given network, the maximum phase lead $\phi_m$ of $V_O$ with respect to $V_i$ is




**[EC ESE - 2000]**

(a) $\sin^{-1}\left(\frac{R}{2R_2}\right)$ (b) $\sin^{-1}\left(\frac{R_1}{R_1+2R_2}\right)$
(c) $\sin^{-1}\left(\frac{R_1}{R_1+3R_2}\right)$ (d) $\sin^{-1}\left(\frac{R_1}{2R_2C_1}\right)$

**113.** The compensator

$$G_C(s) = \frac{5(1+0.3s)}{1+0.1s}$$

Would provide a maximum phase shift of

**[EC ESE - 1999]**

(a) 20° (b) 45°
(c) 30° (d) 60°

**114.** If the transfer function of a phase lead compensator is (s + a)/(s + b) and that of a lag compensator is (s + p)/(s + q), then which one of the following sets of conditions must be satisfied?

**[EC ESE - 1999]**

(a) a > b and p < q (b) a > b and p < q
(c) a < b and p < q (d) a < b and p > q

# SOLUTIONS

**Sol.1. (b)**
It is bode plot of leg – lead compensator

**Sol.2. (d)**
The phase lag network reduces the bandwidth. Hence statement(I) wrong.

**Sol.3. (b)**
The two corner frequencies of lead network are

$$\omega_1 = \frac{1}{0.04} \text{ and } \omega_2 = \frac{1}{0.01}$$

Or, $\omega_1 = 25$ and $\omega_2 = 100$
The maximum phase – lead occurs at mid–frequency

$$\omega_m = \sqrt{\omega_1 \omega_2} = \sqrt{25 \times 100} = \sqrt{2500} = 50 \text{ rad/sec}$$

**Sol.4. (b)**
The steady state error can be reduced by lag compensator.

**Sol.5. (a)**
The given transfer function can be re-written as

$$\frac{K(s+a)}{s+b} = \frac{Ka(1+s/a)}{b(1+s/b)}$$

Now, for this to be a transfer function of lead compensator.

$$\frac{\frac{1}{b}}{\frac{1}{a}} < 1 \text{or } \frac{a}{b} < 1$$

$\therefore$ a < b

**Sol.6. (b)**
Transfer functions having at least one pole or zero in the RHS of s-plane are called non – minimum phase transfer functions. The elements with non – minimum phase transfer functions introduce large phase lags with increasing frequency resulting in complex compensation problems.
The transfer function of transportation lag is

$$G(s) = \frac{1 - sT_1}{1 + sT_2}$$

**Sol.7. (c)**

$$G_C(s) = \frac{s^2 + 10s + 24}{s^2 + 10s + 16}$$

So poles are –2, –8
And zero are –4, –6

$$\text{So } G_C(s) = \frac{(s+4)(s+6)}{(s+2)(s+8)} = \underbrace{\frac{(s+4)}{(s+8)}}_{\text{lead}} \cdot \underbrace{\frac{(s+6)}{(s+2)}}_{\text{lag}}$$

So Gc(s) will work as lead lag or lag lead compensation.

**Sol.8. (a)**
Since lead compensation increases the margin of stability so we use higher order lead compensation.

**Sol.9. (d)**

$$G_C(s) = \frac{1 + 0.5s}{1 + 0.05s}$$

Zero; S = –2; pole; S = –20; $\alpha = \frac{Z}{P} = 0.1$

$$\therefore \ \phi_M = \sin^{-1}\left[\frac{1-\alpha}{1+\alpha}\right] = \sin^{-1}[0.82]$$

$$\therefore \ \omega_M = -\sqrt{Z.P} = \sqrt{40} \approx 6$$

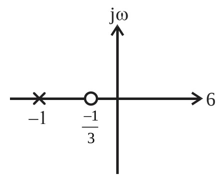
**Sol.10. (d)**
Lead compensator is high pass filter hence it increases bandwidth Lag compensator is low pass filter hence it decreases bandwidth.

**Sol.11. (a)**

$$G_C(s) = \frac{1 + 3s}{1 + s}$$

Lead Compensator;

$$\omega_{max} = \sqrt{\frac{1}{3}}$$




$$\phi_{max} = \sin^{-1}\left(\frac{a-1}{a+1}\right);\ \text{where } a = 3,$$

$$\phi_{max} = \sin^{-1}\left(\frac{2}{4}\right) = 30°.$$

**Sol.12. (c)**

**Sol.13. (d)**

$$\frac{E_2(s)}{E_1(s)} = \frac{R_2 + \frac{1}{sC}}{R_1 + R_2 + \frac{1}{sC}}$$

$$= \frac{1 + sR_2C}{1 + sC(R_1 + R_2)}$$

For lag compensator,

$$\tan^{-1}\left(\frac{\omega C(R_1 + R_2)}{1}\right) \geq \tan^{-1}\left(\frac{\omega CR_2}{1}\right)$$

$\omega C(R_1 + R_2) > \omega CR_2 \Rightarrow R_1 > 0$
which is already given.

**Sol.14. (d)**
Time response of an indicating instrument is decided by Damping system.

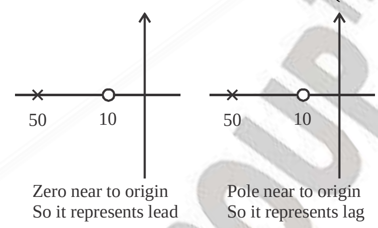
**Sol.15. (a)**

**Sol.16. (d)**
With the effect of integral controller the steady state error as relative stability reduces, because integral controller will add one pole in the system which will the settling time results in reduction in relative stability.

**Sol.17. (d)**
The correct sequence of steps needed to improve system stability is use negative feedback, reduce gain and insert deviation action.

**Sol.18. (c)**

$$G_1(s) = \frac{5(s+10)}{s+50} \qquad G_2(s) = \frac{s+50}{5(s+10)}$$




**Sol.19. (*)**

**Sol.20. (c)**

**Sol.21. (d)**
As the output is speed of a motor, so the output rate control will provide derivable control of the output (which is speed of the motor) or in turn it will control (or limit) the acceleration of the motor.

**Sol.22. (a)**

**Sol.23. (a)**
If the rotor resistance of the servomotor is low then the torque speed characteristics will be non linear and if it is high then characteristic will be linear over wise range of speed and it has better accelerating characteristics.

**Sol.24. (a)**
Larger and more sophisticated aircraft have AC systems operating at 400 Hz if we use higher frequency, the weight of components reduces.

**Sol.25. (c)**

**Sol.26. (a)**
For phase – load compensator, zero is nearer to origin as compared to pole i.e. effect of zero is dominant. Hence option (a) is correct.

**Sol.27. (a)**
Phase lead compensators may be employed to improve system performance and can permit an increased forward gain to reduce steady state error. Another use is to improve damping and thus reduce overshoot and improve settling time.

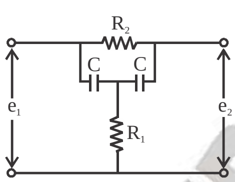
**Sol.28. (b)**
Phase – lead compensation is used to decrease rise time and to decrease overshoot.

**Sol.29. (d)**
The addition for a lag compensator in the system result in an improvement in signal to noise ratio and reduction in bandwidth.

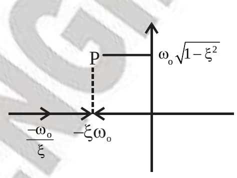
**Sol.30. (a)**
$$G(s) = \frac{10(1+0.04s)}{(1+0.01s)}$$
Comparing with the standard phase lead compensating network.
$$= \frac{\alpha(1+T_1 s)}{(1+\alpha T_1 s)}$$
$T_1 = 0.04$
$\alpha T_1 = 0.01$
So, maximum phase lead occurs at frequency $\omega_m$. i.e.
$$\omega_m = \frac{1}{\sqrt{\alpha T}.\sqrt{T}} = \frac{1}{\sqrt{0.04} \times \sqrt{0.01}} = 50 \text{ rad/sec}$$

**Sol.31. (b)**

**Sol.32. (d)**




Bridge-T network is used for the measurement of resistance at radio frequency.
$$\frac{E_2(s)}{Ei(s)} = \frac{s^2 + \frac{2}{R_2C}s + \frac{1}{R_1R_2C^2}}{s^2 + \frac{2R_1+R_2 s}{R_1R_2C} + \frac{1}{R_1R_2C^2}}$$

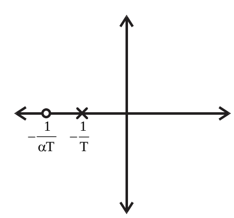



for $\xi = \sqrt{\frac{R_1}{R_2}}, \omega_0 = \frac{1}{C\sqrt{R_1R_2}}$

**Sol.33. (a)**
Transfer function for a phase lead compensator.
$$H(s) = \frac{\beta(1+\tau s)}{(1+\beta\tau s)}$$

**Sol.34. (a)**
Transfer function for a phase lag compensator is
$$H(s) = \frac{1+\tau s}{1+\alpha\tau s}$$
$$\Rightarrow H(s) = \frac{\tau\left(s+\frac{1}{\tau}\right)}{\alpha\tau\left(s+\frac{1}{\alpha\tau}\right)} = \frac{\left(s+\frac{1}{\tau}\right)}{\alpha\left(s+\frac{1}{\alpha\tau}\right)}$$

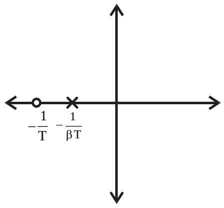
**Sol.35. (b)**
For the given circuit
$$\frac{E_0(s)}{E_i(s)} = \frac{R_2(1+R_1Cs)}{R_1+R_2\left[1+\frac{R_1R_2Cs}{R_1+R_2}\right]}$$
$$\Rightarrow \frac{E_0(s)}{E_i(s)} = \frac{\alpha(1+Ts)}{(1+\alpha Ts)}$$
Where $\alpha = \frac{R_2}{R_1+R_2} < 1$
and $T = R_1C$




Hence given circuit represents a phase lead compensator.

**Sol.36. (b)**
For the given circuit
$$\frac{E_0(s)}{E_i(s)} = \frac{1+R_2Cs}{1+(R_1+R_2)Cs}$$
$$\Rightarrow \frac{E_0(s)}{E_i(s)} = \frac{1+Ts}{1+\beta Ts}$$

Where T = $R_2$ C and β= $\frac{R_1+R_2}{R_2}>1$




Hence given circuit represents a phase lag compensator.

**Sol.37. (b)**
G(s) = s + 1
Put s = jω
G(jω) = jω + 1
$G(j\omega)=\sqrt{\omega^2+1}\angle\tan^{-1}\omega$
At ω = 1 rad/sec
Gain of the system = $\sqrt{1+1}$ = 1.41
Phase of the system = $\tan^{-1}$ 1 = 45°

**Sol.38. (c)**
$e(t)=k\omega=k\frac{d\theta}{dt}$
Taking Laplace transform
E(s) = ks(s)
$\frac{E(s)}{\theta(s)}=ks$
Hence, option (c) is correct.

**Sol.39. (b)**
Let, controller be G(s)
for ramp input
error = A/$K_v$
where, $K_v=\lim_{s\to 0}G(s)\times\frac{9}{s(s+2)}$
$K_v=\lim_{s\to 0}\frac{9G(s)}{s+2}$
for error to be zero G(s) should be of type 1. Hence, option (b) is correct.

**Sol.40. (d)**
$G(s)=\frac{1+3Ts}{1+Ts}$

by comparing, $G(s)=\frac{1+T's}{1+\alpha T's}$
T' = 3T
αT' = T
α(3T) = T
α = 1/3
Maximum phase shift
$\Delta\phi=\sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)=\sin^{-1}\left(\frac{1-1/3}{1+1/3}\right)$
$=\sin^{-1}\left(\frac{1}{2}\right)=30^0$

**Sol.41. (a)**

**Sol.42. (b)**

**Sol.43. (d)**

**Sol.44. (d)**

**Sol.45. (d)**

**Sol.46. (d)**

**Sol.47. (c)**

**Sol.48. (d)**
$G(s)=\frac{\alpha(1+T_s)}{(1+\alpha T_s)}$
∴ At T' = 3T
αT' = T
∴ α × 3T = T ∴ $\alpha=\frac{1}{3}$
$\phi_m=\sin^{-t}\left(\frac{1-\alpha}{1+\alpha}\right)=30^0$

**Sol.49. (a)**
By using lead compensator rise time decreases
$\downarrow_{t_r}\ \infty\frac{1}{B.W.\uparrow}$
Hence B.W. increases

**Sol.50. (c)**
1-ϕ AC supply is applied to synchro transmitter.

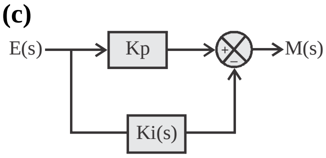
**Sol.51. (c)**

**Sol.52. (b)**

**Sol.53. (b)**

**Sol.54. (d)**

**Sol.55. (c)**




**PI Controller**

$$\frac{M(s)}{E(s)} = K_P + \frac{K_i}{s} = \frac{sK_P + K_i}{s}$$

PI compensator adds one open-loop pole at origin and one open-loop zero at negative real axis.

**Sol.56. (c)**

**Sol.57. (d)**
To have linear torque speed characteristic X/R ratio should be low means R/X ratio high. For fast response inertia should be low.

**Sol.58. (d)**

$$\xi' = \xi + \frac{K_D \omega_n}{2}; t_s = \frac{4}{\xi\omega_n \uparrow} \downarrow$$

Steady state error is reduced by lag compensator so point…… is not correct.

**Sol.59. (d)**

**Sol.60. (b)**
As BW increases so rise time decreases, $\xi$ increases so, $M_p$ decreases.

**Sol.61. (a)**

**Sol.62. (b)**

**Sol.63. (d)**

**Sol.64. (b)**
For stability torque should reduce on increase of speed otherwise due to cumulative effect motor will unstable.

**Sol.65. (d)**

**Sol.66. (c)**

**Sol.67. (a)**

**Sol.68. (d)**
In lead compensator, zero dominates near origin. In lag compensator, pole dominates near origin.

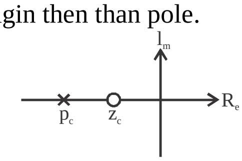
**Sol.69. (a)**

**Sol.70. (d)**

**Sol.71. (a)**
To get higher speed in field controlled dc motor, field current is decreased which decreases the torque.

**Sol.72. (a)**

**Sol.73. (b)**
A phase lead compensating network has zero nearer to origin then than pole.




**Sol.74. (a)**
Phase lag compensation is an integration,> It reduces the steady state error.

Velocity constant = $\frac{1}{\text{steady state error}}$

So, the velocity constant is increased.

**Sol.75. (d)**
Lead compensator is a HPF. So it increases the bandwidth.
In lag compensator,

$$G_C(s) = \frac{1}{\beta}\left(\frac{s+z_c}{s+p_c}\right), \beta = \frac{z_c}{p_c} > 1$$

$\because \beta > 1 \Rightarrow 1/\beta < 1$

$\Rightarrow$ Lag compensator attenuates the signal.

**Sol.76. (d)**

**Sol.77. (b)**

**Sol.78. (c)**

**Sol.79. (c)**

**Sol.80. (d)**

**Sol.81. (d)**
If the rotor axis of synchro transmitter is along the axis of $S_2$ stator winding then maximum voltage is induced in the stator coil $S_2$ and the terminal voltage $V_{s3}$, $V_{s1}$ is zero. This position of the rotor is defined as the electrical zero of the transmitter and is used as reference for specifying the angular position of the rotor.

**Sol.82. (d)**
Offset is inversely $\propto$ to gain

$$e_{ss}\big|_{offset} = \frac{A}{1+k_p}$$

Steady – state error is offset is unaffected by derivative control.

**Sol.83. (b)**

**Sol.84. (a)**
A linear as servomotor has low X/R ratio to make the torque-slip characteristic to be linear.

**Sol.85. (a)**
Permanent magnet dc cannot be used as field control.

**Sol.86. (c)**

**Sol.87. (c)**
Cascade compensation is quite satisfactory and economical in most cases.

**Sol.88. (c)**
It act as a low pass filter.

**Sol.89. (d)**
The maximum phase lead angle occurs at the geometric mean of the corner frequencies of the phase lead network.

**Sol.90. (b)**

$$G_C(s) = \frac{K(1+0.3s)}{(1+0.17s)} = \frac{\alpha(1+\tau s)}{(1+\alpha\tau s)}$$

Where $\tau = 0.3$, $\alpha\tau = 0.17$
From the given network

$$\tau = R_1C \quad \alpha = \frac{R_2}{R_1+R_2}$$

So, $0.3 = 10^{-6} R_1$
$\Rightarrow R_1 = 300\Omega$

$$\alpha = \frac{0.17}{0.3} = \frac{R_2}{300+R_2}$$

$\Rightarrow R_2 = 392.3\ k\Omega \Rightarrow R_2 \approx 400\ k\Omega$

**Sol.91. (c)**
The slope at high frequency range is –20 (n – m) dB/decade.
where n = no. of poles
m = no. of zeros
$\because$ n = 3, m = 0
$\therefore$ slope = –20 × 3= – 60 dB/decade

**Sol.92. (d)**
AC servomotors are essentially induction motor with low X/R ratio for the rotor which has very low inertia (drag-cup type construction).

**Sol.93. (d)**
Tachogenerator feedback has nothing to do with system stability. It simply reduces the damping in the system.

**Sol.94. (c)**

**Sol.95. (d)**

**Sol.96. (c)**

Using phase-lag compensation improves steady state response but speed of time response is deteriorated to a certain extent.

**Sol.97. (c)**
Phase-lead compensation results in increased bandwidth i.e. reduction in setting time and thus speed of the time response is improved.

**Sol.98. (d)**
In a servo system, the gear backlash may cause sustained oscillations or chattering phenomenon, and the system may even turn unstable for large backlash.

**Sol.99. (c)**

**Sol.100. (a)**
In phase-lead compensator, zero is nearer to origin vis–a–vis pole.

**Sol.101. (b)**

**Sol.102. (b)**

**Sol.103. (a)**
Phase lead compensation improves transient response. Phase lag compensation improves steady state response.

