GATE

2019

NETWORK ANALYSIS

ELECTRICAL ENGINEERING

ECG Publications

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Head O ce: S.C.O-121-122-123, 2nd oor, Sector-34/A, Chandigarh-160022

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GATE-2019: Network Analysis| Detailed theory with GATE & ESE previous year papers and detailed solu ons.

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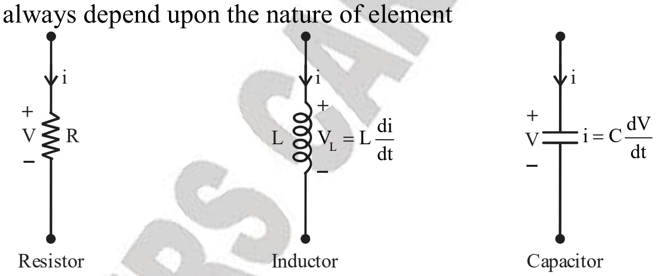
CONTENTS

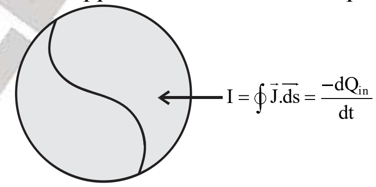
CHAPTER - 1
BASIC CONCEPTS

1.1INTRODUCTION
1.1.1 Charge
Charge can be classified as:
1. Stationary Charge
2. Dynamic Charge

1. Stationary Charge
Stationary charge does not result into electric current because the flow of current means charge moving with net rate across any cross section.
(i) Any electric circuit should always follow law of conservation of charge and law of conservation of energy.
(ii) Circuit theory is analysed always at low frequency and field theory always at high frequency.
(iii) Transit time effect is always neglected at low frequency because
$T >> t_r$
Where T is time period of sinosdual signal
t_r is Transit Time (time taken by signal effect to travel from one point to another point).
(iv) Elemental law is obeyed only at low frequency such as ohm's law. It is not applicable at high frequency because of distributed nature of element.
(v) Elemental law always depend upon the nature of element



For different Element, Different Form of Ohm's Law is present
(i) In time domain, the ohm's law are applicable and also in frequency domain.



Current flowing out of this body is given by equation of continuity as below

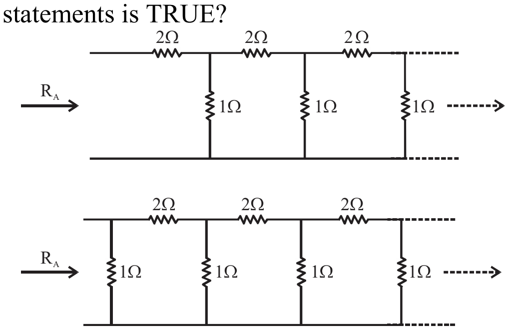
$$I = \oint \vec{J}.\vec{ds} = -\frac{dQ_{in}}{dt} \qquad \ldots(i)$$

This equation gives the law of conservation of charge.

If $\frac{dQ_{in}}{dt} = 0$; means no rate of change of charge within body then eq.(i) become

GATE QUESTIONS

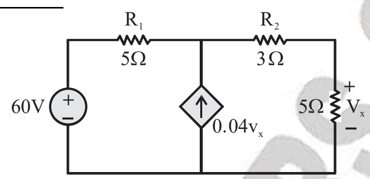
1. R_A and R_B are the input resistances of circuits as shown below. The circuits extend infinitely in the direction shown. Which one of the statements is TRUE?



[GATE - 2016]

(a) $R_A = R_B$ (b) $R_A = R_B = 0$
(c) $R_A < R_B$ (d) $R_B = R_A/(1+R_A)$

2. In the circuit shown in the figure, the magnitude of the current (in amperes) through R_2 is ______

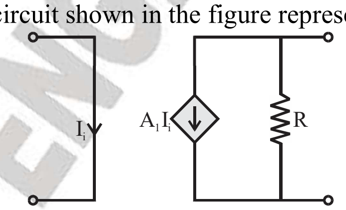


[GATE - 2016]

3. An incandescent lamp is marked 40W, 240V. If resistance at room temperature (26°C) is 120Ω, and temperature coefficient of resistance is 4.8×10^{-3}/°C, then its 'ON' state filament temperature in °C is approximately________

[GATE - 2014]

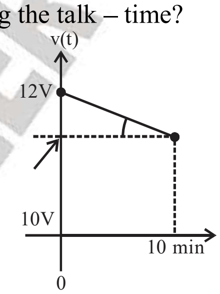
4. The circuit shown in the figure represents a



[GATE - 2014]

(a) Voltage controlled voltage source
(b) Voltage controlled current source
(c) Current controlled current source
(d) Current controlled voltage source

5. A fully charged mobile phone with a 12V battery is good for a 10 minute talk-time. Assume that during the talk – time the battery delivers a constant current of 2A and its voltage drops linearly from 12V to 10V as shown in the figure. How much energy does the battery deliver during the talk – time?

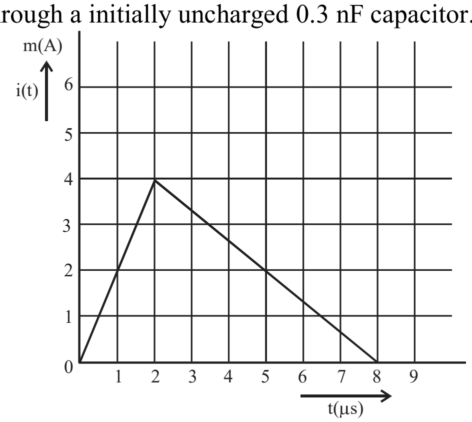


[GATE - 2009]

(a) 220 J (b) 12kJ
(c) 13.2 kJ (d) 14.4 J

Common data for Q. 6 & Q. 7

The current i(t) sketched in the figure flows through a initially uncharged 0.3 nF capacitor.



