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CONTENTS

CHAPTER - 1
BASIC CONCEPT

1.1. INTRODUCTION
Theory of machine is a study of relative motion between various parts of Machine and forces acting upon them. It can be further divided into the two parts.

1.1.1 Kinematics
Kinematics is study of motion without considering external force acting on the machine parts.

1.1.2 Dynamics
Dynamics is study of motion with effect of some external force acting on the machine parts.
It can further divide into two parts

1. Statics

It is a study of forces and their effects on machine and its parts, when it is at rest.

2. Kinetics

It is study of combined effect of mass & motion of Machine and its parts

Momentum $\overrightarrow{F_{ext}} = \frac{d}{dt}(m\vec{v})$

1.1.3 Machine
Is a device for transforming and transforming motion and force from one source to another required end to do some desired work.

Input & input force → Desired work and motion.

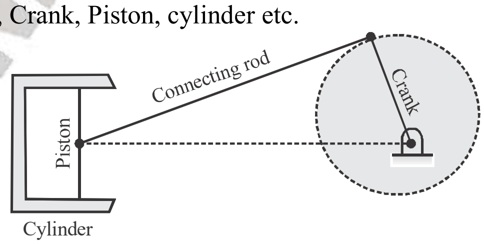
A machine is made of different parts of member which transmit motion and force & these members are called k. links or elements.

1.2 KINEMATIC LINK OR ELEMENT
It is a part of machine which moves relative with respect to some other part of machine. It should be a resistant body so that it is capable of transmitting the motion from one part to other part of machine with negligible deformation.

1.3 CLASSIFICATION OF LINKS
The detailed classification of links is described below:

1.3.1. Rigid Link
It is a link which transmits required motion & force without any significant deformation.

Example. Connecting rod, Crank, Piston, cylinder etc.



Slider Crank Chain Mechanism

IAS OBJ QUESTIONS

1. For one degree of freedom planar mechanism having 6 links, which one of the following is the possible combination?

[IAS - 2007]

(a) Four binary links and two ternary links
(b) Four ternary links and two binary links
(c) Three ternary links and three binary links
(d) One ternary link and five binary links

2. Consider the following statements in respect of four bar mechanism:
1. It is possible to have the length of one link greater than the sum of length of the other three links.
2. If the sum of the lengths of the shortest and the longest links is less than the sum of lengths of the other two, it is known as the Grashoff's linkage.
3. It is possible to have the sum of the lengths of the shortest and the longest links greater than that of the remaining two links.

Which of these statements is/are correct?

[IAS - 2003]

(a) 1, 2 and 3 (b) 2 and 3
(c) 2 only (d) 3 only

3. Assertion (A): The kinematic mechanism shown in figure 1 and figure 2 below are the kinematic inversion of the same kinematic chain.

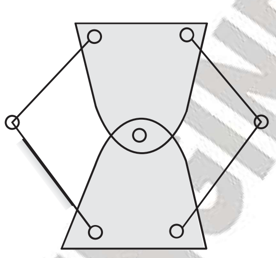


Figure 1

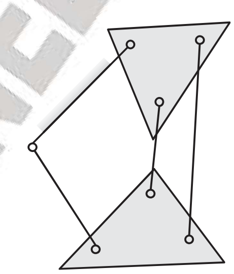


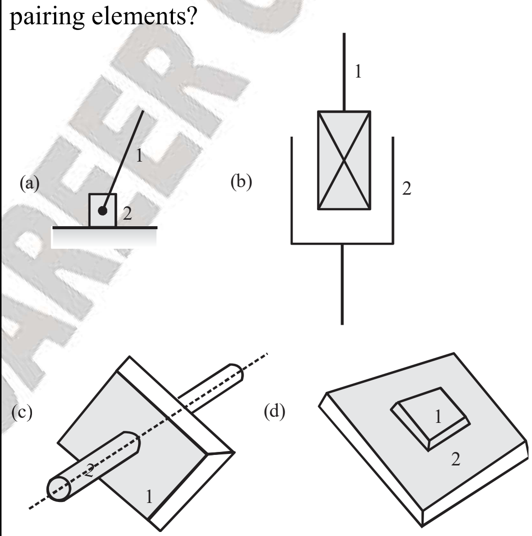
Figure 2

Reason (R): Both the kinematic mechanisms have equal number of links and revolute joints, but different fixed links.

[IAS - 2002]

(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not a correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

4. Which one of the following "Kinematic pairs" has 3 degrees of freedom between the pairing elements?



[IAS - 2002]

5. In a four-link kinematic chain, the relation between the number of links (L) and number of pairs (j) is

[IAS - 2000]

(a) L = 2j + 4 (b) L = 2j – 4
(c) L = 4j + 2 (d) L = 4j – 2

6. The given figure shows a/an

ESE OBJ QUESTIONS

1. Consider the following statements:
1. A kinematic chain is the combination of kinematic pairs joined in such a way that the relative motion between them is completely constrained.
2. The degree of freedom of a kinematic pair is given by the number of independent coordinates required to completely specify the relative movements.
Which of the above statements is/are correct?
[ESE - 2018]
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

2. Which of the following mechanisms are examples of forced closed kinematic pairs?
(i) Cam and roller mechanism
(ii) Door-closing mechanism
(iii) Slider-crank mechanism
Select the correct answer using the code given below.
[ESE - 2017]
(a) (i) and (ii) only (b) (i) and (iii) only
(c) (ii) and (iii) only (d) (i), (ii) and (iii)

3. Assertion (A): Hook's joint connects two non-parallel non–intersecting shafts to transmit motion with a constant velocity ratio.
Reason (R): Hooke's joint connects two shafts the axes of which do not remain in alignment while in motion.
[ESE - 2015]
(a) Both A and R are true and R is the correct explanation of A
(b) Both A and R are true but R is not correct explanation of A
(c) A is true but R is false
(d) A is false but R is true.

4. Which of the following are associated with Ackerman steering mechanism used in automobiles?
1. Has both sliding and turning pairs
2. Less friction and hence long life.
3. Mechanically correct in all positions
4. Mathematically not accurate except in three positions
5. Has only turning pairs
6. Controls movement of two front wheels
[ESE - 2015]
(a) 2, 4, 5 and 6 (b) 1, 2, 3 and 6
(c) 2, 3, 5 and 6 (d) 1, 2, 3 and 5

5. The minimum number of links in a constrained planer mechanism involving revolute pairs and two higher pairs is
[ESE - 2013]
(a) 3 (b) 4
(c) 5 (d) 6

6. Consider the following statements pertaining to an example for a cylindrical pair
1. Piston and cylinder of an IC engine
2. Shaft supported by a foot step bearing
3. Doctor's injection syringe
4. A screw driver operating on a screw
Which of these statements are correct?
[ESE - 2011]
(a) 1 and 4 (b) 2 and 3
(c) 1and 3 (d) 3 and 4

7. The number of degrees of freedom of an epicyclic gear train is
[ESE - 2011]
(a) Zero (b) One
(c) Two (d) Three

8. Consider the following statements:
1. Lower pairs are more resistant than the higher pair in a plane mechanism.
2. In a 4-bar mechanism (with 4 turning pairs), when the link opposite to the shortest link is fixed, a double rocker mechanism results.
Which of the statements given is/are correct?
[ESE - 2006]
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

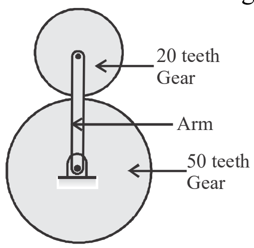
GATE QUESTIONS

1. The number of degrees of freedom in a planner mechanism having n links and j simple hinge joints is

[GATE - 2016]

(a) 3(n – 3) – 2j (b) 3(n – 1) – 2j
(c) 3n-2j (d) 2j–3n+4

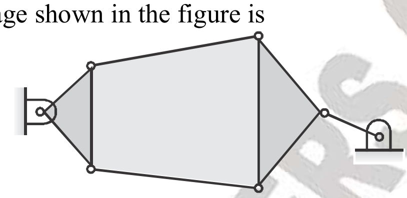
2. The number of degrees of freedom of the planetary gear train shown in the figure is



[GATE - 2015]

(a) 0 (b) 1
(c) 2 (d) 3

3. The number of degrees of freedom of the linkage shown in the figure is



[GATE - 2014]

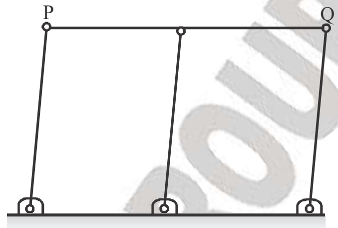
(a) –3 (b) 0
(c) 1 (c) 2

4. In a statically determinate plane truss, the number of joints (j) and the number of members (m) are related by

[GATE - 2014]

(a) j = 3m–3 (b) m = 2j + 1
(c) m = 2j – 3 (d) m = 2j –1

5. A double-parallelogram mechanism in the figure. Note that PQ is a single link. The mobility of the mechanism is



[GATE - 2011]

(a) –1 (b) 0
(c) 1 (d) 2

6. Mobility of a statistically indeterminate structure is

[GATE - 2010]

(a) ≤–1 (b) 0
(c) 1 (d) ≥ 2

7. A planar mechanism has 8 links and 10 rotary joints. The number of degrees of freedom of the mechanism, using Grubler's criterion, is

[GATE - 2008]

(a) 0 (b) 1
(c) 2 (d) 3

8. Match the items in **List-I** and **List-II**

List-I

A. Higher kinematic pair
B. Lower kinematic pair
C. Quick return mechanism
D. Mobility of a linkage

List-II

(i)Grubler's equation
(ii)Line contact
(iii)Euler's equation
(iv)Planer
(v)Shaper
(vi)Surface contact

[GATE - 2006]

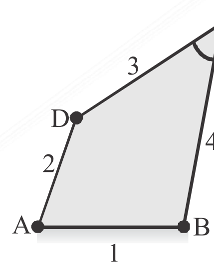
Codes:

(a) A-ii, B-vi, C-iv, D-iii
(b) A-vi, B-ii, C-iv, D-i

CHAPTER - 2
INVERSION OF KINEMATIC CHAINS

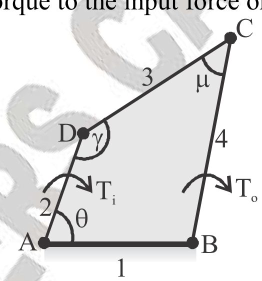
2.1 INTRODUCTION
The method of obtaining different mechanism by fixing different links in a kinematic chain is known as in version of mechanism. Through process of inversion the relative motion between various link is not changed in any manner.

2.2 FOUR BAR CHAIN OR QUADRIC CYCLE CHAIN



As name suggested that, the chain is combination of four kinematic pairs i.e. four Revolute Pair such that the relative motion between the links is completely constrained.
The link AD which is adjacent to the link AB or frame (which is usually fixed) is called as driver (crank) and link BC to which motion is transferred is known as follower (rocker) and link DC which transmits motion from link AD to link BC is known as coupler (connecting rod).

2.2.1 Mechanical Advantage (M.A)
It is the ratio of the output force or torque to the input force or torque at any instant.



From Principle of conservation of energy, we know that:
Power Input = Power output
If T_i is torque applied by input link 2 with angular speed ω_i and T_o is torque obtained by the output link 4 with angular speed ω_o respectively.
Then, $T_i\omega_i = T_o\,\omega_o$ ($\therefore$ P = T ω)

$$M.A = \frac{T_o}{T_i} = \frac{\omega_i}{\omega_o}$$

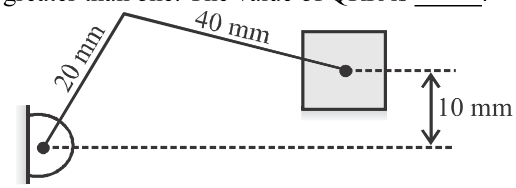
To achieve M.A. to be infinite, the angular velocity ω_0 of the output link 4 becomes zero at extreme positions.
Two extreme condition can be obtained at $\gamma = 0°$ or $\gamma = 180°$. In both conditions the link 2 and link 3 will be in the same line.
These such extreme positions of the mechanism are called as Toggle Position.

GATE QUESTIONS

1. A four bar mechanism has links of length 100mm, 200mm, 300mm and 350 mm. If 350 mm link is fixed, number of links that can fully rotate are?

[GATE - 2018]

2. An offset slider-crank mechanism is shown in the figure at an instant. Conventionally, the Quick Return Ratio (QRR) is considered to be greater than one. The value of QRR is _____.



[GATE - 2014]

3. A 4-bar mechanism with all revolute pairs has link lengths l_f = 20 mm, l_{in} = 40 mm, l_{co} = 50 mm and l_{out} = 60 mm. The suffixes 'f', '*in*', 'co' and 'out' denote the fixed link, the input link, the coupler and output link respectively. Which one of the following statements is true about the input and output links?

[GATE - 2014]

(a)Both links can execute full circular motion
(b)Both links cannot execute full circular motion
(c)Only the output link cannot execute full circular motion
(d)Only the input link cannot execute full circular motion

4. A planar closed kinematic chain is formed with rigid links PQ = 2.0 m, QR = 3.0 m, RS = 2.5 m and SP = 2.7 n with all revolute joints. The link to be fixed to obtain a double rocker (rocker-rocker) mechanism is

[GATE - 2013]

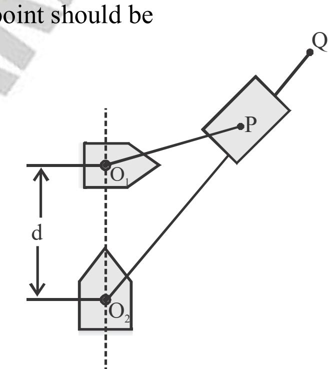
(a) PQ (b) QR
(c) RS (d) SP

5. Which of the following statements is INCORRECT?

[GATE - 2010]

(a) Grashof's rule states that for a planar crank-rocker four bar mechanism, the sum of the shortest and longest link lengths cannot be less than the sum of the remaining two link lengths.
(b)Inversions of a mechanism are created by fixing different links one at a time.
(c)Geneva mechanism is an intermittent motion device.
(d)Gruebler's criterion assumes mobility of a planar mechanism to be one

6. A simple quick return mechanism is shown in the figure. The forward to return time ratio of the quick mechanism is 2:1. If the radius of crank O_1P is 125 mm, then the distance 'd' (in mm) between the crank centre to lever pivot centre point should be



[GATE - 2010]

(a) 144.3 (b) 216.5
(c) 240.0 (d) 250.0

Linked Statement for Q.7 & Q.8

A quick return mechanism is shown below the crank OS is driven at 2 rad/s in counter clockwise direction.

CHAPTER - 3
KINEMATICS OF MACHINE

3.1 INTRODUCTION
Kinematics deals with study of relative motion between the various parts of machine. The motion leads to concept of displacement, velocity and acceleration.

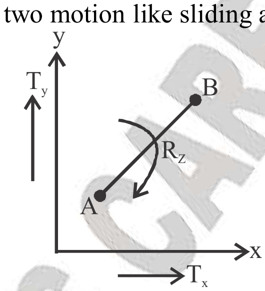
Velocity, $v = \frac{d\vec{s}}{dt}$

Acceleration, $a = \frac{d\vec{v}}{dt}$

$J = \frac{d\vec{a}}{dt}$

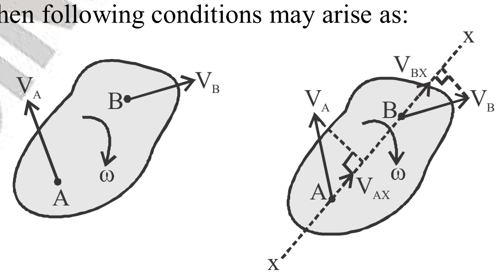
3.2 VELOCITY IN MECHANISM
Velocity analysis of a mechanism can be carried out by following methods:
(i) Relative velocity method
(ii) Instantaneous centre method

3.2.1 Relative Velocity Method
In the planar motion, rigid body has two motion like sliding an rotation as shown in figure



Consider a rigid link AB, which translate in X & Y direction as T_x & T_y respectively. Thus, both end of rigid link translate with same velocity and acceleration along the same path.
Now, consider a link AB, which has rotation along Z direction as R_z. thus, both end of rigid link rotate with same angular velocity and angular acceleration.

