GATE

2019

NETWORK ANALYSIS

ELECTRICAL ENGINEERING

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GATE-2019: Network Analysis| Detailed theory with GATE & ESE previous year papers and detailed solu ons.

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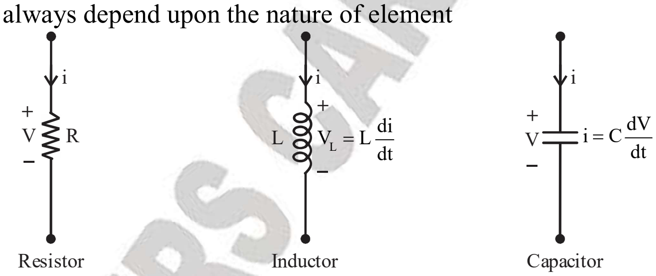
CONTENTS

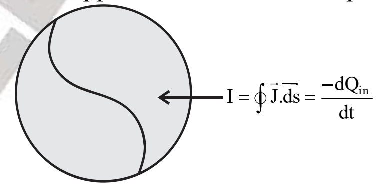
CHAPTER - 1
BASIC CONCEPTS

1.1INTRODUCTION
1.1.1 Charge
Charge can be classified as:
1. Stationary Charge
2. Dynamic Charge

1. Stationary Charge
Stationary charge does not result into electric current because the flow of current means charge moving with net rate across any cross section.
(i) Any electric circuit should always follow law of conservation of charge and law of conservation of energy.
(ii) Circuit theory is analysed always at low frequency and field theory always at high frequency.
(iii) Transit time effect is always neglected at low frequency because
$T>>t_r$
Where T is time period of sinosdual signal
t_r is Transit Time (time taken by signal effect to travel from one point to another point).
(iv) Elemental law is obeyed only at low frequency such as ohm's law. It is not applicable at high frequency because of distributed nature of element.
(v) Elemental law always depend upon the nature of element



For different Element, Different Form of Ohm's Law is present
(i) In time domain, the ohm's law are applicable and also in frequency domain.



Current flowing out of this body is given by equation of continuity as below

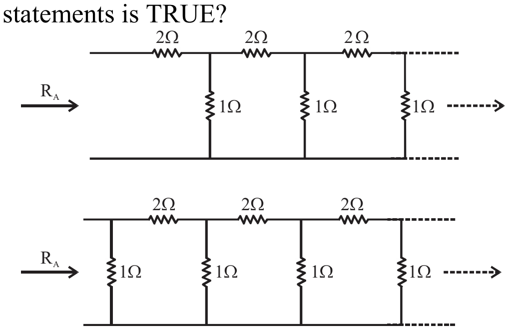
$$I = \oint \vec{J}.\vec{ds} = -\frac{dQ_{in}}{dt} \qquad \ldots(i)$$

This equation gives the law of conservation of charge.

If $\frac{dQ_{in}}{dt} = 0$; means no rate of change of charge within body then eq.(i) become

GATE QUESTIONS

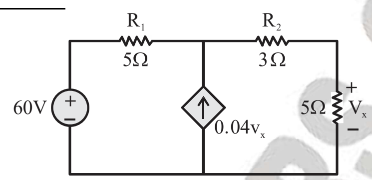
1. R_A and R_B are the input resistances of circuits as shown below. The circuits extend infinitely in the direction shown. Which one of the statements is TRUE?



[GATE - 2016]

(a) $R_A = R_B$ (b) $R_A = R_B = 0$
(c) $R_A < R_B$ (d) $R_B = R_A/(1+R_A)$

2. In the circuit shown in the figure, the magnitude of the current (in amperes) through R_2 is ______

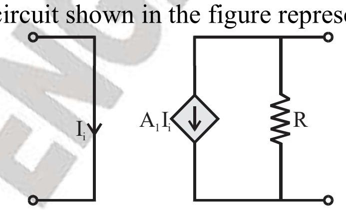


[GATE - 2016]

3. An incandescent lamp is marked 40W, 240V. If resistance at room temperature (26°C) is 120Ω, and temperature coefficient of resistance is 4.8×10^{-3}/°C, then its 'ON' state filament temperature in °C is approximately________

[GATE - 2014]

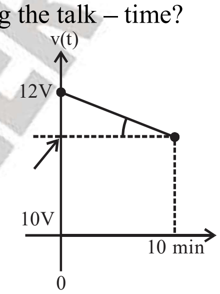
4. The circuit shown in the figure represents a



[GATE - 2014]

(a) Voltage controlled voltage source
(b) Voltage controlled current source
(c) Current controlled current source
(d) Current controlled voltage source

5. A fully charged mobile phone with a 12V battery is good for a 10 minute talk-time. Assume that during the talk – time the battery delivers a constant current of 2A and its voltage drops linearly from 12V to 10V as shown in the figure. How much energy does the battery deliver during the talk – time?

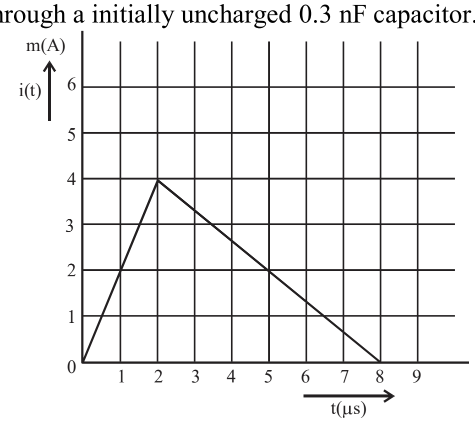


[GATE - 2009]

(a) 220 J (b) 12kJ
(c) 13.2 kJ (d) 14.4 J

Common data for Q. 6 & Q. 7

The current i(t) sketched in the figure flows through a initially uncharged 0.3 nF capacitor.



6. The charge stored in the capacitor at t = 5 μs, will be

[GATE - 2008]

CHAPTER - 2
NETWORK LAWS

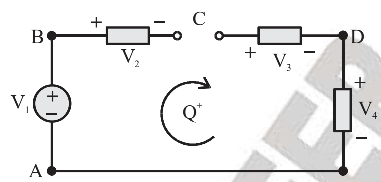
2.1 KIRCHOFF'S VOLTAGE LAW (KVL)

It states that algebraic sum of all voltages in a closed path or loop is zero

$$\sum_{loop} V = 0$$

For writing KVL start from any point in the loop and come to the same point via transversing the path of closed loop. While doing so take voltage rises as positive and voltage drops as negative then

Σ voltage rise + Σ voltage drops = 0



Q^+ positive is moving
A-B is energy is gained say W_1
B-C is energy lost say W_2
C-D is energy lost say W_3
D-A is energy lost say W_4
by consecration of energy
$W_1 = W_2 + W_3 + W_4$ or
$W_1 - W_2 - W_3 - W_4 = 0$

Divide by $Q = \frac{W_1}{Q} - \frac{W_2}{Q} - \frac{W_3}{Q} - \frac{W_4}{Q} = 0$

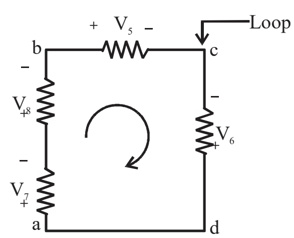
$\Rightarrow V_1 - V_2 - V_3 - V_4 = 0$

$\Sigma V = 0$ (In a loop)

Example. KVL in this loop starting from a in clockwise direction is

$-V_7 - V_8 - V_5 + V_6 = 0$

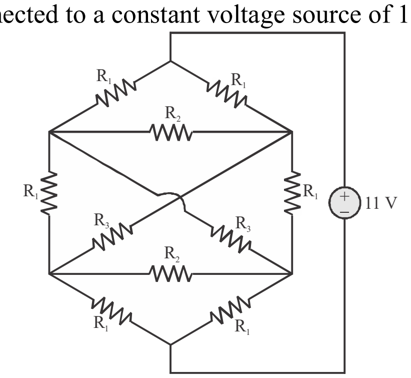
$\Rightarrow V_6 = V_7 + V_8 + V_5$



The basis of the law is that if we start from a particular junction and go round the mesh till we come back to the starting point, then we must be at the same potential with which we started. Hence it means that all the sources of e.m.f. met on the way must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign, plus or minus.

GATE QUESTIONS

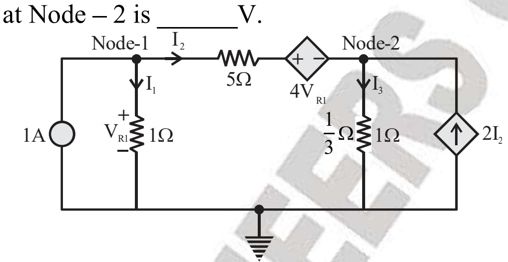
1. Consider the network shown below with $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 3\ \Omega$. The network is connected to a constant voltage source of 11 V.



The magnitude of the current (in amperes, accurate to two decimal places) through the source is __________

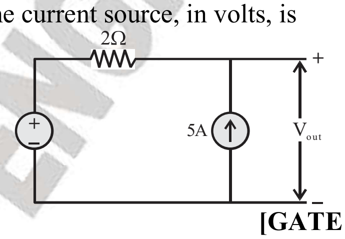
[GATE - 2018]

2. A circuit consisting of dependent and independent sources is shown in the figure. If the voltage at Node -1 is –1V, then the voltage at Node – 2 is ______V.



[GATE - 2017]

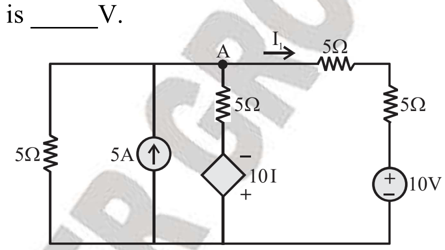
3. In the circuit shown below, the voltage and current sources are ideal. The voltage (V_{out}) across the current source, in volts, is



[GATE - 2016]

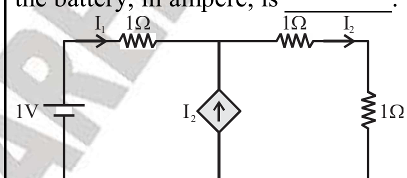
(a) 0 (b) 5
(c) 10 (d) 20

4. In the circuit shown below, the node voltage V_A is _____V.



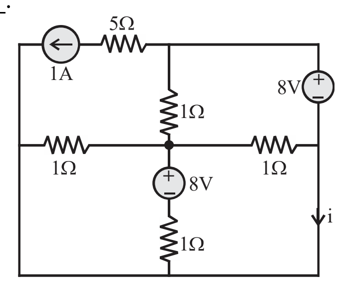
[GATE - 2016]

5. In the given circuit, the current supplied by the battery, in ampere, is ________.



[GATE - 2016]

6. In the figure shown, the current i (in ampere) is _____.

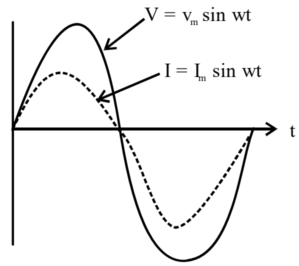


[GATE - 2016]

7. In the given circuit, each resistor has a value equal to 1Ω

CHAPTER - 3
A.C ANALYSIS

3.1 AC THROUGH PURE OHMIC RESISTANCE ALONE



$v = V_m \sin\omega t$

v = iR

$i = \frac{V_m}{R}\sin\omega t$

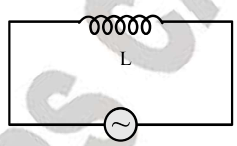
Current is max when sin wt = 1

i.e. $I_m = \frac{V_m}{R}$

$\therefore\ i = I_m \sin\omega t$

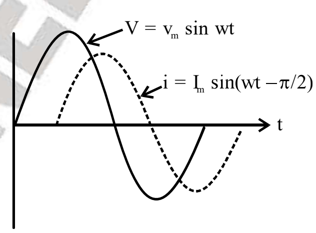
3.2 AC THROUGH PURE INDUCTANCE ALONE

Whenever an alternating voltage is applied to a purely inductive coil, a back emf is produced due to the self-inductance of the coil



$v = L\frac{di}{dt}$

$V_m \sin\omega t = \frac{Ldi}{dt} \Rightarrow di = \frac{v_m}{L}\sin\omega t dt$



$i = \frac{V_m}{L}\int \sin\omega t dt$

ASSIGNMENT

1. The current flowing through a circuit-containing passive elements like R, L or C is I = 15.5 sin (2500t – 145°) with voltage source V = 311 sin (2500t + 170°). The impedance Z is

(a) $20 \angle -25^\circ \ \Omega$ (b) $20 \angle 25^\circ \ \Omega$

(c) $20 + j20\Omega$ (d) $14.14 - j14.14 \ \Omega$

2. There is a pure element in series with R = 25Ω which causes the current to lag the voltage by 20°. It is _____ if frequency is 400 Hz.

