GATE

2019

LINEAR CONTROL SYSTEM

ELECTRICAL ENGINEERING

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E-Mail: ecgpublica ons@gmail.com | info@engineerscareergroup.in

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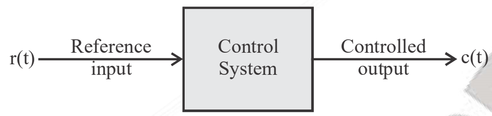
CONTENTS

CHAPTER - 1
INTRODUCTION TO CONTROL SYSTEM

1.1 INTRODUCTION
A control System is a combination of elements arranged in a planned manner where in each element causes an effect to produce a desired output. This cause and effect relationship is governed by a mathematical relation.



If the input of system is controlled in desired manner, the system is called control system.
Any system can be characterized mathematically by
1. Transfer function 2. State model

$$\text{Transfer function} = \frac{\text{L.T.of output}}{\text{L.T.of input}} = \frac{L\left[c(t)\right]}{\left[c(s)\right]} = \frac{C(s)}{R(s)}\bigg|_{\text{initial conditions}=0}$$

Transfer function is also called impulse response of the system.

1. Disturbances
The signal that has some adverse effect on output of system called disturbances if it is generated inside called internal distributes if it is other called out external disturbances.

2. Plant
It is defined as the portion of system when is to be controlled it is also called process.

3. System
A system is an arrangement or component such that it gives proper output to given input e.g. classroom example of physical system.

4. Control System
It is an arrangement of different physical component such that it gives the desired output for the given input by means of regulate or control either direct or indirect.

5. Controllers
It is the element of system it say, may be external to system it controls the plant or process.

6. Performance Specifications
Control system are designed to perform specific task. The requirement imposed on control system are usually spelled out as performance specifications. These specifications may be given transact response requirement maximum overshoot settling time is step response.
1. Steady state requirement (steady state error) or may be given in terms of frequency response.
2. Specification of the control system must be given before the design process begins.
3. Most important part of control system design is to sate the performance specification precisely so that they will yield on optional control system for the given purpose.

Mathematical modeling of control system regular must be able to model dynamic system in mathematical terms and analyse their dynamic characteristics.

GATE QUESTIONS

1. The Nyquist stability criterion and the Routh criterion both are powerful analysis tools for determining the stability of feedback controllers. Identify which of the following statements is FALSE:

[GATE - 2018]

(a) Both the criteria provide information relative to the stable gain range of the system.

(b) The general shape of the Nyquist plot is readily obtained from the Bode magnitude plot for all minimum-phase systems.

(c) The Routh criterion is not applicable in the condition of transport lag, which can be readily handled by the Nyquist criterion

(d) The closed-loop frequency response for a unity feedback system cannot be obtained from the Nyquist plot.

2. A system is described by the following differential equation:

$$\frac{dy(t)}{dt}+2y(t)=\frac{dx(t)}{dt}+x(t),\quad x(0) = y(0) = 0$$

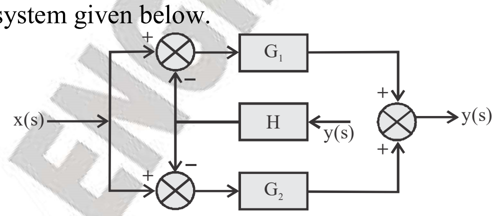
where x(t) and y(t) are the input and output variables respectively. The transfer function of the inverse system is

[GATE - 2017]

(a) $\frac{s+1}{s-2}$ (b) $\frac{s+2}{s+1}$

(c) $\frac{s+1}{s+2}$ (d) $\frac{s-1}{s-2}$

3. Find the transfer function $\frac{Y(s)}{X(s)}$ of the system given below.



[GATE - 2015]

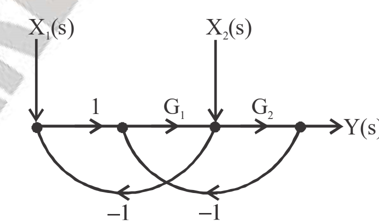
(a) $\frac{G_1}{1+HG_1}+\frac{G_2}{1-HG_2}$

(b) $\frac{G_1}{1+HG_1}+\frac{G_2}{1+HG_2}$

(c) $\frac{G_1+G_2}{1+H(G_1+G_2)}$

(d) $\frac{G_1+G_2}{1-H(G_1+G_2)}$

4. For the signal - flow graph shown in the following expressions is equal to the transfer function $\left.\frac{Y(s)}{X_2(s)}\right|_{X_1(s)=0}$?



[GATE - 2015]

(a) $\frac{G_1}{1+G_2(1+G_1)}$ (b) $\frac{G_2}{1+G_1(1+G_2)}$

(c) $\frac{G_1}{1+G_1G_2}$ (d) $\frac{G_2}{1+G_1G_2}$

5. The impulse response g(t) of a system, G, is a shown in Figure (a). What is the maximum value attained by the impulse response of two cascaded blocks of G as shown in Figure (b)?

ESE OBJ QUESTIONS

1. The open – loop transfer function of a system is $\frac{10K}{1+10s}$

When the system is converted into a closed – loop with unity feedback, the time constant of the system is reduced by a factor of 20. The value of K is

[EE ESE - 2018]

(a) 1.9 (b) 1.6
(c) 1.3 (d) 1.0

2. The effects of feedback on stability and sensitivity are

[EC ESE - 2015]

(a)Negative feedback improves stability and system response is less sensitive to external inputs and parameter variations.
(b)Feedback does not affect stability but system response is sensitive to disturbances and parameter variations.
(c)Feedback does not affect stability response is sensitive to disturbances and parameter variations
(d)Negative feedback affects stability and system response is more sensitive disturbances and parameter variations.

