ECG Publications

AS PER NEW SYLLABUS

2019

GATE

Graduate Aptitude Test in Engineering

Electronics Engineering

Signal & System

Includes

Detailed Theory
Solved Workbook
Practice Assignments
Previous Year Questions
With Chapter Wise Solutions
And much more ...

ENGINEERS CAREER GROUP™

GATE

2019

SIGNAL AND SYSTEM

ELECTRONICS ENGINEERING

ECG Publications

ECG
Publications

A Unit of **ENGINEERS CAREER GROUP**

Head O ce: S.C.O-121-122-123, 2nd Floor, Sector-34/A, Chandigarh-160022

Website: www.engineerscareergroup.in **Toll Free:** 1800-270-4242

E-Mail: ecgpublica ons@gmail.com | info@engineerscareergroup.in

GATE-2019: Signal and System | Detailed theory with GATE & ESE previous year papers and detailed solu ons.

First Edi on: 2016

Price of Book: *INR 715/-*

CONTENTS

CHAPTER		PAGE
1.	INTRODUCTION……………………………………………………………….	1-39
2.	SYSTEM…………………………………………………………………..	40- 79
3.	LINEAR TIME –INVARIANT SYSTEM…….……………………………..	80-123
4.	LINEAR TIME –INVARIANT -2 SYSTEM ..…………………………..	124-142
5.	CONTINOUS –TIME FOURIER SERIES……………………………….	143-180
6.	FOURIER TRANSFORM ……………………………………………..	181-243
7.	CORRELATION AND FILTERING ACTION…………………………	244-263
8.	LAPLACE TRANFORM ………………………………………………	264-336
9.	Z-TRANSFORM………………………………………………………..	337-418
10.	DISCRETE TIME FOURIER TRANSFORM…………………………….	419-454
11.	DISCRETE TIME FOURIER SERIES…………………………………...	455-469
12.	DISCRETE AND FAST FOURIER TRANFORM……………………..	470 -508

CHAPTER - 1
INTRODUCTION

1.1 SIGNAL

Signal is a function of one or more independent variables contain information about some behavior or natural phenomenon.

Example. Speech, Video, Audio, T.V. Signal, Current, Voltage, RF signal etc.

1.2 SYSTEM

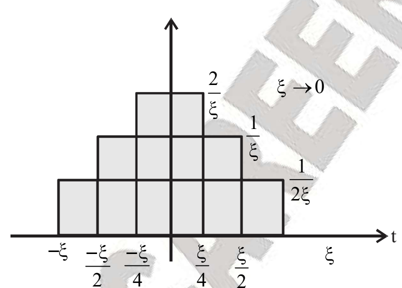
Interconnection of various physical elements which are formed to get the desired response.

Example. High Pass filter, Low pass filter, Automobile Car, Mobile, Tablet, etc.

1.3 IMPORTANT SIGNALS

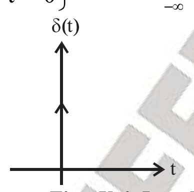
1.3.1 Continuous Time Unit Impulse Signal

The unit Impulse function δ(t) is known as Dirac Delta function.

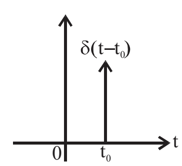


Unity area over an infinitesimal Time interval

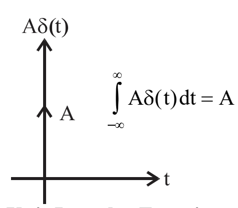
$$\delta(t) = \begin{Bmatrix} 0 & t \neq 0 \\ \infty & t = 0 \end{Bmatrix} \qquad \int_{-\infty}^{\infty} \delta(t)dt = 1$$



Continuous Time Unit Impulse Functior



Time Continuous Unit Impulse Functioı



Scaled Unit Impulse Function

WORKBOOK

Example 1. What are even and odd fart of x(t) = δ(t)

Solution.

$$\delta_e(t) = \frac{\delta(t)+\delta(-t)}{2}$$

$$\delta_0(t) = \frac{\delta(t)-\delta(-t)}{2}$$

δ(t) is even function δ(t) = δ(-t)

$$\delta_e(t) = \frac{2\delta(t)}{2} = \delta(t)$$

$$\delta_0(t) = \frac{\delta(t)-\delta(t)}{2} = 0$$

Example 2. What are even and odd parts of x(t) = u(t)

Solution.

$$u_e(t) = \frac{u(t)+u(-t)}{2} = \frac{1}{2}$$

$$u_0(t) = \frac{u(t)+u(-t)}{2} = \frac{1}{2}\text{Sgn}\, t$$

$$u(t) = \frac{1}{2} + \frac{1}{2}\text{Sgnt}$$

Example 3. What is R.M.S value of $x(t) = A_1 \cos(\omega t+\phi_1) + A_2 \cos(\omega t+\phi_2)$

Solution.

Power of signal x(t) will be

$$P = \frac{A_1^2}{2} + \frac{A_2^2}{2}$$

$$\text{R.M.S value} = \sqrt{\frac{A_1^2}{2} + \frac{A_2^2}{2}}$$

Example 4. Calculate the energy and power of signal

$x(n) = -(0.5)^n u(n)$

Solution.

$$E = \sum_{n=-\infty}^{\infty}\left[-(0.5)^n u(n)\right]^2$$

$$= \sum_{n=0}^{\infty}(0.25)^n = \frac{1}{1-0.25} = \frac{4}{3}$$

$$P = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-\infty}^{\infty}[x(n)]^2$$

$$= \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-0}^{\infty}\left(\frac{1}{4}\right)^n$$

$$P = \frac{1}{2N+1}\cdot\frac{4}{3} = 0$$

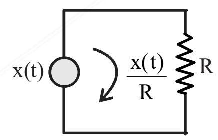
8. For odd function = $\int_{-a}^{a} x(t)dt = 0$

1.5.6 Energy and Power Signals

Energy and power are used to measure size of signal and the energy and power are always calculated by normalized power.