1. Direction of Velocity
Consider a rigid body rotating with angular speed (ω) and it has two points A & B on it. The velocity of point A & B are V_A & V_B respectively shown in the figure. Since, the velocity component V_{AX} & V_{BX} parallel to connecting line should be equal if V_{AX} & V_{BX} are if velocity component are different then following conditions may arise as:



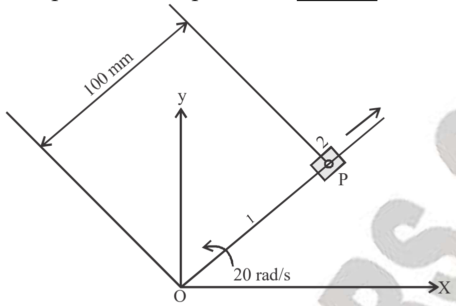
GATE QUESTIONS

1. Which of the statement is correct regarding Oldham coupling?
1. Torsional load is transferred along shaft axis
2. A velocity ratio of 1 : 2 between shaft is obtained without using gears
3. Bending load is transferred perpendicular to shaft axis
4. Rotational movement is transferred along shafts axis

[GATE - 2018]

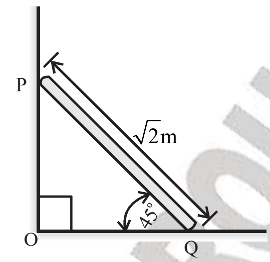
(a) 1 and 3 (b) 1 and 4
(c) 2 and 3 (d) 2 and 4

2. Block 2 slides on link 1 at a uniform of 6 m/s as shown in the figure. Link 1 is rotating at a constant angular velocity f 20 radian/s counterclockwise. The magnitude of the total acceleration (in m/s^2) if point P of the block with respect to fixed point O is______



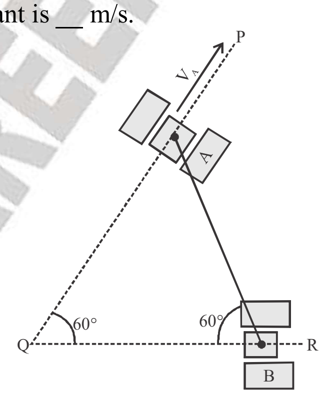
[GATE - 2017]

3. The rod PQ of length L = $\sqrt{2}$m, and uniformly distributed mass of M = 10 kg, is released from rest at the position shown in the figure. The ends slide along the frictionless faces OP and OQ. Assume acceleration due to gravity, g = 10 m/s^2. The mass moment of inertia of the rod about its centre of mass and an axis perpendicular to the plane of the figure is (ML^2/12). At this instant, the magnitude of angular acceleration (in radian/s^2) of the rod is ________.



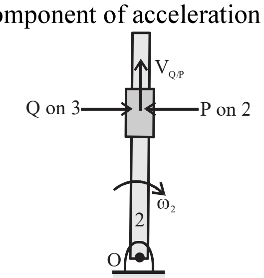
[GATE - 2017]

4. The rod AB, of length 1 m, shown in the figure is connected to two sliders at each end through pins. The sliders can slide along QP and QR. If the velocity V_A of the slider at A is 2 m/s, the velocity of the midpoint of the rod at this instant is __ m/s.



[GATE - 2016]

5. In the figure, link 2 rotates with constant angular velocity ω_2. A slider link 3 moves outwards with a constant relative velocity $V_{Q/P}$, where Q is a point on slider 3 and P is a point on link 2. The magnitude and direction of Corioli's component of acceleration is given by

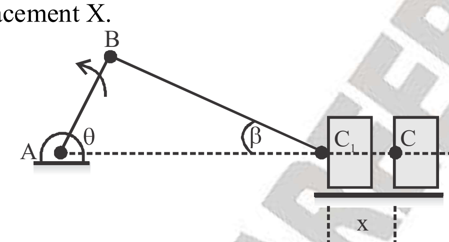


CHAPTER - 4
DYNAMICS OF MACHINE

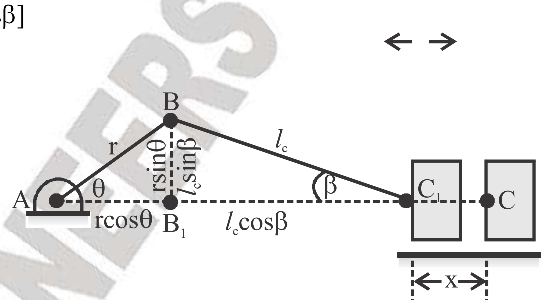
4.1 INTRODUCTION
Dynamic deals with study of forces acting upon the various parts of machine while working order accelerating masses. Dynamic forces are always present when machines working under full or part load condition.

4.1.1 D'Alembert's Principle
It states that, the inertia force and couples, and the external force and torques on a body together gives statically equilibrium.
Thus, $\Sigma F + F_i = 0$
and $\Sigma T + C_i = 0$

4.2 SLIDER-CRANK MECHANISM
Let, the crank AB has turned through angle θ from inner dead centre (IDC) and slider changes its position C to C_1 with displacement X.



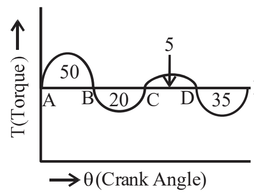
4.2.1 Displacement of Piston
It is a distance travel by piston.
$x = C\,C_1 = CA - C_1A$
$x = (l_c + r) - (r\cos\theta + l_c\cos\beta)$
$= (nr + r) - (r\cos\theta + nr\cos\beta)$ $[\therefore n = l_c/r]$
$= r[n + 1 - \cos\theta - n\cos\beta]$



Where,
$\sin^2\beta + \cos^2\beta = 1$
$\cos\beta = \sqrt{1-\sin^2\beta}$
At BB_1, $r\sin\theta = l_c\sin\beta$
$\sin\beta = \sin\theta \times \frac{r}{l_c} = \frac{\sin\theta}{n}$

IAS OBJ QUESTIONS

1. The crank-effort diagram of an engine running a machine is showing the areas above and below the mean line (in joules). What is the maximum fluctuation of energy in the above diagram?



[IAS - 2009]

(a) 0 J (b) 30 J
(c) 50 J (d) 55 J

2. Which one of the following does a flywheel control?

[IAS - 2009]

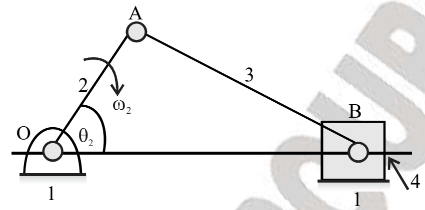
(a)The mean speed of an engine, when load may vary
(b)The cyclic fluctuation of speed with delivering constant output
(c)Variation of load demand of the engine
(d)Mean torque developed by an engine

3. Which one of the following statement true for static balancing of a shaft?

[IAS - 2009]

(a) The net dynamic force acting on shaft is zero
(b) The net couple due to dynamic acting on the shaft is zero
(c) Both (a) and (b)
(d) Neither (a) nor (b)

4. In the figure given below, when is the absolute velocity of end B of the coupler equal to the absolute velocity of the end A of the coupler?



[IAS - 2007]

(a) $\theta_2 = 90°$ (b) $\theta_2 = 45°$
(c) $\theta_2 = 0°$ (d) Never

5. With usual notations for different parameters involved, the maximum fluctuation of energy for a flywheel is given by

[IAS - 2002]

(a) $2EC_s$ (b) $\frac{EC_s}{2}$
(c) $2EC_s^2$ (d) $2E^2C_s$

6. Consider the following statements:
If the fluctuation of speed during a cycle is $\pm$ 5% of mean speed of a flywheel, the coefficient of fluctuation of speed will
1. Increase with increase mean speed of prime mover
2. Decrease with increase of mean speed of prime mover
3. Remain same with increase of mean speed of prime mover

Which of these statements(s) is/are correct?

[IAS - 2001]

(a) 1 and 3 (b) 1 and 2
(c) 3 alone (d) 2 alone

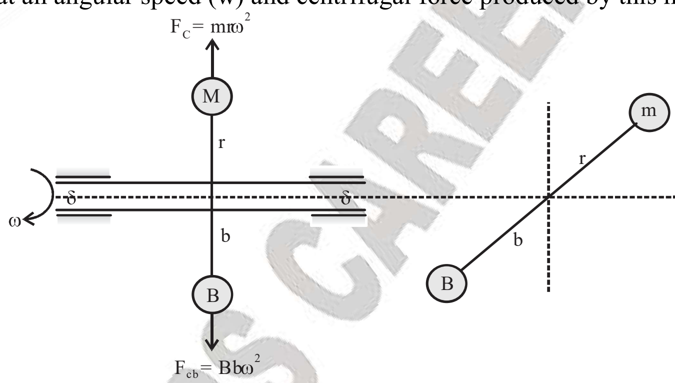
7. Assertion (A): d' Alembert's principle is known as the principle of dynamic equilibrium.
Reason (R): d' Alembert's principle converts a dynamic problem into a static problem.

[IAS - 2000]

CHAPTER - 5
BALANCING

5.1 INTRODUCTION
In an every machine there are two types of parts are commonly used as: rotating part and reciprocating part. It is necessary to balance both types of parts in the machines to avoid unbalance force components. These unbalanced focus case excessive noise, vibrations or and fear of the system.
Meanwhile now days high speed engines and other machines are requirement of each every industry. Thus, it is very essential to balance the machines to reduce or eliminate unbalanced forces.

5.2 BALANCING OF A SINGLE ROTATING MASS
Whenever a certain mass is attached to a rotating shaft, it exists centrifugal force which tends to bend the shaft and produce vibration and also produce lends on bearings. To eliminate the effect of centrifugal force we should attach an another balanced mass in opposite directions in such a way that it eliminate effect of centrifugal force. Let us consider a mass (m) is attached to the shaft which is rotating at an angular speed (w) and centrifugal force produced by this mass is F_C. Thus



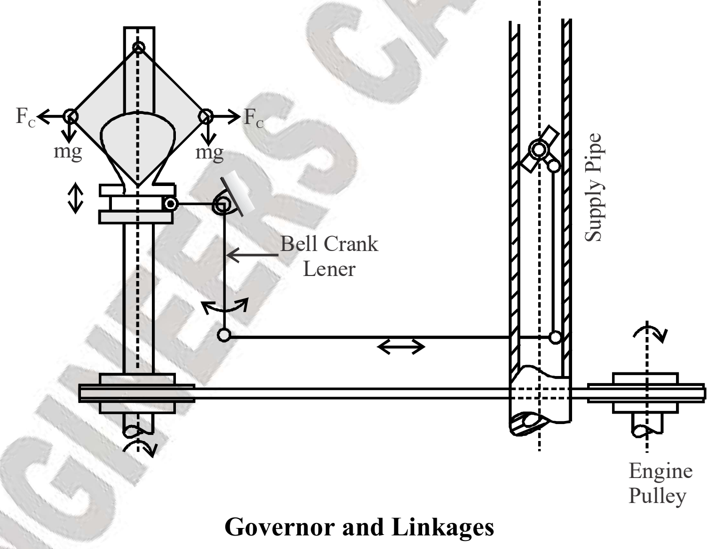
$$F_c = F_{cb}$$
$$mr\omega^2 = Bb\omega^2$$
$$mr = Bb$$

1. Product of mass × radius will be remains same for both original mass and balanced mass respectively.
2. Usually radius of balanced mass is taken larger to reduce the balance mass.
3. Both mass will rotate at same angular speed.

5.3 EXTERNAL BALANCING OF SINGLE ROTATING MASS
If in any case balancing mass cannot possible to place just equal and opposite to original mass then external balancing of mass can be used. In such case, balanced mass to be placed in different

CHAPTER - 6
GOVERNORS

6.1 INTRODUCTION
The flywheel which minimizes fluctuations of speed within the cycle but it cannot minimize fluctuations due to load variation. This means flywheel does not exercise any control over mean speed of the engine. To minimize fluctuations in the mean speed which may occur due to load variation, governor is used. The governor has no influence over cyclic speed fluctuations but it controls the mean speed over a long period during which load on the engine may vary.
When there is change in load, variation in speed also takes place then governor operates a regulatory control and adjusts the fuel supply to maintain the mean speed nearly constant. Therefore, the governor automatically regulates through linkages, the energy supply to the engine as demanded by variation of load so that the engine speed is maintained nearly constant.
The sketch of a governor along with linkages which regulates the supply to the engine. The governor shaft is rotated by the engine shown in the figure. If load on the engine increases the engine speed tends to reduce, as a result of which governor balls move inwards. This causes sleeve to move downwards and this movement is transmitted to the valve through linkages to increase the opening and, thereby, to increase the supply.
On the other hand, reduction in the load increases engine speed. As a result of which the governor balls try to fly outwards. This causes an upward movement of the sleeve and it reduces the supply. Thus, the energy input (fuel supply in IC engines, steam in steam turbines, water in hydraulic turbines) is adjusted to the new load on the engine. Thus the governor senses the change in speed and then regulates the supply instantaneously. Due to this type of action it is simple example of a mechanical feedback control system which senses the output and regulates input accordingly.

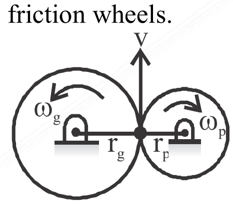


Governor and Linkages

6.2 CLASSIFICATION OF GOVERNORS
The broad classification of governor can be made depending on their operation.
1. Centrifugal governors
2. Inertia and flywheel governors
3. Pickering governors

CHAPTER - 7
GEARS & GEAR TRAINS

7.1 INTRODUCTION
Gears are toothed wheels which are used to transmit rolling and a sliding motion along the tangent at the point of contract. It can be successfully possible by the engagement of teeth.
The concept of gears has been derived from the rolling of two friction cylinders or disc. If there is no slip to be assumed in such case, that can definitely transit motion of one to another and vice-versa. The rotating discs are known as friction wheels.



At a point of contact, the same linear velocity can be obtained as

$$v = \omega_g.r_g = \omega_p.r_p$$

$$v = 2\pi N_g r_g = 2\pi N_p r_p$$

or $\frac{N_g}{N_p} = \frac{r_p}{r_g}$

or $\frac{\omega_g}{\omega_p} = \frac{r_p}{r_g}$

It gives that, the speed of the two rolling discs without slipping is always proportional to the radii of the discs.
The friction wheels can be used to transmit motion at lower speeds. At high speeds it is not possible to transmit continuous motion without slipping. Thus, the concept of gear has been introduced to transmit motion and power at smaller centre distance. This lead to the formation of teeth on the discs and the motion between the surface changes from rolling to sliding. The disc with teeth are called as gear or gear wheel.
To obtain large reduction in velocity sometimes, two or more pair of gears may be used which is called as gear trains.

7.2 ADVANTAGES OF GEARS
Gear has many advantage over belt & chain drive.
1. It can transmit exact velocity ratio with positive drive. (No Slip)
2. It is capable of transmitting higher power.
3. It require less space, which gives compact layout. For large V.R.
4. It gives reliable service with high efficiency. (Tear & wear in less)
5. Can’t use when distance between shaft is more.
6. Costly
7. Special manufacturing m/c required

7.3 CLASSIFICATION OF GEARS
Gear can be classified according to their relative position of axes of the shaft as:

CHAPTER - 8
GYROSCOPE

Gyroscope

If the axis of a spinning or rotating body is given an angular motion about an axis perpendicular to the axis of spin, an angular acceleration acts on the body about third perpendicular axis.

The torque required to produce this acceleration is known as the active gyroscope couple or gyroscopic torque.

A reactive gyroscope couple torque or couple also acts similar to the concept of centripetal and centrifugal forces on a body.

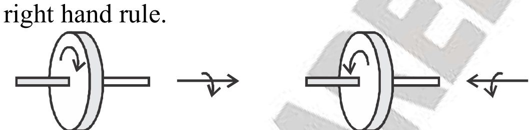
The effect produced by the reactive gyroscope couple is known as gyroscopic effect.