(a) 35 μF capacitor (b) 3.6 mH inductor

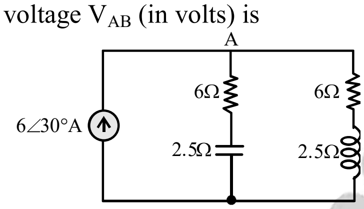
(c) 25μF capacitor (d) 2 mH inductor

3. There is a pure element in series with R = 50Ω which causes the current to lead the voltage by 30°. It is _______ if frequency is 500 Hz.

(a) 11 μF capacitor (b) 22 μF capacitor

(c) 1.2mH (d) 2.4mH inductor

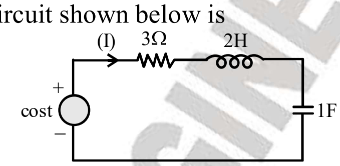
4. In the AC network shown in the figure. The phasor voltage V_{AB} (in volts) is



(a) $7 \angle 30^\circ$ (b) $6 \angle 30^\circ$

(c) $21 \angle 30^\circ$ (d) $11 \angle 30^\circ$

5. The differential equation for the current i(t) in the circuit shown below is



(a) $2\frac{d^2i}{dt^2} + 3\frac{di}{dt} + i(t) = -\sin t$

(b) $2\frac{d^2i}{dt^2} + 3\frac{di}{dt} + i(t) = \sin t$

(c) $2\frac{d^2i}{dt^2} + 3\frac{di}{dt} + i(t) = \cos t$

(d) $2\frac{d^2i}{dt^2} + 3\frac{di}{dt} + i(t) = -\cos t$

Common Data for Q.6 & Q.7

Given below is the current and applied voltage in a series connection of two pure circuit elements.

V = 150 sin (314 t + 10°) volts

i = 15 sin (314 t – 53.4°) amperes

6. The circuit contains a

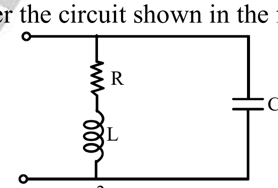
(a) Resistance of 5Ω and capacitor of 1μF

(b) Resistance of 4.47 Ω and inductor of 0.028 H

(c) Resistance of 6.7Ω and capacitor of 0.01 μF

(d) Resistance of 10Ω and inductor of 0.021 H

7. Consider the circuit shown in the figure



In the circuit $4R^2C = 3L$, the resonance frequency ω_0 is

(a) $\sqrt{LC}$ (b) $\frac{1}{\sqrt{LC}}$

(c) $\frac{1}{2\pi\sqrt{LC}}$ (d) $\frac{1}{2\sqrt{LC}}$

8. A voltage V is represented as V = 50 sin(ωt + 30°) – 25 sin(3ωt – 60°) + 16 sin (5ωt + 45°)V R.M.S. value of the voltage is

(a) 41 volt (b) 41.11 volt

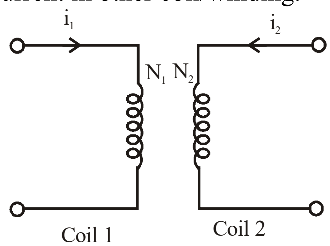
(c) 91 volt (d) 58.14 volt

CHAPTER - 4
MAGNETICALLY COUPLED CIRCUIT

4.1 INTRODUCTION

When two circuits are so placed that a portion of the magnetic flux produced by one links with the turns of both, they are said to be mutually coupled magnetically. This effect is characterized by mutually inductance (M)

Mutual Inductance (M) is the property of magnetic coupling showing an induction of voltage in one coil/winding by a change of current in other coil/winding.



In the above figure two coils 1 and 2 with turns N_1 and N_2 are placed close to each other so that part of flux of one coil links with other coil too. The current i_1 in coil 1 produces flux ϕ_1. Some part of ϕ_1 links only with coil 1 let this is ϕ_{11} this is known as self flux or leakage flux of coil 1. ϕ_{12} is the flux which links with both the coils. ϕ_{12} is called mutual flux. Similarly current i_2 in coil 2 produces ϕ_2 which has ϕ_{22} and ϕ_{21} as its components. ϕ_{22} links only with coil 2 and ϕ_{21} links with both coils.

Now the voltage induced in coil 2 by change in current of coil 1 i_1

$$v_{21} = M_{21}\frac{di_1}{dt}$$

However by Faraday's Law

$$v_{21} = N_2\frac{d\phi_{12}}{dt}$$

$$\Rightarrow M_{21} = N_2\frac{d\phi_{12}}{di_1}$$

$$\Rightarrow M_{21} = N_2\frac{d\phi_{12}}{di_1}$$

If air is the medium between two coils, then magnetization is linear and

$$\frac{d\phi_{12}}{di_1} = \frac{\phi_{12}}{i_1}$$

Hence $M_{21} = \frac{N_2\phi_{12}}{i_1}$

Similarly $M_{12} = \frac{N_1\phi_{21}}{i_2}$

Since the reluctance of both the fluxes i.e. ϕ_{12} & ϕ_{21} is same M_{12} & M_{21} are equal say $M_{12} = M_{21} =$ M.

CHAPTER - 5
NETWORK THEOREMS

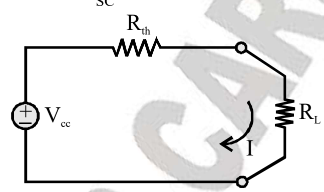
5.1 THEVENIN'S THEORM
Any two terminal bilateral linear circuit can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.

5.1.1 Steps for Solving a Network using Thevenin's Theorem
1. Remove the load resistor (R_L) and find the open circuit voltage (V_{oc}) across the open circuited load terminals.
2. Find the Thevenin's resistance (R_{TH})

1. If Circuit contains only Independent Sources
Deactivate the constant sources (for voltage source remove it by short circuit and for the current source remove it by open circuit) and find the internal resistance (R_{TH}) of the source side looking through the open circuited load terminals.

2. For the circuits containing dependent sources in addition to or in absence of independent sources
Find V_{OC} by open circuiting the load terminals. Then short the load terminals and find the short circuit current (I_{SC}) through the shorted terminals.

Thevenin's resistance is given as: $R_{TH} = \frac{V_{OC}}{I_{SC}}$



(i) Obtain thevenin's equivalent circuit by placing R_{TH} in series with V_{OC}
(ii) Reconnect R_L across the load terminals.

5.1.2 Thevenin's Equivalent Network
$$I(\text{Load current}) = \frac{V_{OC}}{R_{TH} + R_L}$$

If only dependent sources are present in circuit, $R_{Th} = \frac{V_{test}}{I_{test}}; I_{test} = 1A$

V_{test} is calculated across the load by short circuiting it, and current of 1A flows through the short circuited branch as I_{test}. Then $R_{TH} = V_{test}$

CHAPTER - 6
TRANSIENT ANALYSIS

6.1 INTRODUCTION

1. Linear differential equation with constant coefficient obey linearity & superposition theorem.
2. The response of a network excited by a initial energy storage and then left undisturbed is a characteristic of network as with the passage of time networks comes to zero response. This is called natural behavior or its transient response or force +ve behavior. The natural behavior is solution of the network's differential equation with all the sources equated to zero.
3. The response of a network to excitation by an impulse source is very similar to natural behavior. After $t = 0^+$

The impulse response ≡ natural behavior.

4. For forced response steady state value will not be zero. Than steady state value are calculated.

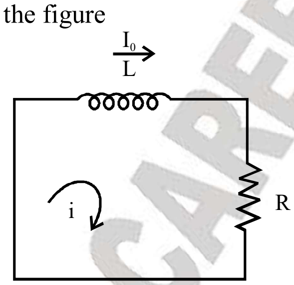
Complete solution = natural behavior + forced solution

Or transient + steady state

Or complementary function + particular integer

6.2 NATURAL BEHAVIOR OF R-L CIRCUIT

Let the initial current in the inductor is I_0 and the inductor is connected to resistance in series so that inductor discharges. As shown in the figure



Writing KVL in the loop

$$\frac{Ldi}{dt} + Ri = 0 \quad \ldots(i)$$

The possible solutions for current (i) is

$i(t) = ke^{st}$, where k is constant

Putting this value in equation (i)

$$L\frac{d(ke^{st})}{dt} + Rke^{st} = 0$$

$$\Rightarrow ksL\,e^{st} + Rke^{st} = 0$$

$$\Rightarrow sL + R = 0$$

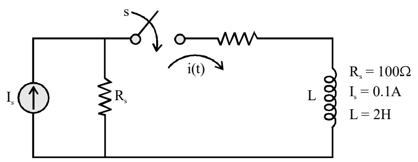
$$\Rightarrow s + \frac{R}{L} = 0 \quad \ldots(ii)$$

Equation (ii) is the characteristic equation of series R-L circuit.

From equation (ii), $s = -\frac{R}{L}$

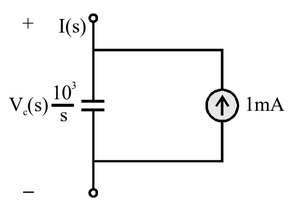
ASSIGNMENT

1. The switch S is closed at t = 0. The rate of change of current $\frac{di}{dt}(0^+)$ is given by



(a) 1A/sec (b) 5A/sec
(c) 2.5 A/sec (d) 3A/sec

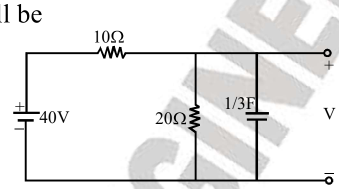
2. A capacitor with some initial voltage can be represented by the shown figure. Where s is laplace transform variable. The value of initial voltage is



(a) 0.5V (b) 2.0V
(c) 1.0 V (d) 0V

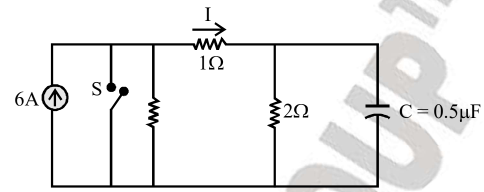
3. A LTI system has an impulse response e^{-2t} for t > 0. If initial conditions are 0 and the input is e^{-3t}, the output for t > 0 is
(a) $e^{-2t} - e^{3t}$ (b) $e^{-2t} - e^{-3t}$
(c) e^{-5t} (d) None of these

4. Consider the network shown below, if the voltage V at a time is 20V, then dV/dt at that time will be



(a) 1V/s (b) –2V/s
(c) 3 V/s (d) – 4 V/s

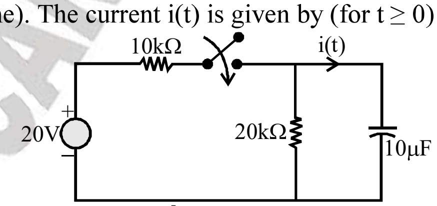
5. In the circuit shown below, the switch S is open for a long time and closed at t = 0



The value of I at t = 0^+ is
(a) – 6A (b) $-\frac{3}{2}$A
(c) 3A (d) $\frac{3}{2}$A

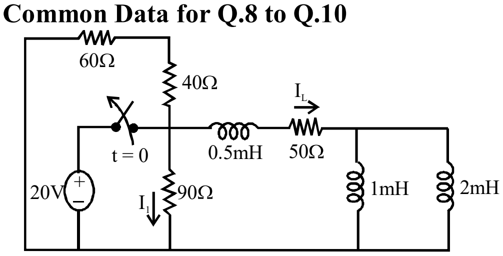
6. $\frac{R}{L}$ has the unit of
(a) Farad (b) $Farad^2$
(c) sec^{-1} (d) sec

7. For the circuit shown, the switch is closed at t = 0 (after having been open for a very long time). The current i(t) is given by (for t ≥ 0)



(a) i(t) = 10 $(1 - e^{-5t})$ mA
(b) i(t) = $(12 - e^{-5t})$ mA
(c) i(t) = $(10 - e^{-10t})$ mA
(d) i(t) = $(2e^{-15t})$ mA

Common Data for Q.8 to Q.10



Switch is opened at t = 0

CHAPTER - 7
LAPLACE TRANSFORMATION AND ITS APPLICATION IN CIRCUIT ANALYSIS

7.1 LAPLACE TRANSFORMATION

The Laplace transformation of a function *f*(t) is defined as

$$F(s) = Lf(t) = \int_0^\infty f(t)e^{-st}dt$$

Where as in complex frequency
s being the intermediate or transformation variable.