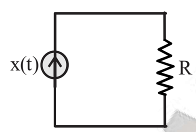
$$P_N = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}|x(t)|^2\,dt$$

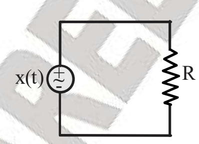
P_a is actual power



$$P_a = \frac{P_N}{R} \qquad P_a = P_N \cdot R$$

If x(t) is current signal   If x(t) is voltage signal





Size of x(t) is given by power and energy

$$p(t) = u(t).i(t) = \frac{x(t)}{R}\cdot x(t)$$

$$p(t) = \frac{x^2(t)}{R}$$

Total instantaneous power dissipated in resistance R.

$$\text{Total Energy } E = \int_{-\infty}^{\infty}\frac{x^2(t)}{2}dt$$, For Normalization, R = 1

Energy dissipated is $E = \int_{-\infty}^{\infty} x^2(t)dt$Real valued signal

$$E = \int_{-\infty}^{\infty}|x(t)|^2\,dt$$ for complex valued signal

Similarly energy for discrete time signal x(n)

Normalized energy can be defined as $E = \sum_{n=-\infty}^{\infty}|x(n)|^2$

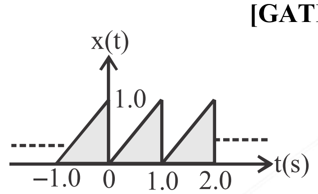
Normalized average power P of x(t) is defined as

$$P = \lim_{N\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}(x(t)^2dt$$ for real as well as compose valued signal

GATE QUESTIONS

1. A periodic signal x(t) is shown in the figure. The fundamental frequency of the signal x(t) in Hz is _____.

[GATE - 2017]



2. If a continuous-time signal x(t) = cos (2πt) is sampled at 4Hz, the value of the discrete time sequence x(n) at n = 5 is

[GATE - 2017]

(a) –0.707 (b) –1
(c) 0 (d) 1

3. Two sequences $x_1[n]$ and $x_2[n]$ have the same energy. Suppose $x_n[n] = \alpha\, 0.5^n\, u[n]$, where α is a positive real number and u[n] is the unit step sequence. Assume

$$x_n[n] = \begin{Bmatrix} \sqrt{1.5} \text{ for } n = 0,1 \\ 0 \quad \text{otherwise.} \end{Bmatrix}$$

Then the value of α is _____.

[GATE - 2015]

4. For a periodic signal v(t) = 30 sin 100t + 10 cos 300t + 6 sin (500 t + π/4), the fundamental frequency in rad/s

[GATE - 2015]

(a) 100 (b) 300
(c) 500 (d) 1500

5. The Dirac delta function δ(t) is given as

[GATE - 2006]

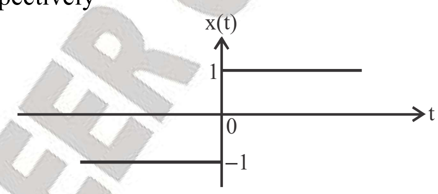
(a) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(b) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(c) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

(d) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

6. The function x(t) is shown in figure. Even and odd part of a unit step function u(t) are respectively



[GATE - 2005]

(a) $\frac{1}{2}, \frac{1}{2}x(t)$ (b) $\frac{1}{2}, \frac{1}{2}x(t)$
(c) $\frac{1}{2}, -\frac{1}{2}x(t)$ (d) $\frac{1}{2}, -\frac{1}{2}x(t)$

7. The power in the signal

$s(t) = 8\cos\left(20\pi t - \frac{\pi}{2}\right) + 4\sin(15\pi t)$ is:

[GATE - 2005]

(a) 40 (b) 41
(c) 42 (d) 82

8. Consider the sequence

$x[n] = (4 - 5j1 + j24)$

The conjugate anti-symmetric part of the sequence is

[GATE - 2004]

(a) [–4 – j2.5, j2, 4 – j2.5]
(b) [– j2.5, 1, j2.5]
(c) [– j2.5, j2, 0]
(d) [–4, 1, 4]

ESE OBJ QUESTIONS

1. A discrete time signal is given as

$$x[n] = \cos\frac{\pi n}{9} + \sin\left[\frac{\pi n}{7} + \frac{1}{2}\right]$$

The period N for the periodic signal is

[EC ESE - 2018]

(a) 126 (b) 32
(c) 252 (d) 64

2. The discrete-time equation y(n+1) +0.5n y(n) = 0.5x (n+1) is NOT attributable to a

[EC ESE - 1999]

(a) Memory less system
(b) Time-carrying system
(c) Linear system
(d) Causal system

3. The period of the function $\cos\pi/4(t-1)$ is

[EC ESE - 1999]

(a) 1/8s (b) 8s
(c) 4s (d) 1/4s

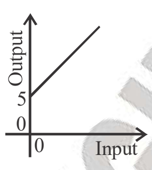
4. The signal $(1+M\cos 4\pi t)\cos(2\pi\times10^3 t)$ contains the frequency component (in Hz)

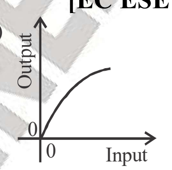
[EC ESE - 1999]

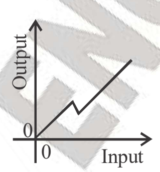
(a) 998, 1000 and 1002
(b) 1000 and 2000
(c) dc2 and 1000
(d) …996, 998, 1000, 1002, 1004

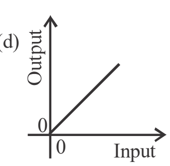
5. Which one of the following input-output relationships is that of a linear system

[EC ESE - 1999]

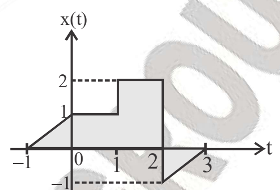
(a)


(b)


(c)


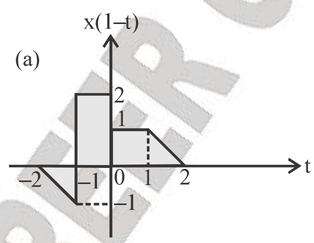
(d)


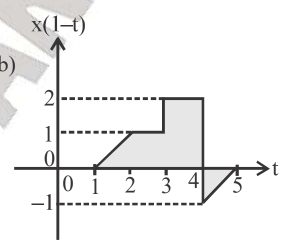
6. If a plot of signal x(t) is as shown in the figure -I

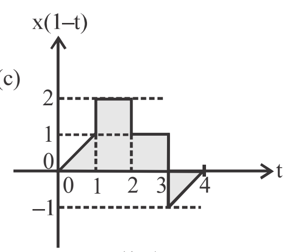


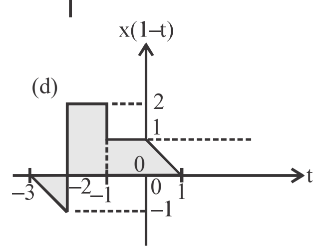
then the plot of the signal x(1-t) will be

[EC ESE - 1999]

(a)


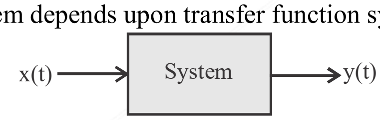
(b)


(c)


(d)


CHAPTER - 2
SYSTEM

2.1 SYSTEM
A system is a mathematical model of physical process that relates the Inputs (or excitation) signal to the output (or response) of signal.
The response or output of system depends upon transfer function system.