**Sol.104. (b)**
Phase lead network has

$$G(s) = \frac{\alpha(1+sT)}{1+s\alpha T}; \alpha < 1$$

**Sol.105. (d)**

$$G(j\omega) = \frac{1+ja\omega T}{1+j\omega T} \text{ where } T > 0, a > 1$$

Phase angle $\phi = \tan^{-1} a\omega T - \tan^{-1} \omega T$

For maximum phase lead, $\frac{d\phi}{d\omega} = 0$

$$\Rightarrow \frac{1}{1+a^2\omega_m^2T^2}.aT - \frac{1}{1+\omega_m^2T^2}.T = 0$$

$\Rightarrow T(a + \omega_m^2 t^2 a - 1 - a^2 \omega_m^2 T^2) = 0$

$\Rightarrow -\omega_m^2 T^2 a (a - 1) + (a - 1) = 0$

$\Rightarrow (a - 1)(1 - \omega_m^2 T^2 a) = 0$

$$\Rightarrow \omega_m^2 = \frac{1}{aT^2} \Rightarrow \omega_m = \frac{1}{T\sqrt{a}}$$

$\phi_m = \tan^{-1} a\,\omega_m T - \tan^{-1} \omega_m T$

$$= \tan^{-1}\sqrt{a} - \tan^{-1}\frac{1}{\sqrt{a}}$$

$$\Rightarrow \phi_m = \tan^{-1}\left(\frac{a-1}{(1+1)\sqrt{a}}\right)$$

$$= \tan^{-1}\left(\frac{a-1}{2\sqrt{a}}\right)$$

$$\Rightarrow \phi_m = \sin^{-1}\left(\frac{a-1}{a+1}\right)$$

**Sol.106. (b)**

$e^{-as} \to$ Transport delay

$\frac{1-s}{1+s} \to$ All – pass filter

$\frac{1+as}{1+bs}$, $a < b \to$ Lag network

$\frac{K}{s(1+as)} \to$ Servomotor

**Sol.107. (c)**
High rotor resistance or low X/R ratio makes the torque-slip characteristic to be linear.

**Sol.108. (a)**

**Sol.109. (d)**

$$\theta_m = \sin^{-1}\left(\frac{1-a}{1+a}\right)$$

**Sol.110. (a)**
$T(s) = e^{-sT}$

**Sol.111. (b)**

$$\frac{V_2(s)}{V_1(s)} = \frac{10s}{s^2+10s+100}$$

For $s \to 0$

$$\frac{V_2(s)}{V_1(s)} = 0$$

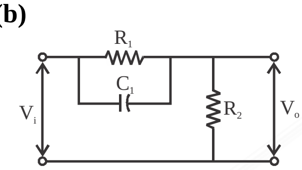
For $s \to \infty$

$$\left.\frac{V_2(s)}{V_1(s)}\right|_{s=\infty} = \frac{10s}{s^2\left(1+\dfrac{10}{s}+\dfrac{100}{s^2}\right)} = 0$$

But for $0 < s < \infty$, $\dfrac{V_2(s)}{V_1(s)} \neq 0$

So, transfer function represents band pass filter.

**Sol.112. (b)**




$$\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + \dfrac{R_1 \dfrac{1}{sC_1}}{R_1 + \dfrac{1}{sC_1}}}$$

$$= \frac{R_2}{R_2 + \dfrac{R_1}{1+sR_1C_1}}$$

$$= \frac{R_2(1+sR_1C_1)}{(R_1+R_2)+sR_1\,R_2C_1}$$

Dividing by $R_1\,R_2\,C_1$.

$$\frac{V_o(s)}{V_i(s)} = \frac{s+\dfrac{1}{R_1C_1}}{s+\dfrac{(R_1+R_2)}{R_1R_2C_1}}$$

$$= \frac{s+\dfrac{1}{R_1C_1}}{s+\dfrac{1}{\left(\dfrac{R_2}{R_1+R_2}\right)R_1C_1}} = \frac{s+\dfrac{1}{\tau}}{s+\dfrac{1}{\alpha\tau}}$$

Where, $\tau = \dfrac{1}{R_1C_1}, \alpha = \dfrac{R_2}{R_1+R_2}$

Maximum phase lead

$$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$$

$$\phi_m = \sin^{-1}\left(\frac{1-\dfrac{R_2}{R_1+R_2}}{1+\dfrac{R_2}{R_1+R_2}}\right)$$

$$\phi_m = \sin^{-1}\left(\frac{R_1}{R_1+2R_2}\right)$$

**Sol.113. (c)**

Maximum phase shift

$$\phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$$

$$\phi_m = \sin^{-1}\left(\frac{1-\dfrac{1}{3}}{1+\dfrac{1}{3}}\right)$$

$$\phi_m = \sin^{-1}\left(\frac{2/3}{4/3}\right)$$

$$\phi_m = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

**Sol.114. (d)**

In phase lead compensator, zero is nearer to origin. In phase lag compensator, pole is nearer to origin.

# CHAPTER - 10
# *STATE VARIABLE APPROACH*

## 10.1 INTRODUCTION

These are minimal set of variables which can completely determine the behavior of system at any given time.

State model:

X = AX + BU  State eqns.

Y = CX + DU  Output eqns.

And both equation combined together is called. State model

| | | |
|---|---|---|
| X – State vector | U – Input vector | Y – Output vector |
| A – System matrix | B – Input matrix | C – Output matrix |
| D – Transmission matrix | Let n ⇒ No. of state variables = order of the system | |
| p ⇒ No. of outputs | m ⇒ No. of inputs | Order [A] = n x n |
| Order [B] = n x m | Oder [C] = p x n | Order [D] = p x m |

## 10.2 DISADVANTAGES OF TRANSFER FUNCTIONS

1.It is defined only under zero initial conditions.

2.It is only applicable to LTI system and there too it is restricted to single input systems.

3.It reveals only the system O/P for a given i/p and provides no information regarding internal states of the system.

4.Classical design methods (roots locus and freq. domain methods) based on transfer function model are trail and error procedures.

## 10.3 ADVANTAGES OF STATE VARIABLE METHOD

1. It is applicable for both LTI and LT varying systems.
2. It takes initial conditions into account.
3. All the internal states of the system can be determined.
4. Applicable for multiple input multiple output.
5. Controllability and observability can be determined easily.

## 10.4 REPRESENTATION OF STAT MODEL

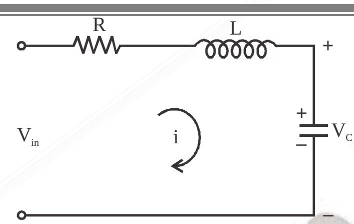
1. Physical variable representation.
2. Phase variable representation
3. Cononical representation.

> State model of a system is not unique property. But transfer function of the system is unique.

## 10.5 PHYSICAL VARIABLE REPRESENTATION

Variables like current, voltage, velocity, distance etc. are taken as state variable.

> In electrical systems, current through each inductor and voltage across each capacitor is taken as state variable.




**Example.** Find out state model for the network shown below.

**Solution.**

$$v_{in} = iR + L\frac{di}{dt}\underbrace{\frac{1}{c}\int i\,dt}_{v_c}$$

$$\Rightarrow L\frac{di}{st} = -R - v_c + v_{in}$$

Applying KVL $\frac{di}{dt} = \frac{-R}{L}i - \frac{1}{L}x_2 + \frac{1}{L}v_{in}$

Put i= $x_1$, $v_c = x_2$, $v_{in}$ = u and $v_0$ = y

$$\frac{dx_1}{dt} = \left(-\frac{R}{L}\right)x_1 - \frac{1}{L}x_2 + \frac{1}{L}u \qquad \text{…(i)}$$

= f($x_1$, $x_2$, u )

From n/w,

$$i = c\frac{dv_c}{dt}$$

$$\therefore \frac{dx_2}{dt} = \frac{1}{c}x_1 + 0.x_2 + 0.u \qquad \text{…(ii)}$$

Also $v_{output} = v_c$

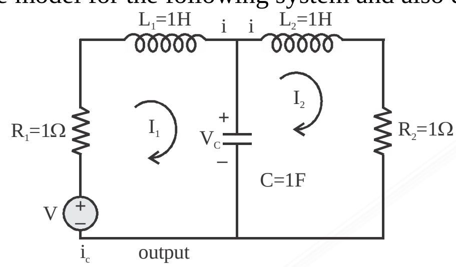
$\Rightarrow$ Y = $x_2$ = (0) $x_1$ + (1) $x_2$ +(0)u …(iii)

Writing equation (I), (II), (III) in matrix form

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{c} & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}u$$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]u$$

**Example.** Find out the state model for the following system and also draw the state diagram.




$$X = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} \times + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Y = [ 1 0]X

Solution.

Controllability

$$AB = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -a \end{bmatrix}$$

$$Q = [B \quad AB] = \begin{bmatrix} 0 & 1 \\ 1 & -9 \end{bmatrix} \therefore |Q_C| \neq 10$$

$\therefore$ System is controllable

Observability

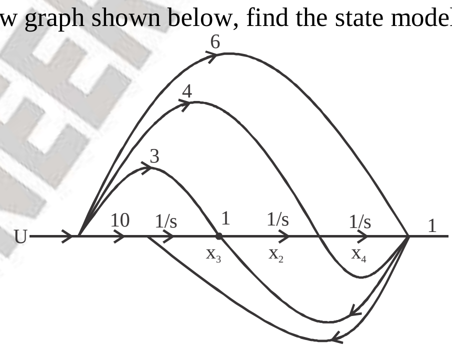
$$CA = [1 \quad 0] \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$\therefore Q_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{and} \, |Q_0| = 1$$

$\therefore Q_0 \neq 0 = 1$

$\therefore$ System is also observable

**Example.** For the signal flow graph shown below, find the state model




y= $x_1$ +6x

$$x_1 = \frac{1}{s}(x_2 - 8y + 4x)$$

xi = $x_2$ -48x-8$x_1$+4x=-8$x_1$+$x_2$-44x

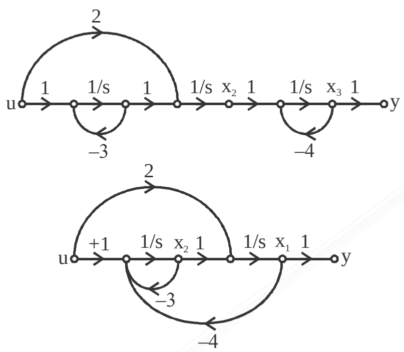
$a_2 = x_3$-4y+3x=-4$x_1$+$x_3$ -$21_x$
$x_3$ = -20y+10xx=-20$x_t$-120x+10x=--20$x_1$ -110x
State model is:

$$\begin{bmatrix} x_1 \\ x_2 \\ xl_3 \end{bmatrix} = \begin{bmatrix} -8 & 1 & 0 \\ -4 & 0 & 1 \\ -20 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_4 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -44 \\ -21 \\ -110 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 6x$$

# ASSIGNMENT

**1.** For the system shown below the state-space equation is $X = Ax + B.u$. The matrix A is




(a) $\begin{bmatrix} 0 & 1 & -4 \\ 1 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & -4 \\ -1 & 0 & 0 \\ +3 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -4 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 3 \end{bmatrix}$

**2.** The state equation of a LTI system is represented by

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} u$$

The Eigen values are

(a) $-1$ and $-2$ (b) $-1$ and 2

(c) 1 and –2 (d) 1 and 2

**3.** A system is described by the dynamic equation

$$\dot{x}(t) = A.x(t) + B.u(t)$$

$$y(t) = C.x(t) \text{where}$$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{and}\, C = [1 \;\; 1]$$

The output transfer function, $Y(s)/U(s)$ is

(a) $\frac{(s+1)}{(s+2)}$ (b) $\frac{s+1}{s+2}$

(c) $\frac{s+2}{s+1}$ (d) $\frac{1}{s+1}$

**4.** For the transfer function:

$$\frac{Y(s)}{U(s)} = \frac{s+3}{(s+1)(s+2)}$$

The state model is given by

$\dot{x} = Ax + Bu$ and $Y = C.x$, The A, B, C are

(a) $\begin{bmatrix} -1 & 0 \\ -2 & 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, [0,1]$

(b) $\begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, [0,1]$

(c) $\begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, [0,1]$

(d) None

**5.** A system is represented by following state and output equations

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u, Y = [4 \;\; 3] x$$

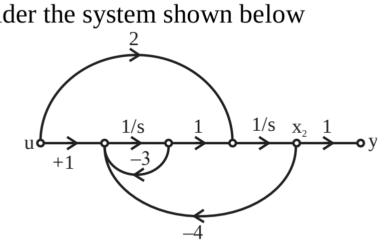
The poles of the system are:

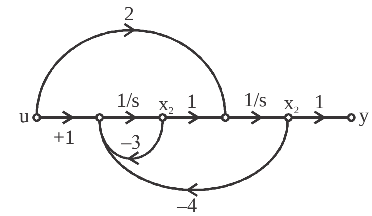
(a) -1 and 2 (b) -1 and -2

(c) 1 and 2 (d) 1 and -2

**Common data for Q.6 to Q.8**

Consider the system shown below






**6.** The controllability matrix is

(a) $\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 \\ 1 & -11 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 \\ 3 & -11 \end{bmatrix}$ (d) None

**7.** The observability matrix is

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$ (d) None

**8.** The system is
(a) Controllable only
(b) Observable only
(c) Controllable and observable
(d) None

**9.** For the system described by X = AX match List-I (Matrix A) with List-II (position of Eigen values) and select the correct answer:

**List-I**

A. $\begin{bmatrix} -1 & 2 \\ 0 & -2 \end{bmatrix}$

B. $\begin{bmatrix} -1 & -2 \\ -2 & -4 \end{bmatrix}$

C. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$

**List-II**
(i) One eigen values at the origin
(ii) Both the eigen values in the LHP
(iii) Both the eigen values in RHP
(iv) Both the eigen values, on the imaginary axis

**Codes:**
(a) A-ii, B-i, C-iii, D-iv
(b) A-ii, B-i, C-iv, D-iii
(c) A-i, B-ii, C-iv, D-iii
(d) A-i, B-ii, C-iii, D-iv

**10.** The system mode described by the state equations

$X = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \end{bmatrix} uY = [1 \;\; 1] \text{is}$

(a) Controllable and observable
(b) Controllable, but not observable
(c) Observable, but not controllable
(d) Neither controllable nor observable

**11.** For the system described by the state equation

$$X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

If the control signal u is given by u[-30.5-3-5] × + v, then the eigen values of the closed-loop system will be

(a) 0, –1, –2 (b) 0, –1, –3
(c) –1, – 1, –2 (d) 0, –1, –1

# SOLUTIONS

**Sol. 1.**
From the SFG

$\dot{x}_1 = -4x_1 + x_2$

$\dot{x}_2 = x_3 + 2u$

$\dot{x}_3 = -3x_3 + u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -4 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} [u]$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -4 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} [u]$$

**Sol. 2.**
Given,

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$(SI - A) = \begin{bmatrix} S & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

$$|(sI - A)| = s(s+3) + 2$$

$= s^2+3s+2$

s= –1, –2

**Sol. 3.**
The transfer function

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1} B$$

$$(sI - A) = \begin{bmatrix} (s+1) & 0 \\ 0 & (s+2) \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{(s+2)(s+1)} \begin{bmatrix} (s+2) & 0 \\ 0 & (s+1) \end{bmatrix}$$

$$C(sI - A)^{-1} B = \frac{1}{(s+2)(s+1)}$$

$$[1 \ \ 1] \begin{bmatrix} (s+2) & 0 \\ 0 & (s+1) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \frac{1}{s+1}$$

**Sol. 4.**
In this type of problem, find the transfer function from options.

**Sol. 5.**

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

$|sI - A| = s(s+3) + 2$

$= s^2 +3s+2$

s = –1, –2

The poles of the system are – 1 and - 2

**Sol. 6.**

$\dot{x}_1 + x_2 + 2u$

$\dot{x}_2 = -4x_1 - 3x_2 + u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} [u]$$

$$A = \begin{bmatrix} 0 & 1 \\ -4 & -3 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 \\ -11 \end{bmatrix}$$

Controllability Matrix,

$C_M$ = [B AB]

$$\begin{bmatrix} 2 & 1 \\ 1 & -11 \end{bmatrix}$$

**Sol. 7.**
$y=x_1$ = [y] = [1 0] [x] C = [1 0]

Observability matrix is

$$(O_M) = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

**Sol. 8.**
For controllable
$|C_M| \neq 0 \ |C_M| = -22 - 1 = 23 \neq 0$
So the system is controllable.
For observable,
$|Q_M| \neq 0$
$|Q_M| \neq 0$
$|Q_M| \neq 1$
So the system is observable
Therefore given system are controllable and observable both.

**Sol. 9. (b)**
A proportional plus derivative controller has the following features:
1.It adds an open loop zero on negative real axis
2.Undamped natural frequency remains same and damping ratio increases
3.Peak overshoot decreases
4.Bandwidth increases
5.Rise time decreases
6.Effect of external noise increase
7.Setting time decreases, i.e. response becomes faster
8.Stability improves

**Sol. 10. (a)**
$Q_c = [B AB A^2B \ldots A^{n-1}B]$

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$C = [1 \quad 1]$

$$AB = \begin{bmatrix} +1 \\ -3 \end{bmatrix}$$

$$\therefore Q_c \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} \neq 0$$

$\therefore$ order 2 , rank 2
$\therefore$ Controllable

$$Q_c = \left[C^T A^T C^T A^{T2} CT\ldots\right]$$

$$C^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, A^T = \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} \neq 0$$

$\therefore$ Rank 2 is observable

**Sol. 11. (a)**

$$x = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} [-0.5 - 3.5] x + v$$

$$\therefore x = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} x + v$$

Characteristic equation
$= \lambda^3 + 3\lambda^2 + 2\lambda + 0 = 0$
$\Rightarrow \lambda = 0, -1, -2$

# GATE QUESTIONS

**1.** The state equation and the output equation of a control system are given below:

$$\dot{x} = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u,$$

$$y = [1.5 \quad 0.625]x$$

The transfer function representation of the system is

**[GATE - 2018]**

(a) $\frac{3s+5}{s^2+4s+6}$ (b) $\frac{3s-1.875}{s^2+4s+6}$

(c) $\frac{4s+1.5}{s^2+4s+6}$ (d) $\frac{6s+5}{s^2+4s+6}$

**2.** Consider the system described by the following state space representation

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = [1 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

If u(t) is a unit step input and $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, the value of output y(t) at t = 1 sec (rounded off to three decimal places) is ______

**[GATE - 2017]**

**3.** The transfer function of the system Y(s)/U(s) whose state – space equations are given below is:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$$

$$y(t) = [1 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

**[GATE - 2017]**

(a) $\frac{(s+2)}{(s^2-2s-2)}$ (b) $\frac{(s-2)}{(s^2+s-4)}$

(c) $\frac{(s-4)}{(s^2+s-4)}$ (d) $\frac{(s+4)}{(s^2-s-4)}$

**4.** A second order LTI system is described by the following state equation.

$$\frac{d}{dt}x_1(t) - x_2(t) = 0$$

$$\frac{d}{dt}x_2(t) + 2x_1(t) + 3x_2(t) = r(t)$$

When $x_1(t)$ and $x_2(t) + 3x_2(t) = r(t)$

When $x_1(t)$ and $x_2(t)$ are the two state variables and r(t) denotes the input. The output c(t) = $x_1(t)$. The system is

**[GATE - 2017]**

(a) Undamped (oscillatory)
(b) Under damped
(c) Critically damped
(d) Over damped

**5.** Consider the state space realization

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 45 \end{bmatrix} u(t)$$

With the initial condition $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; where u(t) denotes the unit step function. The value of $\underset{t\to\infty}{\text{Lt}} \left| x_1^2(t) + x_2^2(t) \right|$ is ______

**[GATE - 2017]**

**6.** Consider the following state-space representation of a linear time-invariant system.

$$x(t) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} x(t), y(t) = cT_x(t), c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

and $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

The value of y(t) for t = loge2 is ___________.