3. The D.C. gain and steady state error for step input for G (s) = $\frac{s+1}{s^2+s+1}$ are:

[EC ESE - 2013]

(a) 1 and 1 (b) 0 and 1
(c) 1 and 0 (d) 0 and 0

4. In control systems, excessive bandwidth is NOT employed because:

[EC ESE - 2013]

(a) Noise is proportional to bandwidth
(b) It leads to low relative stability
(c) It leads to slower response
(d) Noise is proportional to the square of the bandwidth

SOLUTIONS

Sol.1. (a)

$$\text{OLTF} = \frac{10k}{1+10s}$$

$Z_1 = 10$

$$Z_2 = \frac{10}{20} = 0.5$$

$$\text{CLTF} = \frac{10k}{10k+1+10s}$$

$$Z_2 = \frac{10}{10k+1} = 0.5$$

$$\frac{10}{0.5} = 10k+1$$

k = 1.9

Sol.2. (a)

Sol.3. (c)

$$G(s) = \frac{s+1}{s^2+s+1}$$

$$G(s) = \Big|_{s=0} = \frac{0+1}{0+0+1} = 1$$

$$e_{ss} = \text{Steady State Error} = \frac{1}{1+k_p}$$

$$k_p = \lim_{s\to 0} G(s)H(s) = 0$$

Sol.4. (a)

Noise Power = $\eta_0\beta$

Noise Power × Bandwidth × B

CHAPTER - 2
MATHEMATICAL MODES OF PHYSICAL SYSTEMS

2.1 INTRODUCTION
1.A physical system is collection of physical objects connected together to serve an objective.
2.Idealizing assumptions are always made for the purpose of analysis and synthesis of systems. An idealized physical system is called a physical mode.
3.Once a physical model of a physical system is obtained, the next step is to obtain a mathematical model which is the mathematical representation of the physical model through use of appropriate physical laws.

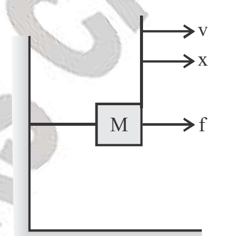
2.2 MECHANICAL SYSTEMS
A mechanical system which is modeled using the three ideal elements would yield a mathematical model which is an ordinary differential equation. All mechanical systems are divided into two parts:

2.2.1 Mechanical Translational System
In this type of mechanical system input is the forced (F) and the output is linear displacement (x) or linear velocity (v). The three ideal elements are:

1. Mass Element

$$F = M\frac{d^2x}{dt^2}$$

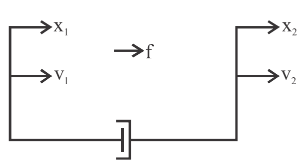
or $F = M\frac{dv}{dt}$



2. Damper Element

$$F = f\frac{d}{dt}(x_1 - x_2) = f\frac{dx}{dt}$$

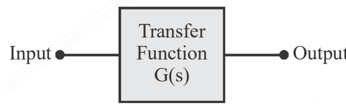
where $x_1 - x_2$ x
or $F = f(v_1 - v_2) = fv$
where $v = v_1 - v_2$



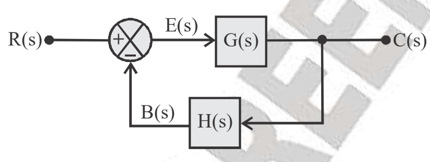
3. Spring Element
$F = K(x_1 - x_{2=}) = Kx$

CHAPTER - 3
BLOCK DIAGRAM ALGEBRA

3.1 BLOCK DIAGRAM
It is a pictorial representation of function performed by each component and of flow of signals. Such a diagram depicts the inter-relationship that exists among various components differing from a purely abstract mathematical model. The block diagram has the advantage of indicating more realistically the signal flows of actual system.

3.2 ELEMENT OF BLOCK DIAGRAM


This represents the elements of a block diagram. The arrow heads pointing towards the block diagram indicate the input and the arrowheads leaving the block represent output. Such arrows are represented to as signal.



Open Loop Transfer Function

1. Open Loop Transfer Function

B(s) = C(s) H(s)

$$\frac{B(s)}{E(s)} = G(s)H(s)$$

2. Feed Forward Transfer Function

$$\frac{C(s)}{E(s)} = G(s)$$

If H(s) = 0 then

G(s) = G(s) H(s) $\therefore$ H(s) = 1

3. Closed Loop Transfer Function

C(s) = G(s) E(s)

E(s) = R(s) – B(s)

E(s) = R(s) – H(s) C(s)

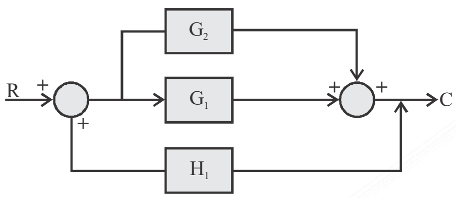
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

4. Branch Point

A branch point is a point from which the signal from the block goes concurrently to other block or summing points.

ASSIGNMENT

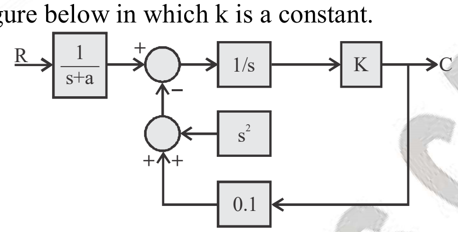
1. Determine C/R from the system shown in figure below:



(a) $\dfrac{G_1 + G_2}{1 - G_1H_1 + G_2H_1}$ (b) $\dfrac{G_1}{1 - G_1H_1}$

(c) $\dfrac{G_1 + G_2}{1 - G_1H_1 - G_2H_1}$ (d) None

2. Find the transfer C/R for the system shown in figure below in which k is a constant.



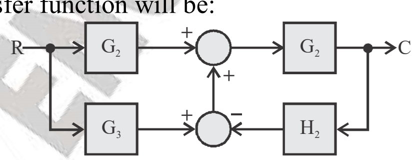
(a) $\dfrac{1}{(s+a)(s^2+s+0.1k)}$

(b) $\dfrac{k}{(s+a)(s^2+s+0.1k)}$

(c) $\dfrac{k}{(s+a)(s^2+s-0.1k)}$

(d) None of these

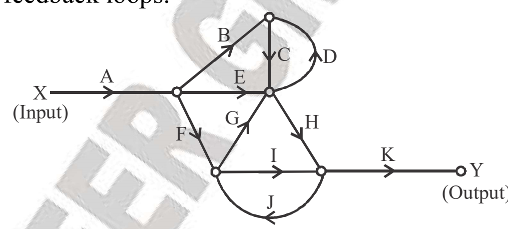
3. Determine C/R for the system given in figure below. Then put $G_3 = G_1\ G_2\ H_2$. Now the new transfer function will be:



(a) $\dfrac{G_1G_2 + G_2G_3}{1 + G_2H_2}$ (b) $\dfrac{G_1G_2 + G_2G_3}{1 - G_2H_2}$

(c) $G_1\ G_3$ (d) $G_1\ G_2$

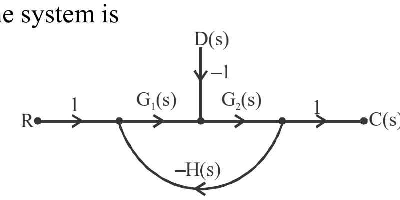
4.The signal flow graph of figure shown below has _______ forward paths and _______ feedback loops.



(a) (4, 4) (b) (4, 5)

(c) (4, 3) (d) (3, 3)

5. The signal flow graph of the system is shown in the given figure. The transfer function $\dfrac{C(s)}{D(s)}$ of the system is



(a) $\dfrac{G_1(s)G_2(s)}{1+G_1(s)H(s)}$

(b) $\dfrac{G_1(s)G_2(s)}{1-G_1(s)G_2(s)H(s)}$

(c) $\dfrac{G_2(s)}{1+G_1(s)G_2(s)H(s)}$

(d) $\dfrac{-G_2(s)}{1-G_1(s)G_2(s)H(s)}$

6. In the signal flow graph of figure y/x equal.

CHAPTER - 4
TIME RESPONSE ANALYSIS OF CONTROL SYSTEM

4.1 INTRODUCTION

4.1.1 Types of System
(No. of open loop poles of the system at origin)
Example.

(i) $G(s)=\frac{K}{(s+1)(s+2)}$, No pole at origin. So it is type 0.

(ii) $G(s)=\frac{K}{s(s+1)(s+1)}$, 1 pole at origin. So type 1.

(iii) $G(s)=\frac{K}{s^2(s+1)(s+2)}$, 2 poles at origin. So type 2

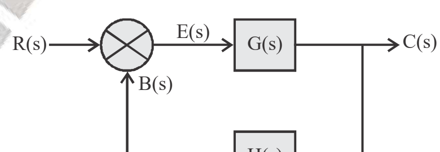
Order is the highest coefficient of s in the denominator of closed loop transfer function.

Example. Consider a unity feedback system whose open loop transfer function is
$G(s)=\frac{K}{(s+1)(s+2)}$ What is the type and order of the system?

Solution.
The closed loop transfer function is
$=\frac{K}{s^2+3s+2}$
So it is a type 0 and order 2 system.

4.2 ERROR ANALYSIS

4.2.1 Steady State Error
A desirable feature of a control system is the faithful following of its input by the output. However, if the actual output of a control system during steady state deviates from the reference input (i.e. desired output, the system is said to possess a steady state error.
As the steady state performance of a control system is assessed by the magnitude of the steady state error possessed by the system and the system input specified as either step or ramp or parabolic.



The magnitude of the steady state error in a closed-loop control system depends on its open-loop transfer function, i.e. G(s) H(s) of the system. The classification of open loop transfer function of a control system is explained below:
GH is loop transfer function
G is open loop transfer function

SOLUTIONS

Sol. 1. (a)

$$e_{ss} = \frac{1}{1+kp}$$

$$1+K_p = \frac{1}{0.2}$$

$k_p = 4$

$$k_p = \lim_{s\to 0} GCS H(s) = 4$$

The error due to step i/p is made to zero so type of system would have increased

$$G(s) = \frac{G(S)H(S)}{S}, \; K_v = \lim_{s\to 0} \;\&\; \frac{GCSH(S)}{s} = 4$$

$$k_v = \frac{1}{4} = 0.25$$

Sol. 2. (b)

$$\text{CE.} \;\; 1+\frac{25}{s(s+6)} = 0$$

$$s^2 + 65 + 25 = 0$$

$\omega n = 5$

$$\xi = \frac{6}{2\times 5} = 0.6$$

Setting time

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{5\times 0.6} = 1.33\,\text{sec}$$

Sol. 3. (d)

$$\text{CE.}1+\frac{k}{s(s+a)} = 0$$

$S^2 + as + k = 0$

$2\xi\omega n = a$

$$\omega_n = \sqrt{k}$$

$$\xi = \frac{a}{2\sqrt{k}}$$

For undreamed system

$\xi < 1$

$$\frac{a}{2\sqrt{k}} < 0 \quad k > \frac{a^2}{4}$$

$$\sqrt{k} > \frac{a}{2}$$

Sol. 4. (b)

Settling time is defined as the time for the response to react and stay within 2% of its final value.