Mathematically, the functional Relationship between I/P and O/P may be written as:

$$y(t) = f\{x(t)\}$$

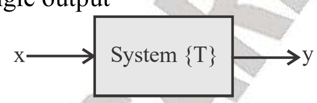
$$y(t) = T[x(f)]$$

Where T implies transformation and gives a mapping to be done on x(t) to get y(t)

2.1.1 Symbolically, we can write

$$x(t) \xrightarrow{s} y(t)$$

Multiple input and/or output signals are possible. But we will restrict our attention for most part in this course to the single Input single output



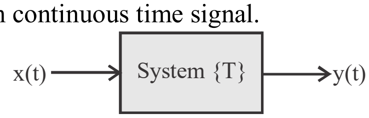
SISO

MIMO

Examples of system. Filters, amplifiers, communication channels, T.V. set are various example of electrical system.

2.2 TYPES OF SYSTEMS
1. Continuous-Time System
2. Discrete-Time System

1. Continuous-Time System
Continuous - Time system may be defined as also continuous. This means that Input and output of continuous time system are both continuous time signal.



Example. Audio, Video Amplifier, Power supplies etc.
Simple Practical example of continuous time – system is Low Pass Filter

WORKBOOK

Example 1. Consider the continuous-Time system with input-output relation. Determine whether the system is time variant or time invariant system.

(a) $y(t) = \frac{1}{\tau}\int_{t-T/2}^{t+T/2} x(\tau)d\tau$

(b) $y(t) = \sum_{k=-\infty}^{\infty} x(t)\delta(t-kt_s)$

(c) y(t) = sin x(t)

(d) $y(t) = x(t)\cos 200\pi t$

(e) y (t) = x (2t)

(f) y (t) = t sin x (t)

Solution.

(a) Time invariant
(b) Time-variant
(c) Time-invariant
(d) Time-variant
(e) Time-variant
(f) Time-variant

Example 2. Consider the discrete-Time system with input - output relation. Determine whether system is time-variant or time-invariant system?

(a) y(n) = n x(n)
(b) y(n) = x(n) – x(n – 1)
(c) y(n) = x(k n)
(d) y(n) = k x(n)
(e) $y(n) = x^2(n)$
(f) $y(n) = x(n)^2$

Solution.

(a) Time -variant
(b) Time-variant
(c) Time-variant
(d) Time-invariant
(e) Time-invariant
(f) Time-variant

Example 3. Check whether the following input -output relationship is invertible or not?

(a) y(t) = |x(t)|
(b) $y(t) = x^2(t)$
(c) $y(t) = \int_{-\infty}^{t} x(\tau)\alpha\tau$
(d) $y(n) = \sum_{k=-\infty}^{n} x(k)$
(e) y(n) = n x(n)

Solution.

(a) Non - invertible
(b) Non - invertible
(c) Invertible
(d) Invertible
(e) Non - invertible

Example 4. Characterize the system y(n) = sinπ n x(n) + sin $\frac{\pi}{2} n \cdot x(n-1)$ in terms of linearity, Time-invariance, causality, stability and memory less system?

Solution.

System is linear

(a) It satisfies Homogeneity and superposition.
(b) System is time - variant
i.e. $y(n-n_0) \neq T\{x(n-n_0)\}$
(c) System has memory
i.e. y(n) depends on x(n – 1)
(d) System is causal
i.e. y(n) depends upon past and present inputs only.
(e) System is stable
Because $y(n) \leq B_y$
If $[x(n) < B_x]$.

GATE QUESTIONS

1. Consider a single input single output discrete- time system with x[n] as input and y[n] as output, where the two are related as

$$y[n]=\begin{cases} n\,|x[n]|, & \text{for } 0\le n\le 10 \\ x[n]-x[n-1], & \text{otherwise}\end{cases}$$

Which one of the following statements is true about the system ?

[GATE - 2017]

(a) It is causal and stable
(b) It is causal but not stable
(c) It is not causal but stable
(d) It is neither causal nor stable

2. Consider and LTI system with magnitude response $|H(f)|=\begin{cases}1-\dfrac{|f|}{20}, & |f|\le 20\\ 0, & |f|>20\end{cases}$ and phase response Arg[H(f)]=2f.

If the input to the system is

$$x(t)=8\cos\left(20\pi t+\frac{\pi}{4}\right)+16\sin\left(40\pi t+\frac{\pi}{8}\right)+24\cos\left(80\pi t+\frac{\pi}{16}\right)$$

Then the average power of the output signal y(t) is___

[GATE - 2017]

3. The transfer function of a causal LTI system is H9s) = 1/s. if the input to the system is x(t) = [sin(t)/πt]u(t); where u(t) is a unit step function. The system output y(t) as $t\to\infty$ is__

[GATE - 2017]

4. The signal x(t) = sin(1400πt), where t is in seconds, is sampled at a rate of 9000 samples per second. The sampled signal is the input to an ideal lowpass filter with frequency response H(f) as follows:

$$H(f)=\begin{cases}1, & |f|\le 12\text{kHz}\\ 0, & |f|>12\text{kHz}\end{cases}$$

What is the number of sinusoids in the output and their frequencies in kHz?