Thus, airplanes, ships, automobiles etc. that have rotating parts in the form of wheel or rotors of engines experience this effect while taking a turn.

Angular Velocity

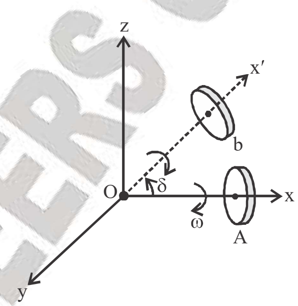
It is defined by Magnitude, direction of axis of rotation (x, y or z) and sense of motion (clockwise (c.c.) or counter clockwise(c.c.w.).

The magnitude of velocity is represented by length of the vector. The direction axis of rotor is represented by drawing a vector parallel to axis of rotor and sense of rotation velocity direction of vector is represented by right hand rule.

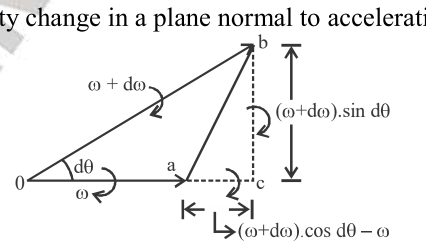


Angular Acceleration

Let a rotor is spinning about horizontal axis OX at a speed ω rad/sec in c.c.w. direction. If the magnitude of the angular velocity changes to (ω + δω) from ω and direction of axis of spin to OX′ in dt time.



1. ac represents angular velocity change in a plane normal to acceleration or x-axis.



2. cb representing angular velocity change in a plane normal to cb or y-axis

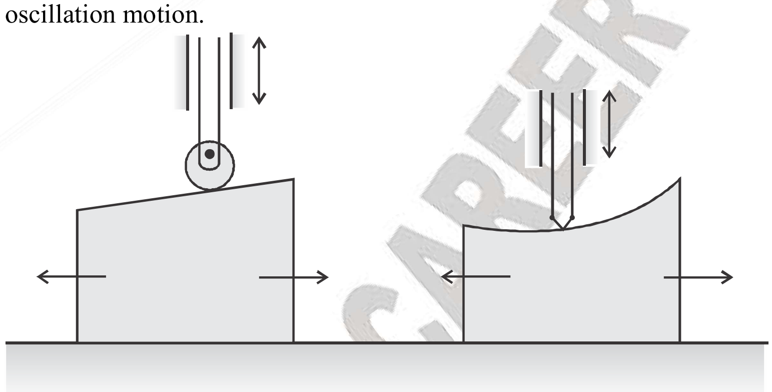
⇒ change in angular velocity w.r.t. time (dt).

CHAPTER - 9
CAMS

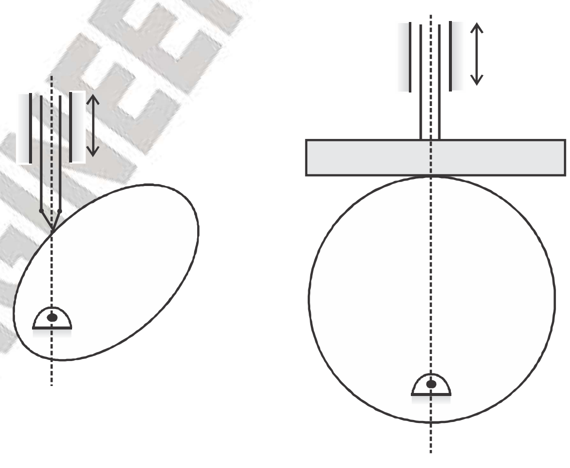
1.1 CAM
Cam is the machine part which is used to impact desired motion to follower by direct contact. The cam may be rotating or reciprocating, while follower can have reciprocating or oscillating motion.

1.2 PARTS OF A CAM
1.Cam-It is driver member in mechanism
2.Follower-It is driven member in mechanism
3.Frame-It supplied cam and guides the follower.

1.2.1 Classifications of Cam
1. On the Basic of Shape
(i) Wedge or Flat Cam- It wedge or flat cam has a translation motion and follower can have either translation or oscillation motion.



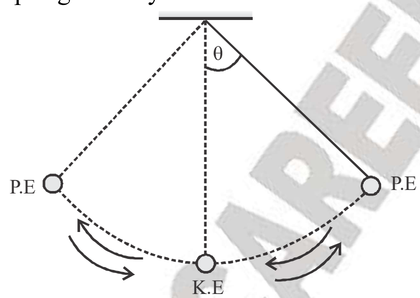
(ii)Radial or Disc Cam
In this type of cam the follower moves radially from the centre rotation of the cam is known as radial disc cam.



CHAPTER - 10
VIBRATION

8.1 INTRODUCTION
All bodies having mass and elasticity are capable of vibration. The mass is inherent of the body and elasticity can relative motion among its parts.
When body particles are displaced by the application of external force, the internal force in the form of elastic energy are present in the body. These forces try to bring the body to its original position.
At equilibrium position, the whole of the elastic energy is converted into kinetic energy and body continues to move in the opposite direction because of it. The whole of the K.E. is again converted into elastic or strain energy due to which the body again returns to the equilibrium position. In this way vibratory motion is repeated indefinitely and exchange of energy takes place.
This, any motion which repeats itself after an internal of time is called vibration or oscillation

Example. Simple pendulum, Spring mass system



The main reasons of vibration are as follows
1. Unbalanced centrifugal force in the system. This is caused because of non uniform material distribution in a rotating machine element.
2. Elastic nature of the system
3. External excitation applied on the system
4. Winds may cause vibrations of certain systems such as electricity lines, telephone lines etc.

8.1.1 Advantages of Vibrations
1. Vibration can be used for useful purposes such as vibration testing equipments, vibrations conveyors, hoppers, sieves and compactors.
2. Vibration is found very fruitful in mechanical workshops such as in improving the efficiency of machining, casting, forging and welding techniques, musical instruments and earth quakes for geological research. Etc.
3. It is very useful for propagation of sound.

8.1.2 Disadvantages of Vibration
1. The vibration causes rapid wear of m/c parts such as bearing and gears.
2. Unwanted vibrations may cause loosening of parts from the machine.
3. Many buildings, structures and bridges fall because of vibration.
4. Mechanical failure of the system if the frequency of excitation coincides with one of the natural frequency of the system, a condition of resonance is reached.
5. Sometimes because of heavy vibrations readings of instruments cannot be taken.

ESE OBJ QUESTIONS

1. Which of the following are the basic building block elements for a mechanical system where forces and straight line displacements are involved without any rotation ?

1. Spring
2. Dashpot
3. Mass
4. Moment of inertia

Select the correct answer using the code given below.

[ESE - 2018]

(a) 1, 2 and 4 (b) 1, 3 and 4
(c) 2, 3 and 4 (d) 1, 2 and 3

2. The equation of motion for a single degree of freedom system is $4\ddot{x}+9\dot{x}+15x=0$
The critical damping coefficient for the system is

[ESE - 2018]

(a) $4\sqrt{2}$ (b) 4
(c) $16\sqrt{2}$ (d) 16

3. The mass of a single degree damped vibrating system is 7.5 kg and it makes 24 free oscillations in 14s when disturbed from its equilibrium position. The amplitude of vibration reduces to 0.25 of its initial value after five oscillations. Then the logarithmic decrement will be

[ESE - 2018]

(a) $\frac{2}{5}\log_e 4$ (b) $\frac{1}{5}\log_e 6$
(c) $\frac{1}{5}\log_e 4$ (d) $\frac{2}{5}\log_e 6$

4. A 20 kg mass is suspended form a spring which deflects 15mm under this load. The value of the critical damping coefficient to make the motion aperiodic will be

[ESE - 2018]

(a) 1010 N/m/s (b) 1013 N/m/s
(c) 1018 N/m/s (d) 1023 N/m/s

5. Consider the following statements:

1. The whirling (critical) speed of a shaft is that rotational speed at which the shaft so runs that the deflection of the shaft from the axis of rotation tends to become infinite.
2. Critical speed is equal to the frequency of transverse vibration of a shaft when the shaft carries a point load or a uniformly distributed load or a combination of both such loads.
3. The whirling of a shaft results from causes such as mass unbalance, hysteresis damping in the shaft, gyroscopic forces and fluid friction in the bearing

Which of the above statements are correct ?

[ESE - 2018]

(a) 1 and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3

6. A simple spring- mass vibrating system has a natural frequency of N. If the spring stiffness is halved and the mass doubled, then the natural frequency will be

[ESE - 2017]

(a) 0.5 N (b) N
(c) 2 N (d) 4 N

7. A car of mass 1450 kg is constructed on a chasis supported by four springs. Each spring has a force constant of 40000 N m. The combined mass of the two people occupying the car is 150 kg. What is the period of execution of two complete vibrations?

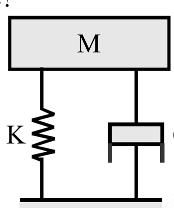
[ESE - 2017]

(a) 0.63 s (b) 1.59 s
(c) 4.96 s (d) 1.26 s

8. Consider the following statements:
Artefacts to prevent harmful effects resulting from vibrations of an unbalanced machine fixed on its foundation include
(i)Mounting the machine on springs thereby minimizing the transmission of forces.
(ii)Using vibration isolating materials to prevent or reduce the transmission of forces.

GATE QUESTIONS

1. In a single degree of freedom underdamped spring-mass-damper system as shown in the figure, an additional damper, is added in parallel such that the system still remains underdamped. Which one of the following statements is ALWAYS true?



[GATE - 2018]

(a) Transmissibility will increase

(b) Transmissibility will decrease

(c) Time period of free oscillations will increase

(d) Time period of free oscillations will decrease

2. The equation of motion for a spring-mass system excited by a harmonic force is

$M\ddot{x} + Kx = F\cos(\omega t)$

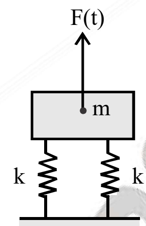
where M is the mass, K is the spring stiffness, F is the force amplitude and ω is the angular frequency of excitation occurs when ω is equal to

[GATE - 2018]

(a) $\sqrt{\frac{M}{K}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{K}{M}}$

(c) $2\pi\sqrt{\frac{K}{M}}$ (c) $\sqrt{\frac{K}{M}}$

3. A machine of mass m = 200 kg is supported on two mounts, with two springs of stiffness k = 10 kN/m and subjected to a harmonic force F(t) = 50 cos 5t kN. Find the magnitude of dynamic force transmitted from each mounting to the ground



[GATE - 2018]

4. The damping ratio for a viscously damped spring mass system, governed by the relationship

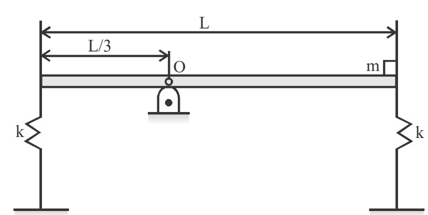
$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$, is given by

[GATE - 2017]

(a) $\sqrt{\frac{c}{mk}}$ (b) $\frac{c}{2\sqrt{km}}$

(c) $\frac{c}{\sqrt{km}}$ (d) $\sqrt{\frac{c}{2mk}}$

5. A thin uniform rigid bar of length L and mass M is hinged at point O, located at a distance of $\frac{L}{3}$ from one of its ends. The bar is further supported using springs, each of stiffness k, located at the two ends. A particle of mass m = $\frac{M}{4}$ is fixed at one end of the bar, as shown in the figure. For small rotations of the bar about O, the natural frequency of the system is



[GATE - 2017]

(a) $\sqrt{\frac{5k}{M}}$ (b) $\sqrt{\frac{5k}{2M}}$

GATE

2019

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CONTENTS

CHAPTER - 1
STATIC AND DYNAMIC LOADING

1.1 INTRODUCTION
Machine design is defined as the use of scientific principles, technical information and imagination in the dissipation of a machine
Or a mechanical system to perform specific functions with maximum economy and efficiency.

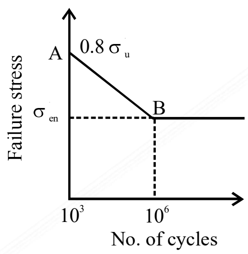
1.2 BASIC PROCEDURE OF MACHINE DESIGN
Example. Gear box assembly

Market Survey
↓
Define specifications of product
↓
Selection of proper mechanism
↓
Prepare general layout & select joining methods between individual components of product.
↓
Design individual components
↓
Prepare assembly

1.3 BASIC REQUIREMENTS OF MACHINE ELEMENTS
1. Strength
2. Rigidity: A machine component should be rigid and it should not deflect or bend too much due to forces or moments that acts on it. For example, a transmission shaft is many times designed on the basis of lateral and torsional rigidities. Therefore, maximum permissible deflection and maximum permissible angle of twist are the criterion of Design.
3. Wear Resistance: Wear is the main reason for putting the machine part out of order. It reduces useful life of the component. Wear also leads to loss accuracy of machine tools. Surface hardening is generally applied to increase wear resistance.
4. Minimum Dimensions & Weight: Material should be strong, hard and rigid with minimum possible dimensions and weight. This will result in minimum material cost.
5. Manufacturability: It is the ease of fabrication and assembly so that labour cost may be minimized.
6. Safety: The shape and dimensions of the machine parts should ensure safety to the operator of the machine.
7. Conformance to Standards: It should confirm to national and international standards covering its possible dimensions, grade and material.
8. Reliability: It is the probability that machine part will perform its intended functions under desired operating conditions over specified period of time.
9. Maintainability: It is case by which a machine part can be serviced or repaired.

GATE QUESTIONS

1. A machine element has an ultimate strength (σ_u) of 600N/mm^2, and endurance limit (σ_{en}) of 250 N/mm^2. The fatigue curve for the element on a log – log plot is shown below. If the element is to be designed for a finite life of 10000 cycles, the maximum amplitude of a completely reversed operating stress is _____ N/mm^2



[GATE - 2017]

2. The principal stresses at a point in a critical section of machine component are σ_1 = 60 MPa, σ_2 = 5 MPa and σ_3 = –40 MPa. For the material of the component, the tensile yield strength is σ_y = 200 MPa. According to the maximum shear theory, the factory of safety is

[GATE - 2017]

(a) 1.67 (b) 2.00
(c) 3.60 (d) 4.00

3. A machine component made of a ductile material is subjected to a variable loading with σ_{min} = –50 MPa and σ_{max} = 50MPa. If the corrected endurance limit and the yield strength for the material are σ'_e = 100MP and σ_y = 300 MPa, respectively, the factor of safety is _____

[GATE - 2017]

4. In a structural member under fatigue loading, the minimum and maximum stresses developed at the critical point are 50 Mpa and 150 MPa, respectively. The endurance, yield, and the ultimate strengths of the material are 200 MPa, 300 MPa, and 400 MPA, respectively. The factor of safety using modified Goodman criterion is

[GATE - 2016]

(a) $\frac{3}{2}$ (b) $\frac{8}{5}$
(c) $\frac{12}{7}$ (d) 2

5. In a linear arc welding process, the heat input per unit length is inversely proportional to

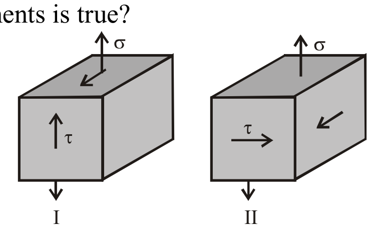
[GATE - 2016]

(a) Welding current
(b) Welding voltage
(c) Welding speed
(d) Duty cycle of the power source

6. A shaft is subjected to pure torsional moment. The maximum shear stress developed in the shaft is 100 MPa. The yield and ultimate strength of the shaft material in tension are 300 MPa and 450 MPa, respectively. The factor of safety using maximum distortion energy (von–Mises) theory is ____________.

[GATE - 2014]

7. Consider the following of stress as shown in configurations I and II in the figure below. From the standpoint of distortion energy (von - Mises) criterion, which one of the following statements is true?