6. The charge stored in the capacitor at t = 5 μs, will be

[GATE - 2008]

CHAPTER - 2
NETWORK LAWS

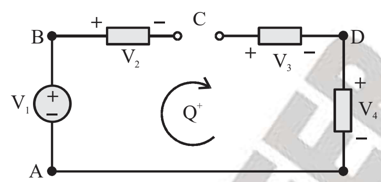
2.1 KIRCHOFF'S VOLTAGE LAW (KVL)

It states that algebraic sum of all voltages in a closed path or loop is zero

$$\sum_{loop} V = 0$$

For writing KVL start from any point in the loop and come to the same point via transversing the path of closed loop. While doing so take voltage rises as positive and voltage drops as negative then

Σ voltage rise + Σ voltage drops = 0



Q^+ positive is moving
A-B is energy is gained say W_1
B-C is energy lost say W_2
C-D is energy lost say W_3
D-A is energy lost say W_4
by consecration of energy
$W_1 = W_2 + W_3 + W_4$ or
$W_1 - W_2 - W_3 - W_4 = 0$

Divide by $Q = \frac{W_1}{Q} - \frac{W_2}{Q} - \frac{W_3}{Q} - \frac{W_4}{Q} = 0$

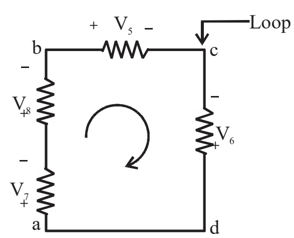
$\Rightarrow V_1 - V_2 - V_3 - V_4 = 0$

ΣV = 0 (In a loop)

Example. KVL in this loop starting from a in clockwise direction is

$- V_7 - V_8 - V_5 + V_6 = 0$

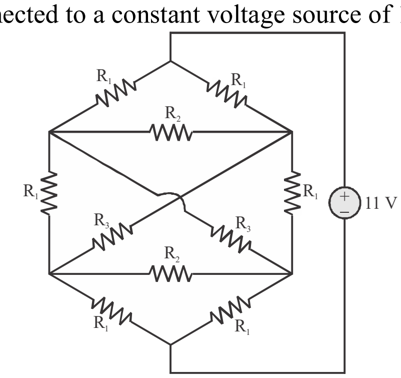
$\Rightarrow V_6 = V_7 + V_8 + V_5$



The basis of the law is that if we start from a particular junction and go round the mesh till we come back to the starting point, then we must be at the same potential with which we started. Hence it means that all the sources of e.m.f. met on the way must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign, plus or minus.

GATE QUESTIONS

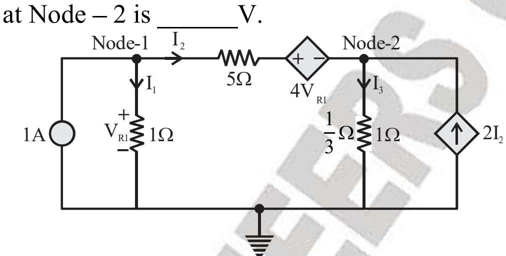
1. Consider the network shown below with $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$ and $R_3 = 3\ \Omega$. The network is connected to a constant voltage source of 11 V.



The magnitude of the current (in amperes, accurate to two decimal places) through the source is __________

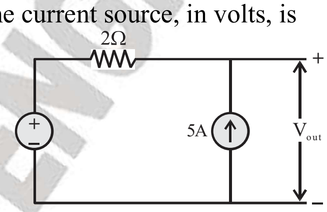
[GATE - 2018]

2. A circuit consisting of dependent and independent sources is shown in the figure. If the voltage at Node -1 is –1V, then the voltage at Node – 2 is ______V.



[GATE - 2017]

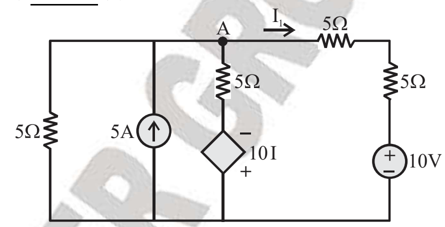
3. In the circuit shown below, the voltage and current sources are ideal. The voltage (V_{out}) across the current source, in volts, is



[GATE - 2016]

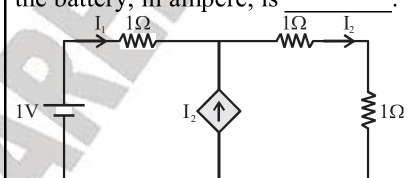
(a) 0 (b) 5
(c) 10 (d) 20

4. In the circuit shown below, the node voltage V_A is _____V.



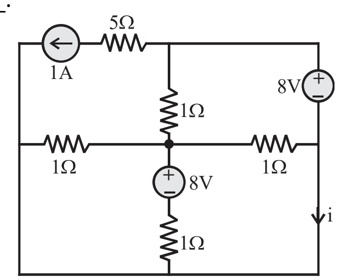
[GATE - 2016]

5. In the given circuit, the current supplied by the battery, in ampere, is ________.



[GATE - 2016]

6. In the figure shown, the current i (in ampere) is _____.

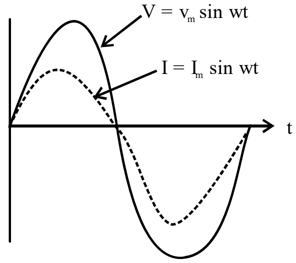


[GATE - 2016]