[GATE - 2017]

(a) Number = 1, frequency = 7
(b) Number = 3, frequencies = 2,7,11
(c) Number = 2, frequencies = 2, 7
(d) Number = 2, frequencies = 7, 11

5. An LTI system with unit sample response h(n) = 5δ[n] – 7δ [n –1] + 7δ[mn–3] – 5δ[n –4] is a

[GATE - 2017]

(a) Low pass filter (b) High pass filter
(c) Band pass filter (d) Band stop filter

6. The input x(t) and the output y(t) of a continuous time system are related as $y(t)=\int_{t-T}^{t}x(u)du$. The system is

[GATE - 2017]

(a) Linear and time variant
(b) Linear and time invariant
(c) Non Linear and time variant
(d) Nonlinear and time invariant

7. Consider $g(t)=\begin{Bmatrix} t-\lfloor t\rfloor, t\ge 0\\ t-\lceil t\rceil, \text{otherwise}\end{Bmatrix}$,

Where $t\in R$

Here, $\lfloor t\rfloor$ represent the largest integer less than or equal to t and $\lceil t\rceil$ denotes the smallest integer greater than or equal to t. The coefficient of the second harmonic component of the fourier series representing g(t) is _____

[GATE - 2017]

8. Consider the signal x(t) = cos(6πt) + sin(8πt), where t is in seconds. The Nyquist sampling

ESE OBJ QUESTIONS

1. Which one of the following relations is not correct ?

[EC ESE - 2011]

(a) $f(t)\,\delta(t) = f(0)\,\delta(t)$

(b) $\int_{-\infty}^{\infty} f(t)\delta(\tau)d\tau = 1$

(c) $\int_{-\infty}^{\infty} \delta(\tau)d\tau = 1$

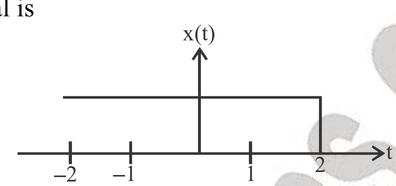
(d) $f(t)\,\delta(t-\tau) = f(\tau)\,\delta(t-\tau)$

2. Assertion (A) : The system described by $y^2(t) + 2y(t) = x^2(t) + x(t) + c$ is linear and static system

Reason (R) : The dynamic system is characterized by differential equation

[EC ESE - 2010]

3. The mathematical model of the below shown signal is



[EC ESE - 2010]

(a) $x(t) = u(2+t)$ (b) $x(t) = u(t-2)$

(c) $x(t) = u(2-t)$ (d) $x(t) = u(t-1)$

4. Decimation is the process of

[EC ESE - 2010]

(a)Retaining sequence values of $X_p[n]$ other than zeroes

(b)Retaining all sequence values of $X_p[n]$

(c)Dividing the sequence value by 10

(d)Multiplying the sequence value by 10

5. What is the period of the sinusoidal signal $x(n) = 5\cos[0.2\pi n]$

[EC ESE - 2009]

(a) 10 (b) 5

(c) 1 (d) 0

6. The output y(t) of a continuous- time system S for the input x(t) is given by

$Y(t) = \int_{-\infty}^{t} x(\lambda)d\lambda$

Which one of the following is correct ?

[EC ESE - 2009]

(a) S is linear and time - invariant

(b) S is linear and time - varying

(c) S is non- linear and time - invariant

(d) S is non- linear and time - varying

7. A function of one or more variables which conveys information on the nature of physical phenomenon is called

Which one of the following is correct ?

[EC ESE - 2009]

(a) Noise (b) Interference

(c) System (d) Signal

8. A signal $x_1(t)$ and $x_2(t)$ constitute the real and imaginary parts respectively of a complex valued signal x(t) .What form of waveform does x(t) possess ?

[EC ESE - 2009]

(a) Real symmetric

(b) Complex symmetric

(c) Asymmetric

(d) Conjugate symmetric

9. A system defined by

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

is an example of

[EC ESE - 2008]

(a) Invertible system

(b) Memory system

(c) Non- invertible system

(d) Averaging system

10. Let x(n) be real - values sequence that is a sample sequence of a wide - sense stationary

CHAPTER - 3
LINEAR TIME-INVARIANT SYSTEM

3.1 INTRODUCTION
A system is a mathematical model of physical process that relates the Inputs (or excitation) signal to the output (or response) of signal. We have discussed the Several Basic Properties of systems, two properties namely Linearity and Time - invariance plays a important role in analysis of signals and system. If a system has both linearity and time - invariance properties, then this system is called Linear -Time Invariant system (LTI system).

> We study LTI system because of the fact that the most of practical and physical system can be modeled in form of Linear-Time (invariant) System

In this chapter we develop the fundamental Input-Output relationships for system having these properties and it will be shown that Input-output Relationship for LTI system is described of a convolution operation.
Importance of convolution operation if one knows the output of unit Impulse then output for general input can be calculated.

3.2 CHARACTERISTICS OF LINEAR TIME - INVARIANT(LTI) SYSTEM
Both continuous-time and discrete-time linear time invariant (LTI) system exhibit one important characteristics that the superposition theorem can be applied to find the response y(t) to a given input x(t).

3.2.1 Important steps to adopted to find response of LTI system using superposition
1. Resolve the input function x(t) in terms of simple or basic function like impulse function for which response can be easily evaluated.
2. Determine Response of LTI system for simple or Basic functional individually.
3. Using superposition theorem, find the sum of individual response which will become overall response y(t) of function x(t) from above, to find the response of LTI system to given function first we have to find the response of LTI system to an unit impulse called as unit impulse response of LTI system.

3.2.2 Unit Impulse Response [$h(t)_n$ or h/n]
Impulse response of continuous time or discrete-time LTI system is output of system due to an unit impulse input applied at time t = 0 or n = 0.

$$y(n) = \begin{cases} \frac{1}{\beta-\alpha}\left(\beta^{n+1} - \alpha^{n+1}\right)U(n) & \alpha \neq \beta \\ \beta^n (n+1)U(n) & \alpha = \beta \end{cases}$$

Example 3. $x(n) = 2^n U(-n)$ and $h(n) = U(n)$

Solution.

$y(n) = 2$ for $n \geq 0$

$= 2^{n+1}$ for $n < 0$

Example 4. $x(n) = e^{-n^2}$ and h(n) = $3n^2$ for all n. Calculate the convolution of x(n) & h(n)

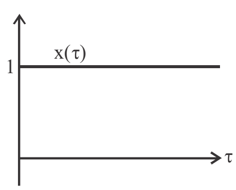
Solution.

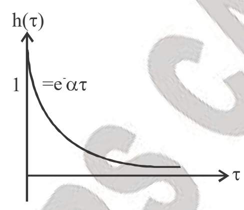
$$y(n) = 3n^2 \sum_{k=-a}^{\infty} e^{-k^2} + 3\sum_{k=-\infty}^{\infty} k^2 e^{-k^2} - 6n \sum_{k=-\infty}^{\infty} k e^{-k^2}$$

Example 5. Calculate output y(t) if input x(t) and impulse response h(t) of a continuous time LTI system are given by x(t) = U(t) & n(t) = $e^{-\alpha t}$ U(t) $\alpha > 0$

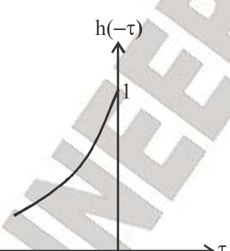
Solution.