[GATE - 2014]

(a) I yields after II
(b) II yields after I
(c) Both Yield simultaneously
(d) Nothing can be said about their relative yielding

8. Which one of the following is not correct?

[GATE - 2014]

ESE OBJ QUESTIONS

1. A machine component is subjected to a flexural stress, which fluctuates between 300 MN/m^2 and –150 MN/m^2. Taking the yield strength = 0.55 of the ultimate strength, endurance strength = 0.50 of the ultimate strength and factor of safety to be 2, the value of the minimum ultimate strength according to modified Goodman relation will be

[ESE - 2017]

(a) 1100 MN/m^2 (b) 1075 MN/m^2
(c) 1050 MN/m^2 (d) 1025 MN/m^2

2. Consider the following statements:
For a component made of ductile material, the failure criterion will be
1. Endurance limit, if the external force is fluctuating
2. Fatigue, if the external force is fluctuating
3. Yield stress, if the external force is static
Which of the above statements are correct

[ESE - 2017]

(a) 1and 2 only (b) 1 and 3 only
(c) 2 and 3 only (d) 1, 2 and 3

3. Consider the following statements:
On heating an elastomer under tensile load, its shrinkage
1. maximizes the enthalpy
2. maximizes the entropy
3. minimizes the free energy
4. avoids breaking
Which of the above statements are correct?

[ESE - 2017]

(a) 1 and 2 (b) 2 and 3
(c) 3 and 4 (d) 1 and 4

4. Statement(I) : Directionally solidified materials have good creep resistance.
Statement (II): Directionally solidified materials may be so loaded that there is no shearing stress along, or tensile stress across, the grain boundaries.

[ESE - 2017]

(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I)
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is not the correct explanation of Statement (i)
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

5. A solid shaft is designed to transmit 100 kW while rotating at *N* r.p.m. If the diameter of the shaft is doubled and is allowed to operate at 2 *N* r.p.m., the power that can be transmitted by the latter shaft is

[ESE - 2016]

(a) 200 kW (b) 400 kW
(c) 800 kW (d) 1600 kW

6. The diameter of a shaft to transmit 25 kW at 1500 r.p.m.. given that the ultimate strength is 150 MPa and the factor of safety is 3, will nearly be

[ESE - 2016]

(a) 12 mm (b) 16 mm
(c) 20 mm (d) 26 mm

7. A shaft of 50 mm diameter transmits a torque of 800 N-m. The width of the rectangular key used is 10 mm. the allowable shear stress of the material of the key being 40 MPa, the required length of the key would be

[ESE - 2016]

(a) 60 mm (b) 70 mm
(c) 80 mm (d) 90 mm

8. The diameter of the pin in a bushed pin type flexible coupling is to be increased for the purposed of

[ESE - 2016]

(a) Higher stress due to shear

CHAPTER - 2
POWER SCREWS

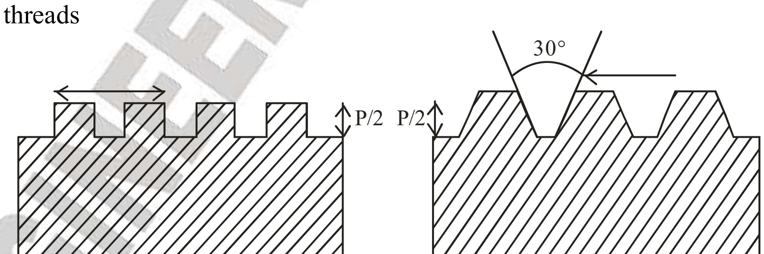
2.1 INTRODUCTION
A power screw is a mechanical device used for converting rotary motion into linear motion and transmitting power. Example; screw jack, lead screw of lathe, vice etc.

2.1.1 Advantages
1. Large head capacity for very smaller dimensions of the power screw resulting in compact design.
2. Simple manufacturing and design.
3. Large mechanical advantage for example, load of 15kN can be raised by applying only 400N.
4. Controlled and accurate linear motion.
5. Smooth and noiseless service.
6. A power screw can be designed with self locking property. In screw jack applications, self locking characteristic is required to prevent the load from falling on its own.

2.1.2 Disadvantages
1. Lower efficiency of 40%
2. High friction in threads causes rapid wear of the screw.

2.1.3 Forms of Threads
1. The threads are used for fastening purpose such as V threads are not suitable for power success. The purpose of fastening threads is to provide high fractional force, which lessons the possibility of loosening the parts assembled by preceded joint.
2. On the other hand, the purpose of power transmission threads is to reduce friction between the screw and nut therefore V threads are not suitable.
3. Screw with smaller angle of thread such as trapezoidal threads are preferred for power transmission.

2.2 TYPES OF POWER SCREW THREADS
There are two mostly used power screw threads are:
1. Square threads
2. Trapezoid threads



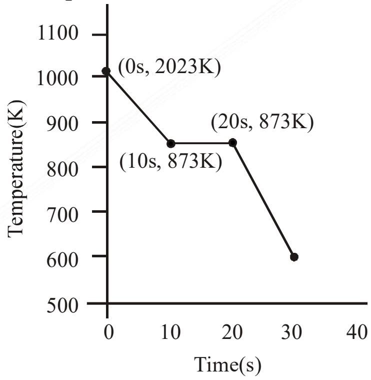
2.2.1 Square Threads
2.2.1.1 Advantages
1. Its efficiency is more than trapezoidal threads.
2. There is no radial pressure or side thrust on the nut.

GATE QUESTIONS

1. Metric thread of 0.8 mm pitch is to be cut on a lathe. Pitch of the lead screw is 1.5mm. If the spindle rotates at 1500 rpm, the speed of rotation of the lead screw (rpm) will be

[GATE - 2017]

2. A hypothetical engineering stress – strain curve shown in the figure has three straight lines PQ, QR, RS with coordinates P(0, 0), Q (0.2, 100), R (0.6, 140) and S (0.8, 130). 'Q' is the yield point, 'r' is the UTS point and s' the fracture point.



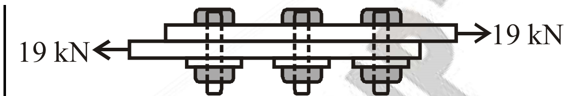
The toughness of the material (in MJ/m^3) is __

[GATE - 2016]

3. A bolt of major diameter 12 mm is required to clamp two steel plates. Cross sectional area of the threaded portion of the bolt is 84.3 mm^2. The length of the threaded portion in grip is 30 mm, while the length of the unthreaded portion in grip is 8 mm. Young's modulus of material is 200 GPa. The effective stiffness (in MN/m) of the bolt in the clamped zone is _______.

[GATE - 2014]

4. For the three bolt system shown in the figure, the bolt material has shear yield strength of 200 MPa. For a factor of safety of 2, the minimum metric specification required for the bolt is



[GATE - 2014]

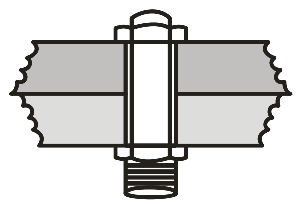
(a) M 8 (b) M 10
(c) M 12 (d) M 16

5. Two threaded bolts A and B of same material and length are subjected to identical tensile load. If the elastic energy stored in bolt A is 4 times that of the bolt B and the mean diameter of bolt A is 12 mm, the mean diameter of bolt B in mm is

[GATE - 2013]

(a) 16 (b) 24
(c) 36 (d) 48

6. In a bolted joint two members are connected with an axial tightening force of 2200 N. If the bolt used has metric threads of 4 mm pitch, the torque required for achieving the tightening force is.



[GATE - 2004]

(a) 0.7 Nm (b) 1.0 Nm
(c) 1.4 Nm (d) 2.8 Nm

7. Bolts in the flanged end of pressure vessel are usually pre-tensioned. Indicate which of the following statements is true.

[GATE - 1998]

(a) Pre-tensioning helps to seal the pressure vessel.
(b) Pre-tensioning increase the fatigue life of the bolts.
(c)Pre-tensioning reduces the maximum tensile stress in the bolts.

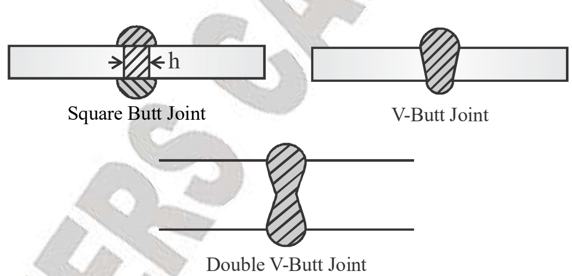
CHAPTER - 3
WELDED JOINTS

3.1 INTRODUCTION
Welding is permanent jointing but un-separable. Riveting is also permanent jointing but separable

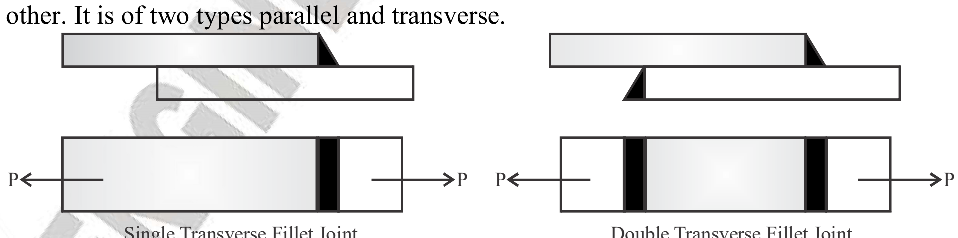
3.1.1 Advantages of Welded Joints
1. Lighter assemblies as compared to riveting where additional cover plates, gussets plates are required
2. Lower cost
3. Changes can be easily made
4. Leak-proof joints
5. Lesser production time
6. Drilling holes in reverted points reduces strength of material.
7. Bad appearance of riveted joints
8. Strength of welded joint is high

3.1.2 Disadvantages
1. Poor vibration damping ability.
2. Thermal distortion due to thermal residual stress therefore stress reliving is a necessity.
3. Quality of weld has to be maintained.

3.2 BUTT JOINTS
A butt joint can be defined as a joint between two components lying approximately in the same plane.



3.3 FILLET JOINTS
It is also called a lap joint, is a joint between two overlapping plates or components. A fillet weld consists of an approximately triangular cross-section joining two surfaces at right angles to each other. It is of two types parallel and transverse.



GATE QUESTIONS

1. A butt weld joint is developed on steel plates having and ultimate tensile strength of 500 MPa and 700 MPa, respectively. The thickness of the plates is 8 mm and width is 20 mm. Improper selection of welding parameters caused an undercut of 3 mm depth along the weld. The maximum transverse tensile load (in kN) carrying capacity of the developed weld joint is _________.

[GATE - 2014]

2. A fillet welded joint is subjected to transverse loading F as shown in figure. Both legs of the fillet are of 10 mm size and the weld length is 30 mm. If the allowable shear stress of the weld is 94 MPa, considering the minimum throat of the weld, the maximum allowable transverse load in kN is

[GATE - 2012]

(a) 14.44 (b) 17.92
(c) 19.93 (d) 22.16

3. A 60 mm long and 6 mm thick fillet weld carries a steady load of 15 kN along the weld. The shear strength of the weld material is equal to 200 MPa. The factor of safety is

[GATE - 2006]

(a) 2.4 (b) 3.4
(c) 4.8 (d) 6.8

4. Weldments in fabricated steel beams are designated for

[GATE - 1987]

(a) Bending stresses at the flange
(b) Shear stresses in transverse plane
(c) Combination of bending and shear
(d) None of these because in fabricated beams welds not to get stressed.

SOLUTIONS

Sol 1. (70kN)

$P_t = \sigma_t (t_{\text{under cut}}) L$
$= 700 \times 20 \times (8 - 3)$
$= 700 \times 100$ N = 70 kN

Sol 2. (*)

Sol 3. (b)

$P_{PPW} = 0.707\, t \times L_e \times \tau_s$
{here PFW → parallel filter weld}

$$\frac{15\times10^3}{0.707\times6\times60} = \tau_s$$

$\tau_S = 58.9344$ MPa
Given shear strength of material
= 200 MPa

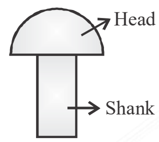
$$\text{FOS} = \frac{200}{58.9344} = 3.4$$

Sol 4. (c)

Steel beams undergoes both bending and shear loads.

CHAPTER - 4
RIVETED JOINTS

4.1 INTRODUCTION



Rivet is specified by the shank diameter. A 20mm rivet means a rivet having 20mm shank diameter.

4.1.1 Applications of Riveted Joints

1. Riveted joints are used where it is necessary to avoid the thermal after effects of welding.
2. Used for metals with poor weld ability such as aluminum alloys.
3. To join different materials like steel and asbestos.
4. Welded joints have poor resistance to vibrations and impact loads.

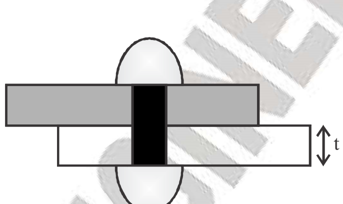
4.1.2 Advantages of Riveted Joint over Welded Joints

1. More reliable in case of vibration and impact loads.
2. Quality of riveted joint can be easily checked.
3. Can be dismantled work without much damaged parent material.

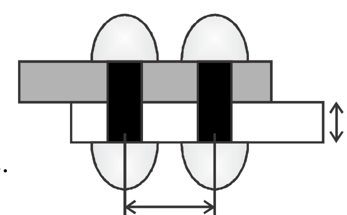
4.1.3 Disadvantages of Riveted Joints Compared to Welded Joint

1. More material cost, holes required for rivets weaker the plate and it is necessary to increase plate thickness to compensate this loss.
2. More labor cost and less productive process.
3. More weight of riveted joints due to overlapping straps requirement.
4. Noisy process
5. Strep concentration is there near holes in plates.

4.2 TYPES OF RIVETED JOINTS

1. 

Single Riveted

2. 

Double Riveted

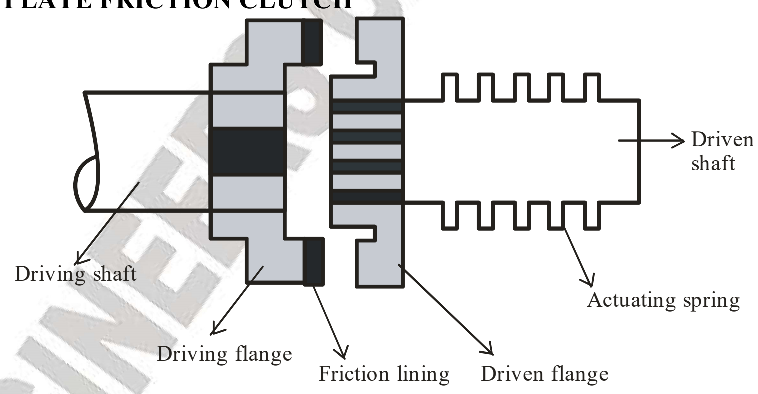
CHAPTER - 5
FRICTION CLUTCHES

5.1 CLUTCH
It is a mechanical device, which is used to connect or disconnect the source of power from the remaining parts of the power transmission system at the will of operator.

5.1.1 Classification of Clutches
1. Positive Contact Clutches: They include square jaw clutches, spiral jaw clutches and toothed clutches. Power transmission is achieved by means of interlocking of jaws or teeth. No slip is there.
2. Friction Clutches: They include single and multi plate clutches, cone clutches and centrifugal clutches. Power transmission is achieved by means of friction between contacting surfaces.
3. Electromagnetic Clutches: They include magnetic particle clutches, magnetic hysteresis clutches and eddy current clutches. Power transmission is achieved by means of magnetic field.
4. Fluid Clutches and Couplings: Power transmission is achieved by means of hydraulic pressure.

5.1.2 Advantages of Jaw Clutches
1. No slip and engagement is positive
2. No heat is generated during engagement or disengagement.

5.1.3 Disadvantages
1. It can be engaged only when both shafts are stationary or rotate with very small speed difference.
2. It cannot be engaged at high speeds

5.2 SINGLE PLATE FRICTION CLUTCH


1. One flange is rigidly hanged to the driving shaft, while the other is connected to the driven shaft by means of splines. The splines permit free axial movement of the driven flange with respect to driven flange shaft.
2. This axial movement is necessary for engagement and disengagement of the clutch.
3. The actually force is provided by a helical spring which forces the driven flange to move towards driving flange.
4. Power is then transmitted from driving flange to driven flange by means of frictional force.