7.1.1 Laplace Transform of a Derivative $\left[\frac{df(t)}{dt}\right]$

$$Lf'(t) = sF(s) - f(0+)$$

7.1.2 Laplace Transform of an Integral $\int f(t)dt$

$$L\left(\int f(t)dt\right) = \frac{1}{s}\int f(t)dt \Big|_{0+} + \frac{1}{s}F(s)$$

$\left[\int f(t)\Big|_{0+} \text{ gives the value of the integral at } t = 0+\right]$

7.1.3 Frequency Shifting

$$L\left(e^{at}f(t)\right) = F(s-a)$$

$$L\left(e^{-at}f(t)\right) = F(s+a)$$

7.2 LAPLACE TRANSFORM OF COMMON FORCING FUNCTIONS

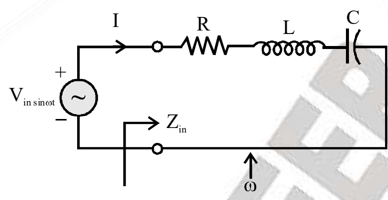
***f*(t)**	**F(s)**	***f*(t)**	**F(s)**
u(t)	$\frac{1}{s}$	$e^{-\alpha t}t^n$	$\frac{\underline{\lvert n}}{(s+\alpha)^{n+1}}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$e^{-\alpha t}\sin\omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$	$e^{-\alpha t}\cos\omega t$	$\frac{s+a}{(s+\alpha)^2+\omega^2}$
$\cos\omega t$	$\frac{\omega}{s^2+\omega^2}$	$\delta(t)$	1
t	$\frac{1}{s^2}$	Sinh θt	$\frac{\theta}{s^2-\theta^2}$

CHAPTER - 8
RESONANCE

8.1 RESONANCE

Resonance in electrical circuits consisting of passive and active elements represents a particular state of the circuit when the current or voltage in the circuit is maximum or minimum with respect to the magnitude of excitation at a particular frequency, the circuit impedance being either minimum of maximum at the power factor unity.
The phenomenon of resonance is observed in both series or parallel a.c. circuits comprising of R, L and C and excited by an a.c. source.

8.2 SERIES RESONANCE

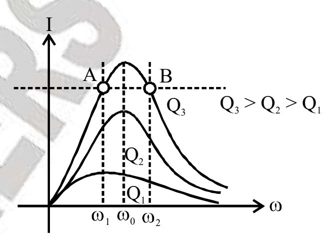


$$Z_{in} = \frac{V}{I} = R + j(\omega L - \frac{1}{\omega c})$$

For resonance V & I must be in same phase
So for some frequency $\omega = \omega_0$

$$Z_{in} = R + j_0 \Rightarrow \omega_0 L - \frac{1}{\omega_0 C} = 0 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$I = \frac{V}{|z|} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega c)^2}} \text{ at } \omega_0,\ I = \frac{V}{R}$$



Points A & B are half power or 3dB points because $20\log_{10}\left(\frac{1}{2}\right)$ = 3dB

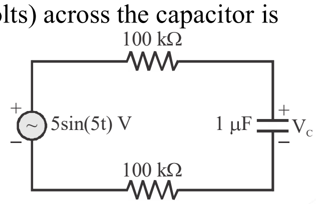
Band width of circuit $\Delta\omega$ = BW = $\omega_2 - \omega_1$
Quality factor

$$Q_0 = 2\pi\left[\frac{\text{Max energy stored}}{\text{Total energy last per perior}}\right]$$

$$Q_0 = 2\pi\left[\frac{\omega_L + \omega_C}{P_R T}\right]$$

GATE QUESTIONS

1. For the circuit given in the figure, the voltage V_C (in volts) across the capacitor is



[GATE - 2018]

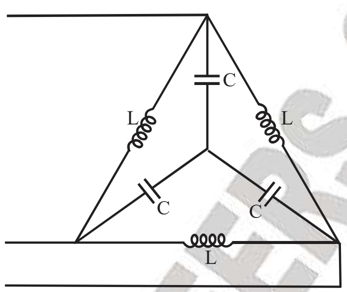
(a) $1.25\sqrt{2}\sin(5t-0.25\pi)$

(b) $1.25\sqrt{2}\sin(5t-0.125\pi)$

(c) $2.5\sqrt{2}\sin(5t-0.25\pi)$

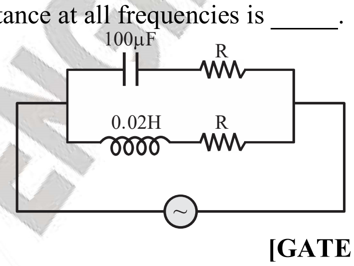
(d) $2.5\sqrt{2}\sin(5t-0.125\pi)$

2. In the balanced 3-phase, 50 Hz, circuit shown below, the value of inductance (L) is 10 mH. The value of the capacitance (C) for which all the line currents are zero, in millifarads, is ______.



[GATE - 2016]

3. The circuit below is excited by a sinusoidal source. The value of R, in Ω for which the admittance of the circuit becomes a pure conductance at all frequencies is _____.



[GATE - 2016]

4. A series RLC circuit is observed at two frequencies. At ω_1 = 1krad/s, are note that source voltage $V_1 = 100\angle 0°$V result in a current $I_1 = 0.03\angle 31°$A. At ω_2 = 2krad/s, the source voltage $V_2 = 100\angle 0°$V results in a current $I_2 = 2\angle 0^{\circ}$ A. The closest values for R, L, C out of the following options are

[GATE - 2014]

(a) R = 50Ω; L = 25mH; C = 10μF;

(b) R = 50Ω; L = 10mH; C=25μF;

(c) R = 50Ω; L = 50mH;C = 5μF;

(d) R = 50Ω; L = 5mH; C = 50μF;

5. Two magnetically uncoupled inductive coils have Q factors q_1 and q_2 at the chosen operating frequency. Their respective resistances are R_1 and R_2. When connected in series, t heir effective Q factor at the same operating frequency is

[GATE - 2013]

(a) $q_1 + q_2$

(b) $(1/q_1)+(1+q_2)$

(c) $(q_1R_1+q_2R_2)/(R_1+R_2)$

(d) $(q_1R_2+q_2R_1)/(R_1+R_2)$

6. For parallel RLC circuit, which one of the following statements is NOT correct?

[GATE - 2010]

(a)The bandwidth of the circuit decreases if R is increased

(b)The bandwidth of the circuit remains same if L is increased

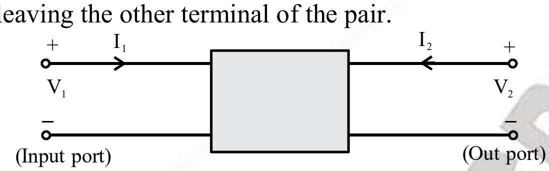
(c)At resonance, input impedance is a real quantity

(d)At resonance, the magnitude of input impedance attains its minimum value

7. The resonant frequency for the given circuit will be

CHAPTER - 9
TWO PORT NETWORKS

9.1 INTRODUCTION
The terminal pair is called as a "port". If the current entering one terminal of a pair is equal and opposite to the current leaving the other terminal of the pair.



9.2 TWO-PORT NETWORK
A two-port network is shown, by which we observe that a two-port network is represented by a black box with four variables, namely, two voltages (V_1, V_2) and two currents (I_1, I_2) which are available for measurements and are relevant for the analysis of two port networks. Of these four variables which two variable may be considered `independent` and which two `dependent` is generally decided by the probable under consideration

Two Port Parameters			
Name	**Express**	**In terms of**	**Matrix Equation**
Open circuit impedance [Z]	V_1, V_2	I_1, I_2	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
Short-circuit admittance [Y]	I_1, I_2	V_1, V_2	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$
Transmission or Chain [T] or [ABCD]	V_1, I_2	V_2, I_2	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$
Inverse Transmission [T']	V_2, I_2	$V_1, -I_1$	$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$
Hybrid (h)	V_1, I_2	I_1, V_2	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$
Inverse hybrid (g)	I_1, V_2	V_1, I_2	$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & h_{12} \\ g_{21} & h_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$

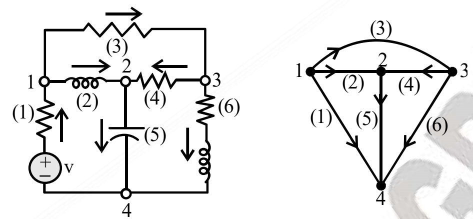
9.3 OPEN CIRCUIT IMPEDANCE (Z) PARAMETERS
Expressing two-port voltages in terms of two-port currents

$$(V_1, V_2) = f(I_1, I_2)$$

CHAPTER - 10
GRAPH THEORY

10.1 IMPORTANT DEFINITIONS

1. Graph: It is the collection of nodes and Branch of a network.



(A) (B)

2. Branch: Each oriented line segment of the graph is called branch.

3. Node: The end point of a branch is called node.

4. Incident Branch: Branch whose end fall on a node is called incident branch.

5. Connected and Non-Connected Graph
If there exists a path between every pair of nodes of a graph, then the graph is called connected graph, otherwise graph is called non-connected graph.

6. Degree of Node: Degree of Node is the number of branches incident on the node.

7. Subgraph: A portion of graph is called subgraph

8. Path: Path is transverse from one node to another node

9. Loop: Loop is a collection of branches in a graph which form a closed path.

10. Tree: The collection of minimum no. of branches connecting all the nodes of a graph without making a loop.
A single graph can have many no. of trees.
The no. of trees for a given graph= $n - 1$
where $n \rightarrow$ no. of nodes

11. Twig: Branch of a tree is called a twig.

12. Cotree: Remaining part of a graph after removal of twigs is called cotree. It is collection of links.

13. Links: are the branches removed from the graph to make a tree.
Total no. of branch of a graph are given by $b = (n-1) + L$
n is no. of nodes
L is No. of links

ASSIGNMENT

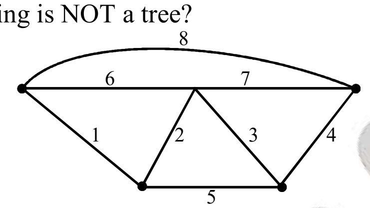
1. For a given network, the incidence matrix is given by

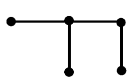
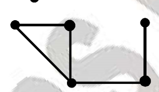
$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

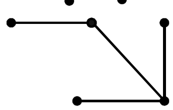
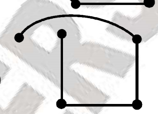
The series branches in the graph are
(a) 3 and 4 (b) 3 and 5
(c) 6 and 4 (d) 3 and 6

2. In Q. 1, the parallel branches are
(a) 3 and 5 (b) 4 and 5
(c) 3 and 6 (d) None

3. Of the graph shown in below, which of the following is NOT a tree?

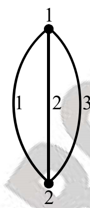


(a)  (b) 

(c)  (d) 

4. If a graph of network has 10 branches and 6 nodes, then number of mesh equations or KVL equation required to solve the network are
(a) 4 (b) 5
(c) 6 (d) 7