Step-I.





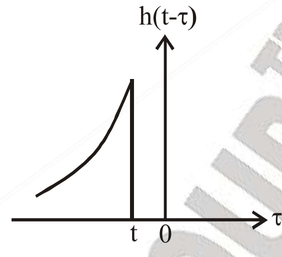
Step-II.



Step-III. h(t - τ)

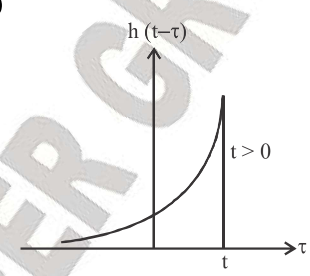
Here x(τ) & h(t − τ) do not overlap

For t < 0



$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$ so, y(t) = 0

For t > 0



Here x(τ) and h(t - τ) overlap from τ = 0 to τ = t

$y(t) = \int_0^t 1 \cdot e^{-\alpha(t-\tau)} \alpha\tau$

$= e^{-\alpha t}\int_0^t e^{\alpha\tau} d\tau = \frac{1}{\alpha}\left(1 - e^{-\alpha t}\right)$

So, output $y(t) = \frac{1}{\alpha}\left(1 - e^{-\alpha t}\right)u(t)$

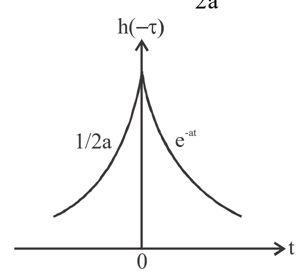
Example 6. What is output y(t) for continuous time LTI system whose impulse response h(t) and input x(t) are given by

$x(t) = e^{-at}U(-t)$ for a > 0

$h(t) = e^{-at}U(t)$

Solution.

$y(t) = \frac{1}{2a}e^{-at} t > 0$ and y(t) = $\frac{1}{2a}e^{at} t < 0$



GATE QUESTIONS

1. Let the input be u and the output be y of a system, and the other parameters are real constants. Identify which among the following systems is not a linear system:

[GATE - 2018]

(a) $\frac{d^3y}{dt^2}+a_1\frac{d^2y}{dt^2}+a_2\frac{dy}{dt}+a_3y$

$$=b_3u+b_2\frac{du}{dt}+b_1\frac{d^2u}{dt^2}$$

(b) $y(t)=\int_0^t e^{\alpha(t-\tau)}\beta u(\tau)d\tau$

(c) y = au + b , $b \neq 0$

(d) y =a u

2. Let z(t) = x(t)*y(t). Where "*" denotes convolution. Let c be a positive real-valued constant.

Choose the correct expression for z(ct).

[GATE - 2017]

(a) c.x(ct)*y(ct) (b) x(ct)*y(ct)
(c) c.x(t)*y(ct) (d) c.x(ct)*y(t)

3. Consider the system with following input-oputput relation $y[n] = (1 + (-1)^n)x[n]$

Where , x[n] is the input and y[n] is the output. The system is

[GATE - 2017]

(a) Invertible and time invariant
(b) Invertible and time varying
(c) Non-invertible and time invariant
(d) Non-invertible and time varying

4. The result of the convolution

$x(-t) * \delta(-t-t_0)$ is

[GATE - 2015]

(a) $x(t+t_0)$ (b) $x(t-t_0)$
(c) $x(-t+t_0)$ (d) $x(-t-t_0)$

5. The impulse response of an LTI system can be obtained by

[GATE - 2015]

(a) Differentiating the unit ramp response
(b) Differentiating the unit step response
(c) Integrating the unit ramp response
(d) Integrating the unit step response

6. For linear tune invariant systems, that are Bounded Input Bounded Output stable, winch one of the following statements is TRUE?

[GATE - 2014]

(a) The impulse response will be integrable, but may not be absolutely integrable.
(b) The unit impulse response will have finite support.
(c) The unit step response will be absolutely integrable.
(d) The unit step response will be bounded.

7. Consider an LTI system with transfer function $H(s)=\frac{1}{s(s+4)}$

If the input to the system is cos(3t) and the steady state output is A sin(3t + α), then the value of A is

[GATE - 2014]

(a) 1/30 (b) 1/15
(c) 3/4 (d) 4/3

8. Consider an LTI system with impulse response $h(t) = e^{-5t}u(t)$. If the output of the system is $y(t) = e^{-3t}u(t) = -e^{-5t}u(t)$ then the input, x(t), is given by

[GATE - 2014]

(a) $e^{-3t}u(t)$ (b) $2e^{-3t}u(t)$
(c) $e^{-5t}u(t)$ (d) $2e^{-5t}u(t)$

9. Two systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by

ESE OBJ QUESTIONS

1. In a two – element series network, the instantaneous voltages across the elements are

$$\sin 314t \text{ and } 3\sqrt{2}\ \sin(314t + 45°)$$

The resultant voltage across the combination is expressed as $V\cos(314t + \theta)$. Then the values of V and θ are

[EC ESE - 2018]

(a) 5 and 36.8° (b) 3.5 and 36.8°
(c) 5 and –53.2° (d) 3.5 and –53.2°

2. A signal f(t) is described as f(t) = [1–|t|] when |t| ≤ 1= 0 when |t| > 1. This represents the unit

[EC ESE - 2012]

(a) Sinc function
(b) Area triangular function
(c) Signum function
(d) Parabolic function

3. Match List-I with List-II and select the correct answer using the code given below the lists

List-I
A. Even signal
B. Causal signal
C. Periodic signal
D. Energy signal

List-II
(i) $x(n) = \left(\frac{1}{4}\right)^n u(n)$
(ii) x (-n) = x(n)
(iii) x(t) u(t)
(iv) x(n) = x(n +N)

[EC ESE - 2012]

Codes:
(a) A -ii, B-iii, C-iv, D-i
(b) A-i, B-iii, C-iv, D-ii
(c) A-ii, B-iv, C-iii, D-i
(d) A-i, B-iv, C-iii, D-ii

4. The impulse response h[n] of an LTI system is
h[n] = u[n+3]+u[n–2] –2u[n–7]
Then the system is
1. Stable 2. Causal
3. Unstable 4. Not causal
Which of these are correct ?