GATE QUESTIONS

1. Single - plate clutch has a friction disc with inner and outer of 20 mm and 40mm, respectively. The friction lining in the disc is made is such a way that the coefficient of friction μ varies radially as μ = 0.01 r, where r is in mm. The clutch needs to transmit a friction torque of 18.85 kN-mm. As per uniform pressure theory, the pressure (in MPa) on the disc is _____

[GATE - 2017]

2. A disc clutch with a single friction surface has coefficient of friction equal to 0.3. The maximum pressure which can be imposed on the friction material is 1.5 MPa. The outer diameter of the clutch plate is 200 mm and its internal diameter is 100 mm. Assuming uniform wear theory for the clutch plate, the maximum torque (in N.m) that can be transmitted is _______.

[GATE - 2014]

3. A clutch has outer and inner diameter 100 mm and 40 mm respectively. Assuming a uniform pressure of 2 MPa and coefficient of friction of liner material 0.4, the torque carrying capacity of the clutch is

[GATE - 2008]

(a) 148 Nm (b) 196 Nm
(c) 372 Nm (d) 490 Nm

4. A disk clutch is required to transmit 5 kW at 2000 rpm. The disk has a friction lining with coefficient of friction equal to 0.25. Bore radius of friction lining is equal to 25 mm. Assume uniform contact pressure of 1 MPa. The value of outside radius of the friction lining is

[GATE - 2006]

(a) 39.4 mm (b) 49.5 mm
(c) 97.9 mm (d) 142.9 mm

5. Axial operation claw clutches having self-locking tooth profile.

[GATE - 1987]

(a) Can be disengaged at any speed
(b) Can be disengaged only unloaded
(c) Can be engages only when unloaded
(d) Can work only with load.

CHAPTER - 6
BRAKES

6.1 BRAKES
A brake is a mechanical device, which is used to absorb energy passed by a moving system or mechanism by means of friction.
Brake capacity depends upon the following three factors.
1. The until presence between braking surfaces.
2. The contacting area of braking surfaces.
3. Radius of brake drum
4. μ
5. Ability of the brake to dissipate heat that is equivalent to the energy being absorbed.

6.2 ENERGY EQUATIONS
Consider a mechanical system of mass m, moving with velocity V_1 is slowed down to velocity V_2,

$\therefore$ During the period of breaking, the KE $= \frac{1}{2}m(V_1^2 - V_2^2)$

Similarly for a rotating body, KE $= \frac{1}{2}I(\omega_1^2 - \omega_2^2)$

$$KE = \frac{1}{2}mk^2(\omega_1^2 - \omega_2^2)$$

Where k is radius of gyration
In certain applications, like hoists, the brake absorbs the potential energy released by the moving weight during the braking period.
PE = mgh
Depending upon the type of applications, the total energy absorbed by the brake is determined by
$E = T \times \theta$
Where θ is angle through which brake drum rotates during the breaking period (rad)

Example. A solid CI disk, 1m in diameter and 0.2m thick is used as flywheel. It is rotating at 350rpm. It is brought to rest in 1.5s by means of a brake calculate
(a) The energy absorbed by the brake
(b) The torque capacity of the brake $P_a = 7200 kg/m^3$
Solution.
D = 1m, t = 0.2m, $N_1 = 350rpm$, $N_2 = 0$
t = 1.5s

(a) $E = \frac{1}{2}mk^2(\omega_1^2 - \omega_2^2)$

$$\omega_1 = \frac{2\pi(350)}{60} = 36.63\ \text{rad/sec}$$

$m = (\pi r^2 h)(7200)$
$m = \pi(.5)^2 \times (.2)(7200) = 1130.97kg$

$k = \frac{d}{\sqrt{8}}$ (for solid disk about its axis of rotation)

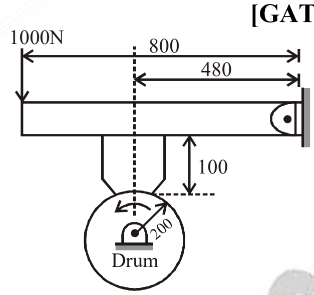
GATE QUESTIONS

1. A four-wheel vehicle of mass 1000 kg moves uniformly in a straight line with the wheels revolving at 10 rad/s. The wheels are identical, each with a radius of 0.2 m. Then a constant braking torque is applied to all the wheels and the vehicle experience a uniform deceleration. For the vehicle to stop in 10 s, the braking torque (in N.m) on each wheel is ________.

[GATE - 2014]

2. A drum brake is shown in the figure. The drum is rotating in anticlockwise direction. The coefficient of friction between drum and shoe is 0.2. The dimensions shown in the figure are in mm. The braking torque (in N-m) for the brake shoe is _________.

[GATE - 2014]



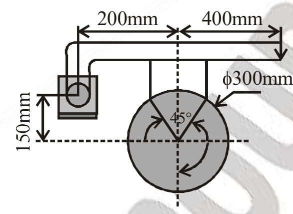
3. A band brake having bandwidth of 80 mm, drum diameter of 250 mm, coefficient of friction of 0.25 and angle of wrap of 270 degrees is required to exert a friction torque of 1000 N m. The maximum tension (in kN) developed in the band is

[GATE - 2010]

(a) 1.88 (b) 3.56
(c) 6.12 (d) 11.56

4. A block – brake shown below has a face width of 300 mm and a mean coefficient of friction of 0.25. For an activating force of 400N, the braking torque in Nm is

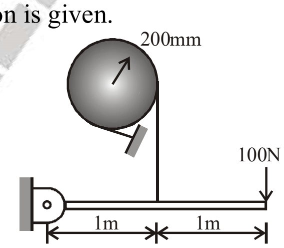
[GATE - 2007]



(a) 30 (b) 40
(c) 45 (d) 60

Linked statement for Q.5 & Q.6

A band brake consists of a lever attached to one end of the band. The other end of the band is fixed to the ground. The wheel has a radius of 200 mm and the wrap angle of the band is 270°. The braking force applied to the lever is limited to 100 N, and the coefficient of friction between the band and the wheel is 0.5 No other information is given.



5. The maximum tension that can be generated in the band during braking is

[GATE - 2005]

(a) 1200 N (b) 2110 N
(c) 3224 N (d) 4420 N

6. The maximum wheel torque that can be completely braked is

[GATE - 2005]

(a) 200 N.m (b) 382 N.m
(c) 604 N.m (d) 844 N.m

7. In a band brake the ratio of tight side band tension to the tension on the slack side is 3. If the angle of overlap of band on the drum is 180° the coefficient of friction required between drum and the band is

[GATE - 2003]

(a) 0.20 (b) 0.25
(c) 0.30 (d) 0.35

CHAPTER - 7
BELTS

7.1 BELT DRIVES
Belt, chain and rope drives are called flexible drives.
Gear drives are rigid drives.
Belts are used to transmit power between two shafts by means of friction.

7.1.1 Advantages of Belt Drives
1. Operation is smooth and silent
2. It can transmit power over considerable distance between the axes of driving and driven shafts.
3. They can transmit only a definite load, which if exceeded, will cause the belt to slip over the pulley.
4. It has ability to absorb shocks and damp vibration
5. It has low cost and simple design.

7.1.2 Disadvantages of Belt Drives
1. It has large dimensions and occupies more space.
2. The VR is not constant due to belt slip.
3. It has low efficiency
4. It has short life

Two types of cross section
(i) Flat belt (ii) V-belt

7.1.3 Advantages of Flat Belts Over V-Belts
1. Relatively cheap and easy to maintain
2. Their maintenance consists of periodic adjustment in the centre distance between shafts in order to compensate stretching.
3. Different VR can be obtained by using a stepped pulley, where the belt is shifts from one step to another, having different diameter.
4. Simple and inexpensive
5. Can be used for long distances up to 15m
6. Efficiency of flat belt is more than efficiency of V-belt

7.1.4 Disadvantages of Flat Belt Drives Over V-Belt Drives
1. The power transmitting capacity of flat belt is low.
2. VR is less than V-belt
3. Flat belts are nosier than V-belts
4. Only horizontal and not vertical.

7.1.5 Advantages of V-Belts
1. Force of friction between the surfaces of the belt and v-grooved pulley is high due to wedge action. This wedging action permits a smaller arc of contact, increases the pulling capacity of the belt and consequently results in increase in power transmitting capacity.
2. Shorter distance belts
3. High VR up to 7: 1
4. Smooth operation

GATE QUESTIONS

1. A wire rope is designated as 6 × 19 standard hoisting. The numbers 6 × 19 represent **[GATE - 2003]**

(a) Diameter in millimeter × length in meter
(b) Diameter in centimeter × length in meter
(c) Number of strands × number of wires in each strand
(d) Number of wires in each strand × number of strands

SOLUTIONS

Sol 1. (c)

CHAPTER - 8
CHAIN DRIVES

8.1 INTRODUCTION
A chain can be defined as a series of links connected by pin joints.
It has some features of belt drive and some of gear drive.

8.1.1 Advantages
1. It can be used for long as well as short distance range.
2. Number of shafts can be driven
3. Small overall dimensions
4. Positive drive and has no slip
5. High efficiency (96% to 98%).
6. No initial tension required
7. Easy to replace.

8.1.2 Disadvantages
1. More wear.
2. Less précised motion
3. Noisy operation

8.2 DESIGN OF SPUR GEARS
A mechanical drive is defined as a mechanism, which is intended to transmit mechanical power over a certain distance, usually involving a change in speed and torque.
Two groups of mechanical drives are
1. Mechanical drives that transmit power by means of friction e.g. belt and rope drive.
2. Mechanical drives that transmit power by means of engagement e.g. chain drive and gear drive.

> **1.** Selection of a proper mechanical drive for a given application depends upon number of factors such as centre distance, VR, shifting arrangement, maintenance and cost.
> **2.** Gear drive is a positive drive and has constant speed.

8.3 CHAIN
The belt drive is not a positive drive because of creep and slip .The chaibn drive is a positive drive .Like belts , chains can be used for larger centre distances .They are made of metal and due to this chain is heavier than the belt but they are flexible like belts .It also requires lubrication from time to time .The lubricant prevents chain from rusting and reduces wear.
The chain and chain drive are shown in figure .The sprockets are used in place of pulleys.The projected teeth of sprockets fit in the recesses of the chain .The distance between roller centers of two adjacent links is known as pitch .The circle passing through the pitch centrers is called pitch circle.

ESE OBJ QUESTIONS

1. For roller chain drive with sprocket having 10 teeth, the velocity of the driven shaft with respect to that of drive will be approximately. **[ESE - 2008]**

(a) Same
(b) 5% above
(c) 5% below
(d) 5% above to 5% below

2. Source of power loss in a chain drive are given below:

1. Friction between chain and sprocket teeth.
2. Overcoming the chain stiffness.
3. Overcoming the friction in shaft bearing.
4. Frictional resistance to the motion of the chain in air or lubricant.

To correct sequence of descending order of power loss due to these sources is **[ESE - 1995]**

(a) 1, 2, 3, 4
(b) 1, 2, 4, 3
(c) 2, 1, 3, 4
(d) 2, 1, 4, 3

SOLUTIONS

Sol 1. (d)

$$V_{max} - V_{min} \; \alpha \left[1 - \cos\left(\frac{180}{z}\right)\right]$$

In order to reduce the variation in chain speed, the number of teeth on the sprocket should be increased. It has been observed that the speed variation is 4% for a sprocket with 11 teeth, 1.6% for sprocket with 17 teeth, and less than 1% for a sprocket with teeth.

For smooth operation at moderate and high speeds, it is considered a good practice to use a driving sprocket with at least 17 teeth. For durability and noise considerations, the minimum number of teeth on the driving sprocket should be 19 or 21.

Sol 2. (a)

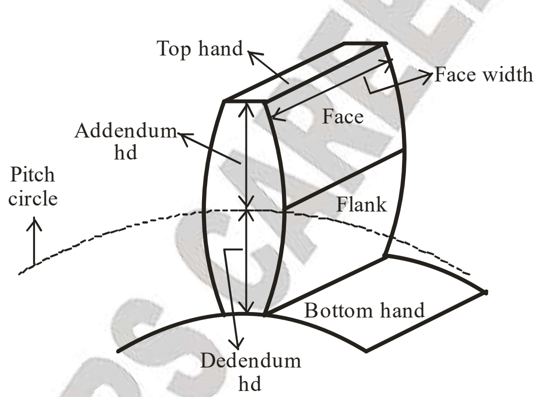
This is the decreasing order in which losses takes place.

CHAPTER - 9
GEARS

9.1 GEAR DRIVES
Gears are defined as toothed wheels or multi lobed comes, which transmit power and motion from one shaft to another by means of successive engagement of teeth.

9.1.1 Advantages
1. It is a positive drive with constant VR
2. CD between shafts is small therefore compact construction
3. It can transmit very large power, even beyond the range of chain and belt drive
4. It can transmit motion at very low velocity which is not possible with belt drives
5. 99% efficiency
6. Provision of gear shifting is there in gear boxes.

9.1.2 Disadvantages
1. Gear drives are costly and their maintenance cost is also higher.
2. Precise alignment is also required.

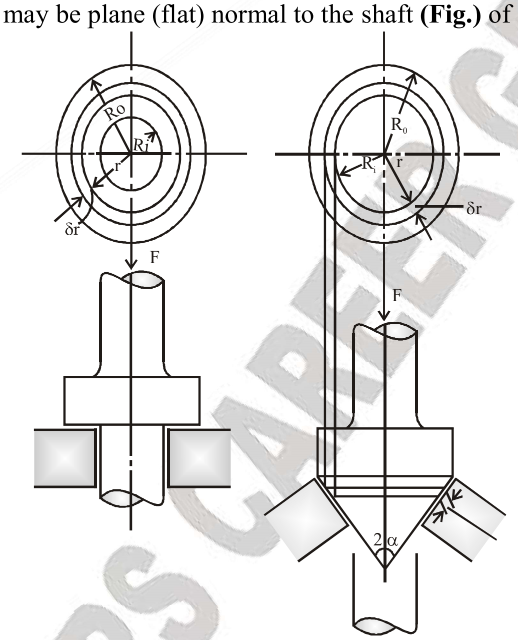
9.2 TERMINOLOGY


1. Pinion: smaller of the two mating gear
2. Gear: larger of the two rotating gear
3. Pitch circle: Pitch circle is the curve of intersection of the pitch surface of revolution and the plane of rotation. It is an imaginary circle that rotates without slipping with the pitch circle of a mating gear corresponding diameter is pitch circle dia. (PCD)
4. Addendum (h_a): height of tooth above PCD
5. Ddendum (h_d): height of tooth below PCD
6. Clearance (C): Clearance is the amount by which dedendum of a given gear exceeds the addendum of its mating tooth.
7. Face width (b): It is width of tooth measured parallel to axis.
8. Tooth space: The width of the space between two adjacent teeth measured along the pitch circle is called the tooth space.
9. Working depth: Sum of addendum of gear is engagement.
10. C.D: It is the distance between centres of pitch circles of mating gears.
11. Pressure angle: It is the angle which the line of action makes with the common tangent to the pitch circles. The pressure angle is also called angle of obliquity.

CHAPTER - 10
BEARING

10.1 INTRODUCTION
When a rotating shaft is subjected to an axial load, the thrust (axial force) is taken either by a pivot or a collar. Examples are the shaft of a steam turbine and propeller shaft of a ship.

