GATE 2019

SIGNAL AND SYSTEM

ELECTRICAL ENGINEERING

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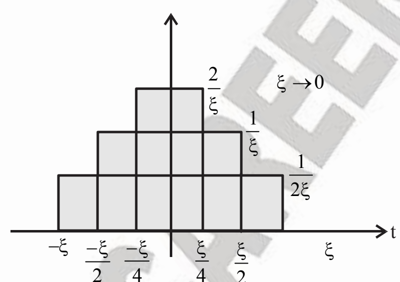
CHAPTER - 1
INTRODUCTION

1.1 SIGNAL
Signal is a function of one or more independent variables contain information about some behavior or natural phenomenon.
Example. Speech, Video, Audio, T.V. Signal, Current, Voltage, RF signal etc.

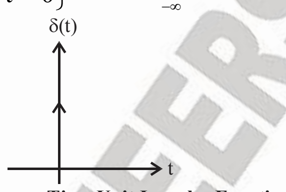
1.2 SYSTEM
Interconnection of various physical elements which are formed to get the desired response.
Example. High Pass filter, Low pass filter, Automobile Car, Mobile, Tablet, etc.

1.3 IMPORTANT SIGNALS
1.3.1 Continuous Time Unit Impulse Signal
The unit Impulse function $\delta(t)$ is known as Dirac Delta function.

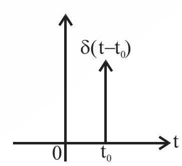


Unity area over an infinitesimal Time interval

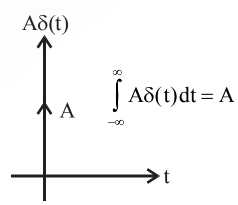
$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \qquad \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$



Continuous Time Unit Impulse Functior



Time Continuous Unit Impulse Functio



Scaled Unit Impulse Function

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WORKBOOK

Example 1. What are even and odd fart of x(t) = δ(t)

Solution.

$$\delta_e(t) = \frac{\delta(t)+\delta(-t)}{2}$$

$$\delta_0(t) = \frac{\delta(t)-\delta(-t)}{2}$$

δ(t) is even function δ(t) = δ(-t)

$$\delta_e(t) = \frac{2\delta(t)}{2} = \delta(t)$$

$$\delta_0(t) = \frac{\delta(t)-\delta(t)}{2} = 0$$

Example 2. What are even and odd parts of x(t) = u(t)

Solution.

$$u_e(t) = \frac{u(t)+u(-t)}{2} = \frac{1}{2}$$

$$u_0(t) = \frac{u(t)+u(-t)}{2} = \frac{1}{2}\text{Sgn}\,t$$

$$u(t) = \frac{1}{2} + \frac{1}{2}\text{Sgnt}$$

Example 3. What is R.M.S value of $x(t) = A_1 \cos(\omega t+\phi_1) + A_2 \cos(\omega t+\phi_2)$

Solution.

Power of signal x(t) will be

$$P = \frac{A_1^2}{2} + \frac{A_2^2}{2}$$

$$\text{R.M.S value} = \sqrt{\frac{A_1^2}{2} + \frac{A_2^2}{2}}$$

Example 4. Calculate the energy and power of signal

$x(n)=-(0.5)^n\, u(n)$

Solution.

$$E = \sum_{n=-\infty}^{\infty}\left[-(0.5)^n u(n)\right]^2$$

$$= \sum_{n=0}^{\infty}(0.25)^n = \frac{1}{1-0.25} = \frac{4}{3}$$

$$P = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-\infty}^{\infty}[x(n)]^2$$

$$= \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-0}^{\infty}\left(\frac{1}{4}\right)^n$$

$$P = \frac{1}{2N+1}\cdot\frac{4}{3} = 0$$

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ASSIGNMENT

1. Random signal can be modeled by
(a) Different equation
(b) Difference equation
(c) Statistical parameters
(d) Integral

2. Even signal satisfies
(a) $x(-t) = x(t)$
(b) $x[-n] = -x[n]$
(c) $x(n+1) = ax[n] + b$
(d) $\frac{dx(t)}{dt} = c$

3. Odd signal satisfies
(a) $x(-t) = x(t)$
(b) $x[-n] = -x[n]$
(c) $x(n+1) = ax[n] + b$
(d) $\frac{dx}{dt}(t) = c$

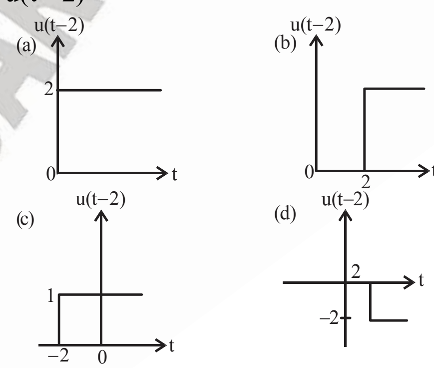
4. Any signal $x(t)$ can be expressed as
(a) $x_e(t) + x_0(t)$
(b) $x_e(t) - x_0(t)$
(c) $\frac{x_e(t)}{x_0(t)}$
(d) $x_e(t) \times x_0(t)$
Where $x_e(t)$ and $x_0(t)$ are even and odd parts of the signal $x(t)$.

5. Periodic signals are
(a) $x(t+T) = x(t)$
(b) $x(t-T) = x(t)$
(c) $x(n+mN) = x[n]$
(d)All the above

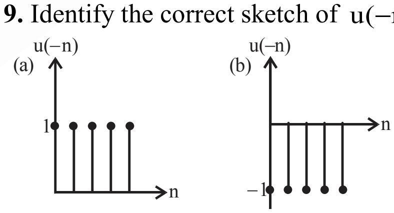
6. Energy signals are the signals with
(a) $0 < E < \infty, P = 0$
(b) $0 < E < \infty, P = \infty$
(c) $0 < P < \infty, E = \infty$
(d) $0 < P < \infty, E = 0$
Where E and P are average energy and power of the signals $x(t)$ or $x[n]$.

7. Power signals are the signals with
(a) $0 < E < \infty, P = 0$
(b) $0 < E < \infty, P = \infty$
(c) $0 < P < \infty, E = \infty$
(d) $0 < P < \infty, E = 0$

8. Identify the correct sketch of unit step signal $u(t-2)$



9. Identify the correct sketch of $u(-n)$



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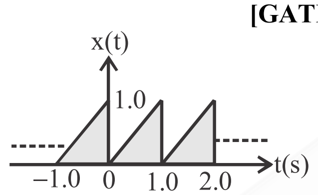
GATE QUESTIONS

1. A periodic signal x(t) is shown in the figure. The fundamental frequency of the signal x(t) in Hz is _____.

[GATE - 2017]



2. If a continuous-time signal $x(t) = \cos(2\pi t)$ is sampled at 4Hz, the value of the discrete time sequence x(n) at n = 5 is

[GATE - 2017]

(a) –0.707 (b) –1
(c) 0 (d) 1

3. Two sequences $x_1[n]$ and $x_2[n]$ have the same energy. Suppose $x_n[n] = \alpha\, 0.5^n\, u[n]$, where α is a positive real number and u[n] is the unit step sequence. Assume

$$x_n[n] = \begin{Bmatrix} \sqrt{1.5} \text{ for } n = 0,1 \\ 0 \quad \text{otherwise.} \end{Bmatrix}$$

Then the value of α is _____.