7. In the given circuit, each resistor has a value equal to 1Ω

CHAPTER - 3
A.C ANALYSIS

3.1 AC THROUGH PURE OHMIC RESISTANCE ALONE



$v = V_m \sin\omega t$

v = iR

$i = \frac{V_m}{R}\sin\omega t$

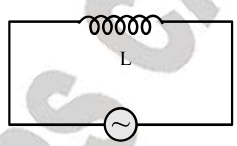
Current is max when sin wt = 1

i.e. $I_m = \frac{V_m}{R}$

$\therefore\ i = I_m \sin\omega t$

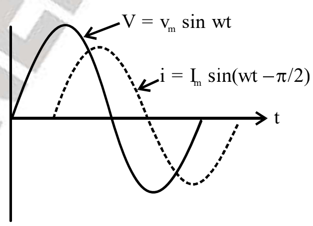
3.2 AC THROUGH PURE INDUCTANCE ALONE

Whenever an alternating voltage is applied to a purely inductive coil, a back emf is produced due to the self-inductance of the coil



$v = L\frac{di}{dt}$

$V_m \sin\omega t = \frac{Ldi}{dt} \Rightarrow di = \frac{v_m}{L}\sin\omega t dt$



$i = \frac{V_m}{L}\int \sin\omega t dt$

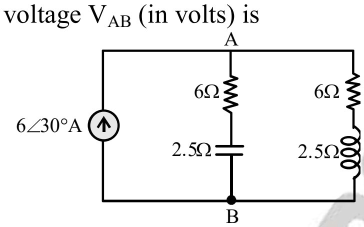
ASSIGNMENT

1. The current flowing through a circuit-containing passive elements like R, L or C is I = 15.5 sin (2500t – 145°) with voltage source V = 311 sin (2500t + 170°). The impedance Z is
(a) $20 \angle -25^o\ \Omega$ (b) $20 \angle 25^o\ \Omega$
(c) $20 + j20\Omega$ (d) $14.14 - j14.14\ \Omega$

2. There is a pure element in series with R = 25Ω which causes the current to lag the voltage by 20°. It is _____ if frequency is 400 Hz.
(a) 35 μF capacitor (b) 3.6 mH inductor
(c) 25μF capacitor (d) 2 mH inductor

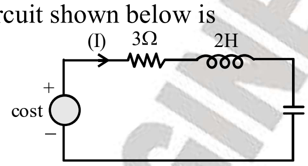
3. There is a pure element in series with R = 50Ω which causes the current to lead the voltage by 30°. It is _______ if frequency is 500 Hz.
(a) 11 μF capacitor (b) 22 μF capacitor
(c) 1.2mH (d) 2.4mH inductor

4. In the AC network shown in the figure. The phasor voltage V_{AB} (in volts) is



(a) $7 \angle 30^o$ (b) $6 \angle 30^o$
(c) $21 \angle 30^o$ (d) $11 \angle 30^o$

5. The differential equation for the current i(t) in the circuit shown below is

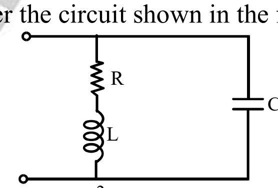


(a) $2\frac{d^2i}{dt^2} + 3\frac{di}{dt} + i(t) = -\sin t$
(b) $2\frac{d^2i}{dt^2} + 3\frac{di}{dt} + i(t) = \sin t$
(c) $2\frac{d^2i}{dt^2} + 3\frac{di}{dt} + i(t) = \cos t$
(d) $2\frac{d^2i}{dt^2} + 3\frac{di}{dt} + i(t) = -\cos t$

Common Data for Q.6 & Q.7
Given below is the current and applied voltage in a series connection of two pure circuit elements.
V = 150 sin (314 t + 10°) volts
i = 15 sin (314 t – 53.4°) amperes

6. The circuit contains a
(a) Resistance of 5Ω and capacitor of 1μF
(b) Resistance of 4.47 Ω and inductor of 0.028 H
(c) Resistance of 6.7Ω and capacitor of 0.01 μF
(d) Resistance of 10Ω and inductor of 0.021 H

7. Consider the circuit shown in the figure



In the circuit $4R^2 C = 3L$, the resonance frequency ω_0 is
(a) $\sqrt{LC}$ (b) $\frac{1}{\sqrt{LC}}$
(c) $\frac{1}{2\pi\sqrt{LC}}$ (d) $\frac{1}{2\sqrt{LC}}$

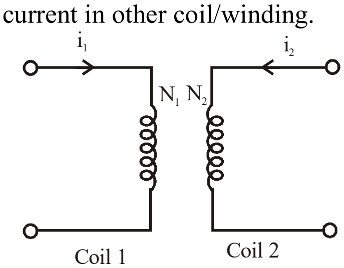
8. A voltage V is represented as V = 50 sin(ωt + 30°) – 25 sin(3ωt – 60°) + 16 sin (5ωt + 45°)V R.M.S. value of the voltage is
(a) 41 volt (b) 41.11 volt
(c) 91 volt (d) 58.14 volt

CHAPTER - 4
MAGNETICALLY COUPLED CIRCUIT

4.1 INTRODUCTION

When two circuits are so placed that a portion of the magnetic flux produced by one links with the turns of both, they are said to be mutually coupled magnetically. This effect is characterized by mutually inductance (M)

Mutual Inductance (M) is the property of magnetic coupling showing an induction of voltage in one coil/winding by a change of current in other coil/winding.



In the above figure two coils 1 and 2 with turns N_1 and N_2 are placed close to each other so that part of flux of one coil links with other coil too. The current i_1 in coil 1 produces flux ϕ_1. Some part of ϕ_1 links only with coil 1 let this is ϕ_{11} this is known as self flux or leakage flux of coil 1. ϕ_{12} is the flux which links with both the coils. ϕ_{12} is called mutual flux. Similarly current i_2 in coil 2 produces ϕ_2 which has ϕ_{22} and ϕ_{21} as its components. ϕ_{22} links only with coil 2 and ϕ_{21} links with both coils.