Sol. 5. (a)

$$k_P = \lim_{s\to 0} G(s)$$

$$= k_P = \lim_{s\to 0} \frac{k(s+2)}{s^2(s^2+75+12)} = \infty$$

$$k_v = \lim_{s\to 0} s.G(s)$$

$$= \lim_{s\to 0} \frac{k(s+2)}{s(s^2+75+12)} = \infty$$

$$K.G = \lim_{s\to 0} s^2 G(s)$$

$$= \lim_{s\to 0} \frac{k(s+2)}{s^2+75+12} = \frac{2k}{12} = \frac{k}{6}$$

Sol. 6. (c)

For open loop T.F.

Poles are lies at s = 0, 0, –2

Hence repeated poles at origin unstable

For close loop system

$$1+\frac{k(s+1)}{s^2(s+2)} = 0$$

$S^3 + 2s^2 + ks + k = 0$

S^3	1	k	
S^2	2	k	
S^1	$\frac{2k-k}{2}$		
S^0	k		k>0

So for k > 0 close loop system is stable.

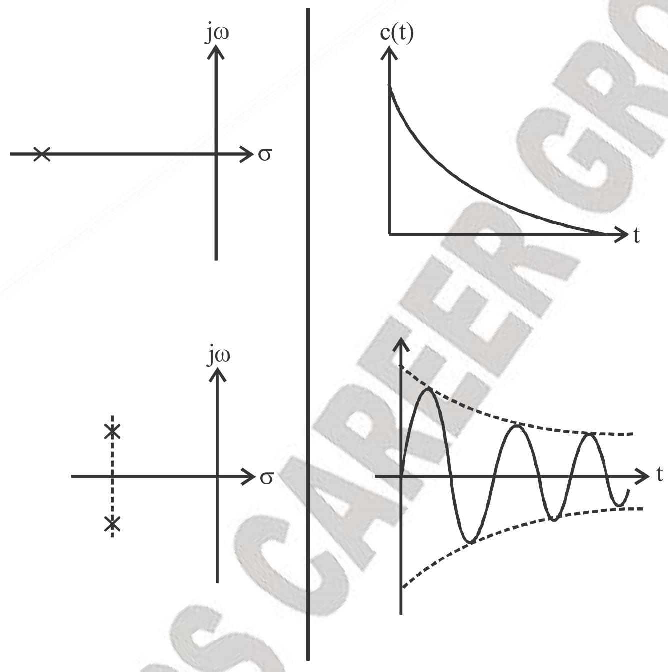
Sol. 7. (b)

Derivative compensation is phase lead compensation so damping factor (δ) increases ω_n (natural frequency) remains unchanged.

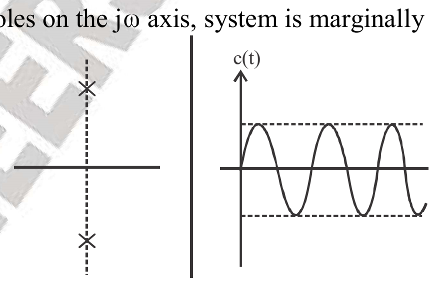
CHAPTER - 5
STABILITY ANALYSIS OF CONTROL SYSTEM

5.1 INTRODUCTION
If all the poles of the system lie in the left half of s plane, then the system is stable.

Location of poles (roots) **Response**



Case-I.
If there are non – repeated poles on the jω axis, system is marginally stable.



Case-II.
If there are repeated poles of the system on jω axis, system is unstable.

CHAPTER - 6
ROOT LOCUS

6.1 INTRODUCTION

The Routh's criterion gives a satisfactory answer to the question of stability but its adoption to determine the relative stability is not satisfactory and requires trial and error procedure even in the analysis problem.

A simple technique, known as the root locus technique, for finding the roots of the characteristic equation, introduced by W.R. Evans, is extensively used in control engineering practice. This technique provides a graphical method of plotting the locus of the roots in the s-plane as a given system parameter is varied over the complete range of values (may be from zero to infinity). The roots corresponding to a particular value of the system parameter can then be located on the locus or the value of the parameter for a desired root location can be determined from the locus.

6.2 RULES OF DRAWING THE ROOT LOCUS

1. Root locus start from open loop poles and ends on open loop zeros or ∞ with $K = \infty$

Let no. of poles = n (open loop poles)

No. of open loop zeros = m

(i) No. of root loci ending on $\infty = n - m,\ n > m$

2. Root locus is always symmetrical about real axis.

3. A point on real axis lies on the root locus if no. of poles + zeros to the right of the point are odd.

4. Asymptotes are the paths along which root locus moves towards ∞.

(i) No. of asymptotes = $(n - m)$

(ii) Angle of asymptotes

$$\theta_A = \frac{(2x+1)180^\circ}{n-m}$$

$x = 0, 1, 2, \ldots\ n - m - 1$

(iii) Centroid : It is the point of intersection of asymptotes with the real axis.

$$\sigma_A = \frac{\sum(\text{real part of poles}) - \sum(\text{real part of zeros})}{n-m}$$

5. Determination of Breakaway or break in point : On the root locus between two adjacent poles the two poles move towards each other with K=0 and move at a point where K is maximum and the root locus will break away into two parts. This point is called the breakaway point and it is determined by:

Put $\left(\frac{dK}{ds} = 0\right)$ and find out the value of 's'

6. Angle of departure or Angle of arrival

angle made by root locus with real axis when it departs from a complex open loop poles is called angle of departure.

$$\begin{Bmatrix} \phi_D\ (\text{angle of departure}) = 180^\circ + \angle GH' \\ \phi_A\ (\text{angle of arrival} = 180^\circ - \angle GH') \end{Bmatrix}$$

GH' is value of function excluding the concerned poles at the poles itself

GATE QUESTIONS

1. The range of K for which all the roots of the equation $s^3 + 3s^2 + 2s + K = 0$ are in the left half of the complex s-plane is

[GATE - 2017]

(a) $0 < K < 6$ (b) $0 < K < 16$
(c) $6 < K < 36$ (d) $6 < K < 16$

2. The root locus of the feedback control system having the characteristic equation $s^2 + 6Ks + 2s + 5 = 0$ where $K > 0$, enters into the real axis at

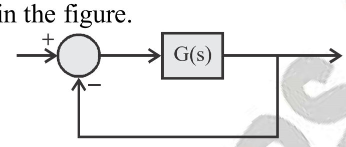
[GATE - 2017]

(a) $s = -1$ (b) $s = -\sqrt{5}$
(c) $s = -5$ (d) $s = \sqrt{5}$

3. A linear time invariant (LTI) system with the transfer function

$$G(s) = \frac{K(s^2 + 2s + 2)}{(s^2 - 3s + 2)}$$

Is connected in unity feedback configuration as shown in the figure.