[GATE - 2015]

4. For a periodic signal $v(t) = 30 \sin 100t + 10 \cos 300t + 6 \sin(500t + \pi/4)$, the fundamental frequency in rad/s

[GATE - 2015]

(a) 100 (b) 300
(c) 500 (d) 1500

5. The Dirac delta function $\delta(t)$ is given as

[GATE - 2006]

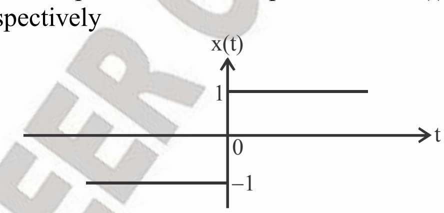
(a) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(b) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(c) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

(d) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

6. The function x(t) is shown in figure. Even and odd part of a unit step function u(t) are respectively



[GATE - 2005]

(a) $\frac{1}{2}, \frac{1}{2}x(t)$ (b) $\frac{1}{2}, \frac{1}{2}x(t)$
(c) $\frac{1}{2}, -\frac{1}{2}x(t)$ (d) $\frac{1}{2}, -\frac{1}{2}x(t)$

7. The power in the signal

$s(t)=8\cos\left(20\pi t - \frac{\pi}{2}\right) + 4\sin(15\pi t)$ is:

[GATE - 2005]

(a) 40 (b) 41
(c) 42 (d) 82

8. Consider the sequence

$x[n]=(4-5j1+j24)$

The conjugate anti-symmetric part of the sequence is

[GATE - 2004]

(a) $[-4-j2.5,\ j2,\ 4-j2.5]$
(b) $[-j2.5,\ 1,\ j2.5]$
(c) $[-j2.5,\ j2,\ 0]$
(d) $[-4,\ 1,\ 4]$

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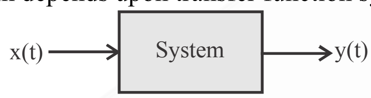
CHAPTER - 2
SYSTEM

2.1 SYSTEM
A system is a mathematical model of physical process that relates the Inputs (or excitation) signal to the output (or response) of signal.
The response or output of system depends upon transfer function system.



Mathematically, the functional Relationship between I/P and O/P may be written as:

$$y(t) = f\{x(t)\}$$

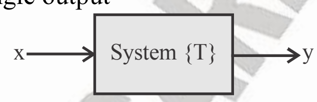
$$y(t) = T[x(f)]$$

Where T implies transformation and gives a mapping to be done on x(t) to get y(t)

2.1.1 Symbolically, we can write

$$x(t) \xrightarrow{s} y(t)$$

Multiple input and/or output signals are possible. But we will restrict our attention for most part in this course to the single Input single output



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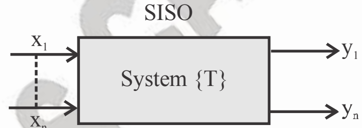
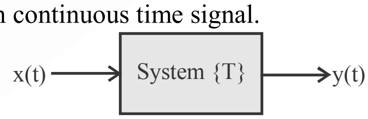


MIMO

Examples of system. Filters, amplifiers, communication channels, T.V. set are various example of electrical system.

2.2 TYPES OF SYSTEMS
1. Continuous-Time System
2. Discrete-Time System

1. Continuous-Time System
Continuous - Time system may be defined as also continuous. This means that Input and output of continuous time system are both continuous time signal.



Example. Audio, Video Amplifier, Power supplies etc.
Simple Practical example of continuous time – system is Low Pass Filter

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GATE QUESTIONS

1. Consider a single input single output discrete- time system with x[n] as input and y[n] as output, where the two are related as

$$y[n]=\begin{cases} n\,|x[n]|, & \text{for } 0\le n\le 10 \\ x[n]-x[n-1], & \text{otherwise}\end{cases}$$

Which one of the following statements is true about the system ?

[GATE - 2017]

(a) It is causal and stable
(b) It is causal but not stable
(c) It is not causal but stable
(d) It is neither causal nor stable

2. Consider and LTI system with magnitude response $|H(f)|=\begin{cases}1-\dfrac{|f|}{20}, & |f|\le 20\\ 0, & |f|>20\end{cases}$ and phase response Arg[H(f)]=2f.
If the input to the system is

$$x(t)=8\cos\left(20\pi t+\frac{\pi}{4}\right)+16\sin\left(40\pi t+\frac{\pi}{8}\right)+24\cos\left(80\pi t+\frac{\pi}{16}\right)$$

Then the average power of the output signal y(t) is___

[GATE - 2017]

3. The transfer function of a causal LTI system is H9s) = 1/s. if the input to the system is x(t) = [sin(t)/πt]u(t); where u(t) is a unit step function. The system output y(t) as $t\to\infty$ is__

[GATE - 2017]

4. The signal x(t) = sin(1400πt), where t is in seconds, is sampled at a rate of 9000 samples per second. The sampled signal is the input to an ideal lowpass filter with frequency response H(f) as follows:

$$H(f)=\begin{cases}1, & |f|\le 12\text{kHz}\\ 0, & |f|>12\text{kHz}\end{cases}$$

What is the number of sinusoids in the output and their frequencies in kHz?

[GATE - 2017]

(a) Number = 1, frequency = 7
(b) Number = 3, frequencies = 2,7,11
(c) Number = 2, frequencies = 2, 7
(d) Number = 2, frequencies = 7, 11

5. An LTI system with unit sample response h(n) = 5δ[n] – 7δ [n –1] + 7δ[mn–3] – 5δ[n –4] is a

[GATE - 2017]

(a) Low pass filter (b) High pass filter
(c) Band pass filter (d) Band stop filter

6. The input x(t) and the output y(t) of a continuous time system are related as

$y(t)=\int_{t-T}^{t} x(u)\,du$. The system is

[GATE - 2017]

(a) Linear and time variant
(b) Linear and time invariant
(c) Non Linear and time variant
(d) Nonlinear and time invariant

7. Consider $g(t)=\begin{Bmatrix} t-\lfloor t\rfloor, t\ge 0\\ t-\lceil t\rceil, \text{otherwise}\end{Bmatrix}$,

Where $t\in R$
Here, $\lfloor t\rfloor$ represent the largest integer less than or equal to t and $\lceil t\rceil$ denotes the smallest integer greater than or equal to t. The coefficient of the second harmonic component of the fourier series representing g(t) is _____

[GATE - 2017]

8. Consider the signal x(t) = cos(6πt) + sin(8πt), where t is in seconds. The Nyquist sampling

CHAPTER - 3
LINEAR TIME-INVARIANT SYSTEM

3.1 INTRODUCTION
A system is a mathematical model of physical process that relates the Inputs (or excitation) signal to the output (or response) of signal. We have discussed the Several Basic Properties of systems, two properties namely Linearity and Time - invariance plays a important role in analysis of signals and system. If a system has both linearity and time - invariance properties, then this system is called Linear -Time Invariant system (LTI system).

> We study LTI system because of the fact that the most of practical and physical system can be modeled in form of Linear-Time (invariant) System

In this chapter we develop the fundamental Input-Output relationships for system having these properties and it will be shown that Input-output Relationship for LTI system is described of a convolution operation.
Importance of convolution operation if one knows the output of unit Impulse then output for general input can be calculated.

3.2 CHARACTERISTICS OF LINEAR TIME - INVARIANT(LTI) SYSTEM
Both continuous-time and discrete-time linear time invariant (LTI) system exhibit one important characteristics that the superposition theorem can be applied to find the response y(t) to a given input x(t).

3.2.1 Important steps to adopted to find response of LTI system using superposition
1. Resolve the input function x(t) in terms of simple or basic function like impulse function for which response can be easily evaluated.
2. Determine Response of LTI system for simple or Basic functional individually.
3. Using superposition theorem, find the sum of individual response which will become overall response y(t) of function x(t) from above, to find the response of LTI system to given function first we have to find the response of LTI system to an unit impulse called as unit impulse response of LTI system.

3.2.2 Unit Impulse Response [$h(t)_n$ or h/n]
Impulse response of continuous time or discrete-time LTI system is output of system due to an unit impulse input applied at time t = 0 or n = 0.

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GATE QUESTIONS

1. Let the input be u and the output be y of a system, and the other parameters are real constants. Identify which among the following systems is not a linear system:

[GATE - 2018]

(a) $\frac{d^3y}{dt^2}+a_1\frac{d^2y}{dt^2}+a_2\frac{dy}{dt}+a_3y$

$$=b_3u+b_2\frac{du}{dt}+b_1\frac{d^2u}{dt^2}$$

(b) $y(t)=\int_0^t e^{\alpha(t-\tau)}\beta u(\tau)d\tau$

(c) y = au + b , $b \neq 0$

(d) y =a u

2. Let z(t) = x(t)*y(t). Where "*" denotes convolution. Let c be a positive real-valued constant.

Choose the correct expression for z(ct).