10.1.1 Collar Bearing
A collar bearing or simply a collar is provided at any position along the shaft and bears the axial load on a mating surface.
The surface of the collar may be plane (flat) normal to the shaft **(Fig.)** of conical shape **(Fig.).**



10.1.2 Pivot Bearing
When the axial load is taken by the end of the shaft which is inserted in a recess to bear the thrust, it is called a *pivot bearing* or simply a *pivot*. It is also known as *footstep bearing*.

$$= \int_{R_i}^{R_o} p \times 2\pi r dr \quad = \int_{R_i}^{R_o} \frac{C}{r} \times 2\pi r dr$$

$$= \int_{R_i}^{R_o} 2\pi C.dr = (2\pi C r)_{R_i}^{R_o} = 2\pi C (R_o - R_i) = 2\pi pr (R_o - R_i)$$

or pressure intensity p at a radius *r* of the collar,

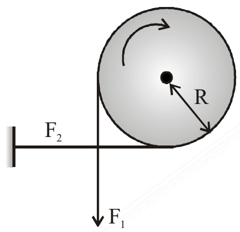
$$p = \frac{F}{2\pi r (R_o - R_i)}$$

In a flat pivot, in which $R_i = 0$, the pressure would be infinity at the centre of the bearing (r = 0), which cannot be true. Thus, the uniform wear theory has a flaw in it.
Collars and pivots, using the above two theories, have been analysed below:

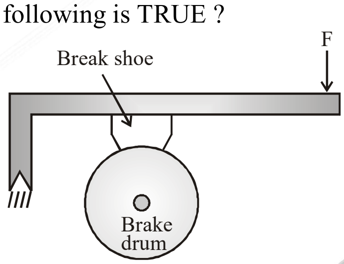
GATE QUESTIONS

1. The force F_1 and F_2 in a brake band and the direction of rotation of the drum are as shown in the figure. The coefficient of friction is 0.25. The angle of wrap is $3\pi/2$ radians. It is given that R = 1m and F_2 = 1N. the torque (in N-m) exerted on the drum is___

[GATE - 2016]



2. For the brake shown in figure, which one of the following is TRUE ?



[GATE - 2016]

(a) Self energizing for clockwise rotation of the drum
(b) Self energizing for anti – clockwise rotation of the drum
(c) Self energizing for rotation in either direction of the drum
(d) not of the self energizing type

3. A hydrodynamic journal is subjected to 2000 N load at a rotational speed of 2000 rpm. Both bearing bore diameter and length are 40 mm. If radial clearance is 20 μm and bearing is lubricated with an oil having viscosity 0.03 Pa.s, the sommerfeld number of the bearing is ___________.

[GATE - 2014]

4. Ball bearings are rated by a manufacturer for a life of 10^6 revolutions. The catalogue rating of particular bearing is 16 kN. If the design load is 2 kN, the life of the bearing will be p × 10^6 revolutions, where p is equal to _________.

[GATE - 2014]

5. A solid circular shaft needs to be designed to transmit a torque of 50 N.m. If the allowable shear stress of the material is 140 MPa, assuming a factor of safety of 2, minimum allowable design diameter in mm is

[GATE - 2012]

(a) 8 (b) 16
(c) 24 (d) 34

6. Two identical ball bearings P and Q are operating at loads 30 kN and 45 kN respectively. The ratio of the life of bearing P to the life of bearing Q is

[GATE - 2011]

7. A lightly loaded full journal bearing has journal diameter of 50 mm, bush bore of 50.05 mm and bush length of 20 mm. If rotational speed of journal is 1200 rpm and average viscosity of liquid lubricant is 0.03 Pa s, the power loss (in W) will be

[GATE - 2010]

(a) 37 (b) 74
(c) 118 (d) 237

8. A journal bearing has a shaft diameter of 40 mm and a length of 40 mm. The shaft is rotating at 20 rad/s and the viscosity of the lubricant is 20 MPa.s. The clearance is 0.020 mm. The loss of torque due to the viscosity of the lubricant is approximately.

[GATE - 2008]

(a) 0.040 Nm (b) 0.252 Nm
(c) 0.400 Nm (d) 0.652 Nm

9. A natural feed journal bearing of diameter 50 mm and length 50 mm operating at 20 revolution/sec. carries a load of 2.0 kN. The lubricant used has a viscosity of 20 MPa/s. The

GATE

2019

STRENGTH OF MATERIALS

MECHANICAL ENGINEERING

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GATE-2019: Strength of Materials | Detailed theory with GATE & ESE previous year papers and detailed solu ons.

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CONTENTS

CHAPTER - 1
MECHANICS

1.1 FORCE
Force may be defined as a push or pull which produces or tends to produce a change in the state of rest or of uniform motion of a body or change in the direction of motion of the body.

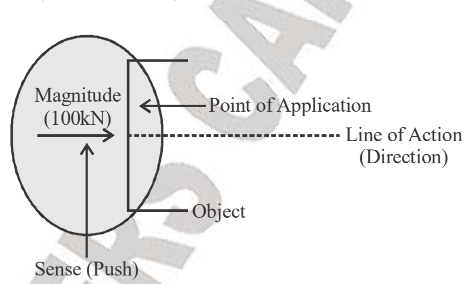
F = ma

$$\text{Force} = \text{mass} \times \text{acceleration}$$

This is the fundamental equation of motion which gives the measurement of force. It is a vector equation.

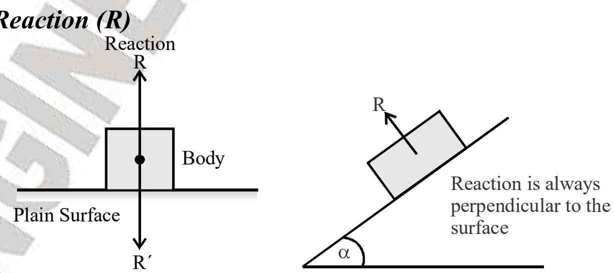
1.1.1 Effects of a Force
1. Change the motion
2. Change the direction
3. Change the size or shape
4. Give rise to internal stresses

1.1.2 Characteristics of Force
1. The magnitude of the force in known units such as Newton's or kilonewtons. (i.e., 50 N, 100 KN etc.)
2. The line of action or direction of the force may be taken with respect to reference lines.
3. Its point of application, that is the point on the body at which forces acts.
4. Sense or nature of the force (Push or Pull)



1.1.3 Classification of Forces
1. According to the Nature of Force

(i) Action (R′) and Reaction (R)



GATE QUESTIONS

1. Two disks A and B with identical mass (m) and radius (R) are initially at rest. They roll down from the top of identical inclined planes without slipping. Disk A has all of its mass concentrated at the rim, while Disk B has its mass uniformly distributed. At the bottom of the plane, the ratio of velocity of the center of disk A to the velocity of the center of disk B is

[GATE - 2017]

(a) $\sqrt{\frac{3}{4}}$ (b) $\sqrt{\frac{3}{2}}$

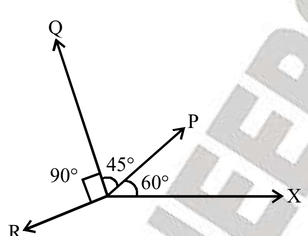
(c) 1 (d) $\sqrt{2}$

2. A particle of unit mass is moving on a plane. Its trajectory, in polar coordinates, is given by $r(t) = t^2$, $\theta(t) = t$, where t is time. The kinetic energy of the particle at titme t = 2 is

[GATE - 2017]

(a) 4 (b) 12
(c) 16 (d) 24

3. The magnitudes of vectors P, Q and R are 100 kN, 250 kN and 150 kN, respectively as shown in the figure.

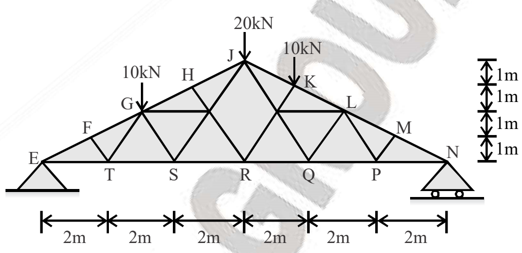


The respective values of the magnitude (in kN) and the direction (with respect to the x-axis) of the resultant vector are

[GATE - 2016]

(a) 290.9 and 96.0° (b) 368.1 and 94.7°
(c) 330.4 and 118.9° (d) 400.1 and 113.5°

4. A plane truss with applied loads is shown in the figure.

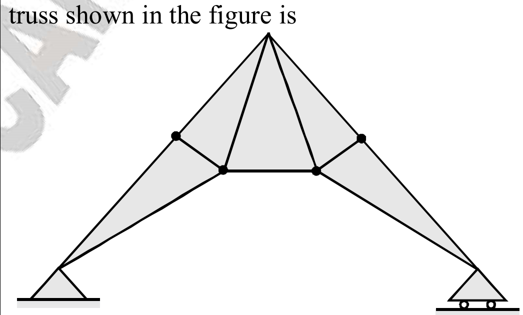


The members which do not carry any force are

[GATE - 2016]

(a) FT, TG, HU, MP, PL
(b) ET, GS, UR, VR, QL
(c) FT, GS, HU, MP, QL
(d) MP, PL, HU, FT, UR

5. The kinematic indeterminacy of the plane truss shown in the figure is



[GATE- 2016]

(a) 11 (b) 8
(c) 3 (d) 0

6. An assembly made of a rigid arm A-B-C hinged at end A and supported by an elastic rope C-D at end C is shown in the figure. The members may be assumed to be weightless and the lengths of the respective members are as shown in the figure.

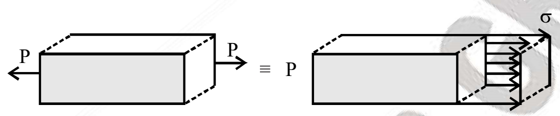
CHAPTER - 2
STRESS AND STRAIN

2.1 STRESS (σ)

When a material is subjected to an external force, a resisting force is set up within the component. The internal resistance force per unit area acting on a material or intensity of the forces distributed over a given section is called the stress at a point.

(i) It uses original cross section area of the specimen and also known as engineering stress or conventional stress.

Therefore, $\sigma = \frac{P}{A}$



Where P is expressed in Newton (N) and A, original area, in square meters (m), the stress σ will be expressed in N/m^2. This unit is called Pascal (Pa).

(ii) As Pascal is a small quantity, in practice, multiples of this unit is used.

$1\,kPa = 10^3\,Pa = 10^3\,N/m^2$ (kPa = Kilo Pascal)

$1MPa = 10^6\,Pa = 10^6\,N/m^2 = 1N/mm^2$ (MPa = Mega Pascal)

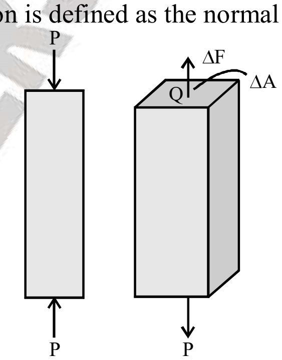
$1GPa = 10^9\,Pa = 10^9\,N/m^2$ (GPa = Giga Pascal)

Let us take an example: A rod 10 mm × 10 mm cross-section is carrying an axial tensile load 10 kN. In this rod the tensile stress developed is given by

$$(\sigma_1) = \frac{P}{A} = \frac{10kN}{(10mm \times 10mm)} = \frac{10 \times 10^3\,N}{100mm^2} = 100N/mm^2 = 100MPa$$

The resultant of the internal forces for an axially loaded member is normal to a section cut perpendicular to the member axis.

The force intensity on the shown section is defined as the normal stress.

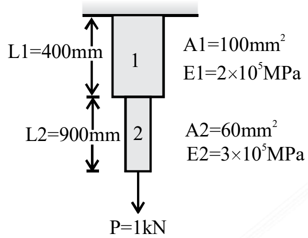


$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \quad \text{and} \quad \sigma_{avg} = \frac{P}{A}$$

GATE QUESTIONS

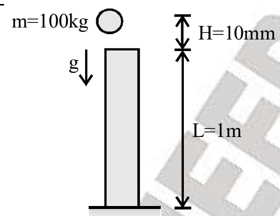
1. Consider the stepped bar made with a linear elastic material and subjected to an axial load of 1 kN, as shown in the figure.

[GATE - 2017]



Segments 1 and 2 have cross-sectional area of 100 mm^2 and 600 mm^2, Young's modoulus of 2×10^5 MPa and 3×10^5 MPa, and length of 400 mm and 900 mm, respectively. The strain energy (in N-mm, up to one decimal place) in the bar due to the axial load is _______.

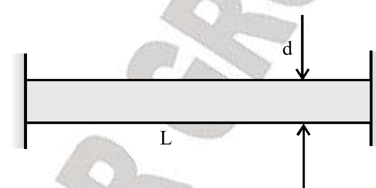
2. A point mass of 100 kg is dropped onto a massless elastic bar (cross-sectional area = 100 mm^2, length = 1 m, Young's modulus = 100 gPa) from a height H of 10 mm as shown (Figure is not to scale). If g = 10 m/s^2, the maximum compression of the elastic bar is _________mm.



3. A horizontal bar, fixed at one end (x = 0), has a length of 1m, and cross-sectional area of 100mm^2. Its elastic modulus varies along its length as given by $E(x) = 100\ e^{-x}$ GPa, where x is the length coordinate (in m) along the axis of the bar. An axial tensile load of 10 kN is applied at the free end (x = 1). The axil displacement of the free end is ______mm.

[GATE - 2017]

4. An initially stree-free massless elastic beam of length L and circular cross-section with diameter d (d<<L) is held fixed between two walls as shown. The beam material has Young's modulus E and coefficient of thermal expansion α.



If the beam is slowly and uniformly heated, the temperature rise required to cause the beam to buckle is proportional to

[GATE - 2017]

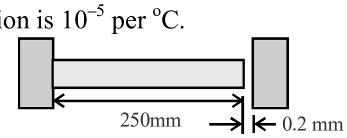
(a) d (b) d^2
(c) d^3 (d) d^4

5. In the engineering stress-strain curve for mild steel, the Ultimate Tensile Strength (UTS) refers to

[GATE - 2017]

(a) Yield stress (b) Proportional limit
(c) Maximum stress (d) Fracture stress

6. A circular metallic rod of length 250 mm is placed between two rigid immovable walls as shown in the figure. The rod is in perfect contact with the wall on the left side and there is a gap of 0.2 mm between the rod and the wall on the right side. If the temperature of the rod is increased by 200oC, the axial stress developed in the rod is ______MPa.
Young's modulus of the material of the rod is 200 GPa and the coefficient of thermal expansion is 10^{-5} per °C.