Now the voltage induced in coil 2 by change in current of coil 1 i_1

$$v_{21} = M_{21}\frac{di_1}{dt}$$

However by Faraday's Law

$$v_{21} = N_2\frac{d\phi_{12}}{dt}$$

$$\Rightarrow M_{21} = N_2\frac{d\phi_{12}}{di_1}$$

$$\Rightarrow M_{21} = N_2\frac{d\phi_{12}}{di_1}$$

If air is the medium between two coils, then magnetization is linear and

$$\frac{d\phi_{12}}{di_1} = \frac{\phi_{12}}{i_1}$$

Hence $M_{21} = \frac{N_2\phi_{12}}{i_1}$

Similarly $M_{12} = \frac{N_1\phi_{21}}{i_2}$

Since the reluctance of both the fluxes i.e. ϕ_{12} & ϕ_{21} is same M_{12} & M_{21} are equal say $M_{12} = M_{21} = M$.

CHAPTER - 5
NETWORK THEOREMS

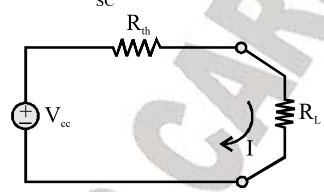
5.1 THEVENIN'S THEORM
Any two terminal bilateral linear circuit can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.

5.1.1 Steps for Solving a Network using Thevenin's Theorem
1. Remove the load resistor (R_L) and find the open circuit voltage (V_{oc}) across the open circuited load terminals.
2. Find the Thevenin's resistance (R_{TH})

1. If Circuit contains only Independent Sources
Deactivate the constant sources (for voltage source remove it by short circuit and for the current source remove it by open circuit) and find the internal resistance (R_{TH}) of the source side looking through the open circuited load terminals.

2. For the circuits containing dependent sources in addition to or in absence of independent sources
Find V_{OC} by open circuiting the load terminals. Then short the load terminals and find the short circuit current (I_{SC}) through the shorted terminals.

Thevenin's resistance is given as: $R_{TH} = \dfrac{V_{OC}}{I_{SC}}$



(i) Obtain thevenin's equivalent circuit by placing R_{TH} in series with V_{OC}
(ii) Reconnect R_L across the load terminals.

5.1.2 Thevenin's Equivalent Network
$$I(\text{Load current}) = \frac{V_{OC}}{R_{TH} + R_L}$$

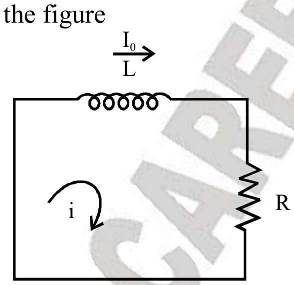
If only dependent sources are present in circuit, $R_{Th} = \dfrac{V_{test}}{I_{test}}; I_{test} = 1A$

V_{test} is calculated across the load by short circuiting it, and current of 1A flows through the short circuited branch as I_{test}. Then $R_{TH} = V_{test}$

CHAPTER - 6
TRANSIENT ANALYSIS

6.1 INTRODUCTION
1. Linear differential equation with constant coefficient obey linearity & superposition theorem.
2. The response of a network excited by a initial energy storage and then left undisturbed is a characteristic of network as with the passage of time networks comes to zero response. This is called natural behavior or its transient response or force +ve behavior. The natural behavior is solution of the network's differential equation with all the sources equated to zero.
3. The response of a network to excitation by an impulse source is very similar to natural behavior.
After $t = 0^+$
The impulse response ≡ natural behavior.
4. For forced response steady state value will not be zero. Than steady state value are calculated.
Complete solution = natural behavior + forced solution
Or transient + steady state
Or complementary function + particular integer

6.2 NATURAL BEHAVIOR OF R-L CIRCUIT
Let the initial current in the inductor is I_0 and the inductor is connected to resistance in series so that inductor discharges. As shown in the figure



Writing KVL in the loop

$$\frac{Ldi}{dt} + Ri = 0 \qquad \ldots(i)$$

The possible solutions for current (i) is

$i(t) = ke^{st}$, where k is constant

Putting this value in equation (i)

$$L\frac{d(ke^{st})}{dt} + Rke^{st} = 0$$

$$\Rightarrow ksL\,e^{st} + Rke^{st} = 0$$

$$\Rightarrow sL + R = 0$$

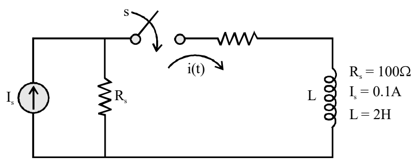
$$\Rightarrow s + \frac{R}{L} = 0 \qquad \ldots(ii)$$

Equation (ii) is the characteristic equation of series R-L circuit.

From equation (ii), $s = -\frac{R}{L}$

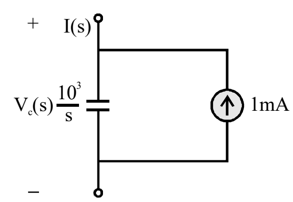
ASSIGNMENT

1. The switch S is closed at t = 0. The rate of change of current $\frac{di}{dt}(0^+)$ is given by



(a) 1A/sec (b) 5A/sec
(c) 2.5 A/sec (d) 3A/sec

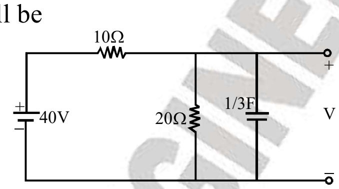
2. A capacitor with some initial voltage can be represented by the shown figure. Where s is laplace transform variable. The value of initial voltage is



(a) 0.5V (b) 2.0V
(c) 1.0 V (d) 0V

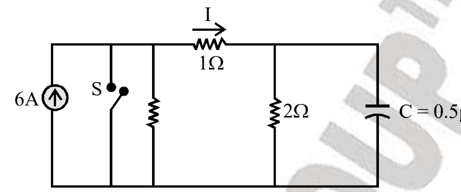
3. A LTI system has an impulse response e^{-2t} for t > 0. If initial conditions are 0 and the input is e^{-3t}, the output for t > 0 is
(a) $e^{-2t} - e^{3t}$ (b) $e^{-2t} - e^{-3t}$
(c) e^{-5t} (d) None of these