5. Of the graph shown in the figure below, the no. of possible trees is/are



(a) 1 (b) 2
(c) 3 (d) 4

6. For given network, the incidence matrix is given by

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

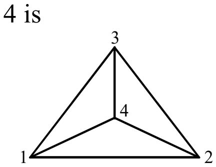
If i_2 = 2A, i_4 = 4A, i_5 2A, where i_k represents current K^{th} branches, then i_6 is given by:
(a) 4A (b) 2A
(c) 0A (d) 6A

7. If the reduced incidence matrix of a given network is given as below, then the no. of possible trees are

$$A = \begin{bmatrix} -1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

(a) 10 (b) 12
(c) 14 (d) 16

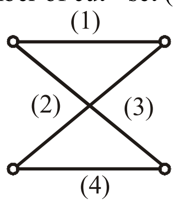
8. This is a network graph. The degree of the nodes 1, 2, 3, 4 is



(a) 2
(b) 3
(c) 2 for 1, 2, 3 and 3 for 4
(d) All 3

GATE QUESTIONS

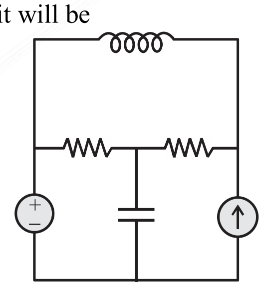
1. In the following graph, the number of trees (P) and the number of cut - set (Q) are



[GATE - 2008]

(a) P = 2, Q = 2
(b) P = 2, Q = 6
(c) P = 4, Q = 6
(d) P = 4, Q = 10

2. The number of chords in the graph of the given circuit will be



[GATE - 2008]

(a) 3
(b) 4
(c) 5
(d) 6

3. The matrix A given below in the node incidence matrix of a network. The columns correspond to branches of the network while the rows correspond to nodes. Let V = $[V_1 V_2 ...... V_6]^T$ denote the vector of branch voltage while I = $[i_1 i_2 .... i_6]^T$ that of branch currents. The vector E = $[e_1\ e_2\ e_3\ e_4]^T$ denotes the vector of node voltage relative to a common ground.

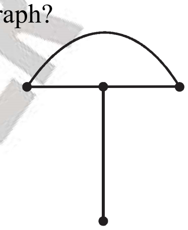
$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{bmatrix}$$

Which of the following statement is true?

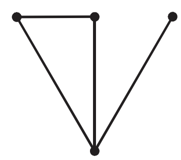
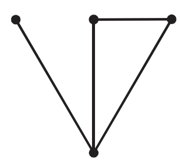
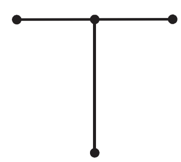
[GATE - 2007]

(a) The equation $V_1 - V_2 + V_3 = 0$, $V_3 + V_4 - V^5 - V_5 = 0$ are KVL equation for the network for some loops
(b) The equations $V_1 - V_s - V_6 = 0$, $V_4 + V_5 - V_6 = 0$ are KVL equations for the network for some loops
(c) E = AV
(d) AV = 0 are KVI equations for the network

4. Consider the network graph shown in the figure. Which one of ht following is NOT a 'tree' of this graph?



(a)

(b)

(c)

(d)


[GATE - 2004]

(a) a
(b) b
(c) c
(d) d

5. The minimum number of equations required to analyze the circuit shown in the figure is

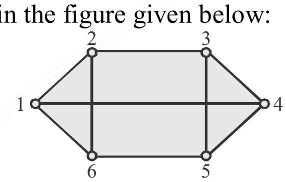
ESE OBJ QUESTIONS

1. Consider the following statements regarding trees:
1. A tree contains all the nodes of the graph.
2. A tree shall contain any one of the loops.
3. Every connected graph has at least one tree.
Which of the above statements are correct?

[EC ESE - 2017]

(a) 1 and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3

2. Consider the following with regards to graph as shown in the figure given below:



1. Regular graph
2. Connected graph
3. Complete graph
4. Non-regular graph
Which of the above are correct ?

[EC ESE - 2017]

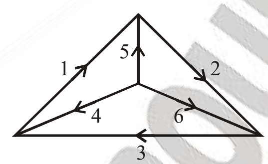
(a) 1 and 4 (b) 3 and 4
(c) 2 and 3 (d) 1 and 2

3. If Q_1 and Q_2 be the sub-matrices of Q_f (fundamental cut-set matrix) corresponding to twigs and links of a connected graph respectively, then
1. Q_t is an identity matrix.
2. Q_t is a rectangular matrix
3. Q_f is of rank $(n - 1)$
Which of the above are correct?

[EE ESE - 2014]

(a) 1and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3

4. For the oriented graph as given above, taking 4, 5, 6 as tree branches the tie set matrix is



[EE ESE - 2013]

(a) $\begin{bmatrix} -1 & 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 1 & 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$

5. For a given connected network and for a fixed tree, the fundamental loop matrix is given by

$$B = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{bmatrix}$$

The fundamental cut-set matrix Q corresponding to the same tree is given by

[EC ESE - 2012]

(a) $Q = \begin{bmatrix} -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

(b) $Q = \begin{bmatrix} -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

CHAPTER - 11
NETWORK FUNCTIONS

11.1 INTRODUCTION

The basic definition of one port and two port network being discussed earlier, here we will discuss about the transform of excitation and response along with their relations. A network function exhibits the relationship between the transform of the source or excitation to the transform of the response for a electrical network. Further to this, we will discuss the stability of the network function mathematically formulating the network function mathematically formulating the network function through "pole-zero" concept.

11.2 DRIVING POINT IMPEDANCE AND ADMITTANCE

The driving point impedance of a one port network is defined as

$$Z(s) = \frac{V(s)}{I(s)} \quad \text{… (i)}$$

While the driving point admittance is given as

$$Y(s) = \frac{I(s)}{V(s)} \quad \text{… (ii)}$$

For the one port network

Similarly, for the two port network, the driving point impedance and admittance at port 1 is defined as

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} \quad \text{…(iii)}$$

and $Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$ … (iv)

While the driving point impedance and admittance at the port 2 are designated as

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)} \quad \text{…(iv(a))}$$

and $Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$ … (v)

11.3 TRANSFER IMPEDANCE AND ADMITTANCE

Transfer impedance is defined as the ratio of transform voltage at output port to the transformed current at the input port of a two port network.

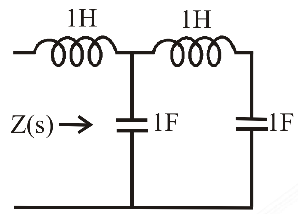
This gives, $Z_{12}(s) = \frac{V_2(s)}{I_1(s)}$ … (vi)

In a similar way, the transfer admittance is defined as the ratio of current transform at output port to the voltage transform at the input port. It is given as

$$Y_{12}(s) = \frac{I_2(s)}{V_1(s)} \quad \text{…(vii)}$$

GATE QUESTIONS

1. The driving point impedance Z(s) for the circuit shown below is

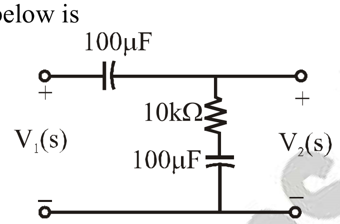


[GATE - 2014]

(a) $\frac{s^4+3s^2+1}{s^3+2s}$ (b) $\frac{s^4+2s^2+4}{s^2+2}$

(c) $\frac{s^2+1}{s^4+s^2+1}$ (d) $\frac{s^3+1}{s^2+s^2+1}$

2. The transfer function $\frac{V_2(s)}{V_1(s)}$ of the circuit shown below is



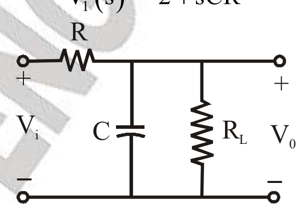
[GATE - 2013]

(a) $\frac{0.5s+1}{s+1}$ (b) $\frac{3s+6}{s+2}$

(c) $\frac{s+2}{s+1}$ (d) $\frac{s+1}{s+2}$

3. If the transfer function of the following network is

$$\frac{V_0(s)}{V_1(s)}=\frac{1}{2+sCR}$$



The value of the load resistance R_L is

[GATE - 2009]

(a) $\frac{R}{4}$ (b) $\frac{R}{2}$

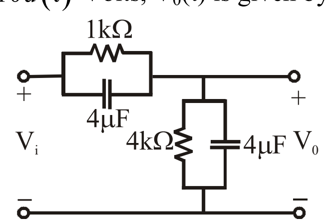
(c) R (d) 2R

4. The first and the last critical frequencies (singularities) of a driving pint impedance function of a passive network having kinds of elements, are a pole and a zero respectively. The above property will be satisfied by

[GATE - 2006]

(a) RL network only
(b) RC network only
(c) LC network only
(d) RC as well as RL networks

5. In the figure shown below, assume that all the capacitors are initially uncharged. If $V_i(t)=10u(t)$ Volts, $V_0(t)$ is given by



[GATE - 2006]

(a) $8e^{-t/0.004}$Volts
(b) $8\left(1-e^{-t/0.004}\right)$Volts
(c) 8u(t) Volts
(d) 8Volts

6. The first and the last critical frequency of an RC-driving point impedance function must respectively be

[GATE - 2006]

(a) A zero and a pole
(b) A zero and a zero
(c) A pole and a pole
(d) A pole and a zero

GATE
2018

POWER SYSTEM

ELECTRICAL ENGINEERING

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CONTENTS

CHAPTER - 1
BASIC CONCEPTS

1.1 INTRODUCTION

1. In electrical engineering the energy is studied in reference to charge (Q). Voltage and current are basic electrical circuit variables.

Voltage is defined as work done on unit charge i.e. $V = \frac{dW}{dQ}\ J/C$ or volts. (V).

Current is defined as time rate of change of charge i.e $I = \frac{dQ}{dt}\ c/s$ or Ampere (A)

Now, $v \times i = \frac{dW}{dQ} \times \frac{dQ}{dt} = \frac{dW}{dt}$

$\frac{dW}{dt}$ is time rate of charge of energy, which basically means power (p)

$p(t) = v(t) \times i(t)$ J/s or watts (w)

2. Current is taken positive in the direction of movement of positive charge and vice-versa.

The voltage at point A (V_A) is voltage with respect to ground.

The voltage between two points V_{AB} is equal to ($V_A – V_B$). Voltage V_{AB} is positive if positive charge looses energy in moving from A to B or negative charge gains energy in moving from A to B.

To calculate V_{AB} start from point B and go towards A, while doing so take voltage rises as positive and voltage drops as negative and add all such voltages.

3. The positive charge will have natural tendency to be repelled by positive charge and attracted by negative charge as shown in the figure (1.1), Q^+ will naturally move towards B and the direction of current will be from A and B as shown figure (1.2). In this case charge is losing energy and device is getting energy from charge such devices are called passive elements or load. Power in this case in positive.

Figure (1.1)

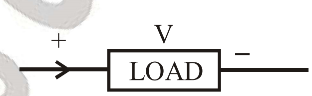


Figure (1.2)

4. In figure (1.3) as shown below Q^+ will have to be supply energy by external medium to move it from B to and direction of current will be from B to in this case charge is gaining energy and device is supplying energy such device are called active elements source or generator. Power in this case is negative.

Figure (1.3)

ESE OBJ QUESTIONS

1. The three non-inductive loads of 5 kW, 3 kW and 2 kW are connected in a star network between R, Y and B phases and neutral. The line voltage is 400V. The current in the neutral wire is nearly

[ESE - 2017]

(a) 11 A (b) 14A
(c) 17 A (d) 21 A

2. A three-phase star-connected load is operating at a power factor angle ϕ with ϕ being the angle between

[ESE - 2017]

(a) Line voltage and line current
(b) Phase voltage and phase current
(c) Line voltage and phase current
(d) Phase voltage and line current

3. Consider the following statements regarding three-phase transformers in Open-Delta (V–V) connections;

1.Being a temporary remedy when one transformer forms of Delta-Delta system is damaged, and removed from service.
2.The Volt Ampere (VA) supplied by each transformer is half of the total VA, and the system is not overloaded.
3.An important precaution is that load shall be reduced by $\sqrt{3}$ times in this case.