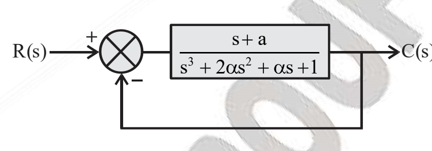
For the closed loop system shown, the root locus for $0 < K < \infty$ intersects the imaginary axis for K = 1.5. the closed loop system is stable for

[GATE - 2017]

(a) $K > 1.5$
(b) $1 < K < 1.5$
(c) $0 < K < 1$
(d) No positive value of K

4. A closed-loop system is shown in the figure. The system parameter α is not known. The condition for asymptotic stability of the closed loop system is

[GATE - 2017]



(a) $\alpha < -0.5$ (b) $-0.5 < \alpha < 0.5$
(c) $0 < \alpha < 0.5$ (d) $\alpha > 0.5$

5. The gain at the breakaway point of the root locus of a unity feedback system with open loop transfer function $G(s) = \frac{Ks}{(s+1)(s-4)}$ is

[GATE - 2016]

(a) 1 (b) 2
(c) 5 (d) 9

6. The forward-path transfer function and the feedback-path transfer function of a single loop negative feedback control system are given as

$$G(s) = \frac{K(s+2)}{s^2 + 2s + 2}$$

and H(s) = 1 respectively. If the variable parameter K is real positive, then the location of the breakaway point on the root locus diagram of the system is ______.

[GATE - 2016]

7. The open-loop transfer function of a unity-feedback control system is

$$G(s) = \frac{K}{s^2 + 5s + 5}$$

The value of K at the breakaway point of the feedback control system's root-locus plot is ________.

[GATE - 2016]

8. The open loop poles of a third order unity feedback system are at 0, −1. −2. Let the frequency corresponding to the point where the root locus of the system transits to unstable

CHAPTER - 7
CONTROLLERS

7.1 INTRODUCTION

While designing a system, the designer selects the reasonable values for the peak overshoot, rise time and the settling time. The designer is never sure of the final design of the system as to whether it is good or not. For example, if the system has been designed for minimum overshoot, the rise time increases and on the other hand if the rise time chosen is small, peak overshoot will be large. A system thus requires modification in order to meet even two independent specifications. This is called compensation and is achieved by the help of proportional, derivative or integral or derivative feedback control. In practice a combination of derivative and integral control is employed.

Let us consider a system whose block diagram is shown in Figure. It has a controller whose output signal will have an effect on the system performance. Its purpose is to measure the error between the output and the desired output.

The transfer function of the controller is

$$K = \frac{Y(s)}{E(s)}$$

Where

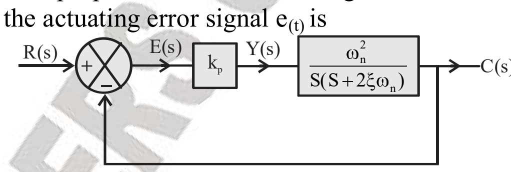
E(s) = R(s) – B(s)

or E(s) = R(s) – H(s)C(s)

this relationship is termed as control action relationship. We will now discuss various control actions as available to the control system engineer for improvement of system performance.

7.2 PROPORTIONAL CONTROL ACTION

In this the actuating signal is proportional to the error signal. The relationship between the output of the controller, $y_{(t)}$ and the actuating error signal $e_{(t)}$ is



$y_{(t)} = Ke_{(t)}$

In Laplace-transform form, it can be written as

$Y_{(s)} = KE_{(s)}$

Or $K_p = \frac{Y(s)}{E(s)}$

7.3 INTEGRAL CONTROL ACTION

In this value of the controller output $y_{(t)}$ is altered at a rate proportional to the error signal $e_{(t)}$. The output $y_{(t)}$. The output $y_{(t)}$ depends upon the integral of the error signed $e_{(t)}$.

ASSIGNMENT

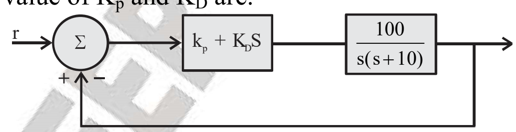
1. Consider the following statements:
1. A Proportional plus derivative controller.
2. Increase the stability of the system
3. Improves the steady-state accuracy
Which of these statements are correct?
(a) 1, 2 and 3 (b) 1 and 2
(c) 2 and 3 (d) 1 and 3

2. The transfer function of simple RC network as a controller is $G_C(s) = \frac{s+z_1}{s+p_1}$. The condition for the RC network to act as a phase lead controller is
(a) $p_1 < z_1$ (b) $p_1 = 0$
(c) $p_1 = z_1$ (d) $p_1 > z_1$

3. The industrial controller having the best steady
state accuracy is
(a) A derivative controller
(b) An integral controller
(c) A rate feed back controller
(d) A proportional controller

4. The transfer function of a phase lead controller is $\frac{1+3Ts}{1+Ts}$. The maximum value of phase provided by this controller is
(a) 90° (b) 60°
(c) 45° (d) 30°

5. A control system with a PD controller is shown in fig. if the velocity error constant K_v = 1000 and the damping ratio ζ = 0.5, then the value of K_p and K_D are.