4. Consider the network shown below, if the voltage V at a time is 20V, then dV/dt at that time will be



(a) 1V/s (b) –2V/s
(c) 3 V/s (d) – 4 V/s

5. In the circuit shown below, the switch S is open for a long time and closed at t = 0

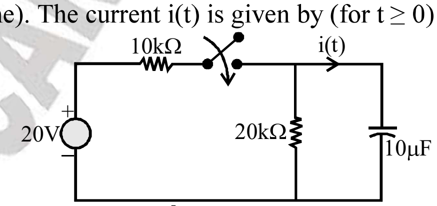


The value of I at t = 0^+ is

(a) – 6A (b) $-\frac{3}{2}$A
(c) 3A (d) $\frac{3}{2}$A

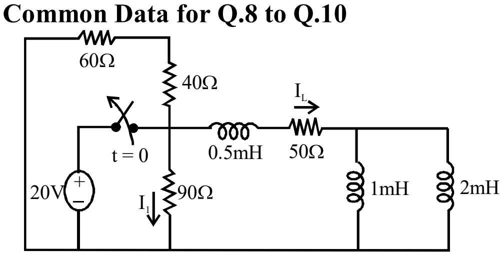
6. $\frac{R}{L}$ has the unit of
(a) Farad (b) Farad^2
(c) $\sec^{-1}$ (d) sec

7. For the circuit shown, the switch is closed at t = 0 (after having been open for a very long time). The current i(t) is given by (for t ≥ 0)



(a) i(t) = 10 $(1 - e^{-5t})$ mA
(b) i(t) = $(12 - e^{-5t})$ mA
(c) i(t) = $(10 - e^{-10t})$ mA
(d) i(t) = $(2e^{-15t})$ mA

Common Data for Q.8 to Q.10



Switch is opened at t = 0

CHAPTER - 7
LAPLACE TRANSFORMATION AND ITS APPLICATION IN CIRCUIT ANALYSIS

7.1 LAPLACE TRANSFORMATION

The Laplace transformation of a function $f(t)$ is defined as

$$F(s) = Lf(t) = \int_0^\infty f(t)e^{-st}dt$$

Where as in complex frequency
s being the intermediate or transformation variable.

7.1.1 Laplace Transform of a Derivative $\left[\frac{df(t)}{dt}\right]$

$$Lf'(t) = sF(s) - f(0+)$$

7.1.2 Laplace Transform of an Integral $\int f(t)dt$

$$L\left(\int f(t)dt\right) = \frac{1}{s}\int f(t)dt\ \Big|_{0+} + \frac{1}{s}F(s)$$

$$\left[\int f(t)\Big|_{0+} \text{ gives the value of the integral at } t = 0+\right]$$

7.1.3 Frequency Shifting

$$L\left(e^{at}f(t)\right) = F(s-a)$$

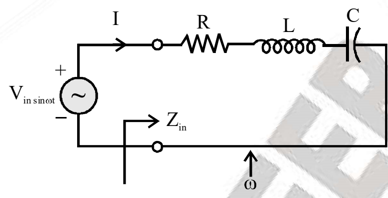
$$L\left(e^{-at}f(t)\right) = F(s+a)$$

7.2 LAPLACE TRANSFORM OF COMMON FORCING FUNCTIONS

$f(t)$	F(s)	$f(t)$	F(s)
u(t)	$\frac{1}{s}$	$e^{-\alpha t}t^n$	$\frac{\lfloor n}{(s+\alpha)^{n+1}}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$	$e^{-\alpha t}\sin\omega t$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$	$e^{-\alpha t}\cos\omega t$	$\frac{s+a}{(s+\alpha)^2+\omega^2}$
$\cos\omega t$	$\frac{\omega}{s^2+\omega^2}$	$\delta(t)$	1
t	$\frac{1}{s^2}$	Sinh θt	$\frac{\theta}{s^2-\theta^2}$

CHAPTER - 8
RESONANCE

8.1 RESONANCE
Resonance in electrical circuits consisting of passive and active elements represents a particular state of the circuit when the current or voltage in the circuit is maximum or minimum with respect to the magnitude of excitation at a particular frequency, the circuit impedance being either minimum of maximum at the power factor unity.
The phenomenon of resonance is observed in both series or parallel a.c. circuits comprising of R, L and C and excited by an a.c. source.

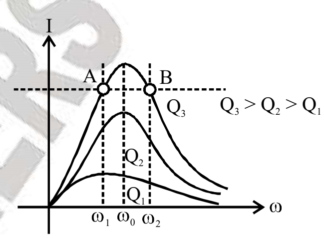
8.2 SERIES RESONANCE


$$Z_{in} = \frac{V}{I} = R + j(\omega L - \frac{1}{\omega c})$$

For resonance V & I must be in same phase
So for some frequency $\omega = \omega_0$

$$Z_{in} = R + j_0 \Rightarrow \omega_0 L - \frac{1}{\omega_0 C} = 0 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$I = \frac{V}{|z|} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega c)^2}} \text{ at } \omega_0,\ I = \frac{V}{R}$$



Points A & B are half power or 3dB points because $20\log_{10}\left(\frac{1}{2}\right) = 3\text{dB}$

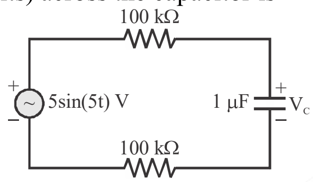
Band width of circuit $\Delta\omega = BW = \omega_2 - \omega_1$
Quality factor

$$Q_0 = 2\pi\left[\frac{\text{Max energy stored}}{\text{Total energy last per perior}}\right]$$

$$Q_0 = 2\pi\left[\frac{\omega_L + \omega_C}{P_R T}\right]$$

GATE QUESTIONS

1. For the circuit given in the figure, the voltage V_C (in volts) across the capacitor is



[GATE - 2018]