[GATE - 2017]

(a) c.x(ct)*y(ct) (b) x(ct)*y(ct)

(c) c.x(t)*y(ct) (d) c.x(ct)*y(t)

3. Consider the system with following input-oputput relation $y[n] = (1 + (-1)^n)x[n]$

Where , x[n] is the input and y[n] is the output. The system is

[GATE - 2017]

(a) Invertible and time invariant

(b) Invertible and time varying

(c) Non-invertible and time invariant

(d) Non-invertible and time varying

4. The result of the convolution

$x(-t) * \delta(-t-t_0)$ is

[GATE - 2015]

(a) $x(t+t_0)$ (b) $x(t-t_0)$

(c) $x(-t+t_0)$ (d) $x(-t-t_0)$

5. The impulse response of an LTI system can be obtained by

[GATE - 2015]

(a) Differentiating the unit ramp response

(b) Differentiating the unit step response

(c) Integrating the unit ramp response

(d) Integrating the unit step response

6. For linear tune invariant systems, that are Bounded Input Bounded Output stable, winch one of the following statements is TRUE?

[GATE - 2014]

(a) The impulse response will be integrable, but may not be absolutely integrable.

(b) The unit impulse response will have finite support.

(c) The unit step response will be absolutely integrable.

(d) The unit step response will be bounded.

7. Consider an LTI system with transfer function $H(s)=\frac{1}{s(s+4)}$

If the input to the system is cos(3t) and the steady state output is A sin(3t + α), then the value of A is

[GATE - 2014]

(a) 1/30 (b) 1/15

(c) 3/4 (d) 4/3

8. Consider an LTI system with impulse response $h(t) = e^{-5t}u(t)$. If the output of the system is $y(t) = e^{-3t}u(t) = -e^{-5t}u(t)$ then the input, x(t), is given by

[GATE - 2014]

(a) $e^{-3t}u(t)$ (b) $2e^{-3t}u(t)$

(c) $e^{-5t}u(t)$ (d) $2e^{-5t}u(t)$

9. Two systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by

[GATE - 2013]

(a) Product of $h_1(t)$ and $h_2(t)$

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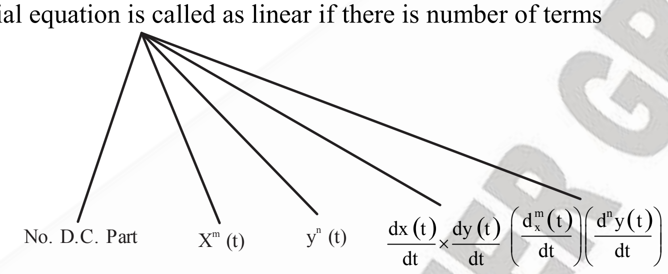
CHAPTER - 4
LINEAR-TIME INVARIANT-2 SYSTEM

4.1 LINEAR CONSTANT CO-EFFICIENT DIFFERENTIAL EQUATIONS (LCC DE)
A general n^{th} -order linear - constant co-efficient differential equation is given by

$$\sum_{K=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{K=0}^{M} b_K \frac{dk_x(t)}{dt^k}$$

Above equation is representation of continuous system.

1. A differential equation is called as linear if there is number of terms



Example. $\frac{d^2 y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 5 = x(t)$

It is Non-Linear differential equation because a d.c. term present in it.

2. Differential equation is said to be time-invariant if all the co-efficient of differential equation are const.

So, $\frac{a_n d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + ...y(t) = b_0 x(t)$

Represents the linear constant co-efficient differential equation.

LCCDE equation is used to analyse the LTI system or we can say if any system is LTI system then it can be represented in differential equation by LCCDE

4.1.1 System described by Difference Equations
The role of differential equation in describing continuous-time system is played by difference equations for discrete-time system.

4.2 LINEAR CONSTANT COEFFICIENT DIFFERENCE EQUATIONS (LCCDE)
The discrete time counterpart of general differential equation is the nth order linear cost. Co-efficient difference equation is given by

$$\sum_{K=0}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$$

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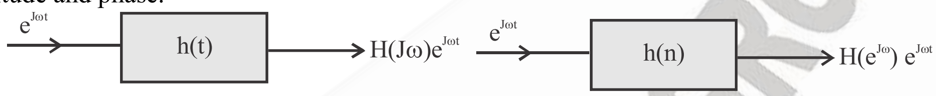
CHAPTER - 5
CONTINUOUS-TIME FOURIER SERIES

5.1 INTRODUCTION

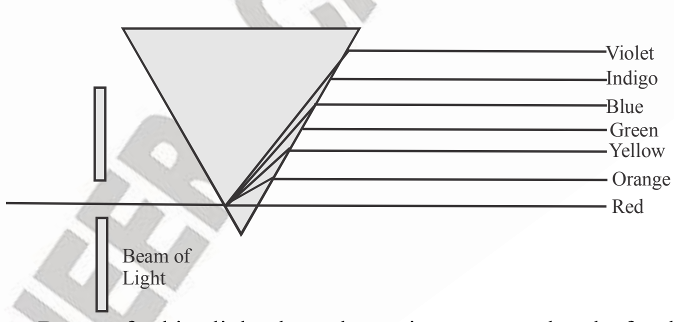
In this chapter we explore an alternative representation of signals and LTI system. Here we represent signal as linear combination of complex exponentials.
The complex exponential in study of LTI system is important from the fact that the response of an LTI system to a complex exponential input is same complex exponential with only change in amplitude and phase.



Till now all signals are draw with respect to time.
That means ‘t’ was considered as variable. The representation of signal w.r.t. to time is called time domain representation.
Since the time domain representation of signal is net sufficient for its analysis. For sake of analysis we will use the frequency domain representation.
In frequency domain representation the variable is frequency ‘f’ rather than ‘t’.
The signal represented in frequency domain is called as “spectrum” the spectrum consists two graphs.

1. Amplitude spectrum occurs in $|M(j\omega)|\,Vs\,\omega$
2. Phase spectrum occurs in $|\phi(j\omega)|\,Vs\,\omega$

Example. Why frequency domain representations is important?
Solution.



Now when we pass Beam of white light through a prism, we get band of colours produced by prism. It means that light Beam passing through a prism analysed it into its colour component without any change. The output colour can be normally specify as spectrum or analysis of light into color is nothing, but frequency analysis.
For analysis the signal, the various parameters are evaluated as

1. Amplitude
2. Frequency context
3. Power and energy densities
4. Periodicity

Periodicity can be obtained in time-domain but frequency content is not evaluated in time-do main.
Hence these signal has to be transformed in frequency domain with the help of fourier series,

CHAPTER - 6
FOURIER TRANSFORM

6.1 INTRODUCTION

In previous chapter we have discussed the Fourier series which is tool used to analyse a periodic-time signal in frequency domain.
Disadvantage of Fourier series is it cannot analyse the non-periodic signals.
So Fourier develops a new tool to analyse the non-periodic or a periodic signal in frequency domain known a Fourier transform.

Fourier transform can be used to analyse both periodic and non-periodic signals.

6.2 ANALYSIS OF NON-PERIODIC FUNCTION OVER ENTIRE INTERVAL

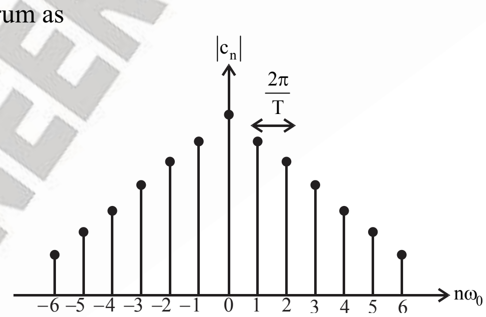
A non-periodic signal may assume as limiting case of periodic signal where the period of signal approaches infinity. Such a signal form by replacing fundamental time period T $\rightarrow \infty$ let us consider a periodic function x(t) having period T. The complex Fourier series representation of function may be written.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{Jn\omega_0 t} \quad \ldots \text{(i)}$$

$$\omega_0 = \frac{2\pi}{T}$$

$$c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-Jn\omega_0 t\,dt} \quad \ldots\text{(ii)}$$

The typical discrete spectrum as



Spacing between successive harmonics will be

$$\omega_0 = \frac{2\pi}{T} = \Delta\omega(\text{say})$$

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ESE OBJ QUESTIONS

1. The Fourier Transform of $e^{-\frac{x^2}{2}}$ is

[EE ESE - 2018]

(a) $\frac{1}{2}.e^{-\frac{\omega^2}{2}}$ (b) $e^{-\frac{\omega^2}{2}}$

(c) $\frac{\pi}{2}$ (d) $\sqrt{\pi}$

2. Fourier series of any periodic signal x(t) can be obtained if

1. $\int_0^T |x(t)|\,dt < \infty$

2. Finite number of discontinuities within finite time interval t

3. Infinite number of discontinuities

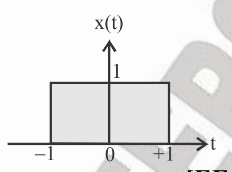
Select the correct answer using the codes given below:

[EE ESE - 2017]

(a) 1, 2 and 3 (b) 1, and 3 only

(c) 1 and 2 only (d) 2 and 3 only

3. The laplace transform of the below function is



[EE ESE - 2017]

(a) $\omega \sin \omega$ (b) $\frac{2\sin\omega}{\omega}$

(c) $\frac{\omega}{\sin\omega}$ (d) $\frac{\cos\omega}{\omega}$

4. Fourier series of any periodic signal x(t) can be obtained if

1. $\int_0^T |x(t)|\,dt < \infty$

2. Finite Number of discontinuities within finite time interval t

3. Infinite number of discontinuities

Select the correct answer using the codes given below:

[EC ESE - 2017]

(a) 1, 2 and 3 (b) 1 and 3 only

(c) 1 and 2 only (d) 2 and 3 only

5. The fourier transform of a rectangular pulse is

[EC ESE - 2012]

(a) Another rectangular pulse

(b) Triangular pulse

(c) sinc function

(d) Impulse function

6. Which one of the following is Dirichelt condition ?

[EC ESE - 2010]

(a) $\int_{t_1}^{\infty} |x(t)| < \infty$

(b)Signal x(t) must have a finite number of maxima and minima in the expansion interval

(c)x(t) can have an infinite number of finite discontinuities in the expansion interval

(d)$x^2(t)$ must be absolutely summable

7. If f(t) is an even function, then what is its Fourier transform $F(j\omega)$?

[EC ESE - 2008]

(a) $\int_0^\infty f(t)\cos(2\omega t)\,dt$

(b) $2\int_0^\infty f(t)\cos(\omega t)\,dt$

(c) $2\int_0^\infty f(t)\sin(\omega t)\,dt$

(d) $2\int_0^\infty f(t)\sin(2\omega t)\,dt$

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CHAPTER - 7
CORRELATION AND FILTERING ACTION

7.1 CORRELATION

Correlation is used to find similarity between two signals.
There are two type of correlation

1. Auto-Correlation
It is used to find similarity between two same signals

2. Cross-Correlation
It is used to find similarity between different signal.

7.1.1 Auto-Correlation

7.1.1.1 Autocorrelation Function

It gives the measure of similarity, match or coherence between a signal and a delayed function.
A signal may be energy signal or power signal.

7.1.1.2 Autocorrelation Function of Energy Signal

Autocorrelation function of this signal may be obtained by integrating the product of signal x(t) and delayed version of its.

$$R_{xx}(\tau)=\int_{-\infty}^{\infty} x(t)\,x(t-\tau)\,dt$$

Where τ is called is searching parameter.

7.1.1.3 Relationship between Auto Correlation and Convolution

$$R_{xx}(\tau)=x(t)\otimes x(-t)\big|_{\text{Replace } t=\tau}$$

7.1.1.4 Properties of Autocorrelation Function

1. Auto correlation is an even function.

$$R_{xx}(\tau)=R_{xx}(-\tau)$$

2. It τ is increased in either direction, the auto correlation reduces, As τ reduces auto correlation increase and it maximum at τ = 0 i.e. at origin mathematically,

$R_{xx}(\tau)<R_{xx}(0)$ for all τ.

and $\lim_{\tau\to\infty} R_{xx}(\tau)=0$

3. Autocorrelation function at τ = 0 gives energy of signal

i.e. $R_{xx}(\tau)=\int_{-\infty}^{\infty} x(t)\,x(t-\tau)\,dt$

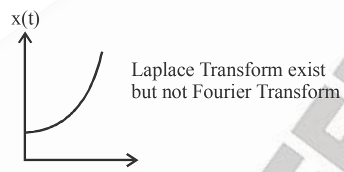
Substituting τ = 0

$$R_{xx}(0)=\int_{-\infty}^{\infty} x^2(t)\,dt$$

$$R_{xx}(0)=\text{Energy of signal}$$

CHAPTER - 8
LAPLACE TRANSFORM

8.1 INTRODUCTION
In previous chapters, we have seen the tools such as Fourier series and Fourier Transform to analyse the signals. Now the Laplace Transform is another mathematical tool which is used for analysis of signals and system. Infect, Laplace Transform provides broader characterization of signal and systems compared to Fourier Transform.
The Laplace Transform can be used where Fourier Transform cannot be used.
Laplace Transform can be used for analysis of unstable systems whereas Fourier Transform has several limitation.
Example.
for given $x(t) = e^{3t.U(t)}$



8.1.1 Definition of Laplace Transform
For general continuous time signal x(t)
The Laplace Transform x(s) is defined as.

$$x(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

Where s is generally a complex variable and is expressed as
$S = \sigma + J\omega$
Also known as complex frequency
Where σ is real part and ω is imaginary part.
For convenience we will sometime denotes the Laplace Transform in operator form al $\angle[x(t)]$ and denote the transform relationship between x(t) and x(s)
As $x(t) \xleftrightarrow{L.T} x(s)$
e^{-st} is kernel of function
It may be noted that integration is taken from 0 to ∞. Therefore, this is called Bilateral Laplace Transform.
Similar if x(t) is zero for $t < 0$
Then Laplace may be defined as

$$\boxed{x(s) = \int_{0}^{\infty} x(t)e^{-st}at}$$

Where $s = \sigma + J\omega$
Integration is taken from 0 to ∞. This is called as unilateral/one-sided Laplace Transform.

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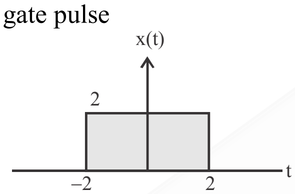
WORKBOOK

Example 1. Find the Laplace of x(t) = e^{-2t} U(t)– e^{-3t} U(–t)

Solution.

Laplace Transform is undefined as no common ROC.

Example 2. Find the Laplace Transform of following gate pulse



Solution.

$$x(t) = 2u(t+2) - 2u(t-2)$$

$$x(s) = \frac{2e^{2s}}{s} - \frac{2e^{-2s}}{s}$$

ROC: Entire S-plane finite duration signal

Example 3. Let x(t) = αs(t) + s(–t)

S(t) = βe^{-3t} U(t) where U(t) is step function and

$$x(s) = \frac{9}{s^2-9} \quad \text{for } -3 < 6 < 3$$

Find α and β ?

Solution.

$$L[s(t)] = \frac{\beta}{s+3} \quad 6 > -3$$

$$L\left[s(-t)\right] = \frac{+\beta}{-s+3} \quad -6 > -3$$

Replace s → –s

$$L[s(-t)] = \frac{\beta}{3-s} \quad \sigma < 3$$

So, x(t) = α s(t) + s(–t)

$$x(s) = \alpha L[s(t)] + L[s(-t)]$$

$$\frac{9}{s^2-9} = \frac{\alpha\beta}{s+3} + \frac{\beta}{3-s} \quad \text{for} -3 < \sigma < 3$$

$$\frac{9}{s^2-9} = \frac{\alpha\beta}{s+3} - \frac{\beta}{s-3}$$

$$\frac{9}{s^2-9} = \frac{\alpha\beta(s-3)}{s^2-9} - \beta(s+3)$$

$$\frac{9}{s^2-9} = \frac{s(\alpha\beta-\beta) - 3\alpha\beta - 3\beta)}{s^2-9}$$

By comparison

$$\alpha\beta - \beta = 0 \quad \& -3(\alpha\beta+\beta) = 9$$

$$\alpha\beta = \beta \text{ ----- (1)}$$

From (1) – 3(2β) = 9

and α = 1 β = –9/6 = –3/2

Example 4. Consider two 2 right sided signal x(t), y(t) related as $\frac{d}{dt}x(t) = -2y(t) + \delta(t)$ and

$$\frac{dy(t)}{dt} = 2x(t)$$

Find x(s) and y(s)?

Solution.

$$\frac{d}{dt}x(t) = -2y(t) + \delta(t)$$

$$sx(s) = -2y(s) + 1$$

$$\frac{d}{dt}y(t) = 2x(t)$$

$$sy(s) = 2x(s)$$

$$sx(s) = \frac{-4}{5}x(s) + 1$$

$$x(s)[s^2+4] = s$$

$$x(s) = \frac{s}{s^2+4}$$

$$y(s) = \frac{2}{s^2+4}$$

Example 5. Find the T.F of system described by given equation.

i.e. $\frac{dy(t)}{dt} + 4y(t) + 3\int_{-\infty}^{t} y(\tau)d\tau = x(t)$

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ASSIGNMENT

1. Laplace transform analysis helps in
(a) Solving integral differential equation.
(b) Converts differential equation into algebraic equation
(c) Converts integral equation into algebraic equation
(d) All the above

2. Laplace transform $x(s)$ of signal $x(t)$ is
(a) $\int_{-\infty}^{\infty} x(t)e^{-st}dt$ (b) $\int_{-\infty}^{\infty} x(t)e^{-st}dt$
(c) $\int_{-\infty}^{\infty} e^{-st}dt$ (d) $\int_{-\infty}^{\infty} x(t)e^{-st}dt$

3. Bilateral and unilateral Laplace transform differs in terms of
(a) Lower limit of integration
(b) Upper limit of integration
(c) They are same
(d) Bilateral transform does not exist.

4. Laplace transform of $u(t)$ is
(a) $\frac{1}{s}$ (b) s
(c) 1 (d) s^2

5. Laplace transform of $x(t) = t$ is
(a) $\frac{2}{s^2}$ (b) $\frac{1}{s^2}$
(c) s^2 (d) $\frac{1}{s}$

6. Region of the convergence of x(S) contain
(a) Zeros (b) Poles
(c) No zeros (d) No pole

7. Inverse Laplace transform of $\frac{1}{(s+a)^2}$ is
(a) $tu(t)$ (b) $te^{-at}u(t)$
(c) $e^{-at}u(t)$ (d) $ae^{-at}u(t)$

8. $x(t) = \cos\omega_0 t\, u(t)$
(a) $\frac{s}{s^2+\omega_0^2}$ (b) $\frac{1}{s^2+\omega_0^2}$
(c) $\frac{\omega_0}{s^2+\omega_0^2}$ (d) $\frac{\omega_0}{s^2}$

9. Laplace transform of signal $x_1(t)$ convolving with $x_2(t)$ is
(a) $x_1(s) * x_2(s)$ (b) $x_1(s)x_2(s)$
(c) $x_1(s)/x_2(s)$ (d) $x_1(t)x_2(t)$

10. For causal continuous-time LTI system, ROC is in the
(a) Left of all system poles
(b) Right of all system poles
(c) Right of all zeros
(d) Left of all zeros

11. If the system is causal and stable, the system poles must lie
(a) On the $j\omega$ axis
(b) On the left half of s-plane
(c) On the right half of s-plane
(d) Both (a) and (c)

12. Laplace transform of $\frac{d}{dt}x(t)$ is
(a) $sx(s)$ (b) $\frac{x(s)}{s} - x(0^-)$
(c) $sx(s) - x(0^-)$ (d) $x(s) - x(0^-)$

13. Laplace transform of $\int_0^t x(\tau)d\tau$ is
(a) $\frac{1}{s}x(s)$ (b) $sx(s)$
(c) $s^2x(s)$ (d) $x(s)/s^2$

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GATE QUESTIONS

1. The solution of the differential equation

$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=0$ with $y(0)=y'(0)=1$ is

[GATE - 2015]

(a) $(2-t)e^{t}$ (b) $(1+2t)e^{-t}$
(c) $(2+t)e^{-t}$ (d) $(1-2t)e^{t}$

2. The bilateral Laplace transform of a function $f(t)=\begin{cases}1 \text{ if } a\le t\le b\\ 0 \text{ otherwise}\end{cases}$ is

[GATE - 2015]

(a) $\frac{a-b}{s}$ (b) $\frac{e^{z}(a-b)}{s}$
(c) $\frac{e^{-az}-e^{-bz}}{s}$ (d) $\frac{e^{z(a-b)}}{s}$

3. Let the signal f(t) = 0 outside the interval $[T_1,T_2]$, where T_1 and T_2 are finite. Furthermore, $|f(t)| < \infty$. The region of convergence (ROC) of the signal's bilateral Laplace transform F(s) is

[GATE - 2015]

(a) A parallel strip containing the jΩ axis
(b) A parallel strip no containing the jΩ axis
(c) The entire s-plane
(d) A half plane containing the jΩ axis

4. Let $x(t) = \alpha\, s(t) + s(-t)$ with $s(t) = \beta e^{-4t}u(t)$, where u(t) is unit step function. If the bilateral Laplace transform of x(t) is

$X(s)=\frac{16}{s^2-16}\ -4<\text{Re}\{s\}<4;$

Then the value of β is ________.

[GATE - 2015]

5. Consider the differential equation $\frac{dx}{dt}=10-0.2x$ with initial condition x(0) = 1. The response x(t) for t > 0 is

[GATE - 2015]

(a) $2-e^{-0.2t}$ (b) $2-e^{0.2t}$
(c) $50-49e^{-0.2t}$ (d) $50-49e^{0.2t}$

6. The output of a standard second-order system for a unit step input is given as $y(t)=1-\frac{2}{\sqrt{3}}e^{-t}\cos\left(\sqrt{3}t-\frac{\pi}{6}\right)$. The transfer function of the system is

[GATE - 2015]

(a) $\frac{2}{(s+2)(s+\sqrt{3})}$ (b) $\frac{1}{s^2+2s+1}$
(c) $\frac{3}{s^2+2s+3}$ (d) $\frac{4}{s^2+2s+4}$

7. Input x(t) and output y(t) of an LTI system are related by the differential equation $y''(t)-y'(t)-6y(t)=x(t)/$. If the system is neither causal nor stable, the impulse response h(t) of the system is

[GATE - 2015]

(a) $\frac{1}{5}e^{3t}u(-t)+\frac{1}{5}e^{-2t}u(-t)$

(b) $-\frac{1}{5}e^{3t}u(-t)+\frac{1}{5}e^{-2t}u(-t)$

(c) $\frac{1}{5}e^{3t}u(-t)-\frac{1}{5}e^{-2t}u(t)$

(d) $-\frac{1}{5}e^{3t}u(-t)-\frac{1}{5}e^{-2t}u(t)$

8. Consider the function $g(t)= e^{-t}\sin(2\pi t)u(t)$ where u(t) is the unit step function. The area under g(t) is _________

[GATE - 2015]

9. The stable linear time invariant (LTI) system has a transfer function $H(s)=\frac{1}{s^2+s-6}$.

ESE OBJ QUESTIONS

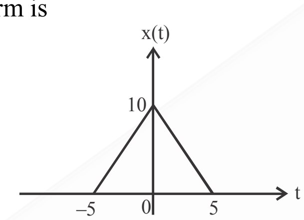
1. The laplace transform of $f(t) = t^n e^{-at} u(t)$ is

[EE ESE - 2018]

(a) $\frac{(n+1)!}{(s+\alpha)^{n+1}}$ (b) $\frac{n!}{(s+\alpha)^{n}}$

(c) $\frac{(n-1)!}{(s+\alpha)^{n+1}}$ (d) $\frac{n!}{(s+\alpha)^{n+1}}$

2. If x(t) is as shown in the figure, its Laplace transform is



[EE ESE - 2018]

(a) $\frac{2e^{+5s}+2e^{-5s}}{s^2}$ (b) $\frac{2e^{+5s}-4+2e^{-5s}}{s^2}$

(c) $\frac{2e^{+5s}-2+2e^{-5s}}{s^2}$ (d) $\frac{2e^{+5s}+4-2e^{-5s}}{s^2}$

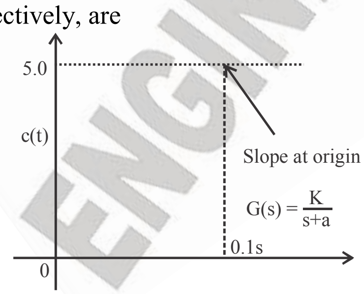
3. The unit – impulse response of a system is $16e^{-2t} - 8e^{-t}$ .Its unit – step response is

[EE ESE - 2018]

(a) $8 + e^{-t} - 4e^{-2t}$ (b) $8 + e^{-t} + 4e^{-2t}$

(c) $8e^{-t} - 8e^{-2t}$ (d) $e^{-t} - 4e^{-2t}$

4. A unit –step input to a first – order system G(s) yields a response as shown in the figure. This can happen when the values of K and a, respectively, are

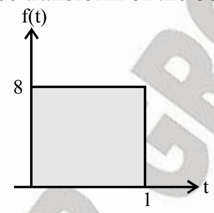


[EE ESE - 2018]

(a) 10 and 10 (b) 5 and 10

(c) 10 and 5 (d) 5 and 5

5. The laplace transform of the below function is



[EE ESE - 2017]

(a) F(s) = 8s $(1 - e^{-s})$ (b) F(s) = $\frac{8}{s}(1+e^{-s})$

(c) $F(s) = 8s(1+e^{-s})$ (d) $F(s) = \frac{8}{s}(1-e^{-s})$

6. The function which has its fourier transform , Laplace transform and Z- transform unity is

[EC ESE - 2012]

(a) Gaussian (b) Impulse

(c) Sinc (d) Pulse

7. $H(e^{j\omega})$ is the frequency response of a discrete time LTI system and $H_1(e^{j\omega})$ is the frequency response of its inverse function. Then

[EC ESE - 2012]

(a) $H(e^{j\omega})H_1(e^{j\omega}) = 1$

(b) $H(e^{j\omega})H_1(e^{j\omega}) = \delta(\omega)$

(c) $H(e^{j\omega})*H_1(e^{j\omega}) = 1$

(d) $H(e^{j\omega})*H_1(e^{j\omega}) = \delta(\omega)$

8. With the following equations, the time invariant systems are

1. $\frac{d^2y(t)}{dt^2} + 2t\frac{d}{dt}y(t) + 5y(t) = x(t)$

2. $y(t) = e^{-2x(t)}$

3. $y(t) = \left[\frac{d}{dt}x(t)\right]^2$

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CHAPTER - 9
Z-TRANSFORM

9.1 INTRODUCTION
Z-Transform which is discrete-time counterpart of Laplace Transform.
It may be observed that Laplace Transform is an extension of continuous-time Fourier. Transform because of fact that Laplace Transform may be applied to broader class of signals than Fourier Transform. Just for instances, there are several signal s for which the Fourier transform does not converges but Laplace Transform converses.
Similarly Z-Transform is introduced to represent discrete-time sequences in Z-domain (Z is complex variable). Also to analyse the difference equations that describes the linear time-invariant (LTI systems) and converts into algebraic equation. Thus simplifying further analysis.
In general Z-Transform of discrete signal x(n) is expressed as

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

Generally denoted a

$$x(z) = z[x(n)]$$

$x(n)$ & $x(z)$ form a Z-Transform pair

$$x(n) \xleftrightarrow{7} x(z)$$

It may be noted that Z-Transform is an infinite power series. It may exists only for those values of Z for which series converges.
x(z) is a complex number and a function of complex variable Z.
In polar form, $Z = re^{J\omega}$
With r gives magnitude of z, $|z|$
ω gives phase of z, $\angle z$

So, $x(re^{J\omega}) = \sum_{n=-\infty}^{\infty} x(n)\left(re^{-J\omega}\right)^{n}$

$$x(re^{J\omega}) = \sum_{n=-\infty}^{\infty} \left[x(n) r^{-n} \right] e^{-J\omega n}$$

We see that $x(re^{J\omega})$ is discrete time Fourier Transform of sequence x(n) multiplied by real exponential r^{-n}
i.e., $x(re^{J\omega}) = F\left[x(n) r^{-n}\right]$
The exponential weighting r^{-n} may decaying or growing with increasing n depended on whether it is greater than or less than unity.
Now if r = 1 or |z| = 1
The expression thus reduces to discrete Fourier Transform of input sequence.

$$x(z)\big|_{z=e^{J\omega}} = x(e^{j\omega}) = \text{DTFT}[x(n)] \qquad \ldots(i)$$

9.2 Z-PLANE OR Z-DOMAIN
Here we transforming discrete time sequence x(n) into x(z)
Where $Z = re^{J\omega}$

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ASSIGNMENT

1. Z-transform helps to convert
(a) Differential equation into algebraic equation
(b) Difference equation into algebraic equation
(c) Solve integral differential equation
(d) All the above

2. For causal signal $x[n]$, z -transform X(z) is

(a) $X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{n}$

(b) $X(z)=\sum_{n=0}^{\infty} x[n]z^{-n}$

(c) $X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}$

(d) $X(z)=\sum_{n=0}^{\infty} x[n]z^{n}$

3. ROC is defined as
(a) Range of values for which z-transform converges
(b) Range of value for which z-transform diverges
(c) Range of values of n for which series converges
(d) Range of values of n for which series diverges.

4. If the lower limit of ROC is less than upper limit of ROC for $X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}$, then the series
(a) Diverges (b) Converges
(c) Can't say (d) Zero

5. ROC of $x[n]$ contains
(a) Poles (b) Zeros
(c) No poles (d) No zero

6. Z-transform of $\delta[n]$ is
(a)1 (b)zero
(c) $\delta[z]1^{-n}$ (d)all the above

7. Z-transform of $x[-n]$ is
(a) $-X(z)$ (b) $X(-z)$
(c) $X\left[\frac{1}{Z}\right]$ (d) $\frac{1}{X(z)}$

8. Z-transform of $x[n-n_0]$ is
(a) $z^{-n_0}X[z]$ (b) $z^{n_0}X[z]$
(c) $X(z+z_0)$ (d) $X[z_0]$

9. ROC of the z-transform of unit step sequence is
(a) $|z|<1$ (b) $|z|>1$
(c) Real part of z (d) $|z|=0$

10. Convolution of two sequences $x_1[n]$ and $x_2[n]$ is
(a) $X_1(z)*X_2(z)$
(b) $X_1(z)X_2(z)$
(c) $X_1(z)+X_2(z)$
(d) $X_1(z)/X_2(z)$

11. Inverse z-transform of $\frac{az}{(z-a)^2}$ is
(a) $a^2u[n]$ (b) $a^n u[n]$
(c) $2a^2u[n]$ (d) $na^n u[n]$

12. Inverse z-transform of $X[z/a]$ is
(a) $x\left[\frac{n}{a}\right]$ (b) $x[n]/a$
(c) $a^n x[n]$ (d) $ax[n]$

13. z-transform of nx is
(a) $\frac{dX(z)}{dz}$ (b) $z\frac{dX(z)}{dz}$

GATE QUESTIONS

1. Consider two discrete-time signals $X_1(n) = \{1,1\}$ and $\{1,2\}$, for n = 0.1
The Z-transform of the convoluted sequence $x(b) = x_1(n) * x_2(n)$ is
[GATE - 2017]
(a) $1 + 2z^{-1} + 3z^{-1}$ (b) $z^2 + 3z + 2$
(c) $1 + 3z^{-1} + 2z^{-2}$ (d) $z^{-2} + 3z^{-3} + 2z^{-4}$

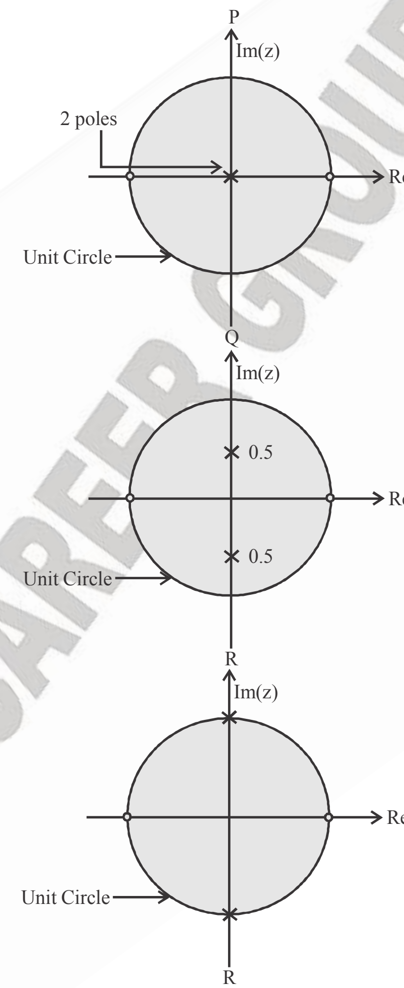
2. Consider a causal and stable LTI(system with rotational transfer function H9z), whose corresponding impulse response begins at n = 0. Further more, $H(1) = \frac{5}{4}$. The poles of H(z) are $P_k = \frac{1}{\sqrt{2}}\exp\left(j\frac{(2k-1)\pi}{4}\right)$ for k = 1, 2, 3, 4. The zeros of H(z) are all at z = 0. Let $g(n) = j^n h(n)$. The value of g(8) equals _____. (Give the answer up to three decimal places).
[GATE - 2017]

3. A cascade system having the impulse responses $h_1(b) = \{\underset{\uparrow}{1}, -1\}$ and $h_2(n) = \{\underset{\uparrow}{1}, 1\}$ is shown in the figure below, where symbol ↑ denotes the time origin.

$$x(n) \longrightarrow \boxed{h_1(n)} \longrightarrow \boxed{h_2(n)} \longrightarrow y(n)$$

The input sequence x(n) for which the casecade system produces an output sequence $y(n) = \{\underset{\uparrow}{1}, 2, 1, -1, -2, -1\}$ is
[GATE - 2017]
(a) $x(n) = \{\underset{\uparrow}{1}, 2, 1, 1\}$ (b) $x(n) = \{\underset{\uparrow}{1}, 1, 2, 2\}$
(c) $x(n) = \{\underset{\uparrow}{1}, 1, 1\}$ (d) $x(n) = \{\underset{\uparrow}{1}, 2, 2, 1\}$

4. The pole-zero plots of three discrete-time systems P, Q and R on the z-plane are shown below.



Which one of the following is TRUE about the frequency selectivity of these systems?
[GATE - 2017]
(a) All three are high-pass filters
(b) All three are band-pass filters
(c) All three are low-pass filters
(d) P is a low-pass filter, Q is a band-pass filter and R is a high - pass filter.

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CHAPTER - 10
DISCRETE TIME FOURIER TRANSFORM

10.1 INTRODUCTION
Basically the Fourier Transform of periodic finite energy signal is called DTFT mathematical.

$$x(e^{J\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-J\omega n}$$

or also denoted as $x(\Omega)$
Where Ω is discrete frequency

10.1.1 Periodic Nature of DTFT
Since Ω is discrete frequency.
Then Substituting $\omega = \omega + 2\pi k$

$$x\left[e^{J(\omega+2\pi k)}\right] = \sum_{n=-\infty}^{\infty} x(n)e^{-J(\omega+2\pi k)n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)e^{-J\omega n} \quad e^{-J2\pi kn}$$

By using Euler's identity

$$e^{-J2\pi kn} = \cos(2\pi kn) - J\sin(2\pi kn)$$

$= 1 - J0$

So, $x\left(e^{J(\omega+2\pi k)}\right) = \sum_{n=-\infty}^{\infty} x(n)e^{-J\omega n}$

So, $x\left(e^{J(\omega+2\pi k)}\right) = x\left(e^{J\omega}\right)$

or
$x(\Omega+2\pi k)=x(\Omega)$
Thus DTFT is periodic nature with a period of 2π. We DTFT is restricted to 0 to 2π or $-\pi$ to π.

> DTFT is continuous frequency Ranging from - ∞ to ∞ because of a periodic time function.

Inverse discrete time Fourier Transform:

$$\boxed{x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\left(e^{J\omega}\right)e^{J\omega n}d\omega}$$

Thus we can say x(n) and $x(e^{J\omega})$ form a Fourier Transform pair.

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GATE QUESTIONS

1. Let X[k] = k + 1, $0 \le k \le 7$ be 8-point DFT of a sequence x[n], where $X[k]=\sum_{n=0}^{N-1} x[n]e^{-j\pi nk/N}$. The value (correct to two decimal places) of $\sum_{n=0}^{3} x[2n]$ is _______

[GATE - 2018]

2. Let h[n] be the impulse response of a discrete-time linear time invariant (LTI) filter. The impulse response is given by

$h(0)=\frac{1}{3}; h[1]=\frac{1}{3}; h[2]=\frac{1}{3};$ and h[n] = 0 for n < 0 and n > 2.

Let H (ω) be the Discrete-Time Fourier transform (DTFT) of h[n], where ω is the normalized angular frequency in radians. Given that $H(\omega_0) = 0$ and $0 < \omega_0 < \pi$, the value of ω_0 (in radians) is equal to _____.

[GATE - 2017]

3. Let $\tilde{x}[n]=1+\cos\left(\frac{\pi n}{8}\right)$ be a periodic signal with period 16. Its DFS coefficients are defined by $a_k=\frac{1}{16}\sum_{n=0}^{15}\tilde{x}[n]\exp\left(-j\frac{\pi}{8}kn\right)$ for all k. The value of the coefficient a_{31} is ________.

[GATE - 2015]

4. The discrete – time signal $x[n]\longleftrightarrow X[z]$ $=\sum_{n=0}^{\infty}\frac{3^n}{2+n}z^{2n}$, where ↔ denotes a transform – pair relationship, is orthogonal to the signal

[GATE - 2008]

(a) $y_1[n] \leftrightarrow Y_1(z) = \sum_{n-0}^{\infty}\left(\frac{2}{3}\right)^n z^{-n}$

(b) $y_2[n]\leftrightarrow Y_2(z)=\sum_{n-0}^{\infty}(5^n-n)z^{-(2n+1)}$

(c) $y_3[n]\leftrightarrow Y_3(z)=\sum_{n-0}^{\infty}2^{-|n|}z^{-n}$

(d) $y_4[n]\leftrightarrow Y_4(z)=2z^4+3z^{-2}+1$

5. A 5-point sequence x [n] is given as

X[−3]=1, x[−2]=1, x[−1]=0, x[0]=5, x[1]=1

Let $X(e^{j\omega})$denote the discrete-time Fourier transform of X[n].The value of $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$ is

[GATE - 2007]

(a) 5 (b) 10π

(c) 16π (d) $5+j10\pi$

6. Let $x(n)=\left(\frac{1}{2}\right)^n u(n), y(n)=x^2(n)$, and $Y(e^{j\omega})$ be the Fourier transform of y(n). Then $Y(e^{j0})$ is.

[GATE - 2006]

(a) $\frac{1}{4}$ (b) 2

(c) 4 (d) $\frac{4}{3}$

7. x[n] = 0; n< - 1, n > 0, x[− 1] = − 1, x[0] = 2 is the input and

y[n] = 0; n < − 1, n > 0, x[−1] = −1, = y[1], y[0] = 3, y[2] = − 2 is the output of a discrete -time LTI system. The system impulse response h[n] will be

[GATE - 2006]

(a) h[n] = 0; n < 0, n > 2, h[0] = 1, h[1] = h[2] = −1

(b) h[n] = 0; n < − 1, n > 1, h [−1] = 1, h[0] = h[1] = 2

(c) h[n] = 0; n > 3, h[0] = −h[1] = 2, h[2] = 1

(d) h[n] = 0; n < - 2, n> 1, h[−2] = h [1] = h[−1] = −[0] = 3

CHAPTER - 11
DISCRETE TIME FOURIER SERIES

11.1 INTRODUCTION
Discrete time Fourier series is used to analyses the discrete periodic signals.
A discrete time x(n) is said to periodic is there is smallest positive integer 'N' for which it is satisfied.
x(n + n) = x(n) for all 'n'

11.1.1 Discrete Fourier Series Representation
The discrete Fourier series Representation of periodic sequence x(n) with fundamental time period N is given by

$$\boxed{x(n)=\sum_{K=0}^{N-1} c_k e^{\frac{J2\pi nk}{N}}}$$

$$(K=0,1,2......N-1)$$

The FS representation of x(n) consists of N harmonically related exponential function.

$e^{\frac{J2\pi kn}{N}}$ K = 0, 1, 2 ……. N – 1

Where c_k is the Fourier Series co-efficient.
It is given by

$$c_k=\frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{\frac{-2\pi kn}{N}}$$

Here input sequence x(n), FS coefficient c_k both are periodic.

11.1.2 Comparison between Continuous Time Fourier Series and Discrete Time Fourier Series

CTFS	DTFS
$x(t)=\sum_{n=-\infty}^{\infty} c_n e^{Jn\omega_0 t}$	$x(n)=\sum_{K=0}^{N-1} c_k e^{\frac{J2\pi kn}{N}}$
$c_n=\frac{1}{T_0}\int_0^{T_0} x(t)e^{-Jn\omega_0 t}$	$c_k=\frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{\frac{-J2\pi nk}{N}}$
C_n is discrete and periodic	C_n is Periodic and Discrete

11.1.3 Convergence of Discrete Fourier Series
Since x(n) is discrete Fourier Series is a Finite series because summation limits are from K = 0 to N – 1. So, I_n compassion to continuous-time case, there is no convergence issue with discrete Fourier series.

11.1.4 Discrete Fourier Series of Arbitrary Periodic Sequence x(n)
W define Twiddle factor) Phase factor

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ASSIGNMENT

1. Discrete-time signal x[n] will be periodic if
(a) x[n+N] = x[n]
(b) x[n+N] = – x[n]
(c) $x[n+N] = \frac{1}{x[n+N]}$
(d) x[n+N] = 1

2. Convergence of discrete Fourier series
(a) Always guaranteed
(b) Conditional convergence
(c) Is not an issue
(d) Non convergent

3. Discrete Fourier series is dual if
(a) $c[n] \xleftrightarrow{DFS} \frac{1}{N_0} x[-k]$
(b) $c[n] \xleftrightarrow{DFS} x[k]$
(c) $c[n] \xleftrightarrow{DFS} x[-k]$
(d) $c[n] \xleftrightarrow{DFS} N_0 x[k]$

4. If x[n] is real and even, then its discrete Fourier series coefficient c_k will be
(a) Real (b) Odd
(c) Both (a)and (b) (d) Imaginary

5. If x[n] is real and odd, then its discrete Fourier series coefficient c_k will be
(a) Real (b) Odd
(c) Imaginary (d) Both (a) & (c)

6. Find the discrete Fourier series for each of the following periodic sequences
(a) $x[n] = \cos(0.1\pi n)$
(b) $x[n] = \sin(0.1\pi n)$
(c) $x[n] = 2\cos(1.6\pi n) + \sin(2.4\pi n)$

7. Consider the sequence
$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$$
(a) Sketch x[n].
(b) Find the Fourier coefficients c_k of x[n].

8. Determine the discrete Fourier series representation for each of the following sequences
(a) $x[n] = \cos\frac{\pi}{4}n$
(b) $x[n] = \cos\frac{\pi}{3}n + \sin\frac{\pi}{4}n$
(c) $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$

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CHAPTER - 12
DISCRETE AND FAST FOURIER TRANSFORM

12.1 INTRODUCTION

In previous chapter we have mainly studied about signal analysis in frequency domain by Fourier Tools and for a discrete time sequence the Fourier Tool used is generally a discrete time Fourier Transform.

$$\boxed{x\left(e^{J\omega}\right)=\sum_{n=-\infty}^{\infty}x\left(n\right)e^{-J\omega n}} \qquad \ldots(i)$$

We know that $x(e^{J\omega})$ is Fourier Transform of discrete time signal x(n).

The frequency analysis of discrete-time signals are usually and most conventional performed on digital computer. To perform frequency analysis of discrete time x(n). We convert time-domain sequence to an equivalent frequency-domain representation such a representation is given by Fourier Transform $x(e^{J\omega})$ or $H(\omega)$.

Since $x(\omega)$ is continuous function of frequency 'ω'. The range of ω is from 0 to 2π or $-\pi$ to π.

Since this calculation is not possible to computer $x(\omega)$ on digital computer because range of summation (Equation (i)) is from "$-\infty$ to ∞".

So, if we make Range finite then it is possible to do these calculation on digital computed.

12.2 FREQUENCY DOMAIN SAMPLING AND RECONSTRUCTION OF DISCRETE-TIME SIGNALS

We recall that a periodic signals have continuous spectrum. If we consider such an a periodic discrete-time sequence x(n) with Fourier Transform.

$$x\left(e^{J\omega}\right)=\sum_{n=-\infty}^{\infty}x\left(n\right)e^{-J\omega n}$$

But DTFT is continuous in nature and periodic with period of 2π and unique information in frequency domain in only one period i.e. "0 to 2π" and DTFT range of from "$-\infty$ to ∞".

A finite range of sequence is obtained by extracting a particular portion from infinite sequence [x(n)] now $x(\omega)$ is continuous and periodic.

A discrete finite sequence is obtained by sampling $x(\omega)$ periodically in frequency at specing $D\omega$ radians between two samples.

For uniqueness of information only samples in fundamental period is necessary.

For convenience, we take N equidistant samples in interval $0 < \omega \le 2\pi$.

Total Range = 2π (one period)

Total samples = N

$D\omega$ (spacing between samples) $=\dfrac{2\pi}{N}$

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GATE QUESTIONS

1. The DFT coefficient out of five DFT coefficients of a five – point real sequence are given as: X(0) = 4, X(1) = 1 – j1 and X(3) = 2 + j2. The zero – the value of the sequence x(n)x(0) is.

[GATE - 2017]

(a) 1 (b) 2
(c) 3 (d) 4

2. The Discrete Fourier Transform (DFT) of the 4-point sequence
X[n] = {x[0], x[1], x[2], x[3]} = {3, 2, 3, 4} is
X[k] = {X[0], X[1], X[2], X[3]} = {12, 2j, 0, –2j}
If $X_1[k]$ is the DFT of the 12-point sequence
$x_1[n]$ = {3, 0, 0, 2, 0, 0, 3, 0, 0, 4, 0, 0},
The value of $\left|\frac{x_1[8]}{X_1[11]}\right|$ is ____.

[GATE - 2016]

3. Two sequence $[a, b, c]$ and $[A, B, C]$ are related as,

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{-1} & W_3^{-2} \\ 1 & W_3^{-2} & W_3^{-4} \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{ where } W_3 = e^{j\frac{2\pi}{3}}$$

If another sequence [p.q.r] is derived as,

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{1} & W_3^{2} \\ 1 & W_3^{2} & W_3^{4} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & W_3^{2} & 0 \\ 0 & 0 & W_3^{4} \end{bmatrix}\begin{bmatrix} A/3 \\ B/3 \\ C/3 \end{bmatrix},$$

then the relationship between the sequences [p.q.r] and [a.b.c] is

[GATE - 2015]

(a) [p.q.r]=[b.a.c] (b) [p.q.r]=[b.c.a]
(c) [p.q.r]=[c.a.b] (d) [p.q.r]=[c.b.a]

4. Consider two real sequences with time-origin marked by the bold value,
$x_1[n]$ = {1,2,3,0}, $x_2[n]$ = {1,3,2,1}
Let $X_1(k)$ and $X_2(k)$ be 4-point DFTs of $x_1[n]$ and $x_2[n]$, respectively.
Another sequence $x_3[n]$ is derived by taking 4-point inverse DFT of $X_3(k)=X_1(k)X_2(k)$.
The value of $x_3[2]$ is ______.

[GATE - 2015]

5. The N – point DFT X of sequence x[n], 0 ≤ n ≤ N – 1 is given by

$$X[k] = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}nk},\ 0 \le k \le N-1.$$

Denote this relation ax X =DFT (x) . For N = 4, which one of the following sequence satisfies DFT (DFT(x)) = x?

[GATE - 2014]

(a) x = [1 2 3 4] (b) x = [1 2 3 2]
(c) x = [1 3 2 2] (d) x = [1 2 2 3]

6. The DFT of a vector [a b c d] is the vector [α β λ δ]. Consider the product

$$[p\,q\,r\,s] = [a\,b\,c\,d]\begin{bmatrix} a & b & c & d \\ d & a & b & c \\ c & d & a & b \\ b & c & d & a \end{bmatrix}.$$

The DFT of the vector [p q r s] is scaled version of

[GATE - 2013]

(a) $[\alpha^2\ \beta^2\gamma^2\delta^2]$
(b) $[\sqrt{\alpha}\sqrt{\beta}\sqrt{\gamma}\sqrt{\delta}]$
(c) $[\alpha+\beta\ \beta+\delta\ \delta+\gamma\ \gamma+\alpha]$
(d) $[\alpha\ \beta\ \gamma\ \delta]$

7. The first six points of the 8-point DFT of a real valued sequence are 5,1-j3,0,3-j4,0 and 3+j4.. The last two points of the DFT are respectively

[GATE - 2011]

(a) 0,1-j3 (b) 0,1+j3