(a) $K_P = 100, K_D = 0.09$
(b) $K_p = 100, K_D = 0.9$
(c) $K_p = 10, K_D = 0.09$
(d) $K_p = 10, K_D = 0.9$

6. A controller transfer function is given by C(s) = (2s + 1)/(09.2s + 1). What is its nature and parameter?
(a) Lag controller, $\alpha = 10$
(b) Lag controller, $\alpha = 2$
(c) Lead controller, $\beta = 0.1$
(d) Lead controller, $\beta = 0.2$

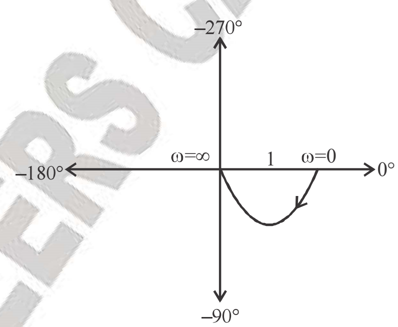
ANSWER KEY

1.	b	**2.**	d	**3.**	b	**4.**	d	**5.**	b	**6.**	c

CHAPTER - 8
FREQUENCY RESPONSE ANALYSIS

8.1 INTRODUCTION
8.1.1 The various Frequency Response Analysis Techniques are
1. Polar plot
2. Nyquist plot
3. Bode plot
4. M & N circles
5. Nicholas chart

8.1.1 Polar Plot
The sinusoidal transfer function $G(j\omega)$ is a complex function and is given by
$G(j\omega) = \text{Re } G(j\omega) + j\, I_m\, G(j\omega)$
Or $G(j\omega) = |G(j\omega)| \surd G(j\omega) = M \surd \phi$
from above equation, it is seen that $G(j\omega)$ may be represented as a phasor of magnitude M and phase angle ϕ. As the input frequency ω is varied from 0 to∞, the magnitude M and phase angle ϕ change and hence the tip of the phasor $G(j\omega)$ traces a locus in the complex plane.
The locus thus obtained is known as polar plot.
When a transfer function consists of 'p' poles and 'Z' zeros, and it doesn't consist poles at origin then the polar plot start from 0^0 with some magnitude and terminates at $-90^0 \times (P - Z)$ with zero magnitude.
When a transfer consists of poles at origin, then the polar plot starts from $-90^0 \times$ no. of poles at origin with '∞' magnitude and ends at $-90^0 \times (P - Z)$ with zero magnitude
Polar coordinates ($|GH| \angle GH$)



Example. Draw the polar plot for the following transfer function:

$$GH = \frac{1}{j\omega + 1} \; |GH| = \frac{1}{\sqrt{\omega^2 + 1}}, \angle GH = -\tan^{-1}\omega$$

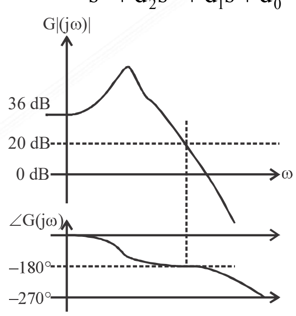
GATE QUESTIONS

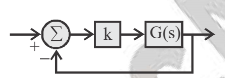
1. For a unity feedback control system with the forward path transfer function $G(s) = \frac{K}{s(s+2)}$.
The peak resonant magnitude M_r of the closed-loop frequency response is 2. The corresponding value of the gain K (correct to two decimal places) is _______

[GATE - 2018]

2. The figure below shows the Bode magnitude and phase plots of a stable transfer function

$$G(s) = \frac{n_0}{s^3 + d_2s^2 + d_1s + d_0}$$



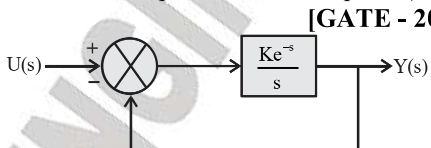


Consider the negative unity feedback configuration with gain k in the feedforward path. The closed loop is stable for $K < k_0$. The maximum value of k_0 is _________.

[GATE - 2018]

3. Consider the unity feedback control system shown. The value of K that results in a phase margin of the system to be 30° is _______. (Give the answer up to two decimal places).

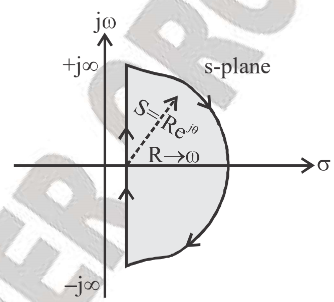
[GATE - 2017]



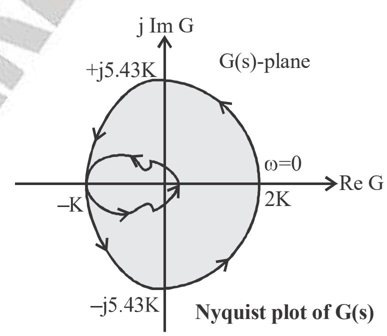
4. A unity feedback control system is characteriszed by the open loop transfer function

$$G(s) = \frac{10K(s+2)}{s^3 + 3s^2 + 10}$$

The Nyquist path and the corresponding Nyquist plot of G(s) are shown in the figures below.