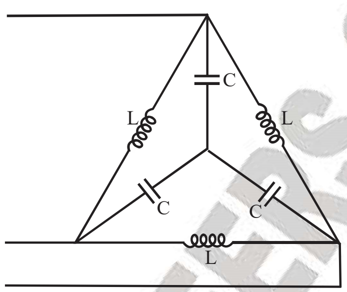
(a) $1.25\sqrt{2}\sin(5t-0.25\pi)$

(b) $1.25\sqrt{2}\sin(5t-0.125\pi)$

(c) $2.5\sqrt{2}\sin(5t-0.25\pi)$

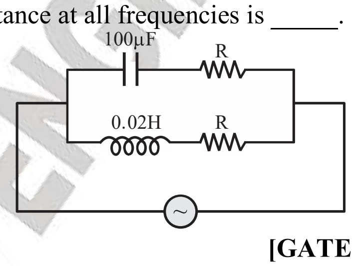
(d) $2.5\sqrt{2}\sin(5t-0.125\pi)$

2. In the balanced 3-phase, 50 Hz, circuit shown below, the value of inductance (L) is 10 mH. The value of the capacitance (C) for which all the line currents are zero, in millifarads, is ______.



[GATE - 2016]

3. The circuit below is excited by a sinusoidal source. The value of R, in Ω for which the admittance of the circuit becomes a pure conductance at all frequencies is _____.



[GATE - 2016]

4. A series RLC circuit is observed at two frequencies. At ω_1 = 1krad/s, are note that source voltage $V_1 = 100\angle 0°$V result in a current $I_1 = 0.03\angle 31°$A. At ω_2 = 2krad/s, the source voltage $V_2 = 100\angle 0°$V results in a current $I_2 = 2\angle 0^{\circ}$ A. The closest values for R, L, C out of the following options are

[GATE - 2014]

(a) R = 50Ω; L = 25mH; C = 10μF;

(b) R = 50Ω; L = 10mH; C=25μF;

(c) R = 50Ω; L = 50mH;C = 5μF;

(d) R = 50Ω; L = 5mH; C = 50μF;

5. Two magnetically uncoupled inductive coils have Q factors q_1 and q_2 at the chosen operating frequency. Their respective resistances are R_1 and R_2. When connected in series, t heir effective Q factor at the same operating frequency is

[GATE - 2013]

(a) $q_1 + q_2$

(b) $(1/q_1)+(1+q_2)$

(c) $(q_1R_1+q_2R_2)/(R_1+R_2)$

(d) $(q_1R_2+q_2R_1)/(R_1+R_2)$

6. For parallel RLC circuit, which one of the following statements is NOT correct?

[GATE - 2010]

(a)The bandwidth of the circuit decreases if R is increased

(b)The bandwidth of the circuit remains same if L is increased

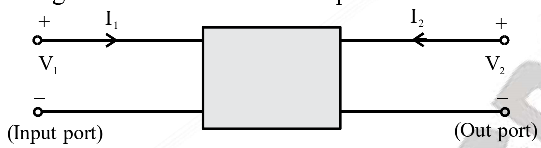
(c)At resonance, input impedance is a real quantity

(d)At resonance, the magnitude of input impedance attains its minimum value

7. The resonant frequency for the given circuit will be

CHAPTER - 9
TWO PORT NETWORKS

9.1 INTRODUCTION
The terminal pair is called as a ″port″. If the current entering one terminal of a pair is equal and opposite to the current leaving the other terminal of the pair.



9.2 TWO-PORT NETWORK
A two-port network is shown, by which we observe that a two-port network is represented by a black box with four variables, namely, two voltages (V_1, V_2) and two currents (I_1, I_2) which are available for measurements and are relevant for the analysis of two port networks. Of these four variables which two variable may be considered `independent` and which two `dependent` is generally decided by the probable under consideration

Two Port Parameters			
Name	**Express**	**In terms of**	**Matrix Equation**
Open circuit impedance [Z]	V_1, V_2	I_1, I_2	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
Short-circuit admittance [Y]	I_1, I_2	V_1, V_2	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$
Transmission or Chain [T] or [ABCD]	V_1, I_2	V_2, I_2	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$
Inverse Transmission [T']	V_2, I_2	$V_1, -I_1$	$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$
Hybrid (h)	V_1, I_2	I_1, V_2	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$
Inverse hybrid (g)	I_1, V_2	V_1, I_2	$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & h_{12} \\ g_{21} & h_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$

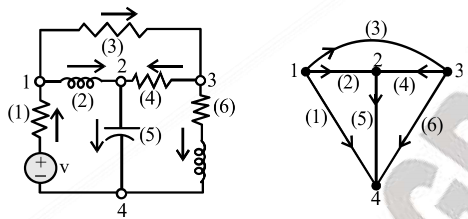
9.3 OPEN CIRCUIT IMPEDANCE (Z) PARAMETERS
Expressing two-port voltages in terms of two-port currents

$(V_1, V_2) = f(I_1, I_2)$

CHAPTER - 10
GRAPH THEORY

10.1 IMPORTANT DEFINITIONS

1. Graph: It is the collection of nodes and Branch of a network.



(A) (B)

2. Branch: Each oriented line segment of the graph is called branch.

3. Node: The end point of a branch is called node.

4. Incident Branch: Branch whose end fall on a node is called incident branch.

5. Connected and Non-Connected Graph
If there exists a path between every pair of nodes of a graph, then the graph is called connected graph, otherwise graph is called non-connected graph.