**[GATE - 2016]**

**7.** Consider a linear time invariant system x= Ax, with initial condition x(0) at t = 0. Suppose $\alpha$ and $\beta$ are eigenvectors of (2 × 2) matrix. A corresponding to distinct eigen values $\lambda_1$ and $\lambda_2$ respectively. Then the response x(t) of the system due to initial condition x(0) = $\alpha$ is

**[GATE - 2016]**

(a) $e^{\lambda_1 t}\alpha$ (b) $e^{\lambda_2 t}\beta$
(c) $e^{\lambda_2 t}\alpha$ (d) $e^{\lambda_1 t}\alpha + e^{\lambda_2 t}\beta$

**8.** A second-order linear time-invariant system is described by the following state equations

$$\frac{d}{dt}x_1(t) + 2x_1(t) = 2u(t)$$

$$\frac{d}{dt}x_2(t) + x_2(t) = u(t)$$

Where $x_1(t)$ and $x_2(t)$ are the two state variables and u(t) denotes the input. If the output c(t) = $x_1(t)$, then the system is

**[GATE - 2016]**

(a) Controllable but not observable
(b) Observable but not controllable
(c) Both controllable and observable
(d) Neither controllable nor observable

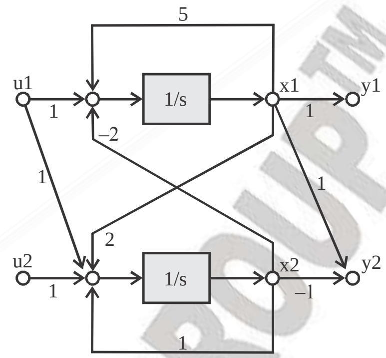
**9.** A sequence x[n] is specified as

$$\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{,for } n \geq 2.$$

The initial conditions are x[0] = 1, x[1] = 1 and x[n] = 0 for n < 0. The value of x[12] is ——

**[GATE - 2016]**

**10.** In the signal flow diagram given in the figure, $u_1$ and $u_2$ are possible inputs whereas $y_1$ and $y_2$ are possible outputs. When would the SISO system derived from this diagram be controllable and observable?




**[GATE - 2015]**

(a) When $u_1$ is the only input and $y_1$ is the only output
(b) When $u_2$ is the only input and $y_1$ is the only output
(c) When $u_1$ is the only input and $y_2$ is the only output
(d) When $u_2$ is the only input and $y_2$ is the only output

**11.** The state variable representation of a system is given as

$$x = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x; x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Y = [0  1]x

The response y(t) is

**[GATE - 2015]**

(a) sin(t) (b) $1-e^t$
(c) 1–cos(t) (d) 0

**12.** An unforced linear time invariant (LTI) system is represented by

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

If the initial conditions are $x_1(0) = 1$ and $x_2(0) = -1$, the solution of the state equation is

**[GATE - 2014]**

(a) $x_1(t) = -1$, $x_2(t) = 2$
(b) $x_1(t) = -e^{-t}$, $x_2(t) = 2e^{-t}$
(c) $x_1(t) = e^{-t}$, $x_2(t) = -e^{-2t}$
(d) $x_1(t) = -e^{-t}$, $x_2(t) = -2e^{-t}$

**13.** The state transition matrix $\phi(t)$ of a system

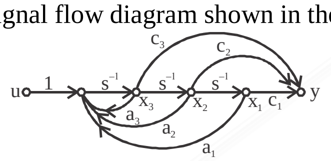
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \text{is}$$

**[GATE - 2014]**

(a) $\begin{bmatrix} t & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 1 & t \end{bmatrix}$ (d) $\begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$

**14.** Consider the state space system expressed by the signal flow diagram shown in the figure.




The corresponding system is

**[GATE - 2014]**

(a) Always controllable
(b) Always observable
(c) Always stable
(d) Always unstable

**15.** The state equation of a second- order linear system is given by

$x(t) = Ax(t)$, $x(0) = x_0$

For $x_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}_1$ $x(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$ and for $x_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$,

$x(t) = \begin{bmatrix} e^{-t} & -e^{-2t} \\ -e^{-t} & 2e^{-2t} \end{bmatrix}$ when $x_0 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, $x(t)$ is

**[GATE - 2014]**

(a) $\begin{bmatrix} -8e^{-t} & +11e^{-2t} \\ 8e^{-t} & -22e^{-2t} \end{bmatrix}$

(b) $\begin{bmatrix} 11e^{-t} & -8e^{-2t} \\ -11e^{-t} & +16e^{-2t} \end{bmatrix}$

(c) $\begin{bmatrix} 3e^{-t} & -5e^{-2t} \\ 3e^{-t} & +10e^{-2t} \end{bmatrix}$

(d) $\begin{bmatrix} -5e^{-t} & -3e^{-2t} \\ -5e^{-t} & +6e^{-2t} \end{bmatrix}$

**16.** The second order dynamic system

$$\frac{dx}{dt} = Px + Qu \quad y = RX$$

has the matrices P, Q and R as follows:

$$P = \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix} Q = \begin{bmatrix} 0 \\ 1 \end{bmatrix} R\begin{bmatrix} 0 & 1 \end{bmatrix}$$

The system has the following controllability and observability properties:

(a) Controllable and observable
(b) Not controllable but observable
(c) Controllable but not observable
(d) Not controllable and not observable

**Common data for Q. 17 & Q. 18**

The state variable formulation of a system is given as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u, x_1(0) = 0,$$

$x_1(0) = 0$ and $y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

**17.** The response y(t) to the unit step input is

**[GATE - 2013]**

(a) $\frac{1}{2} - \frac{1}{2}e^{-2t}$ (b) $1 - \frac{1}{2}e^{-2t} - \frac{1}{2}e^{-t}$

(c) $e^{-2t} - e^{-t}$ (d) $1 - e^{-t}$

**18.** The system is

**[GATE - 2013]**

(a) Controllable but not observable
(b) Not controllable but observable
(c) Both controllable and observable
(d) Both not controllable and not observable

**19.** The state transition matrix $e^{At}$ of the system shown in the figure above is

**[GATE - 2013]**

(a) $\begin{bmatrix} e^{-t} & 0 \\ te^{-t} & e^{-t} \end{bmatrix}$ (b) $\begin{bmatrix} e^{-t} & 0 \\ -te^{-t} & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{-t} & 0 \\ e^{-t} & e^{-t} \end{bmatrix}$ (d) $\begin{bmatrix} e^{-t} & -te^{-t} \\ 0 & e^{-t} \end{bmatrix}$

**20.** The state variable description of an LTI system is given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{pmatrix}0 & a_1 & 0\\ 0 & 0 & a_2\\ a_3 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}+\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}u$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{pmatrix}0 & a_1 & 0\\ 0 & 0 & a_2\\ a_3 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}+\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}u$$

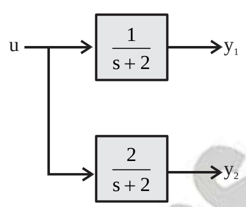
$$y=(1\ \ 0\ \ 0)\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$

Where y is the output and u is the input. The system is controllable for

**[GATE - 2012]**

(a) $a_1 \neq 0, a_2 = 0, a_3 \neq 0$
(b) $a_1 = 0, a_2 \neq 0, a_3 \neq 0$
(c) $a_1 = 0, a_3 \neq 0, a_3 = 0$
(d) $a_1 \neq 0, a_2 \neq 0, a_3 = 0$

**21.** The block diagram of a system with one input u and two outputs $y_1$ and $y_2$ is given below




A State space model of the above system in terms of the state vector x and the output vector $y = [y_1\ \ y_2]^T$ is

**[GATE - 2011]**

(a) $\dot{X}=[2]x+[1]u : y=[1\ \ 2]x$

(b) $\dot{X}=[-2]x+[1]u : y=\begin{bmatrix}1\\ 2\end{bmatrix}x$

(c) $\dot{X}=\begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix}X+\begin{bmatrix}1\\ 1\end{bmatrix}u : y=\begin{bmatrix}1 & 2\end{bmatrix}x$

(d) $\dot{X}=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}x+\begin{bmatrix}1\\ 1\end{bmatrix}u : y=\begin{bmatrix}1\\ 2\end{bmatrix}x$

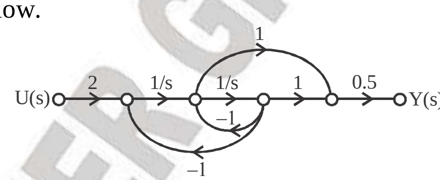
**22.** The system $\dot{X}=AX+Bu$ with $A=\begin{bmatrix}-1 & 2\\ 0 & 2\end{bmatrix}, B=\begin{bmatrix}0\\ 1\end{bmatrix}$ is

**[GATE - 2010]**

(a) Stable and controllable
(b) Stable but uncontrollable
(c) Unstable but controllable
(d) Unstable and uncontrollable

**Common Data for Q. 23 and Q. 24**

The signal flow graph of a system is shown below.




**23.** The state variable representation of the system can be

**[GATE - 2010]**

(a) $\dot{x}=\begin{bmatrix}1 & 1\\ -1 & 0\end{bmatrix}x+\begin{bmatrix}0\\ 2\end{bmatrix}u$

$\dot{y}=\begin{bmatrix}0 & 0.5\end{bmatrix}x$

(b) $\dot{x}=\begin{bmatrix}-1 & 1\\ -1 & 0\end{bmatrix}x+\begin{bmatrix}0\\ 2\end{bmatrix}u$

$\dot{y}=\begin{bmatrix}0 & 0.5\end{bmatrix}x$

(c) $\dot{x}=\begin{bmatrix}1 & 1\\ -1 & 0\end{bmatrix}x+\begin{bmatrix}0\\ 2\end{bmatrix}u$

$\dot{y}=\begin{bmatrix}0.5 & 0.5\end{bmatrix}x$

(d) $\dot{x}=\begin{bmatrix}-1 & 1\\ -1 & 0\end{bmatrix}x+\begin{bmatrix}0\\ 2\end{bmatrix}u$

$\dot{y}=\begin{bmatrix}0.5 & 0.5\end{bmatrix}x$

**24.** The transfer function of the system is

**[GATE - 2010]**

(a) $\frac{s+1}{s^2+1}$
(b) $\frac{s-1}{s^2+1}$
(c) $\frac{s+1}{s^2+s+1}$
(d) $\frac{s-1}{s^2+s+1}$

**25.** Consider the system

$$\frac{dx}{dt} = Ax + Bu \text{ with}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{and } B = \begin{bmatrix} p \\ q \end{bmatrix}$$

Where p and q are arbitrary real numbers. Which of the following statements about the controllability of the system is true?

**[GATE - 2009]**

(a)The system is completely state controllable for any nonzero values of p and q
(b)Only p = 0 and q= 0 result in controllability
(c)The system is uncontrollable for all values of p and q
(d)we cannot conclude about controllability from the given data

**Common Data for Q. 26 and Q. 27**
A system is described by the following state and output equations

$$\frac{dx_1 t}{dt} = -3x_1(t) + x_2(t) + 2u(t)$$

$$\frac{dx_2 t}{dt} = -2x_2(t) + u(t) \quad , y(t) = x_1(t)$$

when u(t) is the input and y(t) is the output

**26.** The system transfer function is

**[GATE - 2009]**

(a) $\frac{s+2}{s^2+5s-6}$ (b) $\frac{s+2}{s^2+5s+6}$
(c) $\frac{2s+5}{s^2+5s+6}$ (d) $\frac{2s-5}{s^2+5s-6}$

**27.** The state-transition matrix of the above system is

**[GATE - 2009]**

(a) $\begin{bmatrix} e^{-3t} & 0 \\ e^{-2t}+e^{-3t} & e^{-2t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{-3t} & e^{-2t}-e^{-3t} \\ 0 & e^{2t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{-3t} & e^{-2t}+e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{3t} & e^{-2t}-e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$

**Common data Q. 28 and Q. 29**
The state space equation of a system is described by $\dot{X} = AX + Bu, Y = CX$ where X is state vector, u is input, Y is output and

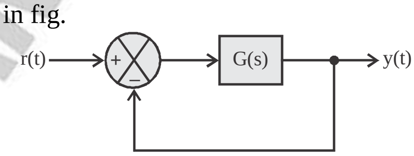
$$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \; 0]$$

**28.** The transfer function G(s) of this system will be

**[GATE - 2008]**

(a) $\frac{s}{(s+2)}$ (b) $\frac{s}{s(s-2)}$
(c) $\frac{s}{(s-2)}$ (d) $\frac{s}{s(s+2)}$

**29.** A unity feedback is provided to the above system G(s) to make it as closed loop system as shown in fig.

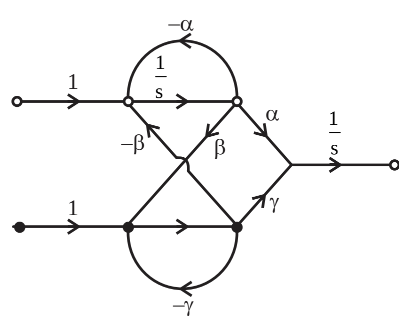



For a unit step input r(t), the steady state error in the input will be

**[GATE - 2008]**

(a) 0 (b) 1
(c) 2 (d) ∞

**30.** A signal flow graph of a system is given below



The set of equalities that corresponds to this signal flow graph is

**[GATE - 2008]**

(a) $\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \beta & -\gamma & 0 \\ \gamma & \alpha & 0 \\ -\alpha & \beta & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

(b) $\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 & \alpha & \gamma \\ \gamma & -\alpha & -\gamma \\ 0 & \beta & -\beta \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

(c) $\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -\alpha & \beta & 0 \\ -\beta & -\gamma & 0 \\ \alpha & \gamma & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

(d) $\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -\alpha & 0 & \beta \\ \gamma & 0 & \alpha \\ -\beta & 0 & -\alpha \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$

**31.** The state space representation of a separately excited DC servo motor dynamics is given as

$$\begin{bmatrix} \frac{d\omega}{dt} \\ \frac{dt_0}{dt} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & -10 \end{bmatrix}\begin{bmatrix} \omega \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix}u$$

Where ω is the speed of the motor, $i_a$ is the armature current and u is the armature voltage. The transfer function $\frac{\omega(s)}{U(s)}$ of the motor is

(a) $\frac{10}{s^2+11s+11}$ (b) $\frac{1}{s^2+11s+11}$

(c) $\frac{10s+10}{s^2+11s+11}$ (d) $\frac{1}{s^2+s+11}$

**Common data Q. 32 and Q. 33**

Consider a linear system whose state space representation is x(t) = Ax(t). if the initial state vector of the system is $x(0)=\begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then the system response is $x(t)=\begin{bmatrix} e^{-2x} \\ -2e^{-2t} \end{bmatrix}$. If the initial state vector of the system changes to $x(0)=\begin{bmatrix} 1 \\ -2 \end{bmatrix}$, then the system response become $x(t)=\begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$

**32.** The eigen value and eigenvector pairs ($\lambda_1$, $V_1$) for the system are

**[GATE - 2007]**

(a) $\left(-1\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(-2\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

(b) $\left(-1\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(2\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

(c) $\left(-1\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(-2\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

(d) $\left(-2\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$ and $\left(1\begin{bmatrix} 1 \\ -2 \end{bmatrix}\right)$

**33.** The system matrix A is

**[GATE - 2007]**

(a) $\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$

**34.** For a system with the transfer function $H(s)=\frac{3(s-2)}{4s^2-2s+1}$ the matrix A in the state space form $\dot{X}=AX+Bu$ is equal to

**[GATE - 2006]**

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -2 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 3 & -2 & 1 \\ 1 & -2 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$

**35.** A linear system is described by the following state equation

$$\dot{X}(t) = AX(t) + BU(t), A - \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The state transition matrix of the system is

**[GATE - 2006]**

(a) $\begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$

(b) $\begin{bmatrix} -\cos t & \sin t \\ -\sin t & -\cos t \end{bmatrix}$

(c) $\begin{bmatrix} -\cos t & -\sin t \\ -\sin t & \cos t \end{bmatrix}$

(d) $\begin{bmatrix} \cos t & -\sin t \\ \cos t & \sin t \end{bmatrix}$

**Common Data for Q. 36 & Q. 37**

A state variable system

$$X(t) = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(T)$$

With the initial condition X(0) = $[-1, 3]^T$ and the unit step input u(t) has

**36.** The state transition matrix

**[GATE - 2005]**

(a) $\begin{bmatrix} 1 & \frac{1}{3}\left(1 - e^{-3t}\right) \\ 0 & e^{-3t} \end{bmatrix}$

(b) $\begin{bmatrix} 1 & \frac{1}{3}\left(e^{-t} - e^{-3t}\right) \\ 0 & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} 1 & \frac{1}{3}\left(e^{3-t} - e^{-3t}\right) \\ 0 & e^{-3t} \end{bmatrix}$

(d) $\begin{bmatrix} 1 & \left(1 - e^{-t}\right) \\ 0 & e^{-t} \end{bmatrix}$

**37.** The state transition equation

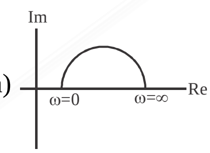
**[GATE - 2005]**

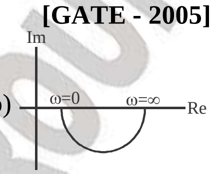
(a) $X(t) = \begin{bmatrix} t - e^{-t} \\ e^{-t} \end{bmatrix}$ (b) $X(t) = \begin{bmatrix} t - e^{-t} \\ 3e^{-3t} \end{bmatrix}$

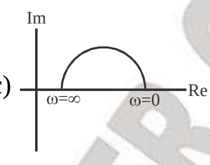
(c) $X(t) = \begin{bmatrix} t - e^{-3t} \\ 3e^{-3t} \end{bmatrix}$ (d) $X(t) = \begin{bmatrix} t - e^{-3t} \\ e^{-3t} \end{bmatrix}$

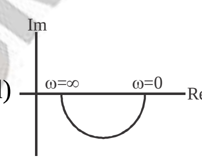
**38.** Which one of the following polar diagrams corresponds to a lag network?

**[GATE - 2005]**

(a) 


(b) 


(c) 


(d) 


**39.** A linear system is equivalently represented by two sets of state equations:

$\dot{X} = AX + BU$ and $\dot{W} = CW + DU$

The eigenvalues of the representations are also computed as [λ] and [μ]. Which one of the following statements is true ?