Nyquist plot of G(s)



Nyquist plot of G(s)

If 0 < K < 1, then number of poles of the closed loop transfer function that lie in the right half of the s-plane is

[GATE - 2017]

(a) 0 (b) 1
(c) 2 (d) 3

5. The Nyquist plot of the transfer function

$$G(S) = \frac{K}{(s^2 + 2s + 2)(s + 2)}$$

Does not encircle the point (–1+j0) for K = 10 but does encircle the point (–1 + j0) for

SOLUTIONS

Sol.1. (a)

Sol.2. (b)
For open loop system no of poles in right half of s plane (P) = 1
$n = p^+ - z^+$
For stability $Z^+ = 0$
N = P = 1

Sol.3. (b)
The T.F. of given Bode plot.

$$\text{T.F.} = \frac{k_1\left(\frac{S}{20}+1\right)}{S\left(\frac{S}{2}+1\right)} = \frac{k(s+20)}{s(s+2)}$$

Sol.4. (c)

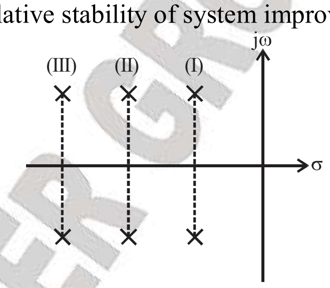
$$\text{T.F.} = \frac{ks^2}{\left(\frac{S}{10}+1\right)^5}$$

Sol.5. (c)
Low – frequency asymptote slope depends upon the poles or zeros at origin.
= (–20) × 2
= –40 dB/decade

Sol.6. (d)
From bode plot we can determine the open loop transfer function but to determine function but to determine the roots of closed – loop control system we have to know G(s) or H(s) separately. So, statement – I is wrong.

Sol.7. (b)
The slop of highest frequency asymptote
= (Z – P) × 20 dB/dec
= (2 – 14) × 20
= –240 dB/dec

Sol.8. (c)
Gain Margin and Phase margin of the system gives relative stability.
Relative stability is analysis of how fast transient has died out in the system. If we moves away from jω axis in left half of s plane then relative stability of system improves.



(iii) is relatively more stable to (ii)
(ii) is relatively more stable to (i).

Sol.9. (d)

Sol.10. (a)

$$G(s)H(s) = \frac{2K}{s(s+1)(s+5)}$$

For marginal stability we need to find frequency of sustained oscillation.
If G(s) H(s) ⇒ s(s + 1) (s + 5) + 2k = 0
⇒ $s^3 + 6s^2 + 5s + 2k = 0$
Now from Rough Huswitz criteria

S^3	1	5
S^2	6	2K
S^1	$\frac{30-2k}{6}$	
S^0	2k	

So k = 15
Now we get that k = 15
So $6s^2 + 30 = 0$

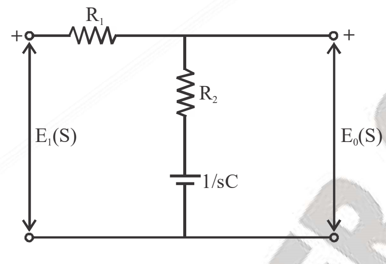
$$\omega_{oscillation} = \sqrt{5}\ \text{rad / sec}$$

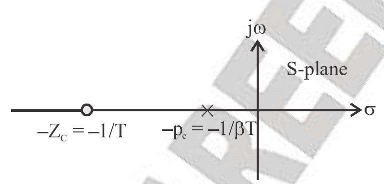
Sol.11. (b)

CHAPTER - 9
COMPENSATORS

9.1 LAG COMPENSATOR

A compensator having the characteristic of a lag network is called a lag compensator. Lag compensation results in a large improvement in steady state performance but results in a slower response due to reduced band width. Lag compensator is essentially a low pass filter and so high frequency noise signals are attenuated.





Transfer function of lag compensator, $G_c(s) = \frac{s+z_c}{s+p_c} = \left(\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}\right)$

9.1.1 Frequency Response of a Lag Compensator

Consider the general form of lag compensator

$$G_c(s) = \frac{s+(1/T)}{s+(1+\beta T)} = \beta\frac{(1+sT)}{(1+s\beta T)}$$

The sinusoidal transfer function of lag compensator is obtained by letting s = jω

$$\therefore G_c(j\omega) = \beta\frac{(1+j\omega t)}{(1+j\omega\beta T)}$$

When ω = 0, $G_c(j\omega)=\beta$

$$G_c(j\omega) = \frac{1+j\omega T}{1+j\omega\beta T} = \frac{\sqrt{1+(\omega T)^2}\angle\tan^{-1}\omega T}{\sqrt{1+(\omega\beta T)^2}\angle\tan^{-1}\omega\beta T} \quad ...(i)$$

The sinusoidal transfer function has two corner frequencies and they are denoted as ω_{c1} and ω_{c2}

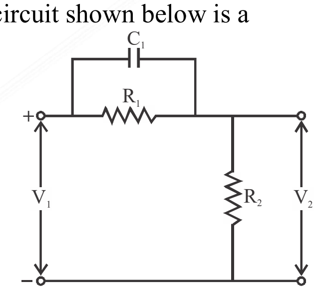
Here, $\omega_{c1} = 1/\beta T$ and $\omega_{c2} = 1/T$

Since, $\beta T > T$, $\omega_{c1} < \omega_{c2}$

ASSIGNMENT

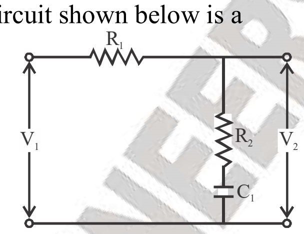
1. A negative feedback control system has a transfer function $G(s) = \dfrac{k}{s+2}$. We select a compensator $G_c(s) = \dfrac{s+a}{s}$ in order to achieve zero steady state error for a step input. Select 'a' and 'k' so that the overshoot to a step is approximately 5% and the settling time (with a 2% criterion) is approximately 1 second.
(a) k=8, a = 5.6 (b) k=8, a = 6.6
(c) k=6, a = 5.6 (d) k=6, a =6.6