6. Degree of Node: Degree of Node is the number of branches incident on the node.

7. Subgraph: A portion of graph is called subgraph

8. Path: Path is transverse from one node to another node

9. Loop: Loop is a collection of branches in a graph which form a closed path.

10. Tree: The collection of minimum no. of branches connecting all the nodes of a graph without making a loop.
A single graph can have many no. of trees.
The no. of trees for a given graph= n – 1
where n $\rightarrow$ no. of nodes

11. Twig: Branch of a tree is called a twig.

12. Cotree: Remaining part of a graph after removal of twigs is called cotree. It is collection of links.

13. Links: are the branches removed from the graph to make a tree.
Total no. of branch of a graph are given by $b = (n-1) + L$
n is no. of nodes
L is No. of links

ASSIGNMENT

1. For a given network, the incidence matrix is given by

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

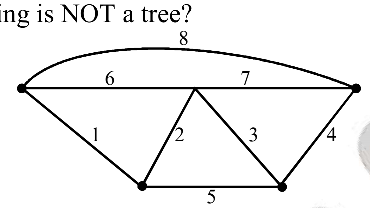
The series branches in the graph are

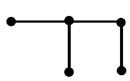
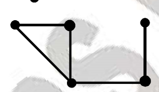
(a) 3 and 4 (b) 3 and 5
(c) 6 and 4 (d) 3 and 6

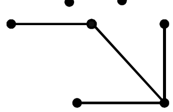
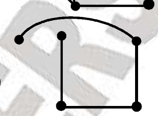
2. In Q. 1, the parallel branches are

(a) 3 and 5 (b) 4 and 5
(c) 3 and 6 (d) None

3. Of the graph shown in below, which of the following is NOT a tree?



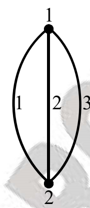
(a)  (b) 

(c)  (d) 

4. If a graph of network has 10 branches and 6 nodes, then number of mesh equations or KVL equation required to solve the network are

(a) 4 (b) 5
(c) 6 (d) 7

5. Of the graph shown in the figure below, the no. of possible trees is/are



(a) 1 (b) 2
(c) 3 (d) 4

6. For given network, the incidence matrix is given by

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

If i_2 = 2A, i_4 = 4A, i_5 2A, where i_k represents current K^{th} branches, then i_6 is given by:

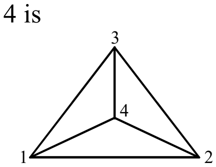
(a) 4A (b) 2A
(c) 0A (d) 6A

7. If the reduced incidence matrix of a given network is given as below, then the no. of possible trees are

$$A = \begin{bmatrix} -1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

(a) 10 (b) 12
(c) 14 (d) 16

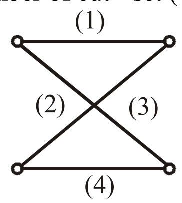
8. This is a network graph. The degree of the nodes 1, 2, 3, 4 is



(a) 2
(b) 3
(c) 2 for 1, 2, 3 and 3 for 4
(d) All 3

GATE QUESTIONS

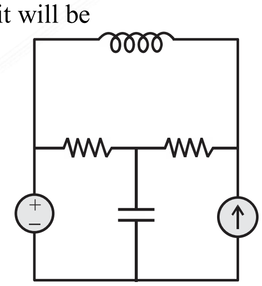
1. In the following graph, the number of trees (P) and the number of cut - set (Q) are



[GATE - 2008]

(a) P = 2, Q = 2 (b) P = 2, Q = 6
(c) P = 4, Q = 6 (d) P = 4, Q = 10

2. The number of chords in the graph of the given circuit will be



[GATE - 2008]

(a) 3 (b) 4
(c) 5 (d) 6

3. The matrix A given below in the node incidence matrix of a network. The columns correspond to branches of the network while the rows correspond to nodes. Let V = $[V_1 V_2 ...... V_6]^T$ denote the vector of branch voltage while I = $[i_1 i_2 .... i_6]^T$ that of branch currents. The vector E = $[e_1\ e_2\ e_3\ e_4]^T$ denotes the vector of node voltage relative to a common ground.

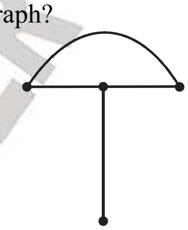
$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{bmatrix}$$

Which of the following statement is true?

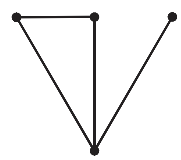
[GATE - 2007]

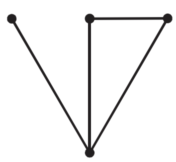
(a) The equation $V_1 - V_2 + V_3 = 0$, $V_3 + V_4 - V^5 - V_5 = 0$ are KVL equation for the network for some loops
(b) The equations $V_1 - V_s - V_6 = 0$, $V_4 + V_5 - V_6 = 0$ are KVL equations for the network for some loops
(c) E = AV
(d) AV = 0 are KVI equations for the network

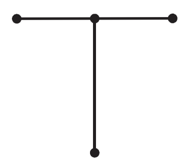
4. Consider the network graph shown in the figure. Which one of ht following is NOT a 'tree' of this graph?



(a) 

(b) 

(c) 

(d) 

[GATE - 2004]

(a) a (b) b
(c) c (d) d

5. The minimum number of equations required to analyze the circuit shown in the figure is

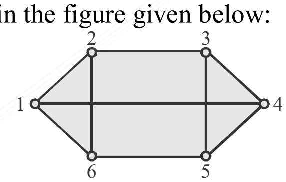
ESE OBJ QUESTIONS

1. Consider the following statements regarding trees:
1. A tree contains all the nodes of the graph.
2. A tree shall contain any one of the loops.
3. Every connected graph has at least one tree.
Which of the above statements are correct?