Which of the above statements are correct?

[ESE - 2017]

(a) 1 and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3

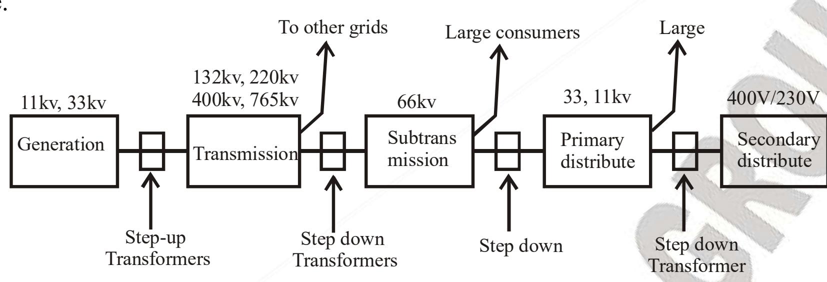
4. In a certain single-phase a.c. circuit the instantaneous voltage is given by $v - V\sin(\omega t + 30°)$ p.u. and the instantaneous current is given by $i = I\sin(\omega t - 30°)$ p.u. Hence the per unit value of reactive power is

[ESE - 2002]

(a) 1/4 (b) 1/2
(c) $\sqrt{3}/4$ (d) $\sqrt{3}/2$

CHAPTER - 2
SINGLE LINE DIAGRAM

2.1 INTRODUCTION
Power system is the study of generation, transmission and distribution of electric energy at very large scale.

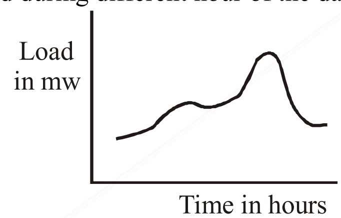


2.2 TRANSMISSION NETWORK
1. Transmission network connect generating plants to consumption point. Transmission is always in 3-ϕ, generally done through over head lines.
2. Transmission network interconnects power pools which are part of grid. Inter connecting power pools has following advantages.
(i) It reduces generation reserve and cost
(ii) It increases reliability
3. Transmission is done at High Voltage (HV). HV transmission has following advantages.
(i) High Voltage for fixed amount of power will have low current which leads to less losses, loss voltage drop.
(ii) Due to reduction in current density conductor cross section area is reduced which leads to less material for same amount of power.
(iii) High voltage also increase power transferable limit and steady state limit.
4. HVAC is called synchronous link
5. HVDC is asynchronous link
6. Voltage levels for HVAC are 765 kv, 400kv, 220 kv and 132kv. Voltage level > 220 kV are extra High voltage (EHV) and voltage levels ≥ 760kv are ultra high voltage (UHV).
7. HVDC up to 500kv is available in India.
8. High voltage needs expenditure on insulating equipments. Hence there is a limit of increasing voltage. High voltage also causes corona interference.
9. In transmission there are problems of corona loss, radio interference, voltage control, load frequency and problem of stability etc.
10. Transmission equipment are
(i) Step up power transformers
(ii) Step down power transformer
(iii) Power transformers are designed with high value of leakage reactance so as to reduce fault current.
(iv) Voltage regulators
(v) Phase shifters to control real power flow
(vi) Transmission lines
(vii) Shunt compensation to maintain voltage profile.

CHAPTER - 3
POWER GENERATION CONCEPT

3.1 INTRODUCTION

1. Load curve
Load curve is the variation of load during different hour of the day, shown on curve.



2. Maximum Demand
The peak load on system is called maximum demand.

3. Connected Load
The sum of the continuous rating of all electrical equipment connected to the supply system is know as connect load.

4. Average Load

$$\text{Daily average load} = \frac{\text{KWh supplied in day}}{24}$$

5. Load Factor

$$\text{Load factor} = \frac{\text{Average load}}{\text{max imum demand}}$$

6. Demand Factor

$$\text{Demand factor} = \frac{\text{Maximum demand}}{\text{Connected load}}$$

7. Diversity Factor

$$\text{Diversity factor} = \frac{\text{Sum of individual max demand}}{\text{max demand of power system}}$$

8. Plant Capacity Factor

$$\text{Capcity or plant factor} = \frac{\text{Averge demand}}{\text{Ratge capacity of plant}}$$

9. Plant Use Factor

$$\text{Plant operating or use factor} = \frac{\text{Total kwh generated}}{\text{Rated capacity} \times \text{number of operating hrs}}$$

CHAPTER - 4
TRANSMISSION LINE PARAMETERS

4.1 INTRODUCTION

4.1.1 Resistance

$$R = \frac{\rho \ell}{A}$$

Where ρ is the resistivity of conductor
ℓ is length of conductor
A is cross sectional area
Resistance changes with temperature

$$R_{T_2} = R_{T_1}\left[1+\alpha\left(T_2 - T_1\right)\right]$$

T_1 & T_2 are temperature is °C
R_{T_1} is resistance at temperature T_1
R_{T_2} is resistance at temperature T_2
α is temperature coefficient of resistance in Ω/°C

4.1.2 Inductance of line

AC flux linkages
$\lambda = N\phi = L\,I$
Where λ & I are in rms
$d\lambda = Nd\phi$
Voltage inducted due to alternating flux linkages

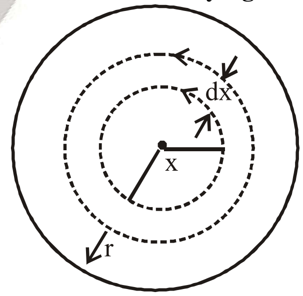
$$V(t) = \frac{d\lambda}{dt}$$

In frequency domain
$V = j\omega\lambda = j\omega LI$
The mutual inductance M_{12} is defined as flux linkages of 1 due to current of 2 as follows

$$M_{12} = \frac{\lambda_{12}}{I_2}$$

Voltage drop in circuit 1 due to current in circuits $V_1 = j\omega\lambda_{12} = j\omega\, M_{12}I_2$

4.1.3 Flux linkages and Inductance of Current carrying Conductor

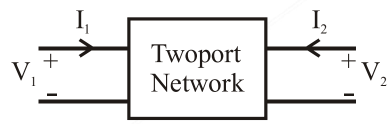


CHAPTER - 5

CHARACTERISTICS AND PERFORMANCE OF POWER TRANSMISSION LINE

5.1 INTRODUCTION

Transmission line can be studies on phase basis under balanced conditions, in this case transmission line can be regarded as two port network. Input port is sending and output port is receiving end.



For transmission line parameter are

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

Here,

$V_1 = V_S$ and $I_1 = I_s$

$V_2 = V_R$ $\quad$ $I_2 = I_R$

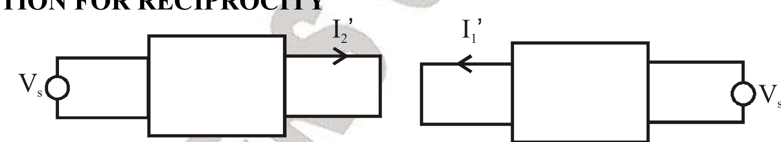
Condition for symmetry

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$\Rightarrow \frac{A}{C} = \frac{D}{C}$$

$\Rightarrow A = D$

5.2 CONDITION FOR RECIPROCITY



$V_s = +BI_2'$

$$-I_1' = CV_s - \frac{A}{D}V_s$$

$$\Rightarrow I_1' = -CV_S + \frac{AD}{B}V_S$$

For reciprocity $I_2' = I_1' \Rightarrow \frac{V_s}{B} = \frac{AD - BC}{B}V_s \Rightarrow AD - BC = 1$

$$\text{\% Efficiency of line} = \frac{\text{Power delivered at receving end}}{\text{Power sent from sending end}} \times 100$$

$$\text{\% Regulation of line} = \frac{V_{r0} - V_r}{V_r} \times 100$$

When V_{r_0} is receiving end voltage at no load

CHAPTER - 6
INSULATED CABLES

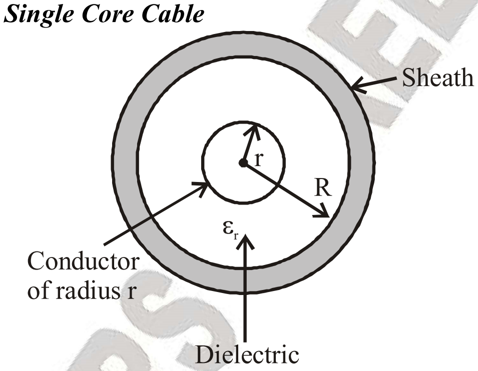
6.1 INTRODUCTION

6.1.1 Electric Cables Consist
1. Conductor for transmitting power
2. Insulation to insulate conductor from direct contact with earth.
3. External protection from mechanical damage etc.

6.1.2 Various Insulting Materials used For Cables
1. Vulcanized Rubber: used for wiring of houses etc for low power usage.
2. Butyl Rubber: More tough can be used for cables without sheath.
3. Neoprene: Used for sheathing material.
4. Poly vinyle chloride PVC:
5. Polythene used for high frequency
6. Impregnated paper

(i) Protective Coverings
These are to protect cable like lead alloy sheath, steel tapes, steel wires etc.

(ii) Electrostatic Stress in Single Core Cable



$$E = g = \frac{q}{2\pi \in x}$$

$$V = \int_r^R E \cdot dx$$

$$= \frac{q}{2\pi \in} \ln \frac{R}{r}$$

$$g = \frac{V}{x \ln(R/r)}$$

g will be maximum at the surface of conductor i.e. x = r

$$g_{max} = \frac{V}{r \ln \frac{R}{r}}$$

Gradient is minimum at the inner radius of the sheath

CHAPTER - 7
OVERHEAD LINE INSULATORS

7.1 INTRODUCTION
The insulators for overhead lines provide insulation to the power conductor from ground mainly made by glazed porcelain or toughened glass.

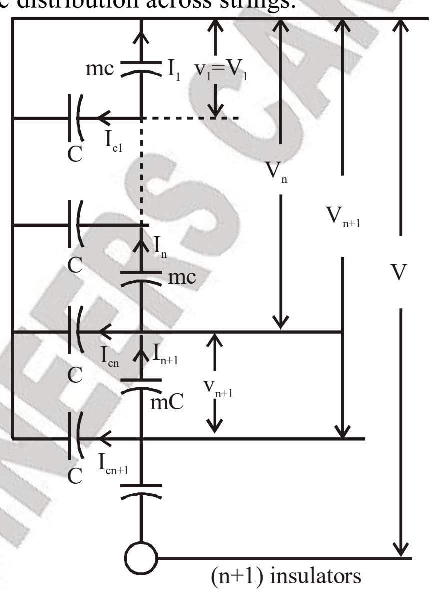
7.1.1 Types
1. Pin type
It is used up to 33kv

2. Suspension type
(i) Each insulator is designed for say 11kv and for higher operating voltages string of insulator disc is used.
(ii) In case of failure of one disc (string) only that need to be replaced.

3. Strain type
These are placed horizontally. These are used to take tension of conductors at line terminals, road crossings etc. They are also known as tension insulators.

4. Shackle type
Used for voltage < 11 kV

7.2 VOLTAGE DISTRIBUTION OVER A STRING OF SUSPENSION INSULATOR
Capacitances are formed between insulator strings and between string and metal parts of structure which causes unequal voltage distribution across strings.



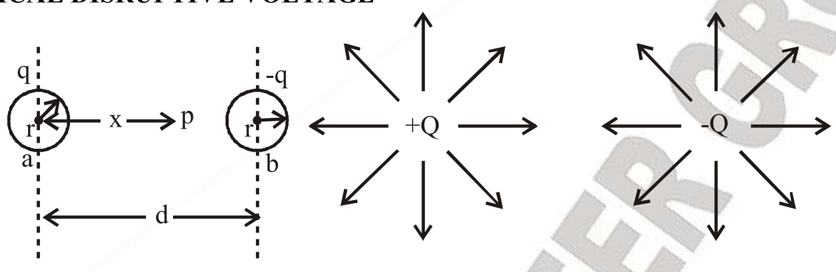
At junction n

$$I_{n+1} = I_{c_n} + I_n$$

CHAPTER - 8
CORONA

8.1 INTRODUCTION
Corona phenomenon is defined as self sustained electric discharge in which the field intensified ionization is localized over a portion of distance between electrodes.
When potential between conductors is increased the gradient around the surface of conductor increases above a voltage higher than critical voltage the nearby air is ionized and there is bluish white glow around the surface of conductor. A hissing noise is also heard along with formation of ozone gas.