**[GATE - 2005]**

(a) [λ] = [μ] and X = W

(b) [λ] = [μ] and X ≠ W

(c) [λ] ≠ [μ] and X = W

(d) [λ] = [μ] and X ≠ W

**40.** The state variable equations of a system are: $\dot{x}_1 = -3x_1 - x_2 = u, \dot{x}_2 = 2x_1$ and $y = x_1 + u$. The system is

**[GATE - 2004]**

(a) Controllable but not observable

(b) Observable but not controllable

(c) Neither controllable nor observable

(d) Controllable and observable

**41.** Given $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the state transition matrix $e^{At}$ is given by

**[GATE - 2004]**

(a) $\begin{bmatrix} 0 & e^{-t} \\ e^{-t} & 0 \end{bmatrix}$ (b) $\begin{bmatrix} e^{t} & 0 \\ 0 & e^{t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix}$ (d) $\begin{bmatrix} 0 & e^{t} \\ e^{t} & 0 \end{bmatrix}$

**42.** The state variable description of a linear autonomous system is, $\dot{X} = AX$ where X is the two dimensional state vector and A is the system matrix given by $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$. The roots of the characteristic equation are

**[GATE - 2004]**

(a) – 2 and +2 (b) –j2 and +j2
(c) –2 and –2 (d) +2 and +2

**43.** A second order system starts with an initial condition of $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ without any external input. The state transition matrix for the system is given by $\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix}$. The state of the system at the end of 1 second is given by

**[GATE - 2003]**

(a) $\begin{bmatrix} 0.271 \\ 1.100 \end{bmatrix}$ (b) $\begin{bmatrix} 0.135 \\ 0.368 \end{bmatrix}$
(c) $\begin{bmatrix} 0.271 \\ 0.736 \end{bmatrix}$ (d) $\begin{bmatrix} 0.135 \\ 1.100 \end{bmatrix}$

**44.** The following equation defines a separately excited dc motor in the form of a differential equation

$$\frac{d^2\omega}{dt} + \frac{B}{J}\frac{d\omega}{dt} + \frac{K^2}{LJ}\omega = \frac{K}{LJ}V_a$$

The above equation may be organized In the state – space form as follows

$$\begin{bmatrix} \dfrac{d^2\omega}{dt^2} \\ \dfrac{d\omega}{dt} \end{bmatrix} = P\begin{bmatrix} \dfrac{d\omega}{dt} \\ \omega \end{bmatrix} + QV_a$$

Where the P matrix is given by

**[GATE - 2003]**

(a) $\begin{bmatrix} -\dfrac{B}{J} & -\dfrac{K^2}{LJ} \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} -\dfrac{K^2}{LJ} & -\dfrac{B}{J} \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ -\dfrac{k^2}{LJ} & -\dfrac{B}{J} \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ -\dfrac{B}{J} & -\dfrac{K^2}{LJ} \end{bmatrix}$

**45.** The zero –input response of a system given by the state – space equation $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $\begin{bmatrix} x_1 & (0) \\ x_2 & (0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is

**[GATE - 2003]**

(a) $\begin{bmatrix} te^{t} \\ t \end{bmatrix}$ (b) $\begin{bmatrix} e^{t} \\ t \end{bmatrix}$
(c) $\begin{bmatrix} e^{t} \\ te^{t} \end{bmatrix}$ (d) $\begin{bmatrix} t \\ te^{t} \end{bmatrix}$

**46.** The transfer function Y(s)/U(s) of system described by the state equation $\dot{x}(t) = -22x(t) + 2u(t)$ and y(t) =0.5x(t) is

**[GATE - 2002]**

(a) $\frac{0.5}{(s-2)}$ (b $\frac{1}{(s-2)}$
(c) $\frac{0.5}{(s+2)}$ (d) $\frac{1}{(s+2)}$

# SOLUTIONS

**Sol. 1. (a)**

Transfer function, $T(s) = \frac{Y(s)}{U(s)} = C[sI - A]^{-1} B$

$A = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

C = [1.5 0.625]

$[sI - A] = \begin{bmatrix} s+4 & 1.5 \\ -4 & s \end{bmatrix}$

$[sI - A]^{-1} = \frac{1}{(s^2 + 4s + 6)} \begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix}$

$[sI - A]^{-1} B = \frac{1}{(s^2 + 4s + 6)} \begin{bmatrix} 2s \\ 8 \end{bmatrix}$

$C[sI - A]^{-1} B = \frac{1}{s^2 + 4s + 6} [1.5 \quad 0.625] \begin{bmatrix} 2s \\ 8 \end{bmatrix}$

$T(s) = \frac{3s + 5}{s^2 + 4s + 6}$

**Sol. 2. (1.284)**

$A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \quad 0], X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Input = $\frac{1}{s}$ (unit step)

Y(t) = cX(t)

$X(t) = L^{-1}[X(s)]$

$X(s) = (SI - A)^{-1} X(0) + (SI - A)^{-1} BU(s)$

$[SI - A] = \begin{bmatrix} s & -1 \\ 0 & s+2 \end{bmatrix}$

$[SI - A]^{-1} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$

$X(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$

$X(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{s}$

$= \begin{bmatrix} \frac{1}{s} + \frac{1}{s^2(s+2)} \\ \frac{1}{s(s+2)} \end{bmatrix}$

$X(t) = L^{-1}[X(s)] = \begin{bmatrix} 1 + 0.5t - 0.25 + 0.25e^{-2t} \\ 0.5 - 0.5e^{-2t} \end{bmatrix}$

$Y(t) = [1 \quad 0]X(t) = [1 + 0.5T - 0.25 + 0.25e^{-2T}]$

Y(1) = 1 + 0.5 – 0.25 + 0.135

= 1.284

**Sol. 3. (d)**

$TF = C(SI - A]^{-1} B$

$SI - A = \begin{bmatrix} s-1 & -2 \\ -2 & s \end{bmatrix}$

$TF = \frac{[1 \quad 0] \begin{bmatrix} s & 2 \\ 2 & s-1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}}{(s-1)(s) - 4}$

$TF = \frac{s+4}{s^2 - s - 4}$

**Sol. 4. (d)**

$\dot{x}_1 - x_2 = 0 \Rightarrow \dot{x}_1 - x_2$ … (i)

$\dot{x}_2 - 2x_1 + 3x_2 = r$

$\dot{x}_2 = r - 2x_1 - 3x_2$ … (ii)

$\begin{bmatrix} \dot{x} \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$

$C = x_1$

$$[C] = [1 \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

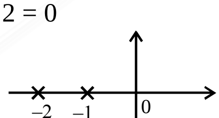
$$TF = C\frac{Adj[SI - A]}{|SI - A|}B + D$$

$$|SI - A| = \begin{bmatrix} S & -1 \\ -2 & S+3 \end{bmatrix} Adj[SI - A] = \begin{bmatrix} S+3 & +1 \\ -2 & S \end{bmatrix}$$

$$TF = \frac{[1 \quad 0]\begin{bmatrix} S+3 & +1 \\ -2 & S \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}}{S(S+3)+2}$$

$$= \frac{[1 \quad 0]\begin{bmatrix} 1 \\ S \end{bmatrix}}{S^2+3S+2} = \frac{1}{S^2+3S+2}$$

CE $S^2 + 3S + 2 = 0$




Over damped system

**Sol. 5. (5)**

$x(t) = ZIR + ZSR$

$$ZIR = e^{At}x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$ZSR = L^{-1}[\phi(S)\ BU(S)]$

$$(SI - A) = \begin{bmatrix} s & 0 \\ 0 & s+9 \end{bmatrix}, Adj(SI - A) = \begin{bmatrix} s+9 & 0 \\ 0 & s \end{bmatrix}$$

$$\phi(s) = (SI - A)^{-1} = \frac{Adj[SI - A]}{|SI - A|} = \begin{bmatrix} \frac{1}{S} & 0 \\ 0 & \frac{1}{S+9} \end{bmatrix}$$

$$ZSR = L^{-1}\left[\begin{bmatrix} \frac{1}{S} & 0 \\ 0 & \frac{1}{S+9} \end{bmatrix}\begin{bmatrix} 0 \\ 45 \end{bmatrix}\begin{bmatrix} \frac{1}{S} \end{bmatrix}\right]$$

$$= L^{-1}\begin{bmatrix} 0 \\ \frac{45}{S(S+9)} \end{bmatrix} = \begin{bmatrix} 0 \\ 5(1 - e^{-9t}) \end{bmatrix}$$

$x_1(t) = 0$, $x_2(t) = 5(1 - e^{-9t})$

$$\underset{t\to\infty}{Lt}\left|\sqrt{x_1^2(t) + x_2^2(t)}\right| = 5$$

**Sol. 6. (6)**

$Y(t) = C^T X(t)$

$X(t) = e^{At}X(0)$

$e^{At} = L^{-1}[(SI-A)^{-1}]$

$$SI\text{-}A = \begin{bmatrix} s-1 & 0 \\ 0 & s-2 \end{bmatrix}$$

$$(SI - A)^{-1} = \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s-2} \end{bmatrix}$$

$$e^{AT} = \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}$$

$$X(t) = \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} e^t \\ e^{2t} \end{bmatrix}$$

$$Y(t) = [1 \quad 1]\begin{bmatrix} e^t \\ e^{2t} \end{bmatrix} = [e^t + e^{2t}]_{t=0.693}$$

$t = \ln_e(2) = 0.693 = [2+4] = 6$

$Y(t) = 6$

**Sol. 7. (a)**

$$A = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$X(t) = e^{At}\times(0)$

$$e^{At} = L^{-1}[(sI-A)^{-1}] = A = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}$$

$X(0) = \alpha$

$$X(t) = \begin{bmatrix} e^{\lambda_1 t} & 0 \\ 0 & e^{\lambda_2 t} \end{bmatrix}\begin{bmatrix} \alpha \\ 0 \end{bmatrix}$$

$X(t) = \alpha e^{\lambda_1 t}$

**Sol. 8. (a)**

$x_1 = -2x_1 + 3U$

$x_2 = -x_2 + U$

$c = x_1$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}U$$

$[c]=[1\ 0]\begin{bmatrix}x_1\\x_2\end{bmatrix}$ By applying Gilbert's test, the system is controllable but not observable.

**Sol. 9. (233)**

$$\begin{bmatrix}x[n]\\x[n-1]\end{bmatrix}=\begin{bmatrix}1&1\\1&0\end{bmatrix}^n\begin{bmatrix}1\\0\end{bmatrix}, n\geq 2\ ,\ n=2$$

$$\begin{bmatrix}x(2)\\x(1)\end{bmatrix}=\begin{bmatrix}1&1\\1&0\end{bmatrix}^2\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}2&1\\1&1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}2\\1\end{bmatrix}$$

x(2) = 2, x(1) = 1 , n=3

$$\begin{bmatrix}x(3)\\x(2)\end{bmatrix}=\begin{bmatrix}1&1\\1&0\end{bmatrix}^2\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}3&2\\2&1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}3\\2\end{bmatrix}$$

x(3) = 3, x(2) = 2

From the above values we can write the recursive relation as

x(n) = x(n–1) + x(n –2)
x(2) = x(1) + x(0) = 1 +1 = 2
x(3) = x(2) + x(1) = 2 +1 = 3
x(4) = x(3) + x(2) = 3 + 2 = 5
x(5) = x(4) + x(3) = 5 + 3 = 8
x(6) = x(5) + x(4) = 8 + 5 = 13
x(7) = x(6) + x(5) = 13+ 8 = 21
x(8) = x(7) + x(6) = 21 + 13 = 34
x(9) = x(8) + x(7) = 34 + 21 = 55
x(10) = x(9) + x(8) = 55 + 34 = 89
x(11) = 89 + 55 = 144
x(12) = 144 + 89 = 233

**Sol. 10. (b)**

$$\dot{x}_1=5x_1-2x_2+u_1$$
$$\dot{x}_2=2x_1+x_2+u_1+u_2$$

$y_1 = x_1$
$y_2 - x_2$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}5&-2\\2&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}1&0\\1&1\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}$$

$$\begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}1&0\\0&-1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}0&0\\0&0\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}$$

$$A=\begin{bmatrix}5&-2\\2&1\end{bmatrix}\quad B=\begin{bmatrix}1&0\\1&1\end{bmatrix}$$

$$C=\begin{bmatrix}1&0\\0&-1\end{bmatrix}\quad D=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

(i) Case 1 when u is input 2 μ o/p

$$A=\begin{bmatrix}5&-2\\2&1\end{bmatrix}B=\begin{bmatrix}1\\1\end{bmatrix}C=[0\ \ -1]D=0$$

Controllability condition [B AB]

$$\left|\begin{bmatrix}1&3\\1&3\end{bmatrix}\right|=0\ \text{. Not controllable}$$

(ii) Case 2 $u_2$ is input & $y_1$ is O/P

$$A=\begin{bmatrix}5&-2\\2&1\end{bmatrix};B=\begin{bmatrix}0\\1\end{bmatrix}c=[1\quad 0]0=0$$

Controllability condition:-

$|[B\quad AB]\neq 0$

$$\left|\begin{bmatrix}0&-2\\1&1\end{bmatrix}\right|=2\neq 0\quad \text{controllable}$$

Observability condition:-

$$\left|\begin{bmatrix}C\\CA\end{bmatrix}\right|\neq 0\left|\begin{bmatrix}i&0\\5&-2\end{bmatrix}\right|=-2\neq 0\ \text{observable}$$

**Sol. 11. (d)**

$$A=\begin{bmatrix}0&1\\0&-1\end{bmatrix};B\begin{bmatrix}0\\0\end{bmatrix};C=[0\ \ 1]$$

$$x(0)=\begin{bmatrix}1\\0\end{bmatrix}$$

$X(t) = \phi(t).x(0) + L^{-1}(\phi(s).Bu(s))$

$= \phi(t)=e^{-At}=L^{-1}(\phi(s).Bs(s))$

$$\Rightarrow L^{-1}\begin{bmatrix}s&-1\\0&s+1\end{bmatrix}^{-1}$$

$$\phi(t)=L^{-1}\left[\frac{1}{s(s+1)}\begin{bmatrix}s+1&1\\0&s\end{bmatrix}\right]$$

$$\Rightarrow L^{-1}\begin{bmatrix}\frac{1}{s}&\frac{1}{s(s+1)}\\0&\frac{1}{s+1}\end{bmatrix}=\begin{bmatrix}1&1-e^{-t}\\0&e^{-t}\end{bmatrix}$$

$$=\ y(t)=[0\ \ 1]\begin{bmatrix}1&1-e^{-t}\\0&e^{-t}\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}$$

$$\Rightarrow [0\quad e^{t}]\begin{bmatrix}1\\0\end{bmatrix}=0$$

**Sol. 12. (c)**

Solution of state equation of

$X(t) = L^{-1}SI–A^{-1}]X(0)$

$$X(0)=\begin{bmatrix}1\\-1\end{bmatrix}A=\begin{bmatrix}-1 & 0\\0 & -2\end{bmatrix}$$

$$[SI-A]^{-1}=\begin{bmatrix}s+1 & 0\\0 & s+2\end{bmatrix}$$

$$=\frac{1}{(S+1)(S+2)}\begin{bmatrix}S+2 & 0\\0 & S+1\end{bmatrix}$$

$$[SI-A]^{-1}=\begin{bmatrix}\frac{1}{S+1} & 0\\0 & \frac{1}{s+2}\end{bmatrix}$$

$$L^{-1}\left[(SI-A)^{-1}\right]=\begin{bmatrix}L^{-1}\left[\frac{1}{S+1}\right] & 0\\0 & L^{-1}\left[\frac{1}{S+2}\right]\end{bmatrix}$$

$$L^{-1}\left[(SI-A)^{-1}\right]=\begin{bmatrix}e^{-t} & 0\\0 & e^{-2t}\end{bmatrix}$$

$$\begin{bmatrix}X_1(t)\\X_2(t)\end{bmatrix}=\begin{bmatrix}e^{-t} & 0\\0 & e^{-2t}\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$\begin{bmatrix}X_1(t)\\X_2(t)\end{bmatrix}=\begin{bmatrix}-e^{t}\\-e^{-2t}\end{bmatrix}\therefore\frac{X_1(t)=e^{-t}}{X_2(t)=-e^{-2t}}$$

**Sol. 13. (c)**

$$\begin{bmatrix}\dot{x}_1\\\dot{x}_2\end{bmatrix}=\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}u$$

Y = AX +Bu

State transmission matrix

$\phi(t) = \mathcal{L}^{-1}\,(si–A)^{-1}$

$$[Si-A]=\begin{bmatrix}s-1 & 0\\1 & s-1\end{bmatrix}$$

So, $\phi(t)=\mathcal{L}^{-1}\frac{1}{(s-1)^2}\begin{bmatrix}s-1 & 0\\1 & s-1\end{bmatrix}$

$$=\mathcal{L}^{-1}\begin{bmatrix}\frac{1}{s-1} & 0\\\frac{1}{(s-1)} & \frac{1}{s-1}\end{bmatrix}$$

$$\phi(t)=\begin{bmatrix}e^{t} & 0\\te^{t} & e^{t}\end{bmatrix}$$

**Sol. 14. (a)**

From the given signal flow graph , the state model is

$$\begin{bmatrix}\dot{X}_1\\\dot{X}_2\\\dot{X}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\a_3 & a_2 & a_1\end{bmatrix}\begin{bmatrix}X_1\\X_2\\X_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}u$$

$$Y=[C_1C_2C_3]\begin{bmatrix}X_1\\X_2\\X_3\end{bmatrix}$$

$$A=\begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\a_3 & a_2 & a_1\end{bmatrix};B=\begin{bmatrix}0\\0\\1\end{bmatrix};C=[C_1C_2C_3]$$

Controllability:

$Q_c = [B \quad AB \quad A^2B]$

$$Q_C=\begin{bmatrix}0 & 0 & 1\\0 & 1 & a_1\\1 & a_1 & a_2+a_1^2\end{bmatrix}$$

$|Q_C| = 1 \neq 0$

Observability

$$Q_0=\begin{bmatrix}C\\CA\\CA^2\end{bmatrix}$$

$\Rightarrow$

$$\begin{bmatrix}C & C_2 & C_3\\a_3c_3 & c_1+a_2c_3 & c_2+a_1c_3\\c_2a_3+c_3(a_1a_3) & a_2c_2+c_3(a_1a_2+a_3 & c_1+a_1c_2+c_3\left(a_1^2+a_2\right)\end{bmatrix}$$

$|Q_0| \Rightarrow$ depends on $a_1$, $a_2$, $a_3$ & $c_1$ & $c_2$ & $c_3$

It is always controllable

**Sol. 15. (b)**

Apply linearity principle,

$$\begin{bmatrix}3\\5\end{bmatrix}=a\begin{bmatrix}1\\1\end{bmatrix}+b\begin{bmatrix}0\\1\end{bmatrix}s$$

a = 3; b =8

$$\Rightarrow x(t)=3\begin{bmatrix}e^{-t}\\-e^{-t}\end{bmatrix}+\begin{bmatrix}e^{-t} & -e^{-2t}\\-e^{-t} & +2e^{-2t}\end{bmatrix}$$

$$\Rightarrow x(t) = \begin{bmatrix} 1Ie^{-t} & -8e^{-2t} \\ -11e^{-t} & +16e^{-2t} \end{bmatrix}$$

$$\Rightarrow x(t) = \begin{bmatrix} 11e^{-t} - 8e^{-2t} \\ -11e^{-t} + 16e^{-2t} \end{bmatrix}$$

**Sol. 16. (c)**

For controllability, we check controllability matrix

$C_M = [B \quad AB]$

or $C_M = [Q \quad PQ]$

$$PQ = \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

So, $C_M = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$

Det $(C_M) = -1 \neq 0$

So, the system is controllable.