2. The circuit shown below is a



(a) Lag network (b) Lead network
(c) Lead-lag network (d) None

3. The circuit shown below is a



(a) Lag network (b) Lead network
(c) Lead-lag network (d) None

4. The transfer function of a simple RC network functioning as a controller is $G_c(s) = \dfrac{(s+z_1)}{(s+p_1)}$. The required condition for the RC network to act as a phase lead controller is
(a) $p_1 < z_1$ (b) $p_1 > z_1$
(c) $p_1 = z_1$ (d) None of these

5. The damping of the system can be increased by using a compensator having a pair of complex roots as
(a) Phase lead (b) Phase lag lead
(c) Phase lag (d) None of these

6. If poles are added in a transfer function it will cause
(a) Lag compensation
(b) Lead compensation
(c) Lead-lag compensation
(d) None of these

7. If zero are added in a transfer function, it will cause
(a) Lag compensation
(b) Lead compensation
(c) Lead-lag compensation
(d) None of these

8. The transfer function of a lead compensator is $G_c(s) = \dfrac{1+0.12s}{1+0.04s}$. The maximum phase shift that can be obtained from this compensator is
(a) 60^o (b) 45^o
(c) 30^o (d) 15^o

9. Consider the following statements in case of phase lead compensation:
(a) Improvement of gain and phase margins
(b) Less rise time and more settling time
(c) Bandwidth is increased
(d) Affect the steady-state error
Which of these statements are correct?
(a) 1, 2 and 3 (b) 1 and 3
(c) 2 and 3 (d) 2 and 4

10. Consider the following statement in case of phase lag compensation:

GATE QUESTIONS

1. The transfer function C(s) of a compensator is given below:

$$C(s) = \frac{\left(1+\frac{s}{0.1}\right)\left(1+\frac{s}{100}\right)}{(1+s)\left(1+\frac{s}{10}\right)}$$

The frequency range in which the phase (lead) introduce by the compensator reaches the maximum is

[GATE - 2017]

(a) $0.1 < \omega < 1$ (b) $1 < \omega < 10$
(c) $10 < \omega < 100$ (d) $\omega > 100$

2. Which of the following statement is incorrect?

[GATE - 2017]

(a)Lead compensator is used to reduce the settling time.
(b)Lag compensator is used to reduce the steady state error.
(c)Lead compensator may increase the order of a system
(d)Lag compensator always stabilizes an unstable system.

Common data for Q. 3 and Q. 4

The transfer function of a compensator is given as $G_c(s) = \frac{s+a}{s+b}$

3. $G_c(s)$ is a lead compensator if

[GATE - 2012]

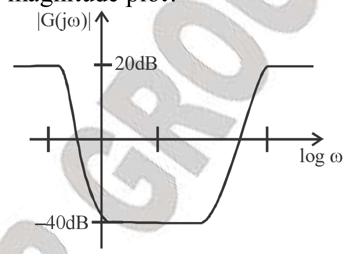
(a) a = 1, b = 2 (b) a = 3, b = 2
(c) a = –3, b = –1 (d) a = 3, b = 1

4. The phase of the above lead compensator is maximum at

[GATE - 2012]

(a) $\sqrt{2}$ rad / s (b) $\sqrt{3}$ rad / s
(c) $\sqrt{6}$ rad / s (d) $1/\sqrt{3}$ rad / s

5. The magnitude plot of a rational transfer function G(s) with real coefficients is shown below. Which of the following compensators has such a magnitude plot?



[GATE - 2009]

(a) Lead compensator
(b) Lag compensator
(c) PID compensator
(d) Lead - lag compensator

6. The transfer functions of two compensators are given below :

$$C_1 = \frac{10(s+1)}{(s+10)}, C_2 = \frac{s+10}{10(s+1)}$$

Which one of the following statements is correct?

[GATE - 2008]

(a) C_1 is lead compensator and C_2 is a lag compensator
(b) C_1 is a lag compensator and C_2 is a lead compensator
(c) Both C_1 and C_2 are lead compensator
(d) Both C_1 and C_2 are lag compensator

7. The open loop transfer function of a plant is given as $G(s) = \frac{1}{s^2-1}$. If the plant is operated in a unity feedback configuration, then the lead compensator that an stabilize this control system is

[GATE - 2007]

(a) $\frac{10(s-1)}{s+2}$ (b) $\frac{10(s+4)}{s+2}$

CHAPTER - 10
STATE VARIABLE APPROACH

10.1 INTRODUCTION

These are minimal set of variables which can completely determine the behavior of system at any given time.

State model:

$X = AX + BU$ State eqns.

$Y = CX + DU$ Output eqns.

And both equation combined together is called. State model

X – State vector U – Input vector Y – Output vector

A – System matrix B – Input matrix C – Output matrix

D – Transmission matrix Let n $\Rightarrow$ No. of state variables = order of the system

p $\Rightarrow$ No. of outputs m $\Rightarrow$ No. of inputs Order [A] = n x n

Order [B] = n x m Oder [C] = p x n Order [D] = p x m

10.2 DISADVANTAGES OF TRANSFER FUNCTIONS

1.It is defined only under zero initial conditions.

2.It is only applicable to LTI system and there too it is restricted to single input systems.

3.It reveals only the system O/P for a given i/p and provides no information regarding internal states of the system.

4.Classical design methods (roots locus and freq. domain methods) based on transfer function model are trail and error procedures.

10.3 ADVANTAGES OF STATE VARIABLE METHOD

1. It is applicable for both LTI and LT varying systems.
2. It takes initial conditions into account.
3. All the internal states of the system can be determined.
4. Applicable for multiple input multiple output.
5. Controllability and observability can be determined easily.

10.4 REPRESENTATION OF STAT MODEL

1. Physical variable representation.
2. Phase variable representation
3. Cononical representation.

State model of a system is not unique property. But transfer function of the system is unique.