8.2 CRITICAL DISRUPTIVE VOLTAGE



Gradient at x

$$E_x = \frac{q}{2\pi \epsilon_0}\left[\frac{1}{x} + \frac{1}{d-x}\right] \text{ and } V_{ab} = \frac{q}{\pi \epsilon_0}\ln\frac{d}{r}$$

After solving

$$E_x = \frac{Vd}{x(d-x)\ln d/r} \quad \text{where v in line to neutral voltage } V = \frac{V_{ab}}{2}$$

Gradient is max when x = r

$$g_{max} = E_r = E_{max} = \frac{Vd}{r(d-r)\ln(d/r)}$$

$$= \frac{V}{r\ln d/r}$$

or $V = r\, g_{max} \ln d/r$

Critical disruptive voltage is the voltage at which disruption or break down of dielectric occurs.
This voltage corresponds to the gradient at the surface of conductor equal to strength of air.
The deflective strength of air at 25°C and 76 cm of Hg pressure is

$g_0 = 30\text{kV/cm}$ Peak

At other temperature or pressure

$g_0' = g_0\delta$

$\delta = \frac{3.92b}{273+\theta}$, where δ is called air density factor.

Where b is pressure in cm of Hg and θ is temperature in °C

CHAPTER - 9
DISTRIBUTION SYSTEMS

9.1 INTRODUCTION

The conductor system by means of which electrical energy is converted from bulk power source to consumers is known as distribution system. Primary distribution is at voltage levels of 11kV, 6.6 kV and 3.3 kV and gives power to bulk consumers like industries.

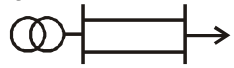
Secondary distribution supply power to households at 400V, 3-ϕ and 230V, single phase.

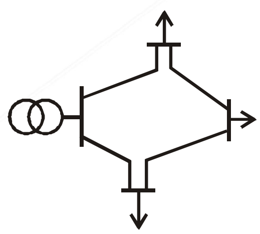
The main criteria for designing of conductors for distribution are voltage drop and current carrying capacity.

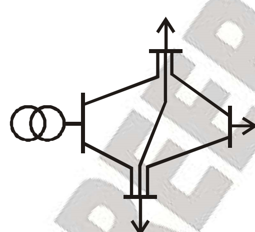
9.1.1 Classification

Distribution systems are classified as per following


Radial feeder

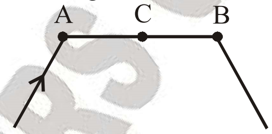

Parallel feeder


Ring main distribution


Inter connected ring main

9.1.2 Uniform Loading

Load in considered uniformly distributed along the length of feeder. For feeder having equal voltage at both ends point of minimum voltage is centre of the feeder.



Max voltage drop from A that $V_{AC} = \frac{2rl^2}{8} = \frac{IR}{8}$

i is current in A/m

r is resistance Ω/m

l is the length of feeder

I = il

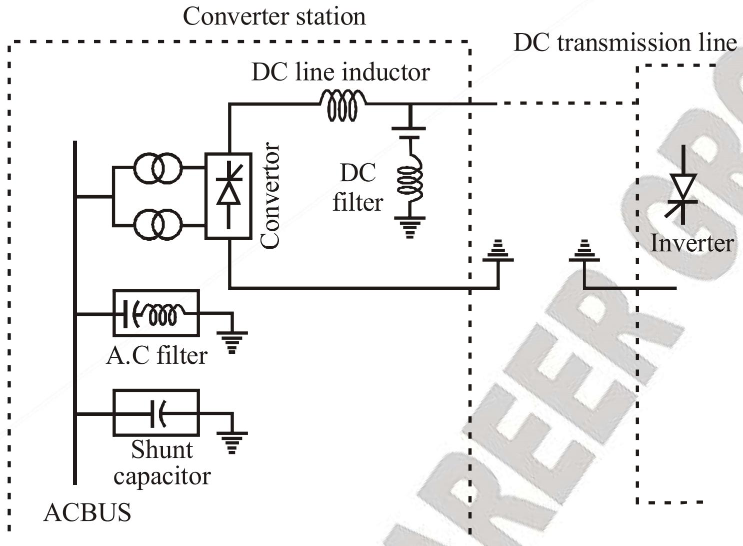
R = rl

Minimum voltage $V_C = \frac{V_A - IR}{8}$

CHAPTER - 10
HIGH VOLTAGE DC TRANSMISSION (HVDC)

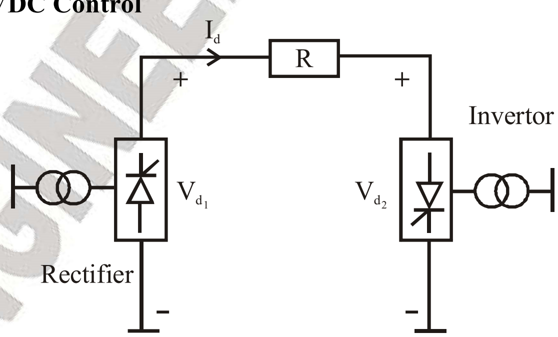
10.1 INTRODUCTION

HVDC transmission consists of two converter stations rectifier and inverter connected to each other by DC cable or overhead line. Main components and arrangement of HVDC transmission is shown below.



Transformer provides suitable voltage ratio to achieve the desired direct voltage, transformer also provide electrical separation between DC and AC systems.
DC line inductor also called smoothing inductor is used to reduce harmonics in current on DC side.
The control of converters introduces phase shift between current and voltage. Thus reactive power is consumed by converters; converter transformers two consume reactive power. This reactive power demand is almost 50-60% of transmitted reactive power. This reactive power at converter stations is supplied by shunt capacitor.

10.1.1 Principle of HVDC Control



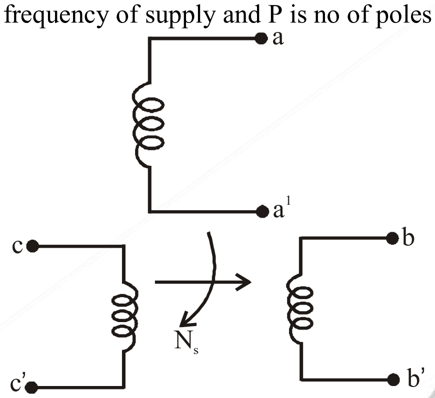
CHAPTER - 11
BASICS OF SYNCHRONOUS MACHINE & R-L CIRCUIT TRANSIENT

11.1 INTRODUCTION

1. When three windings are placed at 120° to each other in space and supply of 3-ϕ, displaced 120° in time to each other is given to such windings than a rotating magnetic field is produced which rotates with synchronous speed in the direction of positive sequence. The synchronous speed.

$$N_s = \frac{120f}{P}$$

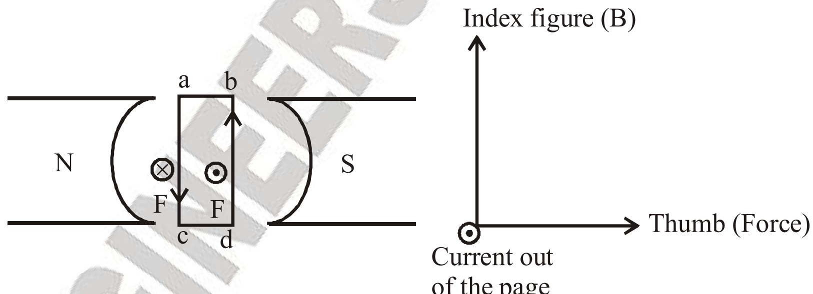
Where f is frequency of supply and P is no of poles



$V_a = V_m \sin\omega t$
$V_b = V_m \sin(\omega t - 120)$
$V_c = V_m \sin(\omega t + 120)$

2. If a conductor moves in a magnetic field than emf is induced in the conductor and if closed path is provided than due to emf current will start to flow in the conductor. The direction of current can be found by Fleming′s Right Hand Rule.

According to Fleming′s Right Hand Rule. Put the thumb of Right Hand in the direction of Force on conductor. The index figure shall be in the direction of magnetic field then middle figure will show direction of current induced.



The synchronous generator has 3-ϕ winding uniformly distributed in slots of stator. The winding are displaced in space 120° to each other.

The rotor is either cylindrical or salient pole type. The rotor is excited By D.C current. The rotor is made to run by prime mover.

CHAPTER - 12
LOAD FLOW

12.1 INTRODUCTION

Load flow is steady state solution of power system network. Load is considered as complex power and not as impedance. The variables are P, Q, |V| & δ at various buses as per nature of bus. Load flow is useful for monitoring power system and for future planning.

Power injection at i^{th} bus $S_i = S_{Gi} - S_{Di}$
Where S_{Gi} is generation at i^{th} bus and S_{Di} is demand at that bus.
In general current injection at i^{th} bus is

$$I_i = \sum_{k=1}^{n} Y_{ik} V_k \; i$$

Where $Y_{ik} = -y_{ik}$ $(i \neq k)$; where y_{ik} is admittance between i^{th} & k^{th} bus

---

$Y_{ik} = \sum_{k=1}^{n} y_{ik}$ When i = k And y_{ii} is admittance between bus and ground or datum bus

$I_{Bus} = Y_{Bus} V_{Bus}$

$$Y_{Bus} = \text{admittance bus matrix} = \begin{bmatrix} Y_{11} & Y_{11} & ....Y_{1n} \\ Y_{21} & Y_{22} & ....Y_{2n} \\ \vdots & & \\ Y_{n1} & Y_{n2} & ....Y_{nn} \end{bmatrix}$$

Y_{BUS} is a symmetric matrix hence only $\frac{n^2 - n}{2} + n = \frac{n(n+1)}{2}$ terms are to be saved.

Y_{BUS} is generally a sparse matrix i.e. having very few non-zero terms.

12.2 BUS INCIDENCE MATRIX

Bus incidence matrix is formed by graph theory and by singular transformation of primitive Y_p matrix if A is Bus incidence matrix then.
$Y_{BUS} = A^T Y_p A$

12.2.1 Effect of Adding or Removing Extra Lines in Y_{BUS}

Assuming no mutual coupling between transmission lines addition of admittance y effects four element if connected between i^{th} & j^{th} bus
Y_{ii} new = $y_{ii\ old} + y$
Y_{jj} new = $y_{jj\ old} + y$
$Y_{ij\ new} = y_{ji\ new} = y_{ij\ old} - y$ and y_{ji} old – y
Addition of element between i^{th} bus and ground effect only
Y_{ii} i.e.
$y_{ii\,new} = y_{ii\,old} + y$

CHAPTER - 13
VOLTAGE CONTROL

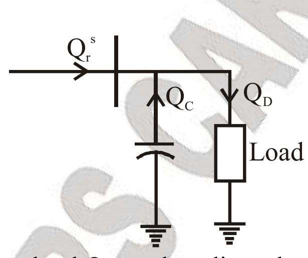
13.1 INTRODUCTION
As we have seen that

$$Q_r = \frac{|V_r|}{X}\Delta V$$

Voltage drop ΔV increases if reactive power demand is increased. Increase in ΔV means that for fixed supply voltage receiving end voltage is decreased.
As we have seen in previous chapter that if loading of line is equal to surge impedance loading (SIL) than voltage remains constant over the line if loading is more than SIL than $V_r < V_s$ and for loading less than SIL, $V_r > V_s$.
Basically the voltage at any node in power system can be controlled by controlling reactive power at that node. If lagging reactive power is supplied at the node by local generator than reactive power taken through line can be maintained at specified value and thus receiving end voltage can be maintained constant.