[EC ESE - 2017]

(a) 1 and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3

2. Consider the following with regards to graph as shown in the figure given below:



1. Regular graph
2. Connected graph
3. Complete graph
4. Non-regular graph
Which of the above are correct ?

[EC ESE - 2017]

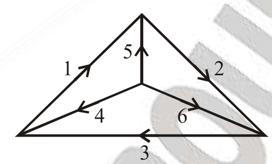
(a) 1 and 4 (b) 3 and 4
(c) 2 and 3 (d) 1 and 2

3. If Q_1 and Q_2 be the sub-matrices of Q_f (fundamental cut-set matrix) corresponding to twigs and links of a connected graph respectively, then
1. Q_t is an identity matrix.
2. Q_t is a rectangular matrix
3. Q_f is of rank $(n - 1)$
Which of the above are correct?

[EE ESE - 2014]

(a) 1and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3

4. For the oriented graph as given above, taking 4, 5, 6 as tree branches the tie set matrix is



[EE ESE - 2013]

(a) $\begin{bmatrix} -1 & 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 1 & 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 & 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$

5. For a given connected network and for a fixed tree, the fundamental loop matrix is given by

$$B = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{bmatrix}$$

The fundamental cut-set matrix Q corresponding to the same tree is given by

[EC ESE - 2012]

(a) $Q = \begin{bmatrix} -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

(b) $Q = \begin{bmatrix} -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

CHAPTER - 11
NETWORK FUNCTIONS

11.1 INTRODUCTION
The basic definition of one port and two port network being discussed earlier, here we will discuss about the transform of excitation and response along with their relations. A network function exhibits the relationship between the transform of the source or excitation to the transform of the response for a electrical network. Further to this, we will discuss the stability of the network function mathematically formulating the network function mathematically formulating the network function through "pole-zero" concept.

11.2 DRIVING POINT IMPEDANCE AND ADMITTANCE
The driving point impedance of a one port network is defined as

$$Z(s) = \frac{V(s)}{I(s)} \quad \text{… (i)}$$

While the driving point admittance is given as

$$Y(s) = \frac{I(s)}{V(s)} \quad \text{… (ii)}$$

For the one port network

Similarly, for the two port network, the driving point impedance and admittance at port 1 is defined as

$$Z_{11}(s) = \frac{V_1(s)}{I_1(s)} \quad \text{…(iii)}$$

and $Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$ … (iv)

While the driving point impedance and admittance at the port 2 are designated as

$$Z_{22}(s) = \frac{V_2(s)}{I_2(s)} \quad \text{…(iv(a))}$$

and $Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$ … (v)

11.3 TRANSFER IMPEDANCE AND ADMITTANCE
Transfer impedance is defined as the ratio of transform voltage at output port to the transformed current at the input port of a two port network.

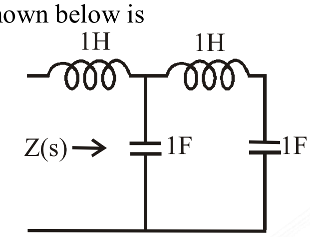
This gives, $Z_{12}(s) = \frac{V_2(s)}{I_1(s)}$ … (vi)

In a similar way, the transfer admittance is defined as the ratio of current transform at output port to the voltage transform at the input port. It is given as

$$Y_{12}(s) = \frac{I_2(s)}{V_1(s)} \quad \text{…(vii)}$$

GATE QUESTIONS

1. The driving point impedance Z(s) for the circuit shown below is

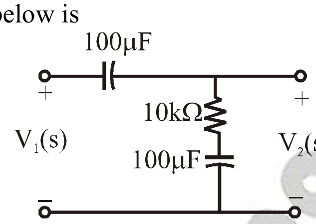


[GATE - 2014]

(a) $\frac{s^4+3s^2+1}{s^3+2s}$ (b) $\frac{s^4+2s^2+4}{s^2+2}$

(c) $\frac{s^2+1}{s^4+s^2+1}$ (d) $\frac{s^3+1}{s^2+s^2+1}$

2. The transfer function $\frac{V_2(s)}{V_1(s)}$ of the circuit shown below is



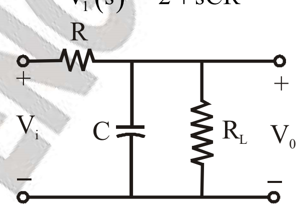
[GATE - 2013]

(a) $\frac{0.5_s+1}{s+1}$ (b) $\frac{3s+6}{s+2}$

(c) $\frac{s+2}{s+1}$ (d) $\frac{s+1}{s+2}$

3. If the transfer function of the following network is

$$\frac{V_0(s)}{V_1(s)}=\frac{1}{2+sCR}$$



The value of the load resistance R_L is

[GATE - 2009]

(a) $\frac{R}{4}$ (b) $\frac{R}{2}$

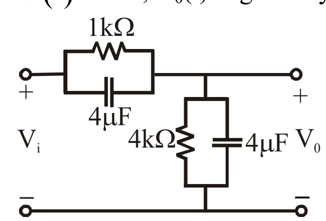
(c) R (d) 2R

4. The first and the last critical frequencies (singularities) of a driving pint impedance function of a passive network having kinds of elements, are a pole and a zero respectively. The above property will be satisfied by

[GATE - 2006]

(a) RL network only
(b) RC network only
(c) LC network only
(d) RC as well as RL networks

5. In the figure shown below, assume that all the capacitors are initially uncharged. If $V_i(t)=10u(t)$ Volts, $V_0(t)$ is given by



[GATE - 2006]

(a) $8e^{-t/0.004}$Volts
(b) $8\left(1-e^{-t/0.004}\right)$Volts
(c) 8u(t) Volts
(d) 8Volts

6. The first and the last critical frequency of an RC-driving point impedance function must respectively be

[GATE - 2006]

(a) A zero and a pole
(b) A zero and a zero
(c) A pole and a pole
(d) A pole and a zero