For Observability, we check Observability matrix

$O_M = \begin{bmatrix} C \\ CA \end{bmatrix}$

or $O_M = \begin{bmatrix} R \\ RP \end{bmatrix}$

$RP = [0 \quad 1] \begin{bmatrix} -1 & 1 \\ 0 & -3 \end{bmatrix} = [0 \quad 3]$

So, $O_M = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}$

det $(O_M) = 0$

**Sol. 17. (a)**

Given, the state variable formulation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}u \quad \ldots \text{(i)}$$

And $y = [1 \quad ]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ … (ii)

from Eq. (i) we get $\dot{x}_1 = 2x_1 + u$

Taking Laplace transform

$$sX_1 - x_1(0) = -2X_1 + \frac{1}{s}$$

(here, $X_1$ denotes Laplace transform of $x_1$)

So, $(s+2)X_1 = \frac{1}{s}$ $(x_1(0) = 0)$ $(x_1(0) = 0)$

or, $X_1 = \frac{1}{s(s+2)}$ … (iii)

Now, from Eq. (ii) we have

$y = x_1$

Taking Laplace transform both the sides

$Y = X_L$

or, $Y = \frac{1}{s(s+2)}$ (from eq. (iii)]

or, $Y = \frac{1}{2}\left[\frac{1}{s} - \frac{1}{s+2}\right]$

Taking inverse Laplace transform

$y = \frac{1}{2}\left[u(t) - e^{-2t}u(t)\right]$

$= \frac{1}{2} - \frac{1}{2}e^{-2t}(\text{for } t > 0)$

**Sol. 18. (a)**

From the given state variable system, we have

$A = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; C = [1 \quad 0]$

Now, we obtain the controllability matrix

$C_M = [B : AB] = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$

And the observability matrix is obtained as

$O_M = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}$

So, we get

Rank of the controllability matrix : Rank $(C_M) = 2$

Rank of the observability matrix : Rank $(O_M) = 1$

Since, the order of state variable is 2($x_1$ and $x_2$). Therefore, we have

Rank$(C_M)$ = order of state variables

But Rank $(O_M)$ < order of state variables

Thus, system is controllable but not observable.

**Sol. 19. (a)**

From the obtained state-variable equations we have

$$A = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

So, $SI - A = \begin{bmatrix} S+1 & 0 \\ -1 & S+1 \end{bmatrix}$

And $(SI-A)^{-1} = \begin{bmatrix} S+1 & 0 \\ -1 & S+1 \end{bmatrix}$

$$= \begin{bmatrix} \frac{1}{s+1} & 0 \\ \frac{1}{(S+1)^2} & \frac{1}{S+1} \end{bmatrix}$$

Hence, the state transition matrix is obtained as $e^{At} = L^{-1}(SI - A)^{-1}$

$$= L^{-1}\left\{\begin{bmatrix} \frac{1}{S+1} & 0 \\ \frac{1}{(S+1)^2} & \frac{1}{S+1} \end{bmatrix}\right\} = \begin{bmatrix} e^{-1} & 0 \\ te^{-t} & e^{-t} \end{bmatrix}$$

**Sol. 20. (d)**

General form of state equation are given as

$\dot{x} = Ax + Bu$

$\dot{y} = Cx + Du$

For the given problem

$$A = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & a_1 & 0 \\ 0 & 0 & a_2 \\ a_3 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a_1a_2 \\ 0 \\ 0 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 0 & 0 & a_1a_2 \\ a_2a_3 & 0 & 0 \\ 0 & a_3a_1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a_1a_2 \\ 0 \\ 0 \end{bmatrix}$$

For controllability it is necessary that following matrix has a tank of n = 3.

$$U = \left[B : AB : A^2B\right] = \begin{bmatrix} 0 & 0 & a_1a_2 \\ 0 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

So, $a_2 \neq 0$

$a_1a_2 \neq 0 \Rightarrow a_1 \neq 0$ ($a_3$ may be zero or note)

**Sol. 21. (b)**

Here $x = y_1$ and $\dot{x} = \frac{dy_1}{dx}$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x \\ 2x \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}\underline{x}$$

Now $y_t = \frac{1}{s+2}u$

$y_t(s+2) = u$

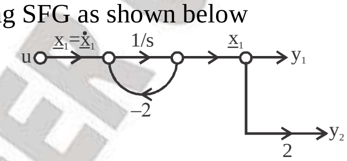
$\dot{y}_1 + 2y_1 = u$

$\dot{x} + 2x = u$

$\dot{x} = -2x = u$

$\underline{\dot{x}} = [-2]\underline{x} + [1]u$

Drawing SFG as shown below




Thus,

$\dot{x}_1 = [-2]\underline{x} + [1]u$

$y_1 = \underline{x}_1; y_2 = 2\underline{x}$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}\underline{x}_1$$

Here $\underline{x}_1 = \underline{x}$

**Sol. 22. (c)**

Stability:

Eigen value of the system are calculated as

$|A - \lambda I) = 0$

$$A - \lambda I = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} -1-\lambda & 2 \\ 0 & 2-\lambda \end{bmatrix}$$

$\Rightarrow \lambda_1, \lambda_2 = -1.2$

Since eigen value of the system are of opposite signs, so it is unstable

Controllability:

$$A = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

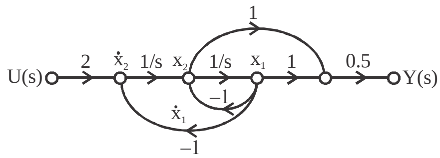
$$AB = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$[B : AB] = \begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}$$

$[B : AB] \neq 0$

So, it is controllable.

**Sol. 23. (d)**
Assign output of each integrator by a state variable




$\dot{x} = -x_1 + x_2$

$\dot{x}_2 = -x_1 + 2u$

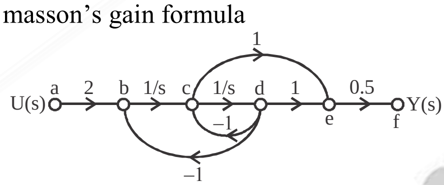
y = $0.5x_1 + 0.5x_2$

State variable representation

$$\dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

$$\dot{y} = [0.5 \;\; 0.5] x$$

**Sol. 24. (c)**
By masson's gain formula




Transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{\Sigma P_k \Delta_k}{\Delta}$$

Forward path given

$P_1$(abcdef) = $2 \times \frac{1}{3} \times 1 \times 0.5$

Loop gain $L_1$ (cdc) = $-\frac{1}{s}$

$$L_2(bcdb) = \frac{1}{s} \times \frac{1}{s} \times -1 = \frac{-1}{s}$$

$\Delta_1 = 1 - [L_1 + L_2] = 1 - \left[-\frac{1}{s} - \frac{1}{s^2}\right] = 1 + \frac{1}{s} + \frac{1}{s^2}$

$\Delta_1 = 1$, $\Delta_2 = 2$

$$\text{So, } H(s) = \frac{Y(s)}{U(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\frac{\frac{1}{s^2}.1 + \frac{1}{s}.1}{1 + \frac{1}{s} + \frac{1}{s^2}} = \frac{(1+s)}{(s^2 + s + 1)}$$

**Sol. 25. (c)**
Here,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{and } B = \begin{bmatrix} p \\ q \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$$

$$S = [B \;\; AB] = \begin{bmatrix} p & q \\ q & p \end{bmatrix}$$

S = pq – pq = 0

Since S is singular, system is completely uncontrollable for all values of p and q.

**Sol. 26. (c)**
Given system equations

$$\frac{dx_1(t)}{dt} = 3x_1(t) + x_2(t) + 2u(t)$$

$$\frac{dx_2(t)}{dt} = -2x_2(t) + u(t)$$

y(t) = $x_1(t)$

Taking Laplace transform on both sides of equations.

$sX_1(s) = -3X_1(s) + X_2 + 2U(s)$ …(i)

$(s + 3)\, X_1(s) = X_2(s) + 2U(s)$

Similarly

$S(+2)X_2(s) = U(s)$ ...(ii)

From equation (i) & (ii)

$$(s+3)X_1(s) = \frac{U(s)}{s+2} + 2U(s)$$

$$X_s(s) = \frac{U(s)}{s+3}\left[\frac{1+2(s+2)}{s+2}\right]$$

$$= U(s)\frac{(2s+5)}{(s+2)(s+3)}$$

From output equation,

$Y(s) = X_1(s)$

$$\text{So, } Y(s) = U(s)\frac{(2s+5)}{(s+2)(s+3)}$$

$$= \frac{(2s+5)}{s^2 + 5s + 6}$$

System transfer function

$$T.F. = \frac{Y(s)}{U(s)} = \frac{(2s+5)}{(s+2)(s+3)}$$

$$= \frac{(2s+5)}{s^2+5s+6}$$

**Sol. 27. (b)**

Given state equations in matrix form can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t)$$

$$\frac{dX(t)}{dt} = AX(t) + Bu(t)$$

State transition matrix is given by

$\phi(t) = L^{-1}[\Phi(s)]$

$\Phi(s) = (sI - A)^{-1}$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{(s+3)(s+2)}\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}$$

$$\text{So } \Phi(s) = (sI\text{-}A)^{-1} = \begin{bmatrix} \frac{1}{(s+3)} & \frac{1}{(s+3)(s+2)} \\ 0 & \frac{1}{(s+2)} \end{bmatrix}$$

$$\phi(t) = L^{-1}[\Phi(S)] = \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

**Sol. 28. (d)**

State space equation of the system is given by,

$\dot{X} = AX + Bu \quad Y = CX$

Taking Laplace transform on both sides of the equations.

$sX(s) = AX(s) + BU(s)$

$(SI–A)X(s) = BU(s)$

$X(s) = (SI – A)^{-1} BU(s)$

$\therefore\ Y(s) = CX(s)$

So $Y(s) = C(sI – A)^{-1} BU(s)$

$$\text{T.F} = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B$$

$$(sI\text{-}A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s+2 \end{bmatrix}$$

$$(sI\text{-}A)^{-1} = \frac{1}{s(s+2)}\begin{bmatrix} s+2 & 1 \\ 0 & s \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{(s+2)} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Transfer function

$$G(s) = C[sI\text{-}A]^{-1} B = [1 \quad 0]\begin{bmatrix} \frac{i}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{(s+2)} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= [1 \quad 0]\begin{bmatrix} \frac{1}{s(s+2)} \\ \frac{1}{(s+2)} \end{bmatrix} = \frac{1}{s(s+2)}$$

**Sol. 29. (a)**

Steady state error is given by

$$e_{ss} = \lim_{s\to 0}\left[\frac{sR(s)}{1+G(s)H(s)}\right]$$

Here $R(s) = L[r(t)] = \frac{1}{s}$(unit step input)

$$G(s) = \frac{1}{s(s+2)}$$

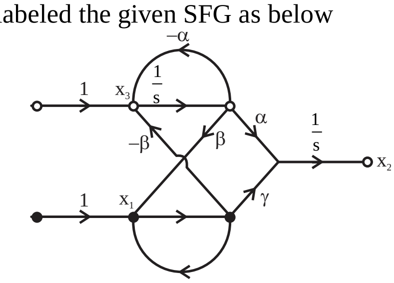
H(s) = 1 (unity feed back)

So,

$$e_{ss} = \lim_{s\to 0}\left[\frac{s\left(\frac{1}{s}\right)}{1+\frac{1}{s(s+2)}}\right] = \lim_{s\to 0}\left[\frac{s(+2)}{s(s+2)+1}\right] = 0$$

**Sol. 30. (c)**

We labeled the given SFG as below




From this SFG we have

$\dot{x}_1 = -\gamma x_1 + \beta x_3 + \mu_1$

$\dot{x}_1 = \gamma x_1 + \alpha x_3$

$\dot{x}_3 = -\beta x_1 - \alpha x_3 + \mu_2$

Thus

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\gamma & 0 & \beta \\ \gamma & 0 & \alpha \\ -\beta & 0 & -\alpha \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

**Sol. 31.(a)**

We have $\begin{bmatrix} \frac{d\omega}{dt} \\ \frac{dt_e}{dt} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & -10 \end{bmatrix}\begin{bmatrix} \omega \\ i_n \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u$

Or $\frac{d\omega}{dt} = -\omega + i_n$ …(i)

and $\frac{di_a}{dt} = -\omega - 10i_a + 10u$ …(ii)

Taking Laplace transform (i) we get

$s\omega(s) = -\omega(s) = I_a(s)$

or $(s + 1)\omega(s) = I_a(s)$ … (iii)

Taking Laplace transform (ii) we get

$sI_a(s) = -\omega(s) - 10I_a(s) + 10U(s)$

or $\omega(s) = (-10-s)I_a(s) + 10U(s)$

$= (-10-s)(s+1)\omega(s) + 10U(s)$ from (iii)

or $\omega(s) = [s^2 + 11s + 10]\ \omega(s) + 10U(s)$

or $(s^2 + 11s + 11)\omega(s) = 10IU(s)$

or $\frac{\omega(s)}{U(s)} = \frac{10}{(s^2 + 11s + 11)}$

**Sol. 32. (a)**

We have $\dot{x}(t) = Ax(t)$

Let A = $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$

For initial state vector $x(0) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ the system

response is x(t) = $\begin{bmatrix} e^{-2t} \\ -2e^{-2t} \end{bmatrix}$

Thus $\begin{bmatrix} \frac{d}{dt}e^{-2t} \\ \frac{d}{dt}\left(-2e^{-2t}\right) \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

or $\begin{bmatrix} -2e^{-2(0)} \\ 4e^{-2(0)} \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} p- & 2q \\ r & -2s \end{bmatrix}$

We get p – 2q = –2 and r – 2s = 4 … (i)

For initials state vector $x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ the system

response is x(t) = $\begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$

Thus $\begin{bmatrix} \frac{d}{dt}e^{-t} \\ \frac{d}{dt}\left(-e^{-t}\right) \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} -e^{-(0)} \\ e^{-t(0)} \end{bmatrix} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} p-q \\ r-s \end{bmatrix}$

We get p – q = –1 and r – s = 1 … (ii)

Solving (1) and (2) set of equations we get

$\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

Thus characteristic equation

$|\lambda I - A| = 0$

$\begin{vmatrix} \lambda & -1 \\ 2 & \lambda+3 \end{vmatrix} = 0$

or $\lambda(\lambda + 3) + 2 = 0$

or $\lambda = -1, -2$

thus Eigen values are –1 and –2

Eigen vectors for $\lambda_1 = -1$

$(\lambda_1 I - A)\ X_1 = 0$

Or $\begin{bmatrix} \lambda_1 & -1 \\ 2 & \lambda+3 \end{bmatrix} = 0$

$\begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = 0$

or $-x_{11} - x_{21} = 0$

or $x_{11} + x_{21} = 0$

We have only one independent equation $x_{11} = -x_{21}$.

Let $x_{11} = K$, then $x_{21} = -K$, the Eigen vector will be

$$\begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = \begin{bmatrix} K \\ -K \end{bmatrix} = K\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Now Eigen vector for $\lambda_2 = -2$

$(\lambda_2 I - A)\, X_2 = 0$

$$\text{or } \begin{bmatrix} \lambda_2 & -1 \\ 2 & \lambda_3 + 3 \end{bmatrix}\begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = 0$$

$$\text{or } \begin{bmatrix} -2 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = 0$$

or $-x_{11} - x_{21} = 0$

or $x_{11} + x_{21} = 0$

We have only one independent equation

$x_{11} = -x_{21}$.

Let $x_{11} = K$, then $x_{21} = -K$, the Eigen vector will be

$$\begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} = \begin{bmatrix} K \\ -2K \end{bmatrix} = K\begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

**Sol. 33. (d)**

As shown in previous solution the system matrix is

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

**Sol. 34. (b)**

In standard form for a characteristic equation give as

$$s^n + a_{n-1}s^{n-1} + ...... + a_1 s + a_0 = 0$$

In its state variable representation matrix A is given as

$$A = \begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -a_0 & -a_1 & -a_2 & ... & -a_{n-1} \end{bmatrix}$$

Characteristic equation of the system is

$4s^2 - 2s + 1 = 0$

So, $a_2 = 4$, $a_1 = -2$, $a_0 = 1$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$$

**Sol. 35. (a)**

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^2+1}\begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix} = \begin{bmatrix} \frac{s}{s^2+1} & \frac{1}{s^2+1} \\ \frac{-1}{s^2+1} & \frac{s}{s^2+1} \end{bmatrix}$$

$$\phi(t) = e^{At} = L^{-1}[(sI - A)]^{-1} = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}$$

**Sol. 36. (a)**

Give state equation

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$\text{Here,} \quad A = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

State transition matrix is given by

$\phi(t) = L^{-1}[(sI\text{-}A)^{-1}]$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s+3 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{s(s+3)}\begin{bmatrix} s+3 & 1 \\ 0 & s \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+3)} \\ 0 & \frac{1}{(s+3)} \end{bmatrix}$$

$\phi(t) = L^{-1}\,[(sI - A)^{-1}]$

$$= \begin{bmatrix} 1 & \frac{1}{3}\left(1 - e^{-3t}\right) \\ 0 & e^{-3t} \end{bmatrix}$$

**Sol. 37. (c)**

State transition equation is given by

$X(s) = \Phi(s)\, X(0) + \Phi(s)\, BU(s)$

Here $\Phi(s) = \Phi(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+3)} \\ 0 & \frac{1}{(s+3)} \end{bmatrix}$

X(0) is initial condition

$X(0)=\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

So

$$X(s)=\begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+3)} \\ 0 & \frac{1}{(s+3)} \end{bmatrix}\begin{bmatrix} -1 \\ 3 \end{bmatrix}+\begin{bmatrix} \frac{1}{s} & \frac{1}{(s+3)s} \\ 0 & \frac{1}{s+3} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\frac{1}{s}$$

$$= \ =\begin{bmatrix} -\frac{1}{s} & \frac{3}{s(s+3)} \\ 0+ & \frac{3}{(s+3)} \end{bmatrix}\begin{bmatrix} \frac{1}{s} \\ 0 \end{bmatrix}\frac{1}{s}=\begin{bmatrix} -\frac{1}{s+3} \\ \frac{3}{s+3} \end{bmatrix}+\begin{bmatrix} \frac{1}{s^2} \\ 0 \end{bmatrix}$$

$$X(s) = \begin{bmatrix} \frac{1}{s^2}-\frac{1}{s+3} \\ \frac{3}{s+3} \end{bmatrix}$$

Taking inverse Laplace transform, we get state transition equation as,

$$X(t) = \begin{bmatrix} t-e^{-3t} \\ 2e^{-3t} \end{bmatrix}$$

**Sol. 38. (d)**

The transfer function of a lag network is

$$T(s)=\frac{1+sT}{1+s\beta T}\ \beta>1; T>0$$

$$|T(j\omega)|=\frac{\sqrt{1+\omega^2T^2}}{\sqrt{1+\omega^2\beta^2T^2}}$$

and $\angle T(j\omega) = \tan^{-1}(\omega T) - \tan^{-1}(\omega\beta T)$

At $\omega = 0$, $|T(j\omega)|=1$

At $\omega = 0$, $\angle T(j\omega) = -\tan^{-1}0 = 0$

At $\omega = \infty$, $|T(j\omega)|=\frac{1}{\beta}$

At $\omega = \infty$, $\angle T(j\omega) = 0$

**Sol. 39. (c)**

We have $\dot{X}=AX+BU$ where $\lambda$ is set of Eigen values

And $\dot{W}=CW+DU$ where $\mu$ is set of Eigen values

If a linear system is equivalently represented by two sets of state equations, then for both sets, states will be same but their sets of Eigen values will not be same

i.e.