13.2 METHOD OF VOLTAGE CONTROL
1. By Changing Excitation of Generator
Sending end voltage V_s can be increased by increasing excitation of alternator.

2. By Static VAR Generators or Shunt Compensation
Capacitors known as shunt capacitors are connected at the receiving end. Capacitor is supplier of reactive power.



$Q_r^s + Q_c = Q_D$; when Q_D varies as per load Q_c can be adjusted to maintain Q_r^s constant.
For capacitor bank in Δ

$$Q_{C3-\phi} = \frac{3|V_r|^2}{X_c}\text{MVAR}$$

Where V_r is line voltage in kV
When capacitor bank in Y

$$Q_{c\,3-\phi} = \frac{|V_r|^2}{X_C}\text{MVAR}$$

Some times at light loads V_r becomes greater then V_s in such situations to keep V_r constant. Inductors are connected at receiving end to lower V_r

$$Q_{L3-\phi} = \frac{3|V_r|^2}{x_L}\text{MVAR} \quad \text{for } \Delta$$

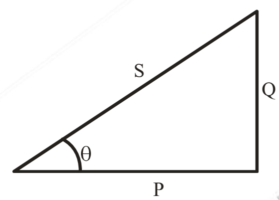
CHAPTER - 14
POWER FACTOR CORRECTION

14.1 INTRODUCTION

Almost 70% load on power system is inductive. The more the load is inductive the poorer the power factor becomes. Low power factor means high current for same amount of real power, High current causes over heating of conductors, large ohmic losers and considerable voltage drop. Hence it is desired to improve the power factor of load.

Inductive load absorbs reactive power. If required reactive power can be supplied locally to the load than power factor of such load will remain same but the power factor of current supplied will change because of less reactive power flow from supply to load.

In terms of powers the power factor angle is shown in power triangle below.



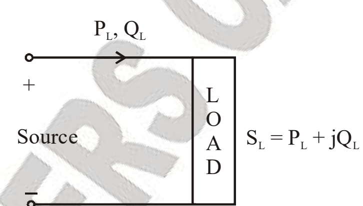
Here S is complex power, P is active or real power and Q is reactive power

$S = p + jQ$

The power factor angle $\theta = \tan^{-1}\frac{Q}{P}$

For large value of Q the angle θ will be more for same P ⇒ Power factor cosθ ↓

Let us consider an inductive load demanding Q_L reactive power. Whole reactive power is supplied through source i.e. $Q_S = Q_L$



Now the power factor angle of source current is

$$\theta = \tan^{-1}\frac{Q_L}{P_L}$$

Capacitor is generator of reactive power. If capacitor is connected across load and supplies Q_C reactive power than p reactive power supplied from source $Q_S = Q_L - Q_C$ and for same real power of load the power factor angle of source current

$$\theta' = \tan^{-1}\frac{Q_L - Q_C}{P_L}$$

We can see $\theta' < \theta \Rightarrow \cos\theta' > \cos\theta$ and hence power of source current in improved.

Reactive power supplied by capacitor C at frequency f is

CHAPTER - 15
ECONOMIC LOAD DISPATCH

15.1 INTRODUCTION
Economic Load dispatch has the aspects

1. Unit commitment

This is pre dispatch issue where in it is required optimal selection of generating units out of pool of generators to meet expected load.

2. Dispatch

where in out of selected generating stations load is distributed in such a way so as reduce overall operating cost.
The cost of generation is not fixed for particular load demand but depend upon the operating constraints of the sources.

15.2 SYSTEM CONSTRAINT
15.2.1 Equality Constraint
$\Sigma P_{G_i} = \Sigma P_{D_i} + P_L$

$\Sigma Q_{G_i} = \Sigma Q_{D_i} + Q_L$

P_L and Q_L are real reactive power loss

15.2.2 Inequality Constraints
1. Generator Constraints

$P_i^2 + Q_i^2 \le S_p^2$

Where s_p is prespecipied KVA loadings
The max P_i is limited by temperature limits and minimum P_i by flame instability of boiler.

$P_{i_{min}} \le P_p \le P_{i_{max}}$

Max Q_i is limited by overheating of rotor winding and min Q_i due to stability limit.

$Q_{i_{min}} \le Q_i \le Q_{i_{max}}$

2. Voltage Constraints

$|V_{i\,min}| \le V_i \le |V_{i\,max}|$

$\delta_{i\,min} \le \delta_i \le \delta_{max}$

3. Running Spare Capacity Constraints

Total Generation G ≥ $P_i + Ps_0$

4. Transmission Line Constraint

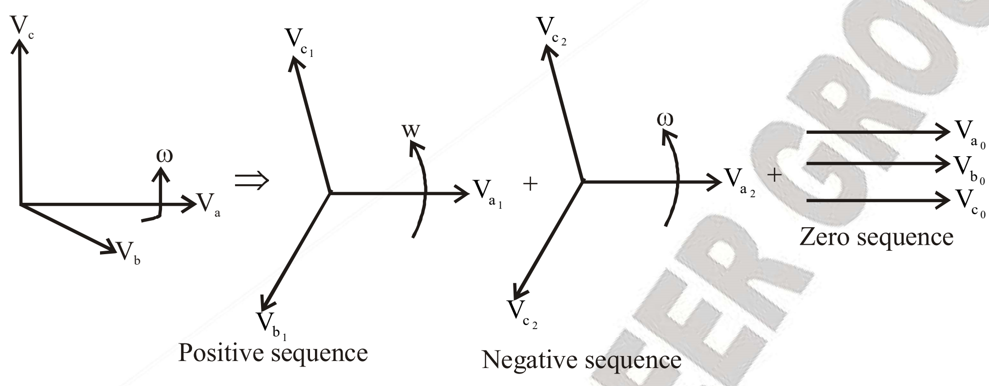
$C_i \le C_{i_{max}}$ where c_i is loading capacity of line

5. Transformer tap settings also are constraints

15.3 GENERATOR OPERATING COST
Only fuel cost for thermal & nuclear is considered. The input energy rate $Fi\left(P_{G_i}\right)$ in Mkcal/h or cost of fuel $C_i\left(P_{G_i}\right)Rs/h$ is function of P_G as sown

CHAPTER - 16
SYMMETRICAL COMPONENTS & SEQUENCE NETWORKS

16.1 FORTESCUE'S THEOREM
An unbalanced set of n phasors may be resolved into (n – 1) balanced n-phase system of different phase sequence and one zero sequence system. As per this theorem the unbalanced three phase may be resolved in 2 balanced 3-phase system one of which is positive sequence (Having same sequence that of unbalanced), Negative sequence (Having sequence positive to that of unbalanced) and zero sequence.



$V_a = V_{a1} + V_{a2} + V_{a0}$

$V_b = V_{b1} + V_{b2} + V_{b0}$

$V_c = V_{c1} + V_{c2} + V_{c0}$

The voltages V_{a1}, V_{a2} & V_{a0} etc are called symmetrical components. The calculation of symmetrical component phasor is made in terms of 'a' as the symmetric phase.

16.2 λ-OPERATOR
The phasor λ is an operator which when operates upon a phasor rotates it by + 120° without changing magnitude.

$\lambda = 1 \angle 120°$

16.2.1 Properties of λ
1. $\lambda = 1 \angle 120° = \cos 120 + j \sin 120$

$\Rightarrow \lambda = -0.5 + j\,\dfrac{\sqrt{3}}{2}$

2. $\lambda^2 = 1 \angle 240° = 1\angle -120° = -0.5 - j\dfrac{\sqrt{3}}{2}$

3. $\lambda^3 = 1 \angle 360° = 1 \angle 0° = 1$

4. $\lambda^3 - 1 = 0$

$\Rightarrow (\lambda - 1)(\lambda^2 + \lambda + 1) = 0$

$\Rightarrow \lambda^2 + \lambda + 1 = 0$

5. $\lambda^* = \lambda^2$

6. $\lambda^4 = \lambda$

CHAPTER - 17
SYMMETRICAL FAULT ANALYSIS

17.1 INTRODUCTION
Generally, the power system works is balanced condition. When a 3-ϕ short circuit occurs, then currents become very high and voltages reduce, still the system remains balanced, such fault are known as symmetrical fault and are severest.

17.1.1 Types of Symmetrical Fault
Symmetrical fault are of the two type.
1. 3-phases coming together or LLL fault.
2. 3-phase touching ground simultaneously or LLLG fault.

The circuit breakers are designed to the limit of short circuit of 3-ϕ.

17.1.2 Purpose of Fault Analysis
The purpose of fault analysis includes:
1. To determine fault voltage and current at different point of power system so that rating of circuit breakers may be determined.
2. Fault analysis helps is selecting appropriate scheme for protective relaying.

Faults may cause severe effects on power system like excessive currents cause heating and rupture of insulation.
In fault analysis certain assumptions are made, which include:
(i) Series resistances of components are neglected.
(ii) Shunt elements of transformers and lines are neglected.
(iii) Normal load currents are neglected i.e. before fault the system is considered open circuited and pre fault voltage at the fault point is taken as $1 \angle 0°$ p.u. This is known as flat profile.

17.2 SHORT CIRCUIT CAPACITY
Short circuit capacity of a network is defined as product of magnitude of pre fault voltage and post fault current.

$SCC \triangleq V^{\circ} |I_F| VA$.

For three phase

$\sqrt{3} V_{lb} I_F = SSC$ in all three phase

If Z_T is the impedance from voltage source to fault point then

$$Z_{T_{p.u}} = \frac{I_b Z_T}{V_b}$$

Now short circuit current $I_{sc} = \frac{V_b}{Z_T}$

$$\therefore \; Z_{T_{p.u}} = \frac{I_b Z_T}{V_b} = \frac{I_b}{I_{sc}} = \frac{V_b I_b}{I_{sc} V_b}$$

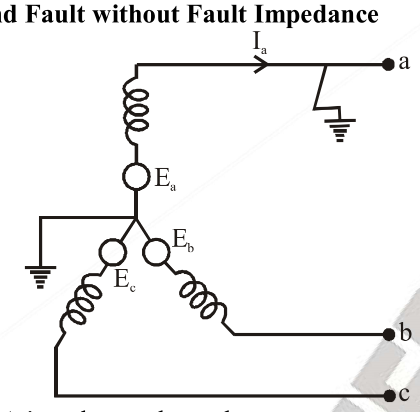
$I_{sc}V_b$ is SCC, hence

$$S.C.C = \frac{S_b}{Z_{T_{p.u.}}}$$

CHAPTER - 18
UNSYMMETRICAL FAULT ANALYSIS

18.1 INTRODUCTION
In unsymmetrical fault the system becomes unbalanced and symmetrical components are used for analysis.

18.1.1 Single Line to Ground Fault without Fault Impedance



Let fault takes place at phase 'a' as shown above then

$V_a = 0$

$I_b = I_c = 0$

Sequence network equation is

$V_{a_0} = -I_{a_0} Z_0$

$V_{a_1} = E_a - I_{a_1} Z_1$

$V_{a_2} = -I_{a_2} Z_2$

$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \lambda & \lambda^2 \\ 1 & \lambda^2 & \lambda \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow I_{a_0} = \frac{I_a}{3} = I_{a_1} = I_{a_2}$

$V_a = 0 = V_{a_1} + V_{a_2} + V_{a_0}$

$\Rightarrow E_a - I_{a_1} Z_1 - I_{a_2} Z_2 - I_{a_0} Z_0 = 0$

$E_a = I_{a_1}(Z_1 + Z_2 + Z_0) \Rightarrow I_{a_1} = \frac{E_a}{Z_1 + Z_2 + Z_0}$

$I_n = I_a = 3I_{a_0}$

If neutral is not grounded then $I_{a_0} = I_{a_1} = I_{a_b} = 0$

Hence if neutral is not grounded then fault current is zero

For neutral grounded through Z_n

CHAPTER - 19
POWER SYSTEM PROTECTION

19.1 INTRODUCTION
1. Protective Relay
A relay is an automatic device which senses an abnormal condition in an electric circuit and closes its contacts, which in turn close the circuit breaker trip coil circuit, thereby, it opens the CB and faulty part of the electric circuit is disconnected from system.