[EC ESE - 2010]

(a) 1 and 2 only (b) 2 and 3 only
(c) 3 and 4 only (d) 1 and 4 only

5. Unit step response of the system described by the equation y(n) +y(n-1) = x(n) is

[EC ESE - 2010]

(a) $\frac{z^2}{(z+1)(z-1)}$ (b) $\frac{z}{(z+1)(z-1)}$
(c) $\frac{z+1}{z-1}$ (d) $\frac{z(z-1)}{(z+1)}$

6. Number of state variables of discrete time system , described by

$Y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n]$ is

[EC ESE - 2010]

(a) 2 (b) 3
(c) 4 (d) 1

7. If the response of LTI continuous time system to unit step input is $\left(\frac{1}{2} - \frac{1}{2}e^{-2t}\right)$, then impulse response of the system is

[EC ESE - 2010]

(a) $\left(\frac{1}{2} - \frac{1}{2}e^{-2t}\right)$ (b) (e^{-2t})
(c) $(1-e^{-2t})$ (d) Constant

8. Which one of the following function is a periodic one ?

[EC ESE - 2008]

(a) sin (10πt) +sin (20πt)

CHAPTER - 4

LINEAR-TIME INVARIANT-2 SYSTEM

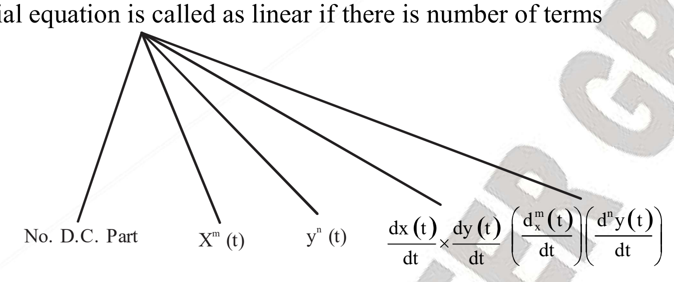
4.1 LINEAR CONSTANT CO-EFFICIENT DIFFERENTIAL EQUATIONS (LCC DE)

A general n^{th} -order linear - constant co-efficient differential equation is given by

$$\sum_{K=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{K=0}^{M} b_K \frac{dk_x(t)}{dt^k}$$

Above equation is representation of continuous system.

1. A differential equation is called as linear if there is number of terms



Example. $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 5 = x(t)$

It is Non-Linear differential equation because a d.c. term present in it.

2. Differential equation is said to be time-invariant if all the co-efficient of differential equation are const.

So, $\frac{a_n d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + ...y(t) = b_0 x(t)$

Represents the linear constant co-efficient differential equation.

LCCDE equation is used to analyse the LTI system or we can say if any system is LTI system then it can be represented in differential equation by LCCDE

4.1.1 System described by Difference Equations

The role of differential equation in describing continuous-time system is played by difference equations for discrete-time system.

4.2 LINEAR CONSTANT COEFFICIENT DIFFERENCE EQUATIONS (LCCDE)

The discrete time counterpart of general differential equation is the nth order linear cost. Co-efficient difference equation is given by

$$\sum_{K=0}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k)$$

ESE OBJ QUESTIONS

1. A waveform is given by $v(t) = 10\sin 2\pi$ 100t.What will be the magnitude of the second harmonic in its Fourier series representation ?

[EC ESE - 2012]

(a) 0V (b) 20V
(c) 100V (d) 200V

2. Which of the following is /are not a property / properties of a power spectral density function $S_x(\omega)$?

[EC ESE - 2007]

(a) $S_x(\omega)$ is a real function of ω
(b) $S_x(\omega)$ is an even function of ω
(c) $S_x(\omega)$ is a non- positive function of ω i.e., $S_x(\omega) \le 0$ for all ω
(d) All of the above

3. Match List-I (Time Domain Property) with List-II (Frequency Domain property pertaining to Fourier Representation Periodicity Properties) and select the correct answer using the codes given below the lists :

List-I
A .Continuous
B. Discrete
C. Periodic
D. Non-periodic

List-II
(i) Periodic
(ii) Continuous
(iii) Non-periodic
(iv) Discrete

[EC ESE - 2004]

Codes:
(a) A-iii, B-iv, C-i, D-ii
(b) A-ii, B-iv, C-i, D-iii
(c) A-ii, B-i, C-iv, D-iii
(d) A-iii, B-i, C-iv, D-ii

4. A square wave is defined by

$$x(t) = \begin{Bmatrix} A, 0 < t < T_0/2 \\ -A, T_0/2 < t < T_0 \end{Bmatrix}$$

It is periodically extended outside this interval. What is the general coefficient a_n in the fourier expansion of this wave ?

[EC ESE - 2004]

(a) 0
(b) $\frac{2A(1-\cos n\pi)}{n\pi}$
(c) $\frac{2A(1-\cos n\pi)}{n\pi}$
(d) $\frac{2A(1-\cos n\pi)}{[(n+1)]\pi}$

CHAPTER - 6
FOURIER TRANSFORM

6.1 INTRODUCTION
In previous chapter we have discussed the Fourier series which is tool used to analyse a periodic-time signal in frequency domain.
Disadvantage of Fourier series is it cannot analyse the non-periodic signals.
So Fourier develops a new tool to analyse the non-periodic or a periodic signal in frequency domain known a Fourier transform.

> Fourier transform can be used to analyse both periodic and non-periodic signals.

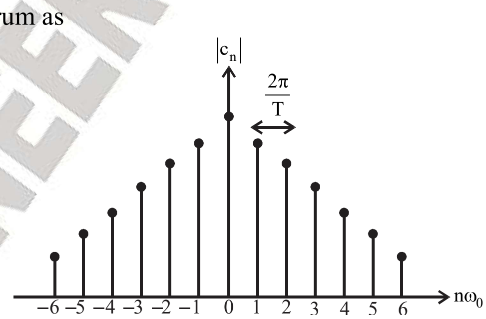
6.2 ANALYSIS OF NON-PERIODIC FUNCTION OVER ENTIRE INTERVAL
A non-periodic signal may assume as limiting case of periodic signal where the period of signal approaches infinity. Such a signal form by replacing fundamental time period T $\rightarrow \infty$ let us consider a periodic function x(t) having period T. The complex Fourier series representation of function may be written.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{Jn\omega_0 t} \quad \ldots \text{(i)}$$

$$\omega_0 = \frac{2\pi}{T}$$

$$c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-Jn\omega_0 t}dt \quad \ldots\text{(ii)}$$

The typical discrete spectrum as



Spacing between successive harmonics will be

$$\omega_0 = \frac{2\pi}{T} = \Delta\omega(\text{say})$$

CHAPTER - 7
CORRELATION AND FILTERING ACTION

7.1 CORRELATION
Correlation is used to find similarity between two signals.
There are two type of correlation
1. Auto-Correlation
It is used to find similarity between two same signals
2. Cross-Correlation
It is used to find similarity between different signal.