$X = W$ but $\lambda \neq \mu$

**Sol. 40. (d)**

We have $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}+\begin{bmatrix} 1 \\ 0 \end{bmatrix}u$

And $Y=\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}+\begin{bmatrix} 1 \\ 2 \end{bmatrix}u$

Here $A=\begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}, B=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $C=\begin{bmatrix} 1 & 0 \end{bmatrix}$

The controllability matrix is

$Q_C= \begin{bmatrix} B & AB \end{bmatrix}=\begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix}$

det $Q_C \neq 0$ Thus controllable

The observability matrix is

$Q_0 = [C^T \quad A^T \quad C^T]$

$=\begin{bmatrix} 1 & -3 \\ 0 & -1 \end{bmatrix}\neq 0$

Det $Q_0 \neq 0$ Thus observable

**Sol. 41. (b)**

$$(sI-A)=\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}-\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} s-1 & 0 \\ 0 & s-1 \end{bmatrix}$$

$$(sI-A)^{-1}=\frac{1}{(s-1)^2}\begin{bmatrix} (s-1) & 0 \\ 0 & (s-1) \end{bmatrix}=\begin{bmatrix} \frac{1}{s-1} & 0 \\ & \frac{1}{s-1} \end{bmatrix}$$

$e^{AT} = L^{-1} [(sI - A)]^{-1}$

$$= \begin{bmatrix} e^t & 0 \\ 0 & e^t \end{bmatrix}$$

**Sol. 42. (a)**

Characteristic equation is given by

$|sI-A|=0$

$$(sI - A) = \begin{vmatrix} s & 0 \\ 0 & s \end{vmatrix} - \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix}$$

$$= \begin{bmatrix} s & -2 \\ -2 & s \end{bmatrix} = s^2 - 4 = 0$$

$s_1$ , $s_2 = \pm 2$

**Sol. 43. (a)**

Since there is no external input, so state is given by

X(t) = ϕ(t) X(0)

ϕ(t) is state transition matrix

X[0] is initial condition

$$\text{So } x(t) = \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$x(t) = \begin{bmatrix} 2e^{-2} \\ 2e^{-1} \end{bmatrix}$$

At t = 1, state of the system

$$x(t)\big|_{t=1} = \begin{bmatrix} 2e^{-2} \\ 2e^{-1} \end{bmatrix} = \begin{bmatrix} 0.271 \\ 1.100 \end{bmatrix}$$

**Sol. 44. (a)**

Given equation can be written as,

$$\frac{d^2\omega}{dt^2} = -\frac{\beta}{J}\frac{d\omega}{dt} - \frac{K^2}{LJ}\omega + \frac{K}{LJ}V_a$$

Here state variables are defined as $\frac{d\omega}{dt} = x_t$

$\omega = x_2$ So State equation is $\dot{x}_2 = \frac{d\omega}{dt} = x_t$

In matrix form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -B/J & -K^2/LJ \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} K/Lj \\ 0 \end{bmatrix} V_a$$

$$\begin{bmatrix} \frac{d^2\omega}{dt^2} \\ \frac{d\omega}{dr} \end{bmatrix} = P\begin{bmatrix} d\omega \\ dt \end{bmatrix} + QV_a$$

$$\text{So matrix P is } \begin{bmatrix} -B/J & -K^2/LJ \\ 1 & 0 \end{bmatrix}$$

**Sol. 45. (c)**

We have

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{and} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{(s-1)^2} \begin{bmatrix} s-1 & 0 \\ +1 & s-1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{+1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix}$$

$$L^{-1}\left[(sI - A)^{-1}\right] = e^{At} = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ te^t \end{bmatrix}$$

**Sol. 46. (d)**

We have $\dot{x}(t) = -2x(t) + 2u(t)$ … (i)

Taking Laplace transform we get

sX(s) = –2X(s) + 2U(s)

or (s +2)X(s) = 2U(s) or $X(s) = \frac{2U(s)}{(s+2)}$

now y(t) = 0.5x(t)

Y(s) = 0.5X(s)

Or $Y(s) = \frac{0.5 \times 2U(s)}{s+2}$ or $\frac{Y(s)}{U(s)} = \frac{1}{(s+2)}$

# ESE OBJ QUESTIONS

**1.** A dynamic system is described by the following equations: $X = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

and Y = [10 0]u

Then the transfer function relating Y and u is given by

**[EE ESE - 2017]**

(a) $\frac{Y(s)}{u(s)} = \frac{10s}{s^2 + 4s + 3}$

(b) $\frac{Y(s)}{u(s)} = \frac{10s}{s^2 + 4s + 3}$

(c) $\frac{Y(s)}{u(s)} = \frac{s}{s^2 + 2s + 1}$

(d) $\frac{Y(s)}{u(s)} = \frac{s}{s^2 + 3s + 1}$

**2.** The system described by the following state equations

$\dot{X} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, Y = [1, 1] X$

1. Completely controllable
2. Completely observable

Which of the above statements is/are correct?

**[EE ESE - 2016]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

**3.** The state -variable formulation of a system is $\dot{x} = Ax + Bu; \quad y = [1 \quad 0]x$

Where

$A = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

The system transformation would be

**[EE ESE – 2015]**

(a) $\frac{s+2}{s^2 + 5s + 6}$ (b) $\frac{2s+5}{s^2 + 5s + 6}$

(c) $\frac{2s-5}{s^2 + 5s - 6}$ (d) $\frac{s+1}{s^2 + 5s + 6}$

**4.** The vector $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ is an eigen value of

$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

One of the eigen values of A is

**[EE ESE - 2015]**

(a) 1 (b) 2
(c) 5 (d) 7

**5. Statement (I):** For radar tracking systems, signals are available in the form of pulse trains.

**Statement (II):** The stability of a discrete – time system is decreased as the sampling period is shortened.

**[EE ESE - 2015]**

(a) Both Statement (I) and Statement (II) are individually true and Statement (ii) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (I).
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

**6.** A discrete time system is stable if all the roots of the characteristic equation lie

**[EE ESE - 2014]**

(a) Outside the circle of unit radius
(b) Within the circle of unit radius
(c) Outside the circle of radius equal to 3 – units
(d) On the circle of finite radius

**7.** Consider the following properties attributed to state model of a system:
(i) State model is unique
(ii) Transfer function for the system is unique
(iii) State model can be derived from transfer function of the system

Which of the above statements are correct?

**[EE ESE - 2014]**

(a) i, ii and iii (b) i and ii only
(c) ii and iii only (d) i and iii only

**8.** This non-linearity represents

**[EE ESE - 2014]**

(a) Dead zone (b) Coulomb friction
(c) Saturation (d) Hysteresis

**9.** The state equations in the phase variable canonical from an be obtained from the transfer function by

**[EE ESE - 2014]**

(a) Cascaded decomposition
(b) Direct decomposition
(c) Inverse decomposition
(d) Parallel decomposition

**10.** The system matrix of a linear time invariant continuous time system is given by

$$A = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}$$

What are the roots of the characteristic equation?

**[EE ESE - 2014]**

(a) – 1, –4 (b) –1, –5
(c) –4, –5 (d) 0, –1

**11.** A non – linear control system is described by the equation

$$\dot{\theta} + K\sin\theta = 0$$

The type of singular point at (0, 0) is

**[EE ESE - 2012]**

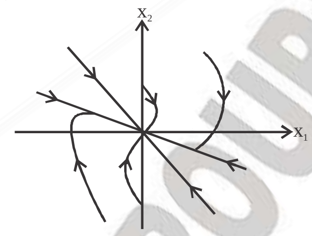
(a) Centre (b) Focus
(c) Saddle point (d) None of above

**12.** The following relation involving state transition matrix $\phi(t)$ does not hold true

**[EE ESE - 2012]**

(a) $\phi(t) = I$
(b) $\phi(t) = \phi[(t)]^{-1}$
(c) $\phi(t_1 - t_2) = \phi(t_1 - t_0)\,\phi(t_2 - t_0)$
(d) $\phi(t_1 + t_2) = \phi(t_1)\,\phi(t_2)$

**13.** The figure shown in a phase-plane representation of trajectories. The singular point shown is a

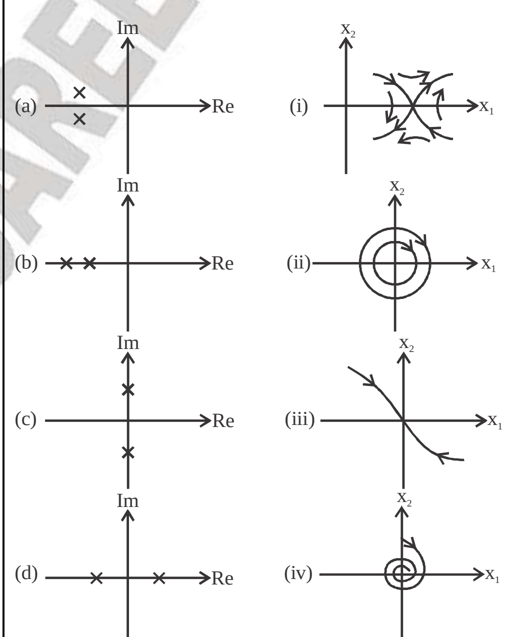



**[EE ESE - 2011]**

(a) Unstable node (b) Saddle point
(c) Stable focus (d) Stable node

**14.** Match List-I with List-II and select the correct answer using the code given below the lists:

**List-I** **List-II**




**[EE ESE - 2011]**

**Codes:**

(a) A-i, B-ii, C-iii, D-iv
(b) A-iv, B-iii, C-ii, D-i
(c) A-i, B-iii, C-ii, D-iv
(d) A-iv, B-ii, C-iii, D-i

**15.** The state variable description of a linear autonomous system is $\dot{x} = Ax$, where x is the two-dimensional state vector and A is given by

$$A = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$$

The poles of the system are located at

**[EE ESE - 2011]**

(a) – 2 and +2 (b) –2j and +2j
(c) –2 and –2 (d) +2 and + 2

**16.** Let $\dot{x} = \begin{bmatrix} 1 & 2 \\ 0 & b \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

where b is an unknown constant. This system is

**[EE ESE - 2011]**

(a) Uncontrollable for b = 1
(b) Uncontrollable for b = 0
(c) Uncontrollable for all values of b
(d) Controllable for all values of b

**17.** System transformation function H(z) for a discrete time LTI system expressed in state variable form with zero initial conditions is variable form with zero initial conditions is

**[EC ESE - 2011]**

(a) $c(zl – A)^{-1} b + d$ (b) $c(zl – A)^{-1}$
(c) $(zl – A)^{-1} z$ (d) $(zl – A)^{-1}$

**18.** The transfer functions for the state representation of continuous time LTI system:
q(t) = Aq(t) + bx (t)
y(t) = cq(t) + dx (t)
is given by

**[EC ESE - 2010]**

(a) $c (sl – A)^{-1} b + d$ (b) $b (sl – A)^{-1} b + d$
(c) $c (sl – A)^{-1} b + d$ (d) $d (sl – A)^{-1} b + c$

**19.** The sate variable description autonomous system is $\dot{X} = AX$ of a linear where X is a two – dimensional vector and A is a matrix given by

$$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}.$$

The poles of the system are located at

**[EE ESE - 2010]**

(a) –2 and –2 (b) –j2 and +j2
(c) –2 and + 2 (d) +2 and +2

**20.** The Z – transform of x(K) is given by

$$x(Z) = \frac{(1-e^{-T})Z^{-1}}{(1-Z^{-1})(1-e^{-T}Z^{-1})}$$

The initial value x(0) is

**[EE ESE - 2010]**

(a) Zero (b) 1
(c) 2 (d) 3

**21.** Consider the following statements with reference to the phase plane
(i) They are general and applicable to a system of any order
(ii) Steady state accuracy and existence of limit cycle can be predicted
(iii) Amplitude and frequency of limit cycle if exists can be evaluated
(iv) Can be applied to discontinuous time system.
Which of the above statements are correct?

**[EE ESE - 2010]**

(a) i, ii, iii and iv (b) ii and iii only
(c) iii and iv only (d)ii, iii and iv

**22.** The system matrix of a continuous time system is given by

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}$$

Then the characteristic equation is

**[EE ESE - 2010]**

(a) $s^2 + 5s + 3 = 0$ (b) $s^2 – 3s – 5 = 0$
(c) $s^2 – 3s + 5 = 0$ (d) $s^2 + 3s + 2 = 0$

**23.** When a transfer function model is converted into state space model, the order of the system may be reduced during which one of the following conditions?

**[EE ESE - 2009]**

(a)Some of the variables are not considered
(b)Some of the variables are hidden
(c)Pole, zero cancellation takes place
(d)The order of the system will never get changed

**24.** A linear system is described by the following state equations:

$$X(t)=\begin{bmatrix}0 & -2\\ 1 & -3\end{bmatrix}X+\begin{bmatrix}2\\ 0\end{bmatrix}Y$$

$Y(t) = [0 \quad 3]X$

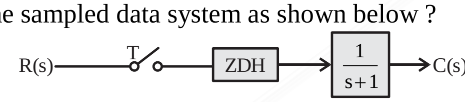
What is the transfer function of the system ?

**[EE ESE - 2009]**

(a) $\dfrac{1}{s^2+2s+3}$ (b) $\dfrac{6}{s^2+3s+2}$

(c) $\dfrac{6}{s^2+2s+3}$ (d) $\dfrac{1}{s^2+3s+2}$

**25.** What is the transfer function C(Z)/R(Z) of the sampled data system as shown below ?




**[EE ESE - 2009]**

(a) $\dfrac{(1-e^{-T})}{(Z-e^{-T})}$ (b) $\dfrac{(Z-e^{-T})}{(Z-e^{-T})}$

(c) $\dfrac{(1-2e^{-T})}{(e^{-T}-Z)}$ (d) $\dfrac{(1-2Ze^{-T})}{(Z-1)}$

**26.** The system matrix of a linear time invariant continuous time system is given by

$$A=\begin{bmatrix}0 & 1\\ -3 & -5\end{bmatrix}$$

What is the characteristic equation ?

**[EE ESE - 2009]**

(a) $s^2 + 5s + 3 = 0$ (b) $s^2 - 3s - 5 = 0$

(c) $s^2 + 3s + 5 = 0$ (d) $s^2 + s + 2 = 0$

**27.** What is the represented by state transition matrix of a system?

**[EE ESE - 2009]**

(a) Free response (b) Impulse response

(c) Step response (d) Forced response

**28.** Transfer function of a certain system is

$$\frac{Y(s)}{U(s)}=\frac{1}{s^4+5s^3+8s^2+6s+3}$$

Which one of the following will be the A, B matrix pair of state variable representation of this system?

**[EC ESE - 2009]**

(a) $\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -3 & -6 & -8 & -5\end{bmatrix}\begin{bmatrix}0\\ 0\\ 0\\ 1\end{bmatrix}$

(b) $\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -5 & -8 & -6 & -3\end{bmatrix}\begin{bmatrix}0\\ 0\\ 0\\ 1\end{bmatrix}$

(c) $\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ -3 & -6 & -8 & -5\end{bmatrix}\begin{bmatrix}1\\ 0\\ 0\\ 0\end{bmatrix}$

(d) $\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -3 & -6 & -8 & -5\end{bmatrix}\begin{bmatrix}1\\ 0\\ 0\\ 0\end{bmatrix}$

**29.** Isocline method is used for which one of the following?

**[EE ESE - 2008]**

(a) Design of nonlinear system

(b) Construction of root loci of nonlinear system

(c) Construction of phase trajectories of nonlinear systems

(d) Stability analysis of nonlinear system

**30. Assertion (A):** Sample–data system requires hold circuit.

**Reason (R):** Hold circuit converts the signal to analog form.

**[EE ESE - 2008]**

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

**31.** The information contained in a signal is preserved in the sampled version if

**[EE ESE - 2008]**

(a) $\omega_s = \omega_m$ (b) $\omega_s = 0.5\ \omega_m$
(c) $\omega_s = 0.1\ \omega_m$ (d) $\omega_s = 2\ \omega_m$
Where $\omega_s$ is the sampling frequency and $\omega_m$ is the maximum frequency contained in the signal.

**32.** The state-variable description of a linear autonomous system is $\dot{X} = AX$ where X is two dimensional state vector and A is a matrix given by $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$.
The poles of the system are located at
**[EE ESE - 2008]**
(a) –2 and +2 (b) –2j and + 2j
(c) –2 and –2 (d) +2 and +2

**33.** Given the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$$

the eigenvalues of A are
**[EE ESE - 2008]**
(a) –1 , –2, –3 (b) –1, 2, –3
(c) 0, 0, –6 (d) –6, –11, –6

**34.** A discrete – time system is stable if all the poles of the Z – transfer function of the system lie
**[EE ESE - 2008]**
(a) Outside the circle of unit radius on the Z – plane
(b) Within a circle of unit radius on the Z – plane
(c) To the left of imaginary axis on the Z – plane
(d) To the right of imaginary axis on the Z – plane

**35.** Match List-I (Properties) with List-II (Effect) and select the correct answer using the code given below the lists :
**List-I**
A.Non linear elements are sometimes intentionally introduced
B. Discrete data control system
C. Feedback can increase system gain
D. Sensitivity considerations are important
**List-II**
(i) Are susceptible to noise
(ii)In one frequency range
(iii)Physical properties may change with environment and ageing
(iv)To impose system stability
**[EE ESE - 2007]**
**Codes:**
(a) A-i, B-ii, C-iii, D-iv
(b) A-iv, B-i, C-ii, D-iii
(c) A-iv, B-i, C-iii, D-ii
(d) A-i, B-ii, C-iv, D-iii

**36.** Consider a system

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t); y = Cx(t)$$

where, $A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}; B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

Which of the statements given below in respect of above system is correct?
**[EE ESE - 2007]**
(a)System is controllable and observable
(b)System is controllable but no observable
(c)System is not controllable but observable
(d)System is not controllable and not observable

**37.** Match List-I with List-II and select the correct answer using the code given below the lists :
**List-I**
A. Relative stability
B. Eigen value
C. Difference equation
D. Corner frequency
**List-II**
(i) State model
(ii) G.M.
(iii) Bode plat
(iv) Sample-data system
**[EE ESE - 2007]**
**Codes:**
(a) A-i, B-ii, C-iii, D-iv
(b) A-i, B-ii, C-iv, D-iii
(c) A-ii, B-i, C-iii, D-iv
(d) A-ii, B-i, C-iv, D-iii

**38. Assertion (A):** The state transition matrix represents the free response of the system.

**Reason (R):** The state transition matrix satisfies the homogenous state equation.

**[EE ESE - 2007]**

(a)Both A and R are true and R is the correct explanation of A
(b)Both A and R are true but R is not the correct explanation of A
(c)A is true but R is false
(d)A is false but R is true

**39.** Which one of the following statements relating to phase plane techniques is not correct?

**[EE ESE - 2007]**

(a)They are general and applicable to system of any order.
(b)Steady - state accuracy and existence of limit cycle can be predicted.
(c)Amplitude and frequency of limit cycle, if exists can be predicted.
(d)It is applicable even to discontinuous time systems.