2. Pick up level
The value of actuating quantity above which relay operates.

3. Reset level
The value of actuating quantity below which relay comes in original position.

4. Operating time
The time between the instant when actuating quantity exceeds pick up value to the instant when relay contacts close.

5. Reach
The area of protection is called reach.

19.2 FUNCTIONAL CHARACTERISTIC OF RELAY
1. Reliability
Relay should operate when required.

2. Selectivity
Relay shall sense as to which part is faulty and which is not.

3. Speed
It must operate at required speed.

4. Sensitivity
It Shall be sensitive.

6. Unit system of protection
In this system protection responds to faults within its own zone and does not make note of condition elsewhere for example Protection of Generator, transformers and BUS bars.

7. Non unit system of protection
In this type selectivity is obtained by current and time grading of the relays at different location for example protection of feeders.

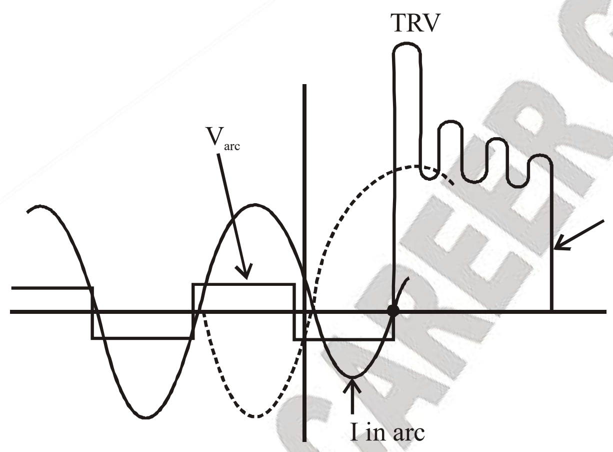
8. Universal torque equation
The universal torque equation for various relays is
$T = K_1 I^2 + K_2 V^2 + K_3 VI\cos(\theta - \tau) + K_4$.

CHAPTER - 20
CIRCUIT BREAKERS

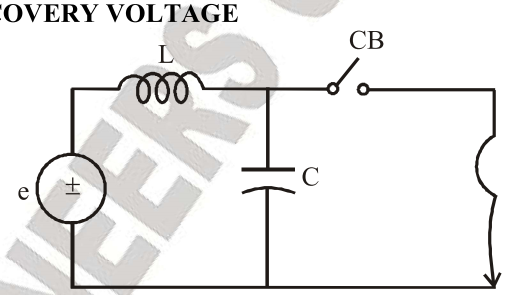
20.1 INTRODUCTION

During fault protective relay energizes the trip circuit of the circuit breaker causing its moving poles to separate from fixed poles. As poles separate electric arc is formed in the air gap feeding the current. For A.C. arc would extinguish at zero current and if it does not strike, the breaker opens successfully. The voltage across the breaker poles is almost constant during arcing phase and after arc is extinguished the ac system voltage appears across poles of circuit breakers.

The voltage opening across the poles when arc extinguishes is known as recovery voltage. Due to inductance and capacitance between source and fault point, transients appear at the time of interruption of current. The voltage across CB in transient period is called transient recovery voltage (TRV). The peak value of TRV may be doubled that of normal voltage, which may cause are to restrike.



20.2 TRANSIENT RECOVERY VOLTAGE



Voltage across CB is voltage across capacitor.
By Laplace transformation the network is redrawn below.

CHAPTER - 21
POWER SYSTEM STABILITY

21.1 INTRODUCTION
Power system stability is the ability of power system to return to normal operating condition after disturbance or in other words ability to remain in synchronism.

1. Steady State Stability

The maximum load which can be supplied when loading is increased gradually is the steady state stability of the system.

2. Dynamic Stability

Dynamic stability is the stability due to small disturbances, amplitude of oscillation is less and die out quickly No fear of loss of synchronism.

3. Transient Stability

Transient stability is under large sudden disturbance like faults. Due to severe disturbance there is fear of loss of synchronism if disturbance not resolved in very short time i.e. with 1 or 2 second.

21.2 DYNAMICS OF SYNCHRONOUS MACHINE
Kinetic energy of rotor

$$K.E. = \frac{1}{2} J\omega_{sm}^2$$

ω_{sm} is synchronous speed

$\omega_s = \frac{P}{2}\omega_{sm}$ where ω_s is speed in elect rad/sec

P = number of poles in the machine

$$K.E = \frac{1}{2}\left(J\left(\frac{2}{P}\right)^2 \omega_s\right)\omega_s$$

$$= \frac{1}{2} M\omega_S$$

Where $M = J\left(\frac{2}{P}\right)^2 \omega_s =$ moment of inertia in J-s/ elect read

Now inertial constant H such that

$$K.E = GH = \frac{1}{2} MW_s$$

Where G is Machine rating in VA

H = inertia constant in J/VA or W –s/VA

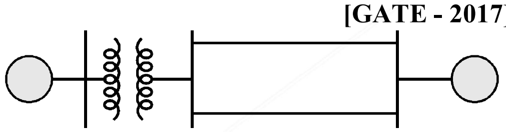
$$M = \frac{2GH}{\omega_s} = \frac{GH}{\pi f} J - s / \text{elect rad}$$

$$M = \frac{GH}{180f} \text{ J-s / elect degree}$$

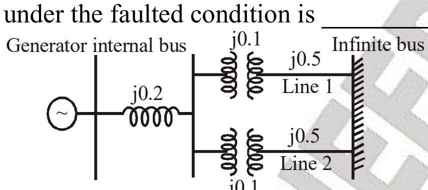
GATE QUESTIONS

1. The figures shows the single line diagram of a power system with a double circuit transmission line. The expression for electrical power is 1.5 sinδ, where δ is the rotor angle. The system is operating at the stable equilibrium point with mechanical power equal to 1 pu. If one of the transmission line circuits is removed, the maximum value of δ, as the rotor swings is 1.221 radian. If the expression for electrical power with one transmission line circuit removed is P_{max} sinδ, the value of P_{max}, in pu is ________.

[GATE - 2017]

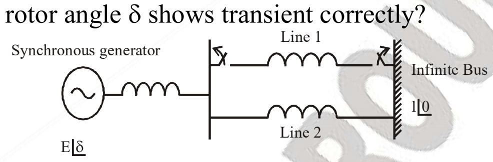


2. The single line diagram of a balanced power system is shown in the figure. The voltage magnitude at the generator internal bus is constant and 1.0 p.u. The p.u. reactances of different components in the system are also shown in the figure. The infinite bus voltage magnitude is 1.0 p.u. A three phase fault occurs at the middle of line 2.
The ratio of the maximum real power than can be transferred during the pre-fault condition to the maximum real power that can be transferred under the faulted condition is __________.

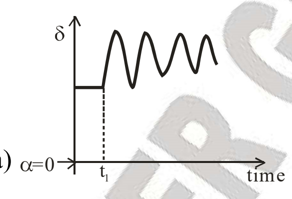


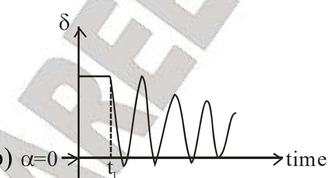
[GATE - 2016]

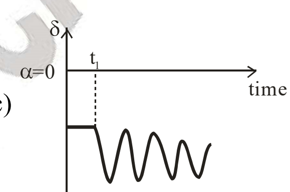
3. The synchronous generator shown in the figure is supplying active power to an infinite bus via short, lossless transmission lines, and is initially in steady state. The mechanical power input to generator and the voltage magnitude E are constant. if one line is tripped at time t_1 by opening circuit breakers at the two ends (although there is no fault), then it is seen that the generator undergoes a stable transient. Which one of the following waveforms of the rotor angle δ shows transient correctly?

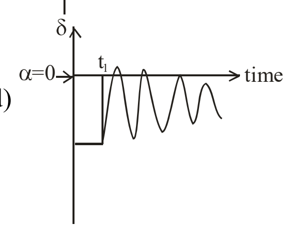


[GATE - 2015]

(a)


(b)


(c)


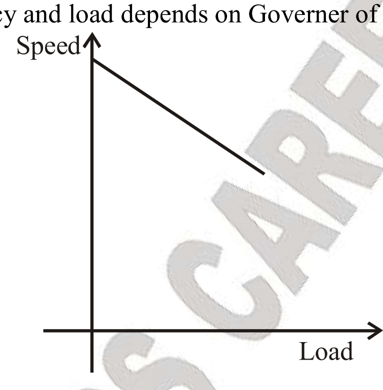
(d)


4. A sustained three phase fault occurs in the power shown in the figure. The current and voltage phasor during the fault (on a common reference), after the natural transients have died

CHAPTER - 22
LOAD FREQUENCY CONTROL

22.1 INTRODUCTION
Due to following reasons system frequency shall remain in striet limits.
1. Speed of A.C. motors is directly related to frequency.
2. The electric clocks are driven by synchronous motors.
3. For frequency outside of range turbine blades may damage.
4. For low frequency the flux required in core increases in transformers and core may saturate.
5. In thermal power plant with reduced frequency the blast of fan decreases ⇒ generation ↓ and this again reduced blast of fan which becomes a cumulative process and the power system may shutdown consequently.

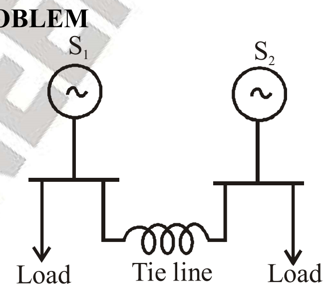
22.1.1 With Increase in Load
$$\frac{Md^2\delta}{dt^2} = P_m - P_e$$
If P_e ↑ then acceleration ↓ and speed reduce thus frequency ↓. This is sensed by load frequency controller and steam input is increased by valve control mechanism thus meeting increased load.
The control of system frequency and load depends on Governer of prime movers.



Governor Characteristic

Governor Characteristic is also called drop characteristic or droop characteristic.

22.2 LOAD-FREQUENCY PROBLEM



SOLUTIONS

Sol. 1. (a)

Sol. 2. (a)

Sol. 3. (b)
Using load frequency controller, the change in frequency and the tie-line real power are sensed which is a measure of the change in rotor angle θ. Hence, Option (b) is correct.

Sol. 4. (b)
$\Delta f_{pu} = -R\Delta P_{pu}$

$= -\frac{0.01}{50}$ $R = 6\% = 0.06$ $= -\frac{0.01}{50} = -0.06 \times \Delta P_{pu}$

$\Delta P_{pu} = 3.33 \times 10^{-3}$ pu

$\Delta P = 120 \times 3.33 \times 10^{-3}$

= 0.4 MW

Hence, option (b) is correct.

Sol. 5. (d)

Sol. 6. (d)

Sol. 7. (b)
Unit of frequency bias coefficient is MW/Hz.

Sol. 8. (d)
Time constant of automatic load frequency control is about 20s.

Sol. 9. (d)

Sol. 10. (b)

Xy + yz = 600 MW, $\frac{xy}{50-f} = \frac{200}{2}$

$\therefore$ xy = 100 (50 – f) …. (i)

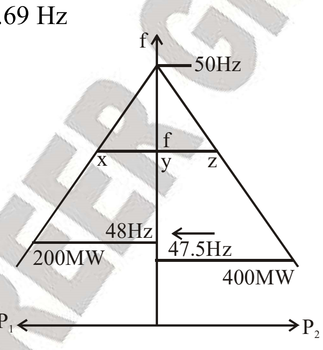
$\frac{yz}{(50-f)} = \frac{400}{2.5} = 160$

$\therefore$ yz = 160 (50 – f) …. (ii)

From equation (i) and (ii),

Xy + yz = (50 – f) 260 = 600

$\Rightarrow$ f = 47.69 Hz



Sol. 11. (a)

$\Delta P_D = -\left(D + \frac{1}{R}\right)\Delta f$

$\therefore\ D + \frac{1}{R} = 2 + \frac{1}{0.025} = 42\ \text{MW/Hz}$

Where, Δf is change in frequency and ΔP_D is change in load demand.