[GATE - 2016]

ESE OBJ QUESTIONS

1. A mild steel bar, circular in cross-section, tapers from 40 mm diameter to 20mm diameter over its length of 800 mm. It is subjected to an axial pull of 20 kN. E = 2×10^5 N/mm^2. The increase in the length of the rod will be

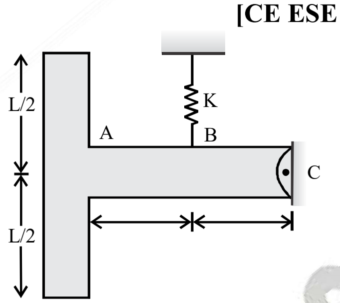
[CE ESE - 2017]

(a) $\frac{1}{10\pi}$ mm (b) $\frac{2}{5\pi}$ mm

(c) $\frac{4}{5\pi}$ mm (d) $\frac{1}{5\pi}$ mm

2. A uniform T-shaped arm of weight W, pinned about a horizontal point c, is support by a vertical spring of stiffness K. The extension of the spring is

[CE ESE - 2017]



(a) $\frac{3W}{4K}$ (b) $\frac{4W}{3K}$

(c) $\frac{3K}{4W}$ (d) $\frac{4K}{3W}$

3. In mild steel specimens subjected to tensile test cycle, the elastic limit in tension is raised and the elastic limit in compression is lowered, This is called

[CE ESE - 2017]

(a) Annealing effect
(b) Bauschinger effect
(c) Strain rate effect
(d) Fatigue effect

4. A solid uniform metal bar of diameter D mm and length ℓ mm hangs vertically from its upper end. The density of the material is ρN/mm^3 and its modulus of elasticity is EN/mm^2. The total extension of the rod due to its own weight would be

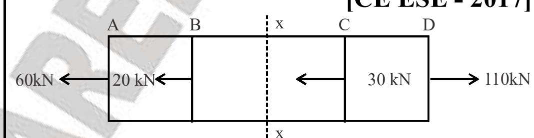
[CE ESE - 2017]

(a) $\frac{\rho\ell^2}{2E}$ (b) $\frac{\rho\ell}{2E}$

(c) $\frac{\rho\ell}{4E}$ (d) $\frac{\rho\ell^2}{4E}$

5. What is the stress at the section x- for the bar ABCD with uniform cross-section 1000 mm^2 ?

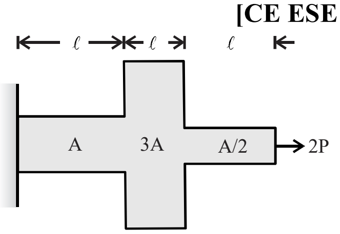
[CE ESE - 2017]



(a) 20 N/mm^2 (Tensile)
(b) 30 N/mm^2 (Compressive)
(c) 80 N/mm^2 (Tensile)
(d) 50 n/mm^2 (compressive)

6. The total elongation of the structural element (fixed at one end, free at the other end, and of varying cross-section) as shown in the figure, when subjected to load 2P at the free end is

[CE ESE - 2017]

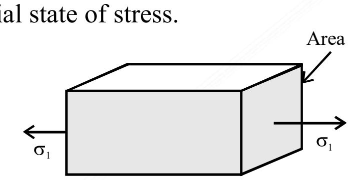


(a) $6.66\frac{P\ell}{AE}$ (b) $5.55\frac{P\ell}{AE}$

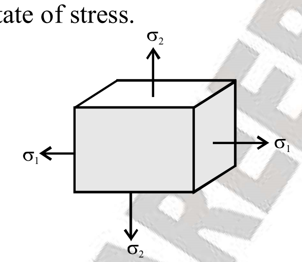
(c) $4.44\frac{P\ell}{AE}$ (d) $3.33\frac{P\ell}{AE}$

CHAPTER - 3
PRINCIPAL STRESS & STRAIN

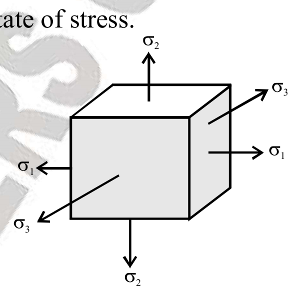
3.1 STATES OF STRESS
3.1.1 Uni-Axial Stress
Only one non-zero principal stress, i.e. σ_1.
Right side figure represents Uni-axial state of stress.



3.1.2 Bi-Axial Stress
One principal stress equals zero, two do not, i.e. $\sigma_1 > \sigma_3$; $\sigma_2 = 0$
Right side figure represents Bi-axial state of stress.



3.1.3 Tri-Axial Stress
Three non-zero principal stresses,
i.e. $\sigma_1 > \sigma_2 > \sigma_3$
Right side figure represents Tri axial state of stress.



3.1.4 Isotropic Stress
Three principal stresses
are equal, i.e. $\sigma_1 = \sigma_2 = \sigma_3$
Right side figure represents Isotropic state of stress.

GATE QUESTIONS

1. A soil sample is subjected to a hydrostatic pressure, σ. The Mohr circle for any point in the soil sample would be

[GATE - 2017]

(a)A circle of radius σ and center at the origin
(b)A circle of radius σ and center at a distance σ from the origin
(c)A point at a distance σ from the origin
(d)A circle of diameter σ and center at the origin

2. A rod of length 20mm is stretched to make a rod of length 40mm. Subsequently, it is compressed to make a rod of final length 10mm. consider the longitudinal tensile strain as positive and compressive strain as negative. The total true longitudinal strain in the rod is

[GATE - 2017]

(a) – 0.5 (b) – 0.69
(c) – 0.75 (d) – 1.0

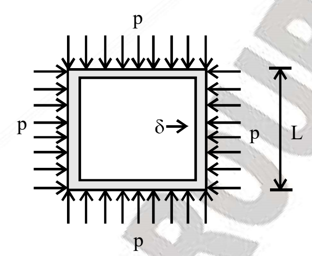
3. The state of stress at a point is $\sigma_x = \sigma_y = \sigma_z = \tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0$ and $\tau_{xy} = \tau_{yx} = 50$Mpa. The maximum normal stress (in MPa) at that point is ____

[GATE - 2017]

4. A rectangular region in a solid is in a state of plane strain. The (x, y) coordinates of the corners of the undeformed rectangle are given by P(0, 0), Q (4, 0), R (4, 3), S (0, 3). The rectangle is subjected to uniform strain, $\varepsilon_{xx} = 0.001$, $\varepsilon_{yy} = 0.002$, $\gamma_{xy} = 0.003$. The deformed length of the elongated diagonal, upto three decimal places, is _______unit.

[GATE - 2017]

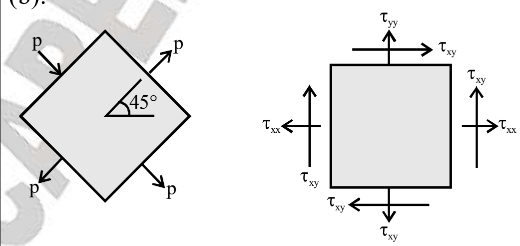
5. A square plate of dimension L × L is subjected to a uniform pressure load p = 250 MPa on its edges as shown in the figure. Assume plane stress conditions. The Young's modulus E = 200 GPa.



The deformed shape is a square of dimension L – 2 δ. If L = 2 m and δ = 0.001 m, the Poisson's ratio of the plate material is ________.

[GATE - 2016]

6. The state of stress at a point on an element is shown in figure (a). The same state of stress is shown in another coordinate system in figure (b).



The components (τ_{xx}, τ_{yy}, τ_{xy}) are given by

[GATE - 2016]

(a) $\left(P/\sqrt{2}, -P/\sqrt{2}, 0\right)$ (b) (0, 0, P)
(c) $\left(P, -P, -P/\sqrt{2}\right)$ (d) $\left(0, 0, P/\sqrt{2}\right)$

7. In a structural member under fatigue loading, the minimum and maximum stresses developed at the critical point are 50 MPa and 150 MPa, respectively. The endurance, yield, and the ultimate strengths of the material are 200 MPa, 300 MPa, and 400 MPA, respectively. The factor of safety using modified Goodman criterion is

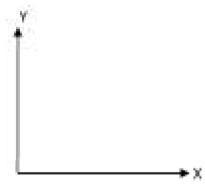
[GATE - 2016]

(a) $\frac{3}{2}$ (b) $\frac{8}{5}$

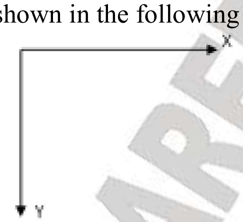
CHAPTER - 4
BENDING MOMENT AND SHEAR FORCE DIAGRAM

4.1 SHEAR FORCE AND BENDING MOMENT

At first we try to understand what shear force is and what is mending moment?
We will not introduce any other co-ordinate system. We use general co-ordinate axis as shown in the figure.

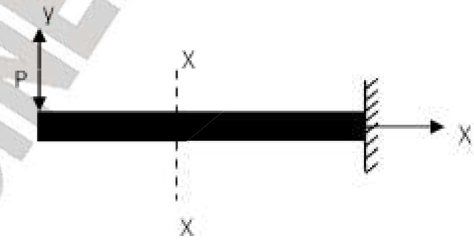


This system will be followed in shear force and bending moment diagram and in deflection of beam. Here downward direction will be negative i.e. negative Y-axis. Therefore downward deflection of the beam will be treated as negative.
Some books fix a co-ordinate axis as shown in the following figure.



Here downward direction will be positive i.e. positive Y-axis. Therefore downward deflection of the beam will be treated as positive. As beam is generally deflected in downward directions and this co-ordinate system treats downward deflection is positive deflection.

Consider a cantilever beam as shown subjected to external load 'P'. If we imagine this beam to be cut by a section X-X, we see that the applied force tend to displace the left-hand portion of the beam relative to the right hand portion, which is fixed in the wall. This tendency is resisted by internal forces between the two parts of the beam. At the cut section a resistance shear force (V_x) and a bending moment (M_x) is induced. This resistance shear force and the bending moment at the cut section is shown in the left hand and right hand portion of the cut beam.

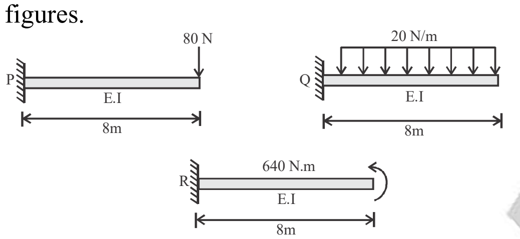


Using the three equations of equilibrium.

GATE QUESTIONS

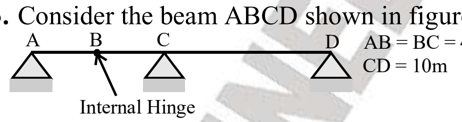
1. A simply supported beam is subjected to a uniformly distributed load. Which one of the following statements is true?
(a) Maximum or minimum shear force occurs where the curvature is zero.
(b) Maximum or minimum bending moment occurs where the shear force is zero.
(c) Maximum or minimum bending moment occurs where the curvature is zero.
(d) Maximum bending moment and maximum shear force occur at the same section.

2. Consider the three prismatic beams with the clamped supports P, Q and R as shown in the figures.



Given that the modulus of elasticity, E is 2.5 $\times$ 10^4 NOa; and the moment of inertia, I is 8 $\times$ 10^3 nn^4, the correct comparison of magnitudes of the shear force S and the bending moment M developed at the support is
(a) $S_P < S_Q < S_R$; $M_P = M_Q = M_R$
(b) $S_P = S_Q > S_R$; $M_P = M_Q > M_R$
(c) $S_F < S_Q < S_R$; $M_P = M_Q = M_R$
(d) $S_P < S_Q < S_R$; $M_F < M_Q < M_R$

3. Consider the beam ABCD shown in figure



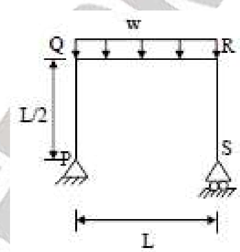
For a moving concentrated load of 50 kN on the beam, the magnitude of the maximum bending moment (in kn-m) obtained at the support C will be equal to _____

[GATE - 2017]

4. For a loaded catilever beam of uniform cross - section, the bending moment (in N-mm) along the length is $M(x) = 5x^2 + 10x$, where x is the distance (in mm) measured from the free end of the beam. The magnitude of shear force (in N) in the cross = section at x = 10mm is _____

[GATE - 2017]

5. The portal frame shown in figure is subjected to a uniformly distributed vertical load w (per unit length)

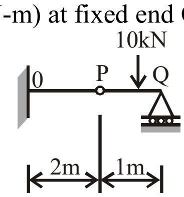


The bending moment in the beam at the joint 'Q' is

[GATE - 2016]

(a) Zero
(b) $\frac{wL^2}{24}$ (hogging)
(c) $\frac{wL^2}{12}$ (hogging)
(d) $\frac{wL^2}{8}$ (sagging)

6. A cantilever beam OP is connected to another beam PQ with a pin joint as shown in the figure. A load of 10kN is applied at the mid-point of PQ. The magnitude of bending moment (in kN-m) at fixed end O is



[GATE - 2015]

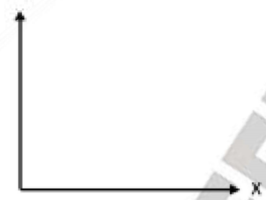
(a) 2.5
(b) 5
(c) 10
(d) 25

7. For the overhanging beam shown in figure, the magnitude of maximum bending moment (in kN-m) is _______.

CHAPTER - 5
DEFLECTION OF BEAM

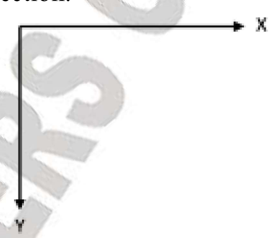
5.1 INTRODUCTION
1.We know that the axis of a beam deflects from its initial position under action of applied forces.
2.In this chapter we will learn how to determine the elastic deflections of a beam.

5.1.1 Selection of Co-ordinate Axis
We will not introduce any other co-ordinate system. We use general co-ordinate axis as shown in the figure. This system will be followed in deflection of beam and in shear force and bending moment diagram. Here downward direction will be negative i.e. negative Y-axis. Therefore downward deflection of the beam subjected to a given loading where we will use the formula,



$$EI\frac{d^2y}{dx^2} = M_x .$$

Some books fix a co-ordinate axis as shown in the following figure. Here downward direction will be positive i.e. positive Y-axis. Therefore downward deflection of the beam will be treated as positive. As beam is generally deflected in downward directions and this coordinate system treats downward deflection is positive deflection.

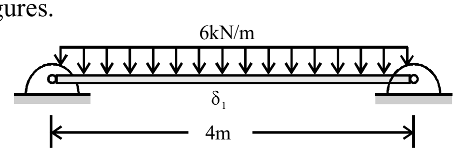


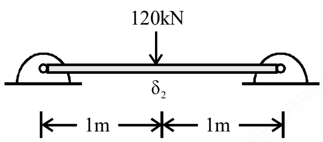
To determine the value of deflection of beam subjected to a given loading where we will use the formula, $EI\frac{d^2y}{dx^2} = -M_x$.

5.1.2 Why to calculate the deflections
1. To prevent cracking of attached brittle materials
2. To make sure the structure not deflected severely and to “appear” safe for its occupants.
3. To help analyzing statically indeterminate structures.
4. Information on deformation characteristics of members is essential in the study of vibrations of machines

GATE QUESTIONS

1. Two prismatic beams having the same flexural rigidity of 1000 kN-m^2 are shown in the figures.





If the mid-span deflections of these beams are denoted by δ_1 and δ_2 (as indicated in the figures), the correct option is

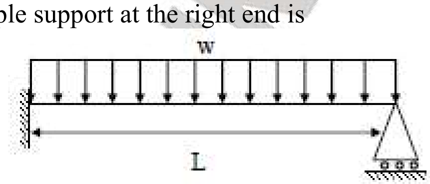
[GATE - 2017]

(a) $\delta_1 = \delta_2$ (b) $\delta_1 < \delta_2$
(c) $\delta_1 > \delta_2$ (d) $\delta_1 >> \delta_2$

2. A simply supported rectangular concrete beam of span 8m has to be span 8m has to be prestressed with a force of 1600 kN. The tendon is of parabolic profile having zero eccentricity at the supports. The beam has to carry an external uniformly distributed load of intensity 30 kN/m. Neglecting the self-weight of the beam, the maximum dip (in meters, up to two decimal places) of the tendon at the mid – span to balance the external load should be ____

[GATE - 2017]

3. A beam of length L is carrying a uniformly distributed load w per unit length. The flexural rigidity of the beam is EI. The reaction at the simple support at the right end is



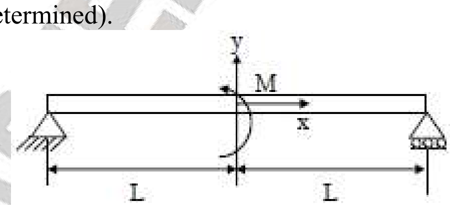
[GATE - 2016]

(a) $\frac{wL}{2}$ (b) $\frac{3wL}{8}$
(c) $\frac{wL}{4}$ (d) $\frac{wL}{8}$

4. A simply supported beam of length 2L is subjected to a moment M at the mid-point x =0 as shown in the figure. The deflection in the domain $0 \le x \le L$ is given by

$$W = \frac{-Mx}{12EIL}(L-x)(x+c)$$

Where E is the Young's modulus, I is the area moment of inertia and c is a constant (to be determined).

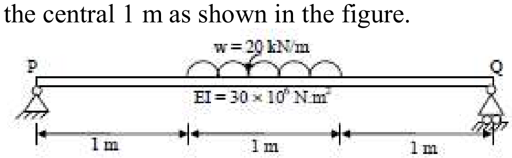


The slope at the center x = 0 is

[GATE - 2016]

(a) ML/(2EI) (b) ML/(3EI)
(c) ML/(6EI) (d) ML/(12EI)

5. A 3 m long simply supported beam of uniform cross section is subjected to a uniformly distributed load of w = 20 kN/m in the central 1 m as shown in the figure.