**40.** Match List-I (Evaluation of the Value of Function) with List-II (Corresponding z-transform expression) and select the correct answer using the code given below the lists:

**List-I**
A. Final value
B. Initial value

**List-II**
(i) $\lim_{z \to 0}(1-z^{-1})F(z)$
(ii) $\lim_{z \to 1}(1-z^{-1})F(z)$
(iii) $\lim_{z \to \infty} F(z)$
(iv) $\lim_{z \to \infty} zF(z)$

**[EE ESE - 2007]**

**Codes:**
(a) A-i, B-iii (b) A-i, B-iv
(c) A-ii, B-iii (d) A-ii, B-iv

**41.** The right hand plane of s-plane, when mapped into z-plane, when the direction of contour is anticlockwise

**[EE ESE - 2007]**

(a) Covers the entire portion of inside of the unit circle
(b) Covers the entire portion of outside of the unit circle
(c) It falls on the unit - circle
(d) It covers the entire portion except the unit circle

**42.** Compared to continuous time system, the discrete system is

**[EE ESE - 2007]**

(a) More accurate but less stable
(b) Less accurate but more stable
(c) More accurate and more stable
(d) Less accurate and less stable

**43.** Which one of the following statements is not related to limit cycles (phenomena) found in non – linear systems?

**[EE ESE - 2006]**

(a)They are oscillations of fixed amplitude and period.
(b)They are undesirable. However, they can be tolerated if magnitude is within desirable limit.
(c)They are independent of initial conditions.
(d)Sight change in parameter, destroys the oscillation.

**44.** Match List-I (Nature of Eigen value) with List-II (Nature of Singular Point) and select the correct answer using the codes given the below the lists:

**List-I**
A. Real, negative and distinct
B. Real, equal but opposite in sign
C. Purely imaginary pair
D. Complex conjugate pair

**List-II**
(i) Centre
(ii) Focus point
(iii) Saddle point
(iv) Stable node
(v) Unstable node

**[EE ESE - 2006]**

**Codes:**
(a) A-i, B-ii, C-v, D-iii
(b) A-iv, B-iii, C-i, D-ii

(c) A-i, B-iii, C-v, D-ii
(d) A-iv, B-ii, C-i, D-iii

**45.** Consider the following statements:
(i) For a linear discrete system to be stable, all the roots of the characteristic equation 1 + GH(z) = 0 should be inside the unit circle.
(ii) The Bode diagram of a sampled data system can be constructed using bilinear transformation.
(iii) The root locus technique can be used for sampled data system without requiring any modifications.
Which of the statements given above is/are correct?

**[EE ESE - 2006]**

(a) Only i (b) Only ii and iii
(c) Only i and iii (d) i, ii and iii

**46.** In order to recover the original signal from the sampled one, what is the condition to be satisfied for sampling frequency $\omega_s$ and highest frequency component $\omega_m$ ?

**[EE ESE - 2006]**

(a) $\omega_m < \omega_s \le 2\omega_m$ (b) $\omega_s \ge 2\omega_m$
(c) $\omega_s < \omega_m$ (d) $\omega_s = \omega_m$

**47.** Given $[\dot{x}] = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}[x] + \begin{bmatrix} 0 \\ k \end{bmatrix}u$

$y = x_1 + x_2$

$[x] = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

What is the transfer function y/x ?

**[EE ESE - 2006]**

(a) $\dfrac{k(s+2)}{s^3+2s^2+s+1}$ (b) $\dfrac{k(s+1)}{s^2+s+1}$
(c) $\dfrac{ks}{s^2+2s+1}$ (d) $\dfrac{k}{s^2+s+1}$

**48.** Match **List-I** with **List-II** and select the correct answer using the code given below the lists:
**List-I**
A. Non-linear system
B. Linear system
C. Time varying system
D. Multiplication in S-domain
**List-II**
(i) Principle of super position and Homogeneity
(ii) Describing – function
(iii) Convolution integral
(iv) Rocket

**[EE ESE - 2005]**

**Codes:**
(a) A-i, B-ii, C-iii, D-iv
(b) A-ii, B-i, C-iv, D-iii
(c) A-ii, B-i, C-iii, D-iv
(d) A-i, B-ii, C-iv, D-iii

**49.** The state equations of a system are given by

$$\dot{x} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

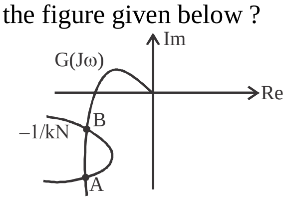
$y = [1 \quad 0 \quad 1]x$

The system is

**[EE ESE - 2005]**

(a) Controllable and observable
(b) Controllable but not completely observable
(c) Neither controllable nor completely observable
(d) Not completely controllable but observable

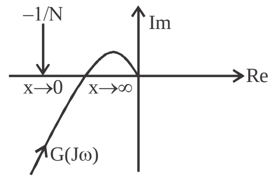
**50.** Which one of the following is correct in respect of the figure given below ?




**[EE ESE - 2005]**

(a) A and B are stable limit cycles
(b) A is stable limit cycle but B is unstable
(c) A is unstable limit cycle but B is stable
(d) Both A and B are unstable

**51.** A unity feedback non-linear control system's plot for-1 /N and G(jω) is shown in the diagram given below:




N is describing function of the non-linear device and G(s) is the transfer function of the linear plant. Which one of the following statements is correct?
The limit cycle is

**[EE ESE - 2005]**

(a) Stable (b) Unstable
(c) Critically stable (d) None of above

**52.** About which one of the following is the phase-plane portrait for the non-linear system given by $\ddot{x}+f(x,\dot{x})$ and satisfying $f(x,\dot{x})$ =, symmetrical?

**[EE ESE- 2004]**

(a) x-axis
(b) $\dot{x}$-axis
(c) Both the x and $\dot{x}$ - axis
(d) Neither x nor $\dot{x}$ - axis

**53.** Match List-I (Singular point) with List-II (Phase portrait) and select the correct answer using the codes given below:

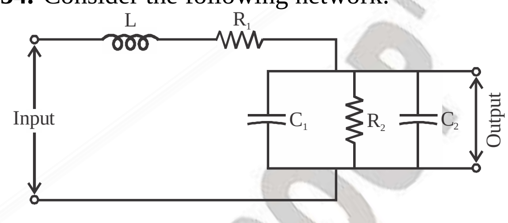
**List-I**
A. Unstable focus
B. Stable focus
C. Stable node
D. Saddle

**List -II**
(i)A logarithmic spiral extending into the singular point
(ii)Trajectories approach singular point adjacent to straight line curve out and leave in vicinity of singular points
(iii)A logarithmic spiral extending out of the singular point
(iv)Trajectories are asymptotic to straight line

**[EE ESE - 2004]**

**Codes:**
(a) A-iii, B-i, C-ii, D-iv
(b) A-i, B-ii, C-iii, D-iv
(c) A-iii, B-iv, C-i, D-ii
(d) A-i, B-iv, C-iii, D-ii

**54.** Consider the following network:




What is the minimum number of states of the network given above in order to determine the complete output of the network over all future time for a given input?

**[EE ESE - 2004]**

(a) 0 (b) 1
(c) 2 (d) 3

**55.** The transfer function of ZOH (Zero Order Hold) is

**[EE ESE - 2005]**

(a) $1-e^{Ts}$ (b) $1-e^{-Ts}$
(c) $\frac{1-e^{Ts}}{s}$ (d) $\frac{1-e^{-Ts}}{s}$

**56.** Which one of the following methods is NOT used for the analysis of nonlinear control systems?

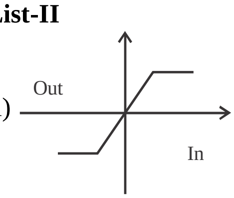
**[EE ESE - 2003]**

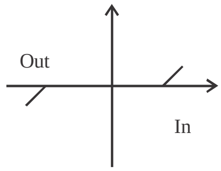
(a) Phase plane method
(b) Describing function method
(c) Liapunov's method
(d) Piecewise linear method

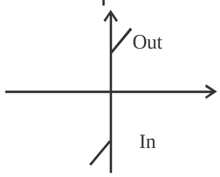
**57.** Match List-I (Nonlinearly) with List-II (Characteristics) and select the correct answer:

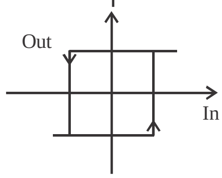
**List-I**
A. Saturation
B. Idealstiction and Coulomb friction
C. Dead Zone
D. Relay with hysteresis

**List-II**

(i) 

(ii) 


(iii) 

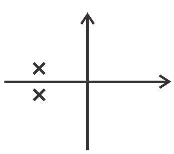

(iv) 

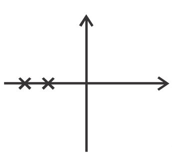

**[EE ESE - 2003]**

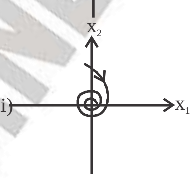
**Codes:**
(a) A-iii, B-i, C-ii, D-iv
(b) A-i, B-iii, C-ii, D-iv
(c) A-iii, B-i, C-iv, D-ii
(d) A-i, B-iii, C-iv, D-ii

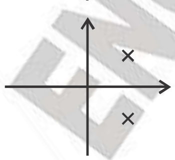
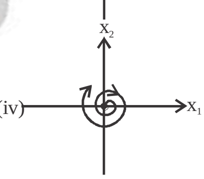
**58.** Match List-I (Root locations) with List-II (Phase-Plane Plots) and select the correct answer:

**List-I** **List-II**

(a)  (i) 


(b)  (ii) 


(c)  (iii) 


(d)  (iv) 


**[EE ESE - 2003]**

**Codes:**
(a) A-iii, B-ii, C-i, D-iv
(b) A-ii, B-iii, C-iv, D-i
(c) A-iii, B-ii, C-iv, D-i
(d) A-ii, B-iii, C-i, D-iv

**59.** The state–space representation of a system is given by

$$\dot{X} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} U \text{ and } Y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T X$$

Then the transfer function of the system is

**[EE ESE - 2003]**

(a) $\frac{1}{s^2+3s+2}$ (b) $\frac{1}{s+2}$
(c) $\frac{s}{s^2+3s+2}$ (d) $\frac{1}{s+1}$

**60.** Consider the following statements with respect to a system represented by its state-space model

$\dot{X} = AX + Bu$ and $Y = CX$

(i) The static vector X of the system is unique
(ii) The Eigen values of A are the poles of the system transfer function
(iii) The minimum number of state variables required is equal to the number of independent energy storage elements in the system
Which of these statements are correct?

**[EE ESE - 2003]**

(a) i and ii (b) ii and iii
(c) i and iii (d) i, ii and iii

**61. Assertion (A):** If any one of the state variables is independent of the control u(t), the process is said to be completely uncontrollable.
**Reason (R):** There is no way of driving this particular state variable to a desired state in finite time by means of a control effort.

**[EE ESE - 2002]**

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is NOT the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

**62.** Match List-I (Elements) with List-II (Digital Control) and select the correct answer:

**List-I**
A. Controller
B. Sampler
C. Hold

**List-II**
(i) A/D converter
(ii) Computer
(iii) D/A converter

**[EE ESE - 2002]**

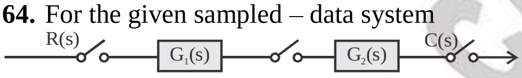
**Codes:**
(a) A-iii, B-i, C-ii
(b) A-ii, B-iii, C-i
(c) A-iii, B-ii, C-i
(d) A-ii, B-i, C-iii

**63.** The output of first order hold between two consecutive sampling instants is a

**[EE ESE - 2002]**

(a) Constant
(b) Quadratic function
(c) Ramp function
(d) Exponential function

**64.** For the given sampled – data system




The z – transform is

**[EE ESE - 2002]**

(a) R(z) ⟶ [$G_2G_1(Z)$] ⟶ C(z)
(b) R(z) ⟶ [$G_2(s)G_1(Z)$] ⟶ C(z)
(c) R(z) ⟶ [$G_2(z)G_1(Z)$] ⟶ C(z)
(d) $RG_1(z)$ ⟶ [$G_2(z)$] ⟶ C(z)

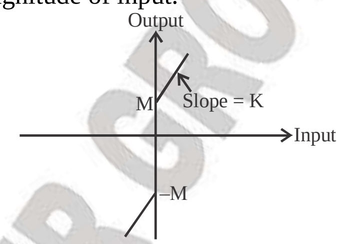
**65.** Consider the following statements:
(i) If the input is a sine wave of radian frequency ω, the output in general is non-sinusoidal containing frequencies which are multiple of ω.
(ii) The jump resonance may occur
(iii) The system exhibits self-sustained oscillation of fixed frequency and amplitude
(iv) The response to a particular test signal is a guide to the behavior to other inputs

Which of the above statements are correct and peculiar to nonlinear system?

**[EE ESE - 2002]**

(a) i, iii and iv
(b) ii, iii and iv
(c) i, ii and iii
(d) i, ii and iv

**66.** The describing function of relay nonlinearity is 4M/πX; M = Magnitude of relay. X = Magnitude of input.




The describing function of given nonlinearity will be

**[EE ESE - 2002]**

(a) $\frac{4MK}{\pi x}$
(b) $K+\frac{4M}{\pi x}$
(c) $\frac{4M\sqrt{1-K^2}}{\pi x}$
(d) $\frac{4M}{\pi K x}$

**67.** Let, $X=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}x+\begin{bmatrix}0\\1\end{bmatrix}U$

$U=[b,\ 0]x$

where b is an unknown constant.
This system is

**[EE ESE - 2002]**

(a) Observable for all values of b
(b) Unobservable for all values of b
(c) Observable for all non- zero values of b
(d) Unobservable for all non -zero values of b.

**68.** The state-space representation in phase-variable form for the transfer function

$$G(s)=\frac{2s+1}{s^2=7s+9}$$

**[EE ESE - 2002]**

(a) $\dot{x}=\begin{bmatrix}0 & 1\\-9 & -7\end{bmatrix}x+\begin{bmatrix}0\\1\end{bmatrix}u;\ y=[1\quad 2]x$

(b) $\dot{x} = \begin{bmatrix} 1 & 0 \\ -9 & -7 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; y = \begin{bmatrix} 0 & 1 \end{bmatrix} x$

(c) $\dot{x} = \begin{bmatrix} -9 & 0 \\ 0 & -7 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; y = \begin{bmatrix} 2 & 0 \end{bmatrix} x$

(d) $\dot{x} = \begin{bmatrix} 9 & -7 \\ 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; y = \begin{bmatrix} 1 & 2 \end{bmatrix} x$

**69.** A linear time invariant system is described by the following dynamic equation
d(x)(t)/dt=Ax(t) + Bu(t) y(t) = Cx(t)

where, $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$

The system is

**[EE ESE - 2002]**

(a) Both controllable and observable
(b) Controllable but unobservable
(c) Observable but uncontrollable
(d) Both uncontrollable and unobservable

**70.** A transfer function of a control system does not have pole- zero cancellation. Which one of the following statements is true?

**[EE ESE - 2002]**

(a) System is neither controllable nor observable
(b)System is completely controllable and observable
(c) System is observable but uncontrollable
(d) System is controllable but unobservable

**71.** The system matrix of a discrete system is given by

$A = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}$

The characteristic equation is given by

**[EE ESE - 2001]**

(a) $z^2 + 5z + 3 = 0$ (b) $z^2 - 3z - 5 = 0$
(c) $z^2 + 3z + 5 = 0$ (d) $z^2 + z + 2 = 0$

**72.** Match List-I (Nature of eigen value) with List-II (Nature of singular point) for linearised autonomous second order system and select the correct answer:

**List-I**
A. Complex conjugate pair
B. Pure imaginary pair
C. Real and equal but with opposite sign
D. Real, distinct and negative

**List-II**
(i) Centre
(ii) Focus point
(iii) Saddle point
(iv) Stable node
(v) Unstable node

**[EE ESE - 2001]**

**Codes:**
(a) A-i, B-v, C-iii, D-iv
(b) A-ii, B-i, C-iii, D-iv
(c) A-ii, B-i, C-iv, D-iii
(d) A-i, B-v, C-iv, D-iii

**73.** A particular control system is described by the following state equations:

$\dot{X} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$ and $Y = \begin{bmatrix} 2 & 0 \end{bmatrix} X$

The transfer function of this system is

**[EE ESE - 2001]**

(a) $\frac{Y(s)}{U(s)} = \frac{1}{2s^2 + 3s + 1}$

(b) $\frac{Y(s)}{U(s)} = \frac{2}{2s^2 + 3s + 1}$

(c) $\frac{Y(s)}{U(s)} = \frac{1}{s^2 + 3s + 2}$

(d) $\frac{Y(s)}{U(s)} = \frac{2}{s^2 + 3s + 2}$

**74.** Consider the single input, single output system with its state variable representation :

$\dot{X} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} U$

$Y = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} X$

The system is

**[EE ESE - 2001]**

(a) Neither controllable nor observable
(b) Controllable but not observable
(c) Uncontrollable but observable
(d) Both controllable and observable

**75.** For the system dynamics described by differential equation $\ddot{y}+3\dot{y}+2y = u(t)$ the transfer function of the system represented in controllable canonical from is $C[sI - A]^{-1}B$. The matrix A would be

**[EC ESE - 2001]**

(a) $\begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix}$

**76.** For the system described by $\dot{X} = AX$ match List-I (Matrix A) with List-II (Position of eignvalues) and select the correct answer:

**List-I**

A. $\begin{bmatrix} -1 & 2 \\ 0 & -2 \end{bmatrix}$

B. $\begin{bmatrix} -1 & -2 \\ -2 & -4 \end{bmatrix}$

C. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$

**List-II**

(i) One eigenvalue at the origin
(ii) Both the eigenvalues in the LHP
(iii) Both the eigenvalues in RHP
(iv) Both the eigenvalues on the imaginary axis.

