GATE 2019

STRENGTH OF MATERIALS

MECHANICAL ENGINEERING

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CONTENTS

CHAPTER - 1
MECHANICS

1.1 FORCE
Force may be defined as a push or pull which produces or tends to produce a change in the state of rest or of uniform motion of a body or change in the direction of motion of the body.

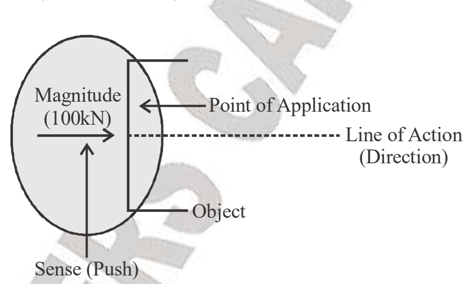
F = ma

$$\text{Force} = \text{mass} \times \text{acceleration}$$

This is the fundamental equation of motion which gives the measurement of force. It is a vector equation.

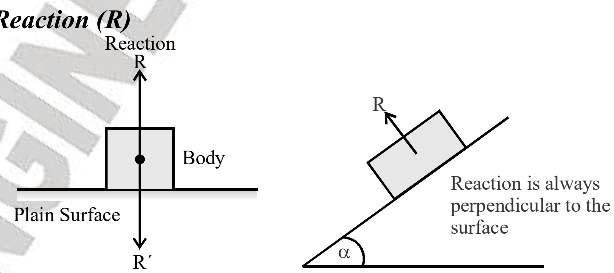
1.1.1 Effects of a Force
1. Change the motion
2. Change the direction
3. Change the size or shape
4. Give rise to internal stresses

1.1.2 Characteristics of Force
1. The magnitude of the force in known units such as Newton's or kilonewtons. (i.e., 50 N, 100 KN etc.)
2. The line of action or direction of the force may be taken with respect to reference lines.
3. Its point of application, that is the point on the body at which forces acts.
4. Sense or nature of the force (Push or Pull)



1.1.3 Classification of Forces
1. According to the Nature of Force

(i) Action (R′) and Reaction (R)



GATE QUESTIONS

1. Two disks A and B with identical mass (m) and radius (R) are initially at rest. They roll down from the top of identical inclined planes without slipping. Disk A has all of its mass concentrated at the rim, while Disk B has its mass uniformly distributed. At the bottom of the plane, the ratio of velocity of the center of disk A to the velocity of the center of disk B is

[GATE - 2017]

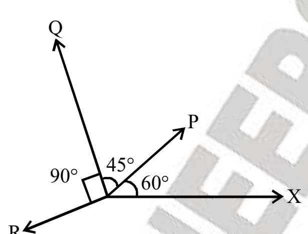
(a) $\sqrt{\frac{3}{4}}$ (b) $\sqrt{\frac{3}{2}}$
(c) 1 (d) $\sqrt{2}$

2. A particle of unit mass is moving on a plane. Its trajectory, in polar coordinates, is given by $r(t) = t^2$, $\theta(t) = t$, where t is time. The kinetic energy of the particle at titme t = 2 is

[GATE - 2017]

(a) 4 (b) 12
(c) 16 (d) 24

3. The magnitudes of vectors P, Q and R are 100 kN, 250 kN and 150 kN, respectively as shown in the figure.

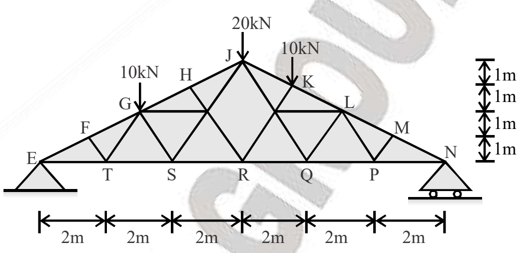


The respective values of the magnitude (in kN) and the direction (with respect to the x-axis) of the resultant vector are

[GATE - 2016]

(a) 290.9 and 96.0° (b) 368.1 and 94.7°
(c) 330.4 and 118.9° (d) 400.1 and 113.5°

4. A plane truss with applied loads is shown in the figure.

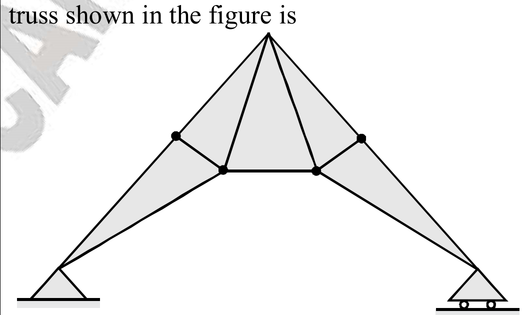


The members which do not carry any force are

[GATE - 2016]

(a) FT, TG, HU, MP, PL
(b) ET, GS, UR, VR, QL
(c) FT, GS, HU, MP, QL
(d) MP, PL, HU, FT, UR

5. The kinematic indeterminacy of the plane truss shown in the figure is



[GATE- 2016]

(a) 11 (b) 8
(c) 3 (d) 0

6. An assembly made of a rigid arm A-B-C hinged at end A and supported by an elastic rope C-D at end C is shown in the figure. The members may be assumed to be weightless and the lengths of the respective members are as shown in the figure.

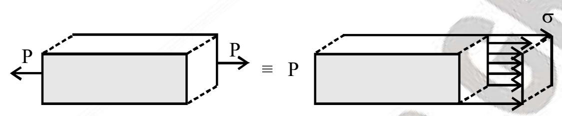
CHAPTER - 2
STRESS AND STRAIN

2.1 STRESS (σ)

When a material is subjected to an external force, a resisting force is set up within the component. The internal resistance force per unit area acting on a material or intensity of the forces distributed over a given section is called the stress at a point.

(i) It uses original cross section area of the specimen and also known as engineering stress or conventional stress.

Therefore, $\sigma = \dfrac{P}{A}$



Where P is expressed in Newton (N) and A, original area, in square meters (m), the stress σ will be expressed in N/m^2. This unit is called Pascal (Pa).

(ii) As Pascal is a small quantity, in practice, multiples of this unit is used.

$1\,kPa = 10^3\,Pa = 10^3\,N/m^2$ (kPa = Kilo Pascal)

$1\,MPa = 10^6\,Pa = 10^6\,N/m^2 = 1\,N/mm^2$ (MPa = Mega Pascal)

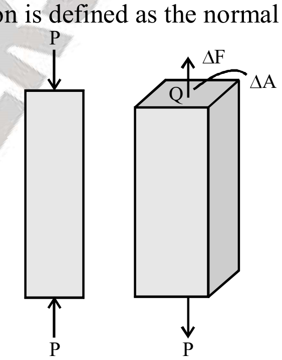
$1\,GPa = 10^9\,Pa = 10^9\,N/m^2$ (GPa = Giga Pascal)

Let us take an example: A rod 10 mm × 10 mm cross-section is carrying an axial tensile load 10 kN. In this rod the tensile stress developed is given by

$$(\sigma_1) = \frac{P}{A} = \frac{10kN}{(10mm \times 10mm)} = \frac{10 \times 10^3\,N}{100mm^2} = 100N/mm^2 = 100MPa$$

The resultant of the internal forces for an axially loaded member is normal to a section cut perpendicular to the member axis.

The force intensity on the shown section is defined as the normal stress.

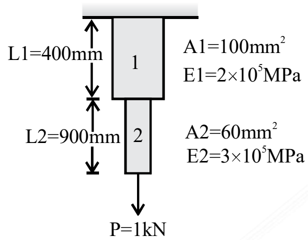


$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \quad \text{and} \quad \sigma_{avg} = \frac{P}{A}$$

GATE QUESTIONS

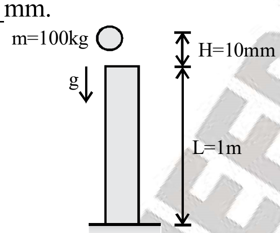
1. Consider the stepped bar made with a linear elastic material and subjected to an axial load of 1 kN, as shown in the figure.

[GATE - 2017]



Segments 1 and 2 have cross-sectional area of 100 mm^2 and 600 mm^2, Young's modoulus of 2×10^5 MPa and 3×10^5 MPa, and length of 400 mm and 900 mm, respectively. The strain energy (in N-mm, up to one decimal place) in the bar due to the axial load is _______.

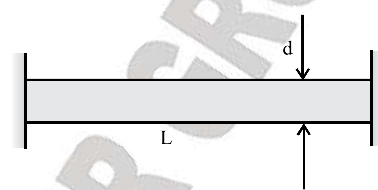
2. A point mass of 100 kg is dropped onto a massless elastic bar (cross-sectional area = 100 mm^2, length = 1 m, Young's modulus = 100 gPa) from a height H of 10 mm as shown (Figure is not to scale). If g = 10 m/s^2, the maximum compression of the elastic bar is _________mm.



3. A horizontal bar, fixed at one end (x = 0), has a length of 1m, and cross-sectional area of 100mm^2. Its elastic modulus varies along its length as given by $E(x) = 100\ e^{-x}$ GPa, where x is the length coordinate (in m) along the axis of the bar. An axial tensile load of 10 kN is applied at the free end (x = 1). The axil displacement of the free end is ______mm.

[GATE - 2017]

4. An initially stree-free massless elastic beam of length L and circular cross-section with diameter d (d<<L) is held fixed between two walls as shown. The beam material has Young's modulus E and coefficient of thermal expansion α.



If the beam is slowly and uniformly heated, the temperature rise required to cause the beam to buckle is proportional to

[GATE - 2017]

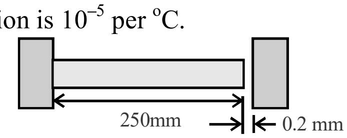
(a) d (b) d^2
(c) d^3 (d) d^4

5. In the engineering stress-strain curve for mild steel, the Ultimate Tensile Strength (UTS) refers to

[GATE - 2017]

(a) Yield stress (b) Proportional limit
(c) Maximum stress (d) Fracture stress

6. A circular metallic rod of length 250 mm is placed between two rigid immovable walls as shown in the figure. The rod is in perfect contact with the wall on the left side and there is a gap of 0.2 mm between the rod and the wall on the right side. If the temperature of the rod is increased by 200oC, the axial stress developed in the rod is ______MPa.
Young's modulus of the material of the rod is 200 GPa and the coefficient of thermal expansion is 10^{-5} per °C.



[GATE - 2016]

ESE OBJ QUESTIONS

1. A mild steel bar, circular in cross-section, tapers from 40 mm diameter to 20mm diameter over its length of 800 mm. It is subjected to an axial pull of 20 kN. E = 2×10^5 N/mm^2. The increase in the length of the rod will be

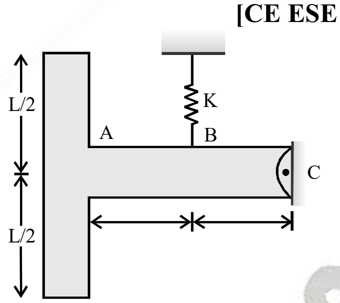
[CE ESE - 2017]

(a) $\frac{1}{10\pi}$ mm (b) $\frac{2}{5\pi}$ mm

(c) $\frac{4}{5\pi}$ mm (d) $\frac{1}{5\pi}$ mm

2. A uniform T-shaped arm of weight W, pinned about a horizontal point c, is support by a vertical spring of stiffness K. The extension of the spring is

[CE ESE - 2017]



(a) $\frac{3W}{4K}$ (b) $\frac{4W}{3K}$

(c) $\frac{3K}{4W}$ (d) $\frac{4K}{3W}$

3. In mild steel specimens subjected to tensile test cycle, the elastic limit in tension is raised and the elastic limit in compression is lowered, This is called

[CE ESE - 2017]

(a) Annealing effect
(b) Bauschinger effect
(c) Strain rate effect
(d) Fatigue effect

4. A solid uniform metal bar of diameter D mm and length ℓ mm hangs vertically from its upper end. The density of the material is ρN/mm^3 and its modulus of elasticity is EN/mm^2. The total extension of the rod due to its own weight would be

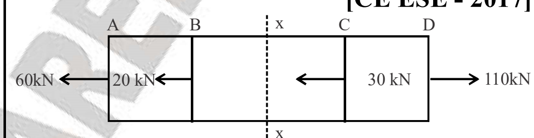
[CE ESE - 2017]

(a) $\frac{\rho\ell^2}{2E}$ (b) $\frac{\rho\ell}{2E}$

(c) $\frac{\rho\ell}{4E}$ (d) $\frac{\rho\ell^2}{4E}$

5. What is the stress at the section x- for the bar ABCD with uniform cross-section 1000 mm^2 ?

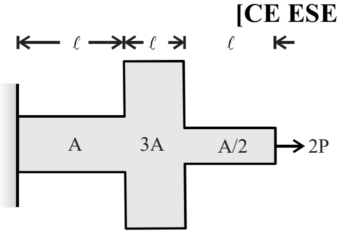
[CE ESE - 2017]



(a) 20 N/mm^2 (Tensile)
(b) 30 N/mm^2 (Compressive)
(c) 80 N/mm^2 (Tensile)
(d) 50 n/mm^2 (compressive)

6. The total elongation of the structural element (fixed at one end, free at the other end, and of varying cross-section) as shown in the figure, when subjected to load 2P at the free end is

[CE ESE - 2017]

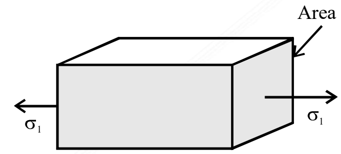


(a) $6.66\frac{P\ell}{AE}$ (b) $5.55\frac{P\ell}{AE}$

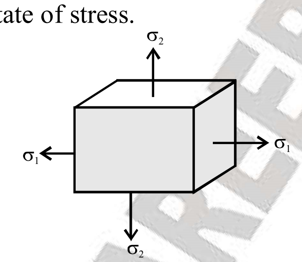
(c) $4.44\frac{P\ell}{AE}$ (d) $3.33\frac{P\ell}{AE}$

CHAPTER - 3
PRINCIPAL STRESS & STRAIN

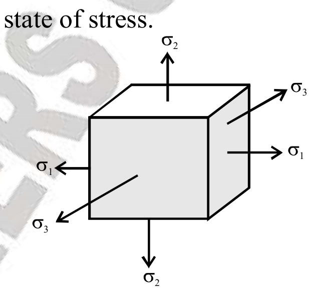
3.1 STATES OF STRESS
3.1.1 Uni-Axial Stress
Only one non-zero principal stress, i.e. σ_1.
Right side figure represents Uni-axial state of stress.



3.1.2 Bi-Axial Stress
One principal stress equals zero, two do not, i.e. $\sigma_1 > \sigma_3$; $\sigma_2 = 0$
Right side figure represents Bi-axial state of stress.



3.1.3 Tri-Axial Stress
Three non-zero principal stresses,
i.e. $\sigma_1 > \sigma_2 > \sigma_3$
Right side figure represents Tri axial state of stress.



3.1.4 Isotropic Stress
Three principal stresses
are equal, i.e. $\sigma_1 = \sigma_2 = \sigma_3$
Right side figure represents Isotropic state of stress.

GATE QUESTIONS

1. A soil sample is subjected to a hydrostatic pressure, σ. The Mohr circle for any point in the soil sample would be

[GATE - 2017]

(a)A circle of radius σ and center at the origin
(b)A circle of radius σ and center at a distance σ from the origin
(c)A point at a distance σ from the origin
(d)A circle of diameter σ and center at the origin

2. A rod of length 20mm is stretched to make a rod of length 40mm. Subsequently, it is compressed to make a rod of final length 10mm. consider the longitudinal tensile strain as positive and compressive strain as negative. The total true longitudinal strain in the rod is

[GATE - 2017]

(a) – 0.5 (b) – 0.69
(c) – 0.75 (d) – 1.0

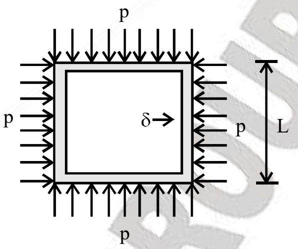
3. The state of stress at a point is $\sigma_x = \sigma_y = \sigma_z = \tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0$ and $\tau_{xy} = \tau_{yx} = 50$Mpa. The maximum normal stress (in MPa) at that point is ____

[GATE - 2017]

4. A rectangular region in a solid is in a state of plane strain. The (x, y) coordinates of the corners of the undeformed rectangle are given by P(0, 0), Q (4, 0), R (4, 3), S (0, 3). The rectangle is subjected to uniform strain, $\varepsilon_{xx} = 0.001$, $\varepsilon_{yy} = 0.002$, $\gamma_{xy} = 0.003$. The deformed length of the elongated diagonal, upto three decimal places, is _______unit.

[GATE - 2017]

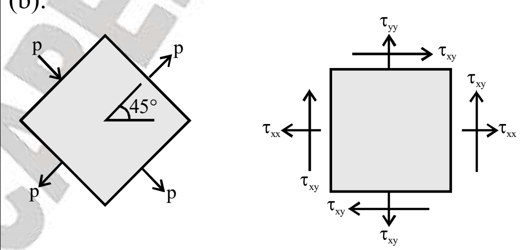
5. A square plate of dimension L × L is subjected to a uniform pressure load p = 250 MPa on its edges as shown in the figure. Assume plane stress conditions. The Young's modulus E = 200 GPa.



The deformed shape is a square of dimension L – 2 δ. If L = 2 m and δ = 0.001 m, the Poisson's ratio of the plate material is ________.

[GATE - 2016]

6. The state of stress at a point on an element is shown in figure (a). The same state of stress is shown in another coordinate system in figure (b).



The components (τ_{xx}, τ_{yy}, τ_{xy}) are given by

[GATE - 2016]

(a) $\left(P/\sqrt{2}, -P/\sqrt{2}, 0\right)$ (b) (0, 0, P)
(c) $\left(P, -P, -P/\sqrt{2}\right)$ (d) $\left(0, 0, P/\sqrt{2}\right)$

7. In a structural member under fatigue loading, the minimum and maximum stresses developed at the critical point are 50 MPa and 150 MPa, respectively. The endurance, yield, and the ultimate strengths of the material are 200 MPa, 300 MPa, and 400 MPA, respectively. The factor of safety using modified Goodman criterion is

[GATE - 2016]

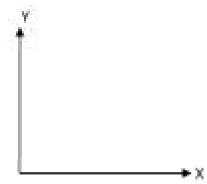
(a) $\frac{3}{2}$ (b) $\frac{8}{5}$

CHAPTER - 4

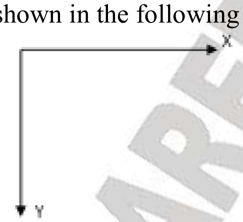
BENDING MOMENT AND SHEAR FORCE DIAGRAM

4.1 SHEAR FORCE AND BENDING MOMENT

At first we try to understand what shear force is and what is mending moment?
We will not introduce any other co-ordinate system. We use general co-ordinate axis as shown in the figure.

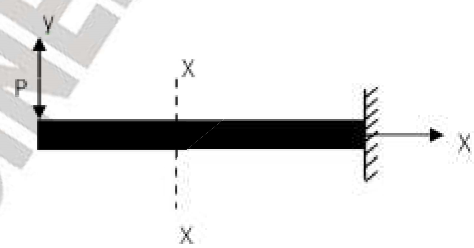


This system will be followed in shear force and bending moment diagram and in deflection of beam. Here downward direction will be negative i.e. negative Y-axis. Therefore downward deflection of the beam will be treated as negative.
Some books fix a co-ordinate axis as shown in the following figure.



Here downward direction will be positive i.e. positive Y-axis. Therefore downward deflection of the beam will be treated as positive. As beam is generally deflected in downward directions and this co-ordinate system treats downward deflection is positive deflection.

Consider a cantilever beam as shown subjected to external load 'P'. If we imagine this beam to be cut by a section X-X, we see that the applied force tend to displace the left-hand portion of the beam relative to the right hand portion, which is fixed in the wall. This tendency is resisted by internal forces between the two parts of the beam. At the cut section a resistance shear force (V_x) and a bending moment (M_x) is induced. This resistance shear force and the bending moment at the cut section is shown in the left hand and right hand portion of the cut beam.



Using the three equations of equilibrium.

GATE QUESTIONS

1. A simply supported beam is subjected to a uniformly distributed load. Which one of the following statements is true?

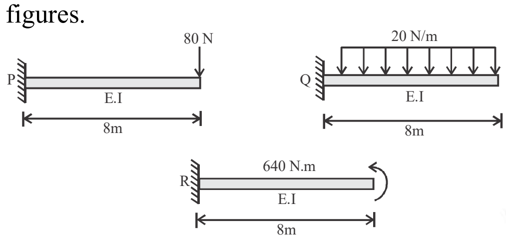
(a) Maximum or minimum shear force occurs where the curvature is zero.

(b) Maximum or minimum bending moment occurs where the shear force is zero.

(c) Maximum or minimum bending moment occurs where the curvature is zero.

(d) Maximum bending moment and maximum shear force occur at the same section.

2. Consider the three prismatic beams with the clamped supports P, Q and R as shown in the figures.



Given that the modulus of elasticity, E is 2.5×10^4 NOa; and the moment of inertia, I is 8×10^3 nn^4, the correct comparison of magnitudes of the shear force S and the bending moment M developed at the support is

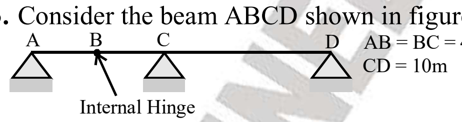
(a) $S_P < S_Q < S_R$; $M_P = M_Q = M_R$

(b) $S_P = S_Q > S_R$; $M_P = M_Q > M_R$

(c) $S_F < S_Q < S_R$; $M_P = M_Q = M_R$

(d) $S_P < S_Q < S_R$; $M_F < M_Q < M_R$

3. Consider the beam ABCD shown in figure



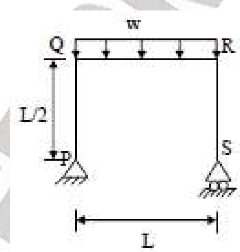
For a moving concentrated load of 50 kN on the beam, the magnitude of the maximum bending moment (in kn-m) obtained at the support C will be equal to _____

[GATE - 2017]

4. For a loaded catilever beam of uniform cross - section, the bending moment (in N-mm) along the length is $M(x) = 5x^2 + 10x$, where x is the distance (in mm) measured from the free end of the beam. The magnitude of shear force (in N) in the cross = section at x = 10mm is _____

[GATE - 2017]

5. The portal frame shown in figure is subjected to a uniformly distributed vertical load w (per unit length)



The bending moment in the beam at the joint 'Q' is

[GATE - 2016]

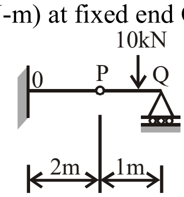
(a) Zero

(b) $\frac{wL^2}{24}$ (hogging)

(c) $\frac{wL^2}{12}$ (hogging)

(d) $\frac{wL^2}{8}$ (sagging)

6. A cantilever beam OP is connected to another beam PQ with a pin joint as shown in the figure. A load of 10kN is applied at the mid-point of PQ. The magnitude of bending moment (in kN-m) at fixed end O is



[GATE - 2015]

(a) 2.5

(b) 5

(c) 10

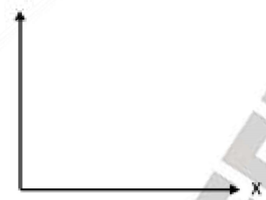
(d) 25

7. For the overhanging beam shown in figure, the magnitude of maximum bending moment (in kN-m) is _______.

CHAPTER - 5
DEFLECTION OF BEAM

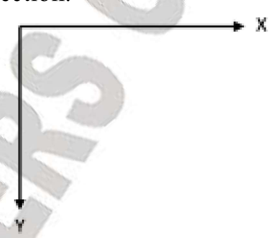
5.1 INTRODUCTION
1.We know that the axis of a beam deflects from its initial position under action of applied forces.
2.In this chapter we will learn how to determine the elastic deflections of a beam.

5.1.1 Selection of Co-ordinate Axis
We will not introduce any other co-ordinate system. We use general co-ordinate axis as shown in the figure. This system will be followed in deflection of beam and in shear force and bending moment diagram. Here downward direction will be negative i.e. negative Y-axis. Therefore downward deflection of the beam subjected to a given loading where we will use the formula,



$EI\frac{d^2y}{dx^2} = M_x$.

Some books fix a co-ordinate axis as shown in the following figure. Here downward direction will be positive i.e. positive Y-axis. Therefore downward deflection of the beam will be treated as positive. As beam is generally deflected in downward directions and this coordinate system treats downward deflection is positive deflection.

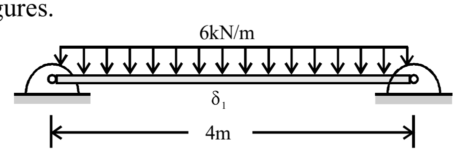


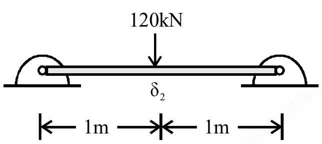
To determine the value of deflection of beam subjected to a given loading where we will use the formula, $EI\frac{d^2y}{dx^2} = -M_x$.

5.1.2 Why to calculate the deflections
1. To prevent cracking of attached brittle materials
2. To make sure the structure not deflected severely and to "appear" safe for its occupants.
3. To help analyzing statically indeterminate structures.
4. Information on deformation characteristics of members is essential in the study of vibrations of machines

GATE QUESTIONS

1. Two prismatic beams having the same flexural rigidity of 1000 kN-m^2 are shown in the figures.





If the mid-span deflections of these beams are denoted by δ_1 and δ_2 (as indicated in the figures), the correct option is

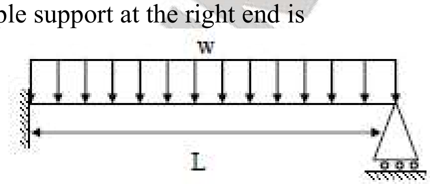
[GATE - 2017]

(a) $\delta_1 = \delta_2$ (b) $\delta_1 < \delta_2$
(c) $\delta_1 > \delta_2$ (d) $\delta_1 >> \delta_2$

2. A simply supported rectangular concrete beam of span 8m has to be span 8m has to be prestressed with a force of 1600 kN. The tendon is of parabolic profile having zero eccentricity at the supports. The beam has to carry an external uniformly distributed load of intensity 30 kN/m. Neglecting the self-weight of the beam, the maximum dip (in meters, up to two decimal places) of the tendon at the mid – span to balance the external load should be ____

[GATE - 2017]

3. A beam of length L is carrying a uniformly distributed load w per unit length. The flexural rigidity of the beam is EI. The reaction at the simple support at the right end is



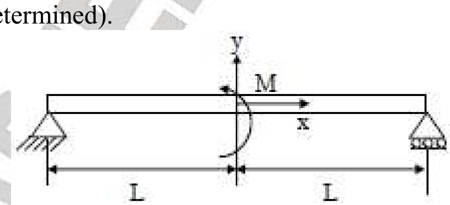
[GATE - 2016]

(a) $\frac{wL}{2}$ (b) $\frac{3wL}{8}$
(c) $\frac{wL}{4}$ (d) $\frac{wL}{8}$

4. A simply supported beam of length 2L is subjected to a moment M at the mid-point x =0 as shown in the figure. The deflection in the domain $0 \le x \le L$ is given by

$$W = \frac{-Mx}{12EIL}(L-x)(x+c)$$

Where E is the Young's modulus, I is the area moment of inertia and c is a constant (to be determined).

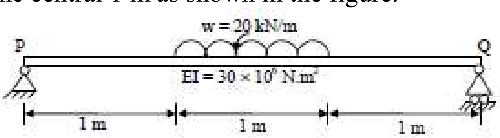


The slope at the center x = 0 is

[GATE - 2016]

(a) ML/(2EI) (b) ML/(3EI)
(c) ML/(6EI) (d) ML/(12EI)

5. A 3 m long simply supported beam of uniform cross section is subjected to a uniformly distributed load of w = 20 kN/m in the central 1 m as shown in the figure.



If the flexural rigidity (EI) of the beam is 30×10^6 N-m^2, the maximum slope (expressed in radians) of the deformed beam is

[GATE - 2016]

(a) 0.681×10^{-7} (b) 0.943×10^{-7}
(c) 4.310×10^{-7} (d) 5.910×10^{-7}

CHAPTER - 6
BENDING & SHEAR STRESSES

6.1 EULER'S BERNOULLI'S EQUATION or (Bending Stress Formula) or Bending Equation

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

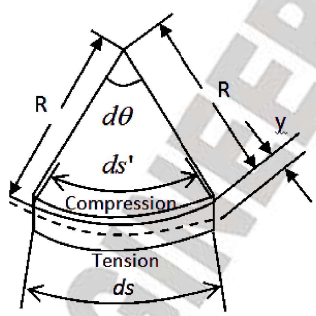
Where σ is bending Stress
M is bending moment
I is moment of inertia
E is modulus of elasticity
R is radius of curvature
y is distance of the fibre from NA (Neutral axis)

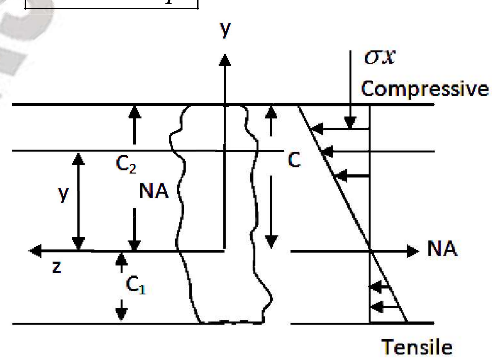
6.2 ASSUMPTIONS IN SIMPLE BENDING THEORY

All of the foregoing theory has been developed for the case of pure bending i.e. constant B.M. along the length of the beam. In such case

1. The shear force at each c/s is zero.
2. Normal stress due to bending is only produced.
3. Beams are initially straight.
4. The material is homogenous and isotropic i.e. it has a uniform composition and its mechanical properties are the same in all directions.
5. The stress strain relationship is linear and elastic.
6. Young's Modulus is the same in tension as in compression.
7. Sections are symmetrical about the plane of bending.
8. Sections which are plane before bending remain plane after bending.

6.3 $\sigma_{max} = \sigma_t = \dfrac{Mc_1}{I}$; $\sigma_{min} = \sigma_c = \dfrac{Mc_2}{I}$ (Minimum in sense of sign)





6.4 SECTION MODULUS (Z)

$$Z = \frac{I}{y}$$

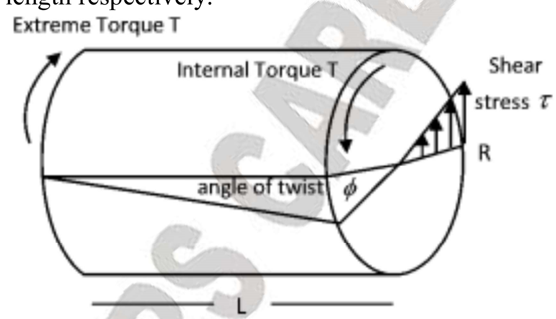
CHAPTER - 7
TORSION, THIN & THICK CYLINDER

7.1 INTRODUCTION
1. In machinery, the general term "shaft" refers to a member, usually of circular cross-section, which supports gears, sprockets, wheels, rotors, etc. and which is subjected to torsion and to transverse or axial loads acting singly or in combination.
2. An "axle" is a non-rotating member that supports wheels, pulleys,… and carries no torque.
3. A "spindle" is a short shaft. Terms such as line shaft, head shaft, stub shaft, transmission shaft, countershaft, and flexible shaft are names associated with special usage.

7.2 TORSION OF CIRCULAR SHAFTS
1. Equation for shafts subjected to torsion "T"

Torsion Equation, $\frac{\tau}{R} = \frac{T}{J} = \frac{G\theta}{L}$

Where j is polar moment of inertia
τ is shear stress induced due to torsion T
G is modulus of rigidity
θ is angular deflection or shaft
R, L is shaft radius and length respectively.



7.2.1 Assumptions
1. The bar is acted upon by a pure torque
2. The section under consideration is remote from the point of application of the load and from a change in diameter.
3. Adjacent cross sections originally plane and parallel remain and parallel after twisting, and any radial line remains straight.
4. The material obeys Hooke's law.
5. Cross-sections rotate as if rigid, i.e. every diameter rotates through the same angle.

GATE QUESTIONS

1. A motor driving a solid circular steel shaft transmits 40 kW of power at 500 rpm. If the diameter of the shaft is 40mm, the maximum shear stress in the shaft is __________MPa.

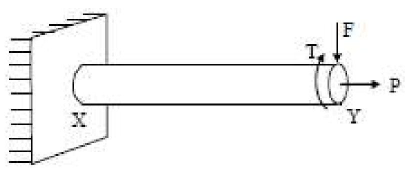
[GATE - 2017]

2. Two circular shafts made of same material, one solid (S) and one hollow (H), have the same length and polar moment of inertia. Both are subjected to same torque. Here. θ_S is the twist and τ_S is the maximum shear stress in the solid shaft, whereas θ_H is the twist and τ_H is the maximum shear stress in the hollow shaft. Which one of the following TRUE?

[GATE - 2016]

(a) $\theta_S = \theta_H$ and $\tau_S = \tau_H$
(b) $\theta_S > \theta_H$ and $\tau_S > \tau_H$
(c) $\theta_S < \theta_H$ and $\tau_S < \tau_H$
(d) $\theta_S = \theta_H$ and $\tau_S < \tau_H$

3. A machine element XY, fixed at end X, is subjected to an axial load P, transverse load F, and a twisting moment T at its free end Y. The most critical point from the strength point of view is



[GATE - 2016]

(a) A point on the circumference at location Y
(b) A point at the center at location Y
(c) A point on the circumference at location X
(d) A point at the center at location X

4. A shaft with a circular cross-section is subjected to pure twisting moment. The ratio of the maximum shear stress to the largest principal stress is

[GATE - 2016]

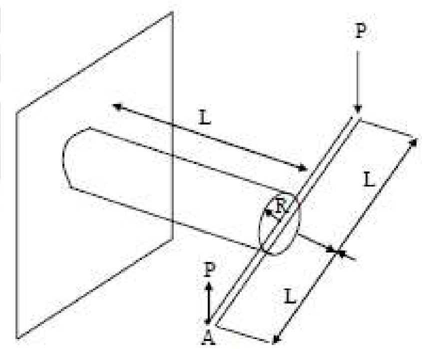
(a) 2.0 (b) 1.0
(c) 0.5 (d) 0

5. A thin cylindrical pressure vessel with closed-ends is subjected to internal pressure. The ratio of circumferential (hoop) stress to the longitudinal stress is

[GATE - 2016]

(a) 0.25 (b) 0.50
(c) 1.0 (d) 2.0

6. A rigid horizontal rod of length 2L is fixed to a circular cylinder of radius R as shown in the figure. Vertical forces of magnitude P are applied at the two ends as shown in the figure. The shear modulus for the cylinder is G and the Young's modulus is E.



The vertical deflection at point A is

[GATE - 2016]

(a) $PL^3/(\pi R^4 G)$ (b) $PL^3/(\pi R^4 E)$
(c) $2PL^3/(\pi R^4 E)$ (d) $4PL^3/(\pi R^4 G)$

7. A hollow shaft $d_0 = 2d_1$ where d_0 and d_1 are the outer and inner diameters respectively) needs to transmit 20kW power at 3000 RPM. If the maximum permissible shear stress is 30 MPa, d_0 is

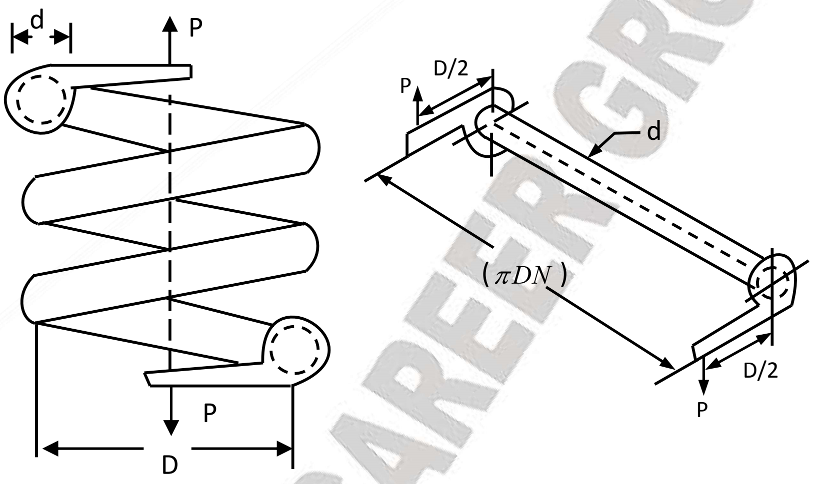
[GATE - 2015]

(a) 11.29 (b) 22.58 mm
(c) 33.87 mm (d) 45.16 mm

CHAPTER - 8
SPRINGS

8.1 INTRODUCTION
A spring is a mechanical device which is used for the efficient storage for the efficient and release of energy.

8.2 HELICAL SPRING-STRESS EQUATION
Let us a close-coiled helical spring has coil diameter D, wire diameter d and number of turn n.



The spring material has a shearing modulus G. The spring index, $C=\frac{D}{d}$. If a force 'P' is exerted in both ends as shown.

The work done by the axial force 'P' is converted into strain energy and stored in the spring.

U = (average torque) x (angular displacement) = $\frac{T}{2}\times\theta$

From the figure we get, $\theta=\frac{TL}{GJ}$

Torque ,T = $\frac{PD}{2}$

Length of wire, L = πDn

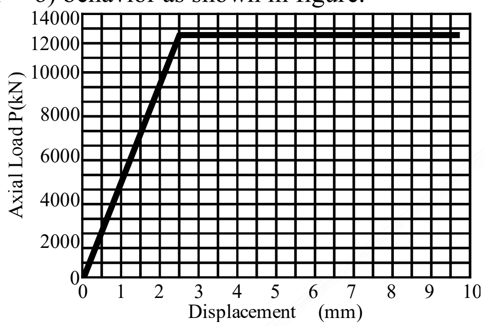
Polar moment of inertia, $(J)=\frac{\pi d^4}{32}$

Therefore $U=\frac{4P^2D^3n}{Gd^4}$

Accordingly to Castigilano's theorem, the displacement corresponding to force P is obtained by partially differentiating strain energy with respect to that force.

GATE QUESTIONS

1. A 2m long, axially loaded mild steel rod of 8mm diameter exhibits the load - displacement $(P - \delta)$ behavior as shown in figure.



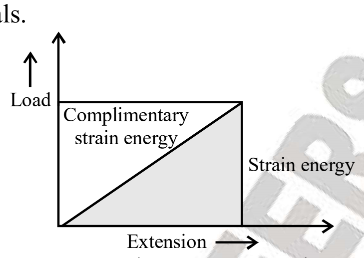
Assume the yield stress of steel as 250 MPa. The complementary strain energy (in N-mm) stored in the bar up to its linear elastic behaviour will be ___

[GATE - 2017]

SOLUTIONS

Sol. 1. (15,707.963)

Complimentary strain energy for linear materials.



Complimentary strain energy = strain energy

$= \frac{1}{2}\sigma_y \times \varepsilon \times A \times L$

$= 12 \times 250 \times \frac{2.5}{2000} \times \frac{\pi}{4} \times 8^2 \times 2000$

$\left(\because \varepsilon = \frac{\delta\ell}{\ell}\right)$

= 15,707.963 N-mm

CHAPTER - 9
THEORY OF COLOUMN & STRAIN ENERGY

9.1. INTRODUCTION
1.Strut: A member of structure which carries an axial compressive load.
2.Column: If the strut s vertical it is known as column.
3.A long, slender column becomes unstable when its axial compressive load reaches a value called the critical buckling load.
4.If a beam element is under a compressive load and its length is an order of magnitude larger than either of its other dimensions such a beam is called as columns.
5.Due to its size its axial displacement is going to be very small compared to its lateral deflection called buckling.
6.Buckling does not vary linearly with load it occurs suddenly and is therefore dangerous
7.Slenderness Ratio: The ratio between the length and least radius of gyration.
8.Elastic Buckling: Buckling with no permanent deformation
9.Euler's buckling is only valid for long, slender objects in the elastic region.
10.For short columns, a different set of equations must be used.

9.2. WHICH IS THE CRITICAL LOAD?
1. At this value the structure is in equilibrium regardless of the magnitude of the angle (provided it stays small)
2. Critical load is the only load for which the structure will be in equilibrium in the disturbed position
3. At this value, restoring effect of the moment in the spring matches the buckling effect of the axial load represents the boundary between the stable and unstable conditions.
4. If the axial load is less than P_{cr} the effect of the moment in the spring dominates and the structure returns to the vertical position after a small disturbance-stable condition.
5. If the axial load is larger than P_{cr} the effect of the axial force predominates and the structure buckles-unstable condition.
6. Because off the large deflection caused by buckling, the least moment of inertia/can be expressed as, $I = Ak^2$
7. Where A is the cross sectional area and r is the radius of gyration of the cross sectional area, i.e.

$$k_{min} = \sqrt{\frac{I_{min}}{A}}$$

8. Note that the smallest radius of gyration of the column, i.e. the least moment of inertia l should be taken in order to find the critical stress. L/k is called the slenderness ratio, it is a measure of the column's flexibility.

9.3. EULER'S CRITICAL LOAD FOR LONG COLUMN
9.3.1 Assumption
1. The column is perfectly straight and of uniform cross-section
2. The material is homogenous and isotropic
3. The material behaves elasticity3
4. The load is perfectly axial and passes through the Centroid of the column section.
5. The weight of the column is neglected.

CHAPTER - 10
THEORIES OF FAILURE

10.1 INTRODUCTION
When some external load is applied on a body, the stresses and strains are produced in the body. The stresses are directly proportional to the strains within the elastic limit. This means when the load is removed, the body will return to its original shape. There is no permanent deformation in the body.
However, If the stress produced in the body due to the application of the load, is beyond the elastic limit, the permanent deformations occur in the body. This means if the load is removed, the body will not retain its original shape. There are some permanent deformations in the body. Whenever permanent deformations occur in the body, the body is said to have "failed". This should be clear that failure does not mean rupture of the body.
The failure takes place when a certain limiting value is reached by one of following:
1. The maximum principal stress
2. The maximum principal strain.
3. The maximum shear stress.
4. The maximum strain energy
5. The maximum shear strain energy.

In all the above cases,
$\sigma_1, \sigma_2, \sigma_3$ = principal stresses in any complex system
σ^* = tensile or compressive stress at the elastic limit.

1. Maximum Principal Stress Theory
According to this theory, the failure of a material will occur when the maximum principal tensile stress (σ_1) in the complex system reaches the value of the maximum stress at the elastic limit in simple tension or the minimum principal stress (i.e, the maximum principal compressive stress) reaches the value of the maximum stress at the elastic limit in simple compression .
Let in a complex three dimensional stress system.
σ_1, σ_2 and σ_3 = principal stresses at a point in three perpendicular directions. The stresses σ_1 and σ_2 are tensile and σ_3 is compressive. Also σ_1 is more than σ_2 .
σ_t^* = tensile stress at elastic limit in simple tension.
σ_c^* = compressive stress at elastic limit in simple compression.
Then according to this theory, the failure will take place if
$\sigma_1 \geq \sigma_t^*$ in simple tension …(1)
Or $|\sigma_3| \geq \sigma_c^*$ in simple compression …(1.1)
Where $|\sigma_3|$ represents the absolute value of σ_3 .
This is the simplest and oldest theory of failure and is known as Rankine's theory. If the maximum principal stress (σ_1) is the design criterion, then maximum principal stress must not exceed the permissible stress (σ_t) for the given material.
Hence, $\sigma_1 = \sigma_t$ …(1.2)
Where σ_t = permissible stress and given by