7.1.1 Auto-Correlation
7.1.1.1 Autocorrelation Function
It gives the measure of similarity, match or coherence between a signal and a delayed function.
A signal may be energy signal or power signal.

7.1.1.2 Autocorrelation Function of Energy Signal
Autocorrelation function of this signal may be obtained by integrating the product of signal x(t) and delayed version of its.

$$R_{xx}(\tau)=\int_{-\infty}^{\infty} x(t)x(t-\tau)dt$$

Where τ is called is searching parameter.

7.1.1.3 Relationship between Auto Correlation and Convolution

$$R_{xx}(\tau)=x(t)\otimes x(-t)\big|_{\text{Replace } t=\tau}$$

7.1.1.4 Properties of Autocorrelation Function
1. Auto correlation is an even function.

$$R_{xx}(\tau)=R_{xx}(-\tau)$$

2. It τ is increased in either direction, the auto correlation reduces, As τ reduces auto correlation increase and it maximum at τ = 0 i.e. at origin mathematically,

$R_{xx}(\tau)<R_{xx}(0)$ for all τ.

and $\lim_{\tau\to\infty} R_{xx}(\tau)=0$

3. Autocorrelation function at τ = 0 gives energy of signal

i.e. $R_{xx}(\tau)=\int_{-\infty}^{\infty} x(t)x(t-\tau)dt$

Substituting τ = 0

$$R_{xx}(0)=\int_{-\infty}^{\infty} x^2(t)dt$$

$$R_{xx}(0)=\text{Energy of signal}$$

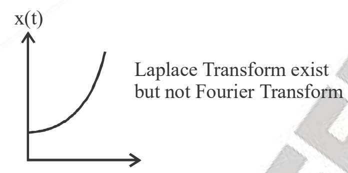
CHAPTER - 8
LAPLACE TRANSFORM

8.1 INTRODUCTION

In previous chapters, we have seen the tools such as Fourier series and Fourier Transform to analyse the signals. Now the Laplace Transform is another mathematical tool which is used for analysis of signals and system. Infect, Laplace Transform provides broader characterization of signal and systems compared to Fourier Transform.
The Laplace Transform can be used where Fourier Transform cannot be used.
Laplace Transform can be used for analysis of unstable systems whereas Fourier Transform has several limitation.

Example.

for given $x(t) = e^{3t.U(t)}$



8.1.1 Definition of Laplace Transform

For general continuous time signal x(t)
The Laplace Transform x(s) is defined as.

$$x(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

Where s is generally a complex variable and is expressed as
$S = \sigma + J\omega$
Also known as complex frequency
Where σ is real part and ω is imaginary part.
For convenience we will sometime denotes the Laplace Transform in operator form al $\angle[x(t)]$ and denote the transform relationship between x(t) and x(s)

As $x(t) \xleftrightarrow{L.T} x(s)$

e^{-st} is kernel of function
It may be noted that integration is taken from 0 to ∞. Therefore, this is called Bilateral Laplace Transform.
Similar if x(t) is zero for t < 0
Then Laplace may be defined as

$$\boxed{x(s) = \int_{0}^{\infty} x(t)e^{-st}at}$$

Where $s = \sigma + J\omega$
Integration is taken from 0 to ∞. This is called as unilateral/one-sided Laplace Transform.

CHAPTER - 9
Z-TRANSFORM

9.1 INTRODUCTION
Z-Transform which is discrete-time counterpart of Laplace Transform.
It may be observed that Laplace Transform is an extension of continuous-time Fourier. Transform because of fact that Laplace Transform may be applied to broader class of signals than Fourier Transform. Just for instances, there are several signal s for which the Fourier transform does not converges but Laplace Transform converses.
Similarly Z-Transform is introduced to represent discrete-time sequences in Z-domain (Z is complex variable). Also to analyse the difference equations that describes the linear time-invariant (LTI systems) and converts into algebraic equation. Thus simplifying further analysis.
In general Z-Transform of discrete signal x(n) is expressed as

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

Generally denoted a

$$x(z) = z[x(n)]$$

$x(n)\ \&\ x(z)$ form a Z-Transform pair

$$x(n) \xleftrightarrow{7} x(z)$$

It may be noted that Z-Transform is an infinite power series. It may exists only for those values of Z for which series converges.
x(z) is a complex number and a function of complex variable Z.
In polar form, $Z = re^{J\omega}$
With r gives magnitude of z, $|z|$
ω gives phase of z, $\angle z$

So, $x(re^{J\omega}) = \sum_{n=-\infty}^{\infty} x(n)\left(re^{-J\omega}\right)^{n}$

$$x(re^{J\omega}) = \sum_{n=-\infty}^{\infty} \left[x(n) r^{-n}\right] e^{-J\omega n}$$

We see that $x(re^{J\omega})$ is discrete time Fourier Transform of sequence x(n) multiplied by real exponential r^{-n}
i.e., $x(re^{J\omega}) = F\left[x(n) r^{-n}\right]$
The exponential weighting r^{-n} may decaying or growing with increasing n depended on whether it is greater than or less than unity.
Now if r = 1 or |z| = 1
The expression thus reduces to discrete Fourier Transform of input sequence.

$$x(z)\Big|_{z=e^{J\omega}} = x(e^{j\omega}) = \text{DTFT}[x(n)] \qquad \ldots(i)$$

9.2 Z-PLANE OR Z-DOMAIN
Here we transforming discrete time sequence x(n) into x(z)
Where $Z = re^{J\omega}$

CHAPTER - 10

DISCRETE TIME FOURIER TRANSFORM

10.1 INTRODUCTION

Basically the Fourier Transform of periodic finite energy signal is called DTFT mathematical.

$$x(e^{J\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-J\omega n}$$

or also denoted as $x(\Omega)$

Where Ω is discrete frequency

10.1.1 Periodic Nature of DTFT

Since Ω is discrete frequency.