**[EC ESE - 2001]**

**Codes**:

(a) A-ii, B-i, C-iii, D-iv
(b) A-ii, B-i, C-iv, C-iii
(c) A-i, B-ii, C-iv, D-iii
(d) A-i, B-ii, C-iii, D-iv

# SOLUTIONS

**Sol.1. (b)**
Given

$$[A]=\begin{bmatrix}0 & 1\\ -3 & -4\end{bmatrix},[B]=\begin{bmatrix}0\\1\end{bmatrix}\text{and}[C]=[10 \quad 0]$$

$$[s\,|-A]=\begin{bmatrix}s & -1\\ 3 & s+4\end{bmatrix}$$

$$[s\,|-A]=\frac{1}{s(s+4)+3}\begin{bmatrix}s+4 & 1\\ -3 & s\end{bmatrix}$$

$$[C][s\,|-A]^{-1}=\frac{1}{s^2+4s+3}[10 \quad 0]\begin{bmatrix}s+4 & 1\\ -3 & s\end{bmatrix}$$

$$=\frac{1}{s^2+4s+3}[10(s+4) \quad 10]$$

$$[C][S\,|-A]^{-1}[B]=\frac{1}{s^2+4s+3}[10(s+4) \quad 10]\begin{bmatrix}0\\1\end{bmatrix}$$

$$=\frac{10}{s^2+4s+3}$$

**Sol.2. (c)**

$$\dot{X}=\begin{bmatrix}0 & 1\\ 2 & -3\end{bmatrix}X+\begin{bmatrix}0\\1\end{bmatrix}U$$

Y = [1 1] X

$$Q_C=[A \quad AB]=\begin{bmatrix}0 & 1\\ 1 & -3\end{bmatrix}|Q_C|=-1\neq 0$$

For controllability $|Q_C| \neq 0$

$$Q_0=\begin{bmatrix}C\\ CA\end{bmatrix}=\begin{bmatrix}1 & 1\\ 2 & -2\end{bmatrix}$$

$|Q_0| = -4 \neq 0$
For observability $|Q_0| \neq 0$

**Sol.3. (b)**

$$\frac{Y(s)}{V(s)}=c(sI-A)^{-1}.B+D$$

$$\because \quad [sI-A]=\begin{bmatrix}s+3 & 1\\ 0 & s+2\end{bmatrix}$$

$$[sI-A]^{-1}=\frac{1}{(s+2)(s+3)}\begin{bmatrix}s+2 & 1\\ 0 & s+3\end{bmatrix}$$

$$\frac{Y(s)}{U(s)}=[1 \quad 0]\times\frac{1}{(s+2)(s+3)}\begin{bmatrix}s+2 & 1\\ 0 & s+3\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}$$

$$=[1 \quad 0]\times\frac{1}{(s+2)(s+3)}\begin{bmatrix}2s+5\\ s+3\end{bmatrix}$$

$$=\frac{2s+5}{(s+2)(s+3)}$$

**Sol.4. (c)**

$$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$$

$$\hat{X}=\begin{bmatrix}1\\2\\-1\end{bmatrix}$$

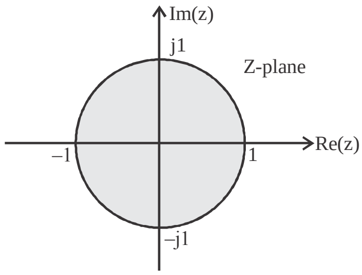
$$\nabla[A-\lambda I][\hat{X}]=0$$

$$\begin{bmatrix}-2-\lambda & 2 & -3\\ 2 & 1-\lambda & -6\\ -1 & -2 & -\lambda\end{bmatrix}\begin{bmatrix}1\\2\\-1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

After solving equation
$-1(2+\lambda)\times 1+4+3=0;\lambda=6$ …(i)
$2+2(1-\lambda)+6=0$
$\Rightarrow \lambda=5$ …(ii)
$-1-4+\lambda=0$
$\lambda=5$ …(iii)

**Sol.5. (b)**

**Sol.6. (b)**
A discrete time system is stable if all the roots of the characteristic equation lie within the circle of unit radius.




**Sol.7. (c)**
The state model of a system is not unique. But where as transfer function for the system is unique and state model can be derived from transfer function of the system.

**Sol.8. (a)**

**Sol.9. (b)**

**Sol.10. (a)**

$$A = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}$$

$$\therefore\ [sI - A] = \begin{bmatrix} s & -1 \\ 4 & s+5 \end{bmatrix}$$

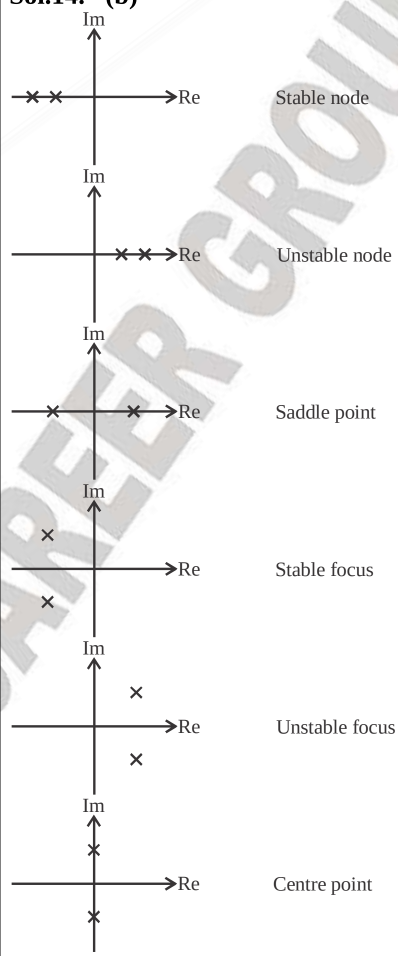
Characteristic equation
$|sI - A| = 0$
i.e. $s(s + 5) - (-1) \times 4 = 0$
$s^2 + 5s + 4 = 0$
$\therefore$ Eigen values are $-4$ and $-1$.

**Sol.11. (a)**

**Sol.12. (a, b, c)**
State transition matrix
$\phi(t) = e^{At}$
$\phi(t_1 - t_0)\ \phi(t_2 - t_0) = e^{A(t1 - t0)}\ e^{A(t2 - t0)}$
$= e^{A(t1 + t2 - 2t0)}$
$\neq e^{A(t1 - t2)}$
Therefore option (c) is not true.
$\phi(t_1 + t_2) = e^{At1}.e^{At2} = e^{A(t1 + t2)}$
$= \phi(t_1 + t_2)$
Therefore option (d) is true.
Relations given in the options (a) and (b) are also wrong because
1.$\phi$(0_ = I not $\phi(t) = I$
2.$\phi^{-1}(t) = \phi(-t)$

**Sol.13. (d)**

**Sol.14. (b)**




**Sol.15. (a)**

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 2 \\ 2 & s \end{bmatrix} = \frac{\begin{bmatrix} s & -2 \\ -2 & s \end{bmatrix}}{s^2 - 4}$$

Poles of the system $s = \pm 2$
Hence, option (a) is correct.

**Sol.16. (d)**

$Q_c = [B \quad AB]$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 2 \\ 0 & b \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 \\ 0 & b \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ b \end{bmatrix}$$

$$Q_C = \begin{bmatrix} 0 & 2 \\ 1 & b \end{bmatrix}$$

For controlling

$|Q_C| \neq 0$

$|Q_C| = 2 \neq 0$

**Sol.17. (a)**

If state variable equations are as follows:

$\dot{x} = AX + bu$ and

$y = cX + du$ then

System transformation function H(z) for a discrete LTI system is $= C(z1 - A)^{-1} b + d$

**Sol.18. (a, c)**

$\dot{q}(t) = Aq(t) + bx(t)$

$y(t) = cq(t) + dx(t)$

Taking Laplace transform of above equations

$sq(s) = Aq(s) + bx(s)$

$y(s) = cq(s) + dx(s)$

$q(s)[sl - A] = bx(s)$

$q(s) = [sl - A]^{-1} \times bx(s)$

$\therefore$ $y(s) = c[sl - A]^{-1} \times bx(s) + dx(s)$

$\Rightarrow y(s) = [c[sl - A]^{-1} \times b + d] \times x(s)$

$$\Rightarrow \frac{y(s)}{x(s)} = c[sl - A]^{-1} b + d$$

**Sol.19. (b)**

$$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

Poles of system = eigen values of [A]

$$[A - \lambda I] = \begin{bmatrix} -\lambda & 2 \\ -2 & -\lambda \end{bmatrix} = 0$$

$\lambda^2 + 4 = 0$

$\lambda = \pm 2j$

**Sol.20. (a)**

$$X(z) = \frac{(1 - e^{-T})Z^{-1}}{(1 - Z^{-1})(1 - e^{-T}Z^{-1})}$$

$$x(z) = \frac{Z(1 - e^{-T})}{(Z - 1)(Z - e^{-T})}$$

so, $\quad x(0) = 0$

**Sol.21. (b)**

**Sol.22. (a)**

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}$$

Characteristic equation $\Rightarrow [sI - A] = 0$

$$\begin{bmatrix} s & -1 \\ 3 & s + 5 \end{bmatrix} = 0$$

$s(s + 5) + 3 = 0$

$s^2 + 5s + 3 = 0$

**Sol.23. (d)**

**Sol.24. (b)**

**Sol.25. (a)**

**Sol.26. (a)**

$|s| - A| = 0$

$$\left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix} \right| = 0$$

$s^2 + 5s + 3 = 0$

**Sol.27. (a)**

**Sol.28. (a)**

$(s^4 + 5s^3 + 8s^2 + 6s + 3)\ Y(s) = u(s)$ …(i)

$X_1 = Y$ …(ii)

$X_2 = \dot{X}_1$ …(iii)

$X_3 = \dot{X}_2$ …(iv)

$X_4 = \dot{X}_3$ …(v)

So, transfer function equation can be written as

$\dot{X}_4 + 5X_4 + 8X_3 + 6X_2 + 3X_1 = U(s)$

$\dot{X}_4 = -3X_1 - 6X_2 - 8X_3 - 5X_4 + U(s)$ …(vi)

Writing above equations in matrix from,

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & -6 & -8 & -5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} U$$

**Sol.29. (c)**
It is a graphical method.

**Sol.30. (a)**
Hold circuit convert signal to analog form.

**Sol.31. (d)**
Sampling frequency should be ≥ 2 × highest frequency of input signal.
or $\omega_s \geq 2\ \omega_m$.

**Sol.32. (a)**
By solving $(\lambda I - A) = 0$

$$\begin{bmatrix} \lambda & -2 \\ -2 & \lambda \end{bmatrix} = 0 \Rightarrow \lambda^2 \pm 4$$

$\lambda = \pm 2$

**Sol.33. (a)**
By solving $(\lambda I - A)$

$$\begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 6 & 11 & \lambda+6 \end{bmatrix} = 0$$

$$\lambda \begin{bmatrix} \lambda & -1 \\ 11 & \lambda+6 \end{bmatrix} - 1 \begin{bmatrix} 0 & -1 \\ 6 & \lambda+6 \end{bmatrix} + 0 = 0$$

$$\lambda(\lambda^2 + 6\lambda + 11) - 6 = 0$$

$$\lambda^3 + 6\lambda^2 + 11\lambda - 6 = 0$$

$\Rightarrow \lambda = -1, -2, -3$

**Sol.34. (b)**

**Sol.35. (b)**
Discrete system are suspectable to noise. Sensitivity may change with environment and ageing.

**Sol.36. (b)**
For controllability
$Q_c = [B: AB: A^2B\ldots\ldots]$

$$AB = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -1 \end{bmatrix}$$

$$Q_C = \begin{bmatrix} 3 & -6 \\ 1 & -1 \end{bmatrix} = 3 \times (-1) - (-6) \times 1 \neq 0$$

Hence controllable

For observability $Q_0 = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$

$$CA = [1 \quad 0] \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} = [-2 \quad 0]$$

$$Q_0 = \begin{bmatrix} -2 & 0 \\ -2 & 0 \end{bmatrix} = 0$$

Hence not observable.

**Sol.37. (d)**
Gain margin is used for study of relative stability. Eigen value roots of system matrix (A) hence in state space model. Corner frequency is the frequency from where slope of Bode plot changes.

**Sol.38. (a)**

**Sol.39. (a)**
Phase plane technique is applicable to system upto second order.

**Sol.40. (c)**
As per definition of initial value and final value theorem.

**Sol.41. (b)**
Unit circle in z – plane represents left hand side of s – plane.

**Sol.42. (c)**

**Sol.43. (c)**

**Sol.44. (b)**

**Sol.45. (c)**
Bilinear transformation is used for Routh stability criteria.

**Sol.46. (b)**
It should satisfy nyquist criteria
$\omega_s \geq 2\omega_m$

**Sol.47. (b)**
$$\text{T.F.} = C(sI - A]^{-1}B \;\; \frac{y}{x} = \frac{k(s+1)}{s^2+s+1}$$

**Sol.48. (b)**

**Sol.49. (d)**

**Sol.50. (c)**
Refer stability analysis by describing function method.

**Sol.51. (b)**

**Sol.52. (b)**

**Sol.53. (a)**

**Sol.54. (c)**
Because both capacitors are in parallel hence simple addition, they act as single source.

**Sol.55. (d)**

**Sol.56. (c)**
Liapunov's method is used for stability analysis of LTI control system. Piecewise linear method is also used for general investigation of non – linear system in addition to phase – plane and describing function method.

**Sol.57. (b)**

**Sol.58. (a)**
Refer phase – trajectory (s) (phase – portrait).

**Sol.59. (d)**
$$T(s) = C[sI - A]^{-1}B$$
$$= \begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}(s+1) & 0 \\ 0 & (s+2)\end{bmatrix}^{-1}\begin{bmatrix}1 \\ 0\end{bmatrix}$$
$$= \begin{bmatrix}1 & 1\end{bmatrix} \times \frac{1}{(s+1)(s+2)}\begin{bmatrix}s+2 & 0 \\ 0 & s+1\end{bmatrix}\begin{bmatrix}1 \\ 0\end{bmatrix}$$
$$= \frac{1}{(s+1)}$$

**Sol.60. (b)**
The state vector X of the system is never unique. It is a characteristic of the state space representation. Eigenvalues of A are given by the equation |s ||A | = 0 which is the characteristic equation of the system. i.e. denominator of the transfer function.

**Sol.61. (a)**
Refer, the definition of state controllability

**Sol.62. (d)**

**Sol.63. (c)**
In a first – order hold, the last two signal samples are used to reconstruct the signal for the current sampling period.

**Sol.64. (a)**

**Sol.65. (c)**
Refer the peculiar characteristics shown by a non – linear system.

**Sol.66. (b)**
For ideal relay $K_N(x) = \dfrac{4M}{\pi X}(K=0)$

**Sol.67. (c)**
$$\left[C^T : A^T C^T\right] = \begin{bmatrix}b & b \\ 0 & 2b\end{bmatrix} = 2b^2 \neq 0 \text{ for } b \neq 0$$

**Sol.68. (a)**
T.F. = $C(sI - A)^{-1} B$
We check for option (a)
$$\text{T.F.} = \begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}s & -1 \\ 9 & s+7\end{bmatrix}^{-1}\begin{bmatrix}0 \\ 1\end{bmatrix}$$
$$= \begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}\frac{s+7}{s^2+7s+9} & \frac{1}{s^2+7s+9} \\ \frac{-9}{s^2+7s+9} & \frac{s}{s^2+7s+9}\end{bmatrix}\begin{bmatrix}0 \\ 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}\frac{1}{s^2+7s+9} \\ \frac{s}{s^2+7s+9}\end{bmatrix}$$

$$\text{T.F.}=\frac{2s+1}{s^2+7s+9}$$

**Sol.69. (a)**
To check for controllable

$$F=\begin{bmatrix}B & AB\end{bmatrix}=\begin{bmatrix}0 & 1 \\ 1 & -3\end{bmatrix}\neq 0$$

$\therefore$ system is controllable
For observable

$$F=\begin{bmatrix}C^T & A^TC^T\end{bmatrix}=\begin{bmatrix}1 & -2 \\ 1 & -2\end{bmatrix}=0$$

$\therefore$ System is unobservable

**Sol.70. (b)**
If the input – output transfer function of a linear time-invariant system has pole-zero cancellation, the system will be either not state controllable or observable depending on how, the state variables are defined. If the transfer function does not have pole-zero cancellation the system can always be represented by completely controllable and observable state model.

**Sol.71. (a)**

$$\begin{vmatrix}z & -1 \\ 3 & (z+5)\end{vmatrix}=0$$ is the characteristic equation.

**Sol.72. (b)**
Refer singular points under non – linear systems.

**Sol.73. (d)**
$G(s) = C(sI – A)^{-1} B$

**Sol.74. (a)**

**Sol.75. (c)**
General representation of phase variable representation:

$$\dot{X}=AX+BU \quad Y=CX+DU$$

Where

$$A=\begin{bmatrix}0 & 1 & 0 & 0 & ....0 \\ 0 & 0 & 1 & 0 & ....0 \\ 0 & 0 & 0 & 1 & ....0 \\ \vdots & \vdots & \vdots & & \\ -a_n & -a_{n-1} & -a_{n-2} & .... & ....0\end{bmatrix}$$

$$B=\begin{bmatrix}0 \\ 0 \\ \vdots \\ b\end{bmatrix}$$

C = [1 0 0 ….. 0]
D = [0]
Differential equation is

$$\frac{d^ny}{dt^n}+a_1\frac{d^{n-1}y}{dt^{n-1}}+a_2\frac{d^{n-2}y}{dt^{n-2}}+.....+a_ny=bx$$

Comparing the given equation

$$\ddot{y}+3\dot{y}+2y=u(t)$$

With $\ddot{y}+a_1\dot{y}+a_2y=bx$

$$A=\begin{bmatrix}0 & 1 \\ -a_2 & -a_1\end{bmatrix}\Rightarrow A=\begin{bmatrix}0 & 1 \\ -2 & -3\end{bmatrix}$$

**Sol.76. (b)**
Eigen values are the roots of |sl – A| = 0

Let $A=\begin{bmatrix}-1 & 2 \\ 0 & -2\end{bmatrix}$

$$sl-A=\begin{bmatrix}s+1 & -2 \\ 0 & s+2\end{bmatrix}$$

|sl – A| = (s + 1) (s + 2)
|sl – A| = 0
⇒ (s + 1) (s + 2) = 0
⇒ s = –1, –2
Thus both the eigen values are in the LHP.

Let $A=\begin{bmatrix}-1 & -2 \\ -2 & -4\end{bmatrix}$

$$sl-A=\begin{bmatrix}s+1 & 2 \\ 2 & s+2\end{bmatrix}$$

|sl – A| = 0
⇒ (s + 1) (s + 4) – 4 = 0

$\Rightarrow s^2 + 5s + 4 - 4 = 0$
$\Rightarrow s(s + 5) = 0$
$\Rightarrow s = 0, -5$
Thus one eigen value is at the origin.

Let $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$sI - A = \begin{bmatrix} s & 1 \\ -1 & s \end{bmatrix}$

$|sI - A| = 0$
$\Rightarrow s^2 + 1 = 0$
$\Rightarrow s = \pm j1$

Thus both the eigen values are on the imaginary axis.

Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$

$sI - A = \begin{bmatrix} s-1 & 0 \\ -2 & s-4 \end{bmatrix}$

$|sI - A| = 0$
$\Rightarrow (s - 1)(s - 4) = 0$
$\Rightarrow s = 1, 4$
Thus both the eigen values are in the RHP.