If the flexural rigidity (EI) of the beam is 30×10^6 N-m^2, the maximum slope (expressed in radians) of the deformed beam is

[GATE - 2016]

(a) 0.681×10^{-7} (b) 0.943×10^{-7}
(c) 4.310×10^{-7} (d) 5.910×10^{-7}

CHAPTER - 6
BENDING & SHEAR STRESSES

6.1 EULER'S BERNOULLI'S EQUATION or (Bending Stress Formula) or Bending Equation

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

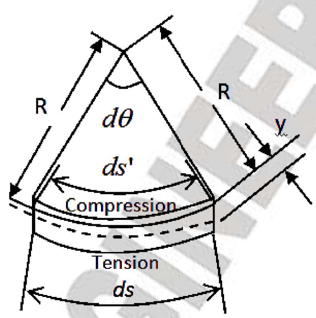
Where σ is bending Stress
M is bending moment
I is moment of inertia
E is modulus of elasticity
R is radius of curvature
y is distance of the fibre from NA (Neutral axis)

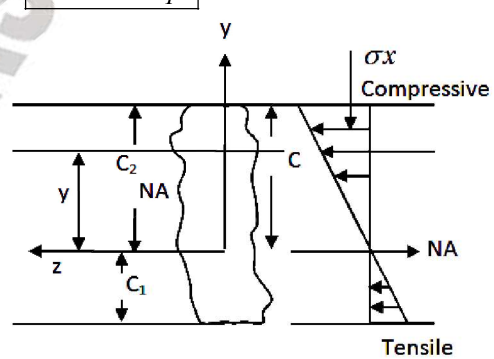
6.2 ASSUMPTIONS IN SIMPLE BENDING THEORY

All of the foregoing theory has been developed for the case of pure bending i.e. constant B.M. along the length of the beam. In such case

1. The shear force at each c/s is zero.
2. Normal stress due to bending is only produced.
3. Beams are initially straight.
4. The material is homogenous and isotropic i.e. it has a uniform composition and its mechanical properties are the same in all directions.
5. The stress strain relationship is linear and elastic.
6. Young's Modulus is the same in tension as in compression.
7. Sections are symmetrical about the plane of bending.
8. Sections which are plane before bending remain plane after bending.

6.3 $\boxed{\sigma_{max} = \sigma_t = \frac{Mc_1}{I}}$; $\boxed{\sigma_{min} = \sigma_c = \frac{Mc_2}{I}}$ (Minimum in sense of sign)





6.4 SECTION MODULUS (Z)

$$Z = \frac{I}{y}$$

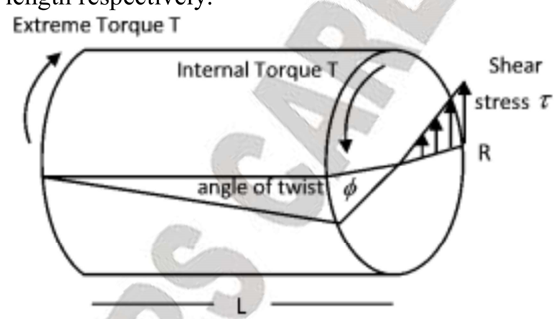
CHAPTER - 7
TORSION, THIN & THICK CYLINDER

7.1 INTRODUCTION
1. In machinery, the general term "shaft" refers to a member, usually of circular cross-section, which supports gears, sprockets, wheels, rotors, etc. and which is subjected to torsion and to transverse or axial loads acting singly or in combination.
2. An "axle" is a non-rotating member that supports wheels, pulleys,… and carries no torque.
3. A "spindle" is a short shaft. Terms such as line shaft, head shaft, stub shaft, transmission shaft, countershaft, and flexible shaft are names associated with special usage.

7.2 TORSION OF CIRCULAR SHAFTS
1. Equation for shafts subjected to torsion "T"

Torsion Equation, $\frac{\tau}{R}=\frac{T}{J}=\frac{G\theta}{L}$

Where j is polar moment of inertia
τ is shear stress induced due to torsion T
G is modulus of rigidity
θ is angular deflection or shaft
R, L is shaft radius and length respectively.



7.2.1 Assumptions
1. The bar is acted upon by a pure torque
2. The section under consideration is remote from the point of application of the load and from a change in diameter.
3. Adjacent cross sections originally plane and parallel remain and parallel after twisting, and any radial line remains straight.
4. The material obeys Hooke's law.
5. Cross-sections rotate as if rigid, i.e. every diameter rotates through the same angle.

GATE QUESTIONS

1. A motor driving a solid circular steel shaft transmits 40 kW of power at 500 rpm. If the diameter of the shaft is 40mm, the maximum shear stress in the shaft is __________MPa.

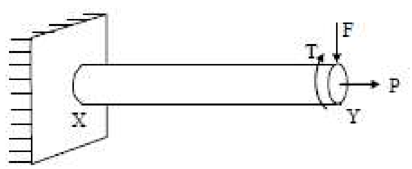
[GATE - 2017]

2. Two circular shafts made of same material, one solid (S) and one hollow (H), have the same length and polar moment of inertia. Both are subjected to same torque. Here. θ_S is the twist and τ_S is the maximum shear stress in the solid shaft, whereas θ_H is the twist and τ_H is the maximum shear stress in the hollow shaft. Which one of the following TRUE?

[GATE - 2016]

(a) $\theta_S = \theta_H$ and $\tau_S = \tau_H$
(b) $\theta_S > \theta_H$ and $\tau_S > \tau_H$
(c) $\theta_S < \theta_H$ and $\tau_S < \tau_H$
(d) $\theta_S = \theta_H$ and $\tau_S < \tau_H$

3. A machine element XY, fixed at end X, is subjected to an axial load P, transverse load F, and a twisting moment T at its free end Y. The most critical point from the strength point of view is



[GATE - 2016]

(a) A point on the circumference at location Y
(b) A point at the center at location Y
(c) A point on the circumference at location X
(d) A point at the center at location X

4. A shaft with a circular cross-section is subjected to pure twisting moment. The ratio of the maximum shear stress to the largest principal stress is

[GATE - 2016]

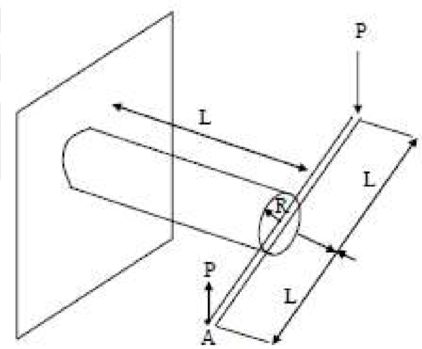
(a) 2.0 (b) 1.0
(c) 0.5 (d) 0

5. A thin cylindrical pressure vessel with closed-ends is subjected to internal pressure. The ratio of circumferential (hoop) stress to the longitudinal stress is

[GATE - 2016]

(a) 0.25 (b) 0.50
(c) 1.0 (d) 2.0

6. A rigid horizontal rod of length 2L is fixed to a circular cylinder of radius R as shown in the figure. Vertical forces of magnitude P are applied at the two ends as shown in the figure. The shear modulus for the cylinder is G and the Young's modulus is E.



The vertical deflection at point A is

[GATE - 2016]

(a) $PL^3/(\pi R^4 G)$ (b) $PL^3/(\pi R^4 E)$
(c) $2PL^3/(\pi R^4 E)$ (d) $4PL^3/(\pi R^4 G)$

7. A hollow shaft $d_0 = 2d_1$ where d_0 and d_1 are the outer and inner diameters respectively) needs to transmit 20kW power at 3000 RPM. If the maximum permissible shear stress is 30 MPa, d_0 is

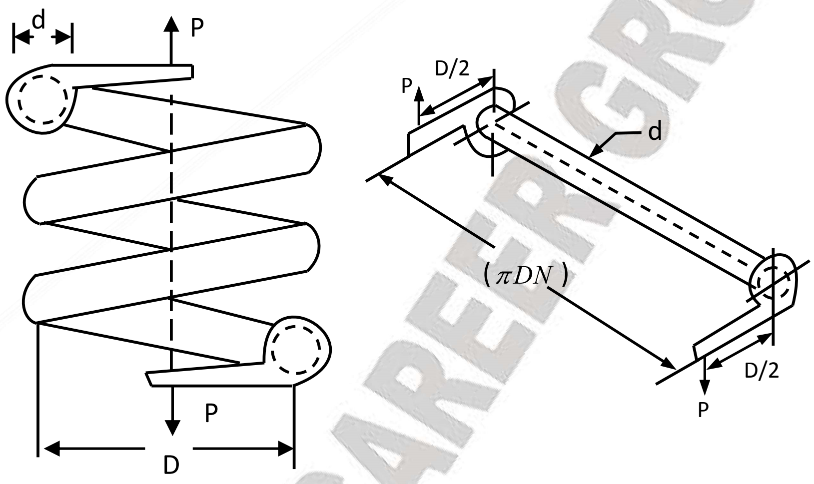
[GATE - 2015]

(a) 11.29 (b) 22.58 mm
(c) 33.87 mm (d) 45.16 mm

CHAPTER - 8
SPRINGS

8.1 INTRODUCTION
A spring is a mechanical device which is used for the efficient storage for the efficient and release of energy.

8.2 HELICAL SPRING-STRESS EQUATION
Let us a close-coiled helical spring has coil diameter D, wire diameter d and number of turn n.



The spring material has a shearing modulus G. The spring index, $C=\frac{D}{d}$. If a force 'P' is exerted in both ends as shown.

The work done by the axial force 'P' is converted into strain energy and stored in the spring.

U = (average torque) x (angular displacement) = $\frac{T}{2}\times\theta$

From the figure we get, $\theta=\frac{TL}{GJ}$

Torque ,T = $\frac{PD}{2}$

Length of wire, L = πDn

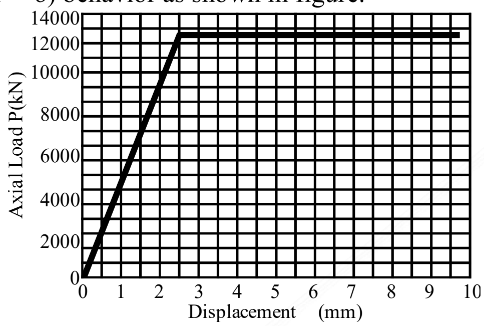
Polar moment of inertia, $(J)=\frac{\pi d^4}{32}$

Therefore $U=\frac{4P^2D^3n}{Gd^4}$

Accordingly to Castigilano's theorem, the displacement corresponding to force P is obtained by partially differentiating strain energy with respect to that force.

GATE QUESTIONS

1. A 2m long, axially loaded mild steel rod of 8mm diameter exhibits the load - displacement $(P - \delta)$ behavior as shown in figure.



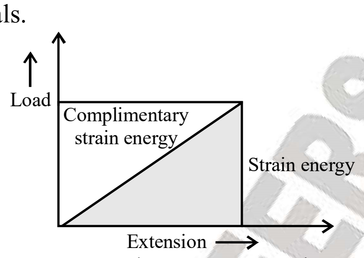
Assume the yield stress of steel as 250 MPa. The complementary strain energy (in N-mm) stored in the bar up to its linear elastic behaviour will be ___

[GATE - 2017]

SOLUTIONS

Sol. 1. (15,707.963)

Complimentary strain energy for linear materials.



Complimentary strain energy = strain energy

$$= \frac{1}{2}\sigma_y \times \varepsilon \times A \times L$$

$$= 12 \times 250 \times \frac{2.5}{2000} \times \frac{\pi}{4} \times 8^2 \times 2000$$

$$\left(\because \varepsilon = \frac{\delta \ell}{\ell}\right)$$

= 15,707.963 N-mm

CHAPTER - 9
THEORY OF COLOUMN & STRAIN ENERGY

9.1. INTRODUCTION
1.Strut: A member of structure which carries an axial compressive load.
2.Column: If the strut s vertical it is known as column.
3.A long, slender column becomes unstable when its axial compressive load reaches a value called the critical buckling load.
4.If a beam element is under a compressive load and its length is an order of magnitude larger than either of its other dimensions such a beam is called as columns.
5.Due to its size its axial displacement is going to be very small compared to its lateral deflection called buckling.
6.Buckling does not vary linearly with load it occurs suddenly and is therefore dangerous
7.Slenderness Ratio: The ratio between the length and least radius of gyration.
8.Elastic Buckling: Buckling with no permanent deformation
9.Euler's buckling is only valid for long, slender objects in the elastic region.
10.For short columns, a different set of equations must be used.

9.2. WHICH IS THE CRITICAL LOAD?
1. At this value the structure is in equilibrium regardless of the magnitude of the angle (provided it stays small)
2. Critical load is the only load for which the structure will be in equilibrium in the disturbed position
3. At this value, restoring effect of the moment in the spring matches the buckling effect of the axial load represents the boundary between the stable and unstable conditions.
4. If the axial load is less than P_{cr} the effect of the moment in the spring dominates and the structure returns to the vertical position after a small disturbance-stable condition.
5. If the axial load is larger than P_{cr} the effect of the axial force predominates and the structure buckles-unstable condition.
6. Because off the large deflection caused by buckling, the least moment of inertia/can be expressed as, $I = Ak^2$
7. Where A is the cross sectional area and r is the radius of gyration of the cross sectional area, i.e.

$$k_{min} = \sqrt{\frac{I_{min}}{A}}$$

8. Note that the smallest radius of gyration of the column, i.e. the least moment of inertia l should be taken in order to find the critical stress. L/k is called the slenderness ratio, it is a measure of the column's flexibility.

9.3. EULER'S CRITICAL LOAD FOR LONG COLUMN
9.3.1 Assumption
1. The column is perfectly straight and of uniform cross-section
2. The material is homogenous and isotropic
3. The material behaves elasticity3
4. The load is perfectly axial and passes through the Centroid of the column section.
5. The weight of the column is neglected.

CHAPTER - 10
THEORIES OF FAILURE

10.1 INTRODUCTION

When some external load is applied on a body, the stresses and strains are produced in the body. The stresses are directly proportional to the strains within the elastic limit. This means when the load is removed, the body will return to its original shape. There is no permanent deformation in the body.

However, If the stress produced in the body due to the application of the load, is beyond the elastic limit, the permanent deformations occur in the body. This means if the load is removed, the body will not retain its original shape. There are some permanent deformations in the body. Whenever permanent deformations occur in the body, the body is said to have "failed". This should be clear that failure does not mean rupture of the body.

The failure takes place when a certain limiting value is reached by one of following:
1. The maximum principal stress
2. The maximum principal strain.
3. The maximum shear stress.
4. The maximum strain energy
5. The maximum shear strain energy.

In all the above cases,

$\sigma_1, \sigma_2, \sigma_3$ = principal stresses in any complex system

σ^* = tensile or compressive stress at the elastic limit.

1. Maximum Principal Stress Theory

According to this theory, the failure of a material will occur when the maximum principal tensile stress (σ_1) in the complex system reaches the value of the maximum stress at the elastic limit in simple tension or the minimum principal stress (i.e, the maximum principal compressive stress) reaches the value of the maximum stress at the elastic limit in simple compression .

Let in a complex three dimensional stress system.

σ_1, σ_2 and σ_3 = principal stresses at a point in three perpendicular directions. The stresses σ_1 and σ_2 are tensile and σ_3 is compressive. Also σ_1 is more than σ_2 .

σ_t^* = tensile stress at elastic limit in simple tension.

σ_c^* = compressive stress at elastic limit in simple compression.

Then according to this theory, the failure will take place if

$\sigma_1 \geq \sigma_t^*$ in simple tension …(1)

Or $|\sigma_3| \geq \sigma_c^*$ in simple compression …(1.1)

Where $|\sigma_3|$ represents the absolute value of σ_3 .

This is the simplest and oldest theory of failure and is known as Rankine's theory. If the maximum principal stress (σ_1) is the design criterion, then maximum principal stress must not exceed the permissible stress (σ_t) for the given material.

Hence, $\sigma_1 = \sigma_t$ …(1.2)

Where σ_t = permissible stress and given by