Then Substituting $\omega = \omega + 2\pi$ k

$$x\left[e^{J(\omega+2\pi k)}\right] = \sum_{n=-\infty}^{\infty} x(n)e^{-J(\omega+2\pi k)n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)e^{-J\omega n} \quad e^{-J2\pi kn}$$

By using Euler's identity

$$e^{-J2\pi kn} = \cos(2\pi kn) - J\sin(2\pi kn)$$

$= 1 - J0$

So, $x\left(e^{J(\omega+2\pi k)}\right) = \sum_{n=-\infty}^{\infty} x(n)e^{-J\omega n}$

So, $x\left(e^{J(\omega+2\pi k)}\right) = x\left(e^{J\omega}\right)$

or

$x(\Omega+2\pi k)=x(\Omega)$

Thus DTFT is periodic nature with a period of 2π. We DTFT is restricted to 0 to 2π or $-\pi$ to π.

> DTFT is continuous frequency Ranging from - ∞ to ∞ because of a periodic time function.

Inverse discrete time Fourier Transform:

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\left(e^{J\omega}\right)e^{J\omega n}d\omega$$

Thus we can say x(n) and $x(e^{J\omega})$ form a Fourier Transform pair.

CHAPTER - 11
DISCRETE TIME FOURIER SERIES

11.1 INTRODUCTION
Discrete time Fourier series is used to analyses the discrete periodic signals.
A discrete time x(n) is said to periodic is there is smallest positive integer 'N' for which it is satisfied.
x(n + n) = x(n) for all 'n'

11.1.1 Discrete Fourier Series Representation
The discrete Fourier series Representation of periodic sequence x(n) with fundamental time period N is given by

$$x(n) = \sum_{K=0}^{N-1} c_k e^{\frac{J2\pi nk}{N}}$$

$(K = 0,1,2......N-1)$

The FS representation of x(n) consists of N harmonically related exponential function.

$e^{\frac{J2\pi kn}{N}}$ K = 0, 1, 2 ……. N – 1

Where c_k is the Fourier Series co-efficient.
It is given by

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{\frac{-2\pi kn}{N}}$$

Here input sequence x(n), FS coefficient c_k both are periodic.

11.1.2 Comparison between Continuous Time Fourier Series and Discrete Time Fourier Series

CTFS	DTFS
$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{Jn\omega_0 t}$	$x(n) = \sum_{K=0}^{N-1} c_k e^{\frac{J2\pi kn}{N}}$
$c_n = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-Jn\omega_0 t}$	$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{\frac{-J2\pi nk}{N}}$
C_n is discrete and periodic	C_n is Periodic and Discrete

11.1.3 Convergence of Discrete Fourier Series
Since x(n) is discrete Fourier Series is a Finite series because summation limits are from K = 0 to N – 1. So, I_n compassion to continuous-time case, there is no convergence issue with discrete Fourier series.

11.1.4 Discrete Fourier Series of Arbitrary Periodic Sequence x(n)
W define Twiddle factor) Phase factor

SOLUTIONS

Sol. 1. (a)

Sol. 2. (c)

Sol. 3. (a)

Sol. 4. (a)

Sol. 5. (c)

Sol. 6.

(a) $x[n]=\frac{1}{2}e^{j\Omega_0 n}+\frac{1}{2}e^{j19\Omega_0 n}, \Omega_0=0.1\pi$

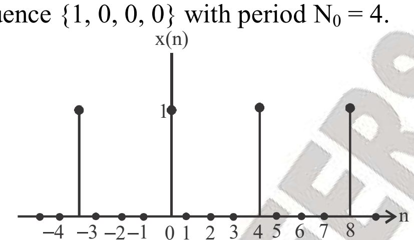
(b) $x[n]=\frac{1}{2j}e^{j\Omega_0 n}-\frac{1}{2j}e^{j19\Omega_0 n}, \Omega_0=0.1\pi$

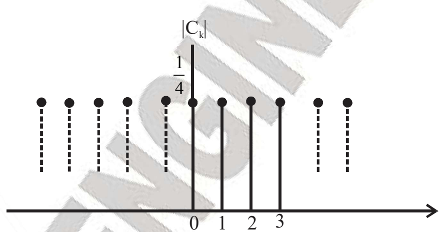
(c) $x[n]=(1-j0.5)e^{j\Omega_0 n}+\left(1+j0.5\right)e^{j4\Omega_0 n}$

$,\Omega_0 = 0.4\pi$

Sol. 7.

(a) The sequence x[n] is sketched in fig. It is seen that x[n]is the periodic extension of the sequence {1, 0, 0, 0} with period $N_0 = 4$.





(b) $x[n]=\sum_{k=0}^{3}c_k e^{jk(\pi/4)n}=\sum_{k=0}^{3}c_k e^{jk(\pi/2)n}$

Sol. 8.

(a)The fundamental period of x[n] is $N_0 = 8$ and $\Omega_0 = 2\pi/N_0 = \pi/4$. Rather than to evaluate the Fourier coefficients c_k, we use Euler's formula and get

$$\cos\frac{\pi}{4}n=\frac{1}{2}(e^{j(\pi/4)n}+e^{-j(\pi/4)n})$$

$$\frac{1}{2}e^{j\Omega_0 n}+\frac{1}{2}e^{-j\Omega_0 n}$$

Thus, the Fourier coefficients for x[n] are

$$c_1=\frac{1}{2}, c_{-1}=c_{-1+8}=c_7=\frac{1}{2}$$

and all other. $C_k = 0$. Hence, the discrete Fourier series of x[n] is

$$x[n]=\cos\frac{\pi}{4}n=\frac{1}{2}e^{j\Omega_0 n}+\frac{1}{2}e^{j7\Omega_0 n}$$

$$\Omega_0=\frac{\pi}{4}$$

(b)The fundamental period of x[n] is $N_0 = 24$ and $\Omega_0 = 2\pi/N_0 = \pi/12$. Again by Euler's formula we have

$$x[n]=\frac{1}{2}\left(e^{j(\pi/3)n}+e^{-j(\pi/3)n}\right)$$

$$+\frac{1}{2j}(e^{j(\pi/4)n}-e^{j(\pi/4)n})$$

$$+\frac{1}{2j}e^{-j4\Omega_0 n}+j\frac{1}{2}e^{-j3\Omega_0 n}$$

$$-j\frac{1}{2j}e^{-j3\Omega_0 n}+\frac{1}{2}e^{-j4\Omega_0 n}$$

Thus,

$$c_3=-j\left(\frac{1}{2}\right), c^4=\frac{1}{2}, c_{-4}=c_{-4+24}=c_{20}$$

$$=\frac{1}{2}, c_{-3}=c_{-3+24}=c_{21}=j\left(\frac{1}{2}\right)$$

and all other $c_k = 0$. Hence, the discrete Fourier